Chapter 5

Noninertial Reference System

Example 3

A wheel of radius b rolls along the ground with constant forward speed V_0. Find the acceleration, relative to the ground, of any point on the rim

Let us choose a coordinate system fixed to the rotating wheel, and let the moving origin be at the center with the x'-axis passing through the point in question

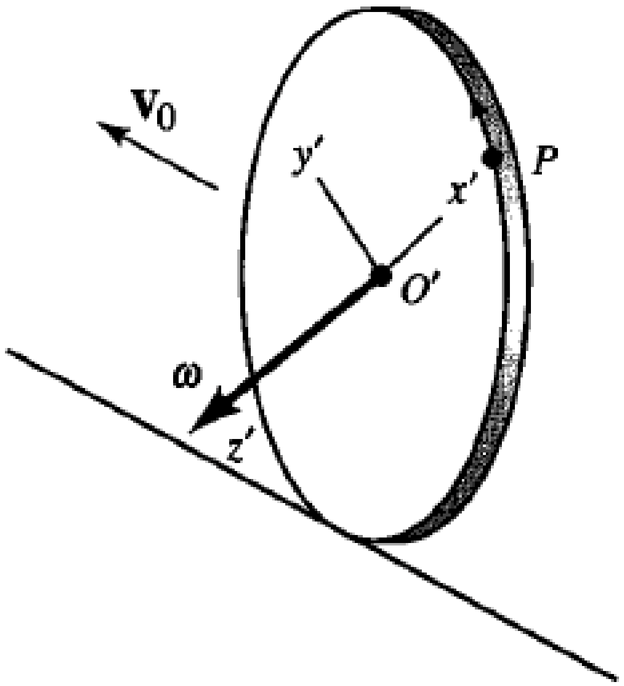
$$r' = bi'$$

$$\omega = \omega k'$$

$$\dot{r}' = 0$$

$$\omega = \frac{V_0}{b} k'$$

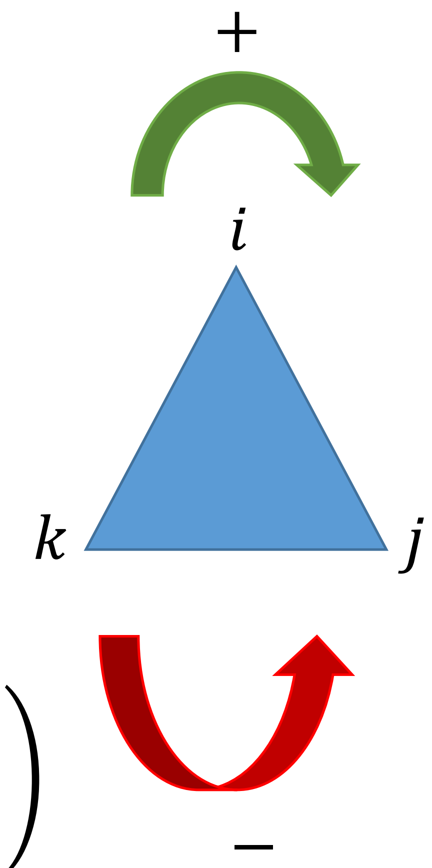
$$\ddot{r}' = 0$$



$$a = \underbrace{a'}_{\text{zero}} + \underbrace{\dot{\omega} \times r'}_{\text{zero}} + \underbrace{2\omega \times v'}_{\text{zero}} + \omega \times (\omega \times r') + \underbrace{A_0}_{\text{zero}}$$

$$\omega \times (\omega \times r') = \omega k' \times (\omega k' \times b i') = \frac{V_0}{b} k' \times \left(\frac{V_0}{b} k' \times b i' \right)$$

$$= \frac{V_0^2}{b} k' \times (k' \times i') = \frac{V_0^2}{b} k' \times j' = \frac{V_0^2}{b} (-i')$$



Example 4

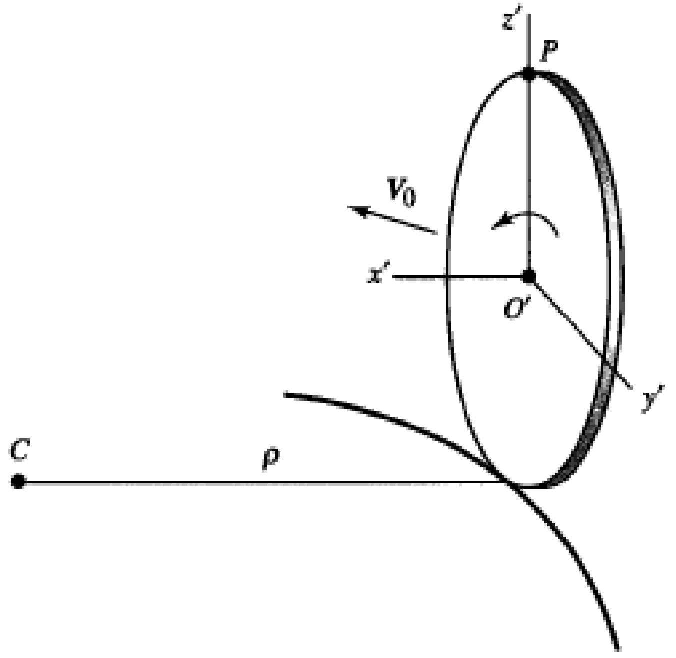
A bicycle travels with constant speed around a track of radius ρ. What is the acceleration of the highest point on one of its wheels? Let V_0 denotes the speed of the bicycle and b is the radius of the wheel.

Origin at the center of the wheel.

x' point toward C .

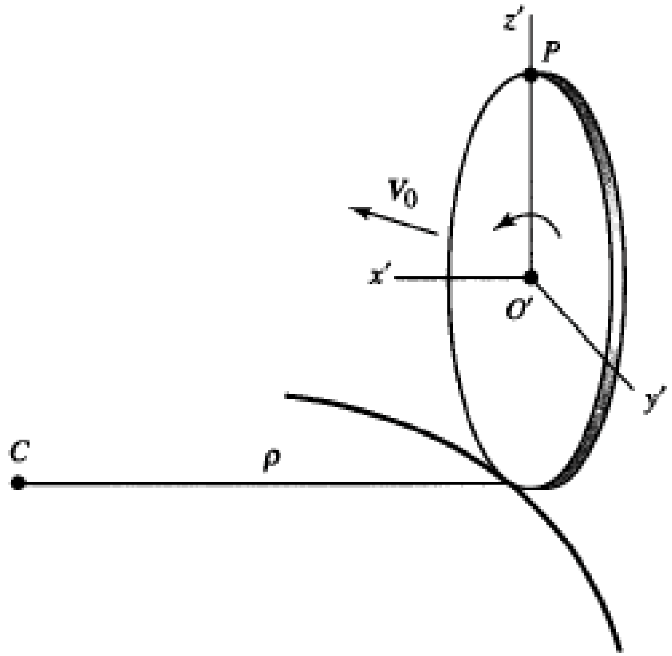
z' axis is vertical .

Rotating coordinate system with the wheel.



Rotating around C with angular velocity

$$\omega = k' \frac{v}{\rho}$$



The acceleration of the moving origin A_0

$$A_0 = i' \frac{V_0^2}{\rho}$$

Each point on the wheel (moving of radius b)

$$a = \ddot{r}' = -k' \frac{V_0^2}{b}$$

Each point on the wheel (The velocity of this point)

$$v' = -j' V_0$$

$$a = a' + \dot{\omega} \times r' + 2\omega \times v' + \omega \times (\omega \times r') + A_0$$

$$-k' \frac{V_0^2}{b}$$

?

$$i' \frac{V_0^2}{\rho}$$

$$=0 \quad [\dot{\omega}=0]$$

$$\omega \times (\omega \times r') = \frac{V_0^2}{\rho} k' \times (k' \times bk') = 0$$

$$2\omega \times v' = 2\left(\frac{V_0}{\rho} k'\right) \times (-j' V_0) \quad = 2\frac{V_0^2}{\rho} i'$$

$$a = a' + \dot{\omega} \times r' + 2\omega \times v' + \omega \times (\omega \times r') + A_0$$

$$a = a' + 0 + 2\omega \times v' + 0 + A_0$$

$$a = -k' \frac{V_0^2}{b} + 2\frac{V_0^2}{\rho} i' + \frac{V_0^2}{\rho} i'$$

$$\Rightarrow$$

$$\boxed{a = -k' \frac{V_0^2}{b} + 3\frac{V_0^2}{\rho} i'}$$

3. Dynamics of a particle in a Rotating Coordinate system.

$$F = ma$$

Where F is the vector sum of all real, physical forces acting on the particle.

$$ma' = F - mA_0 - 2m(\omega \times v) - m\dot{\omega} \times r' - m\omega \times (\omega \times r')$$

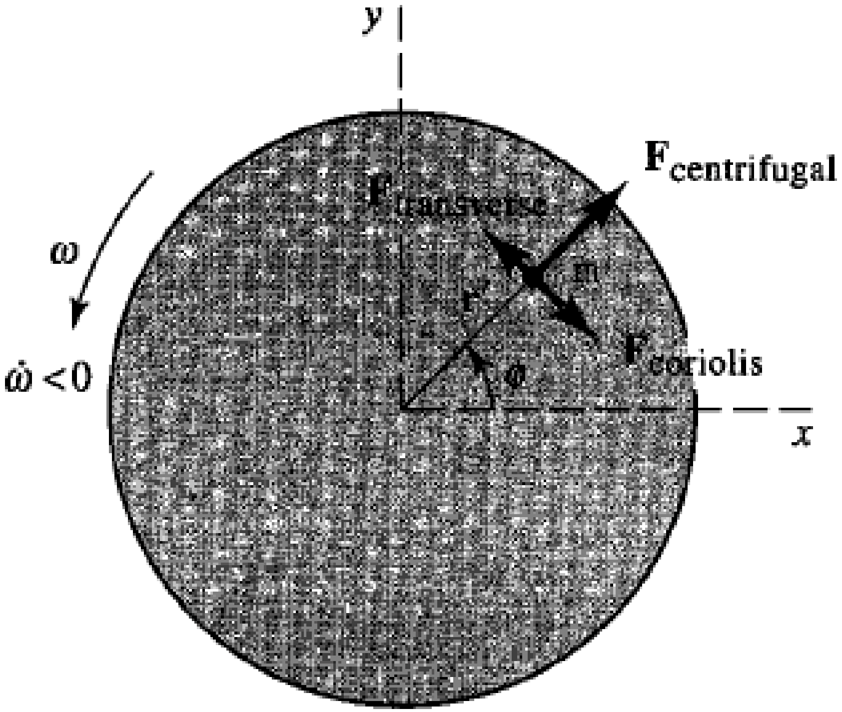
Transvers force $\quad F'_{trans} = -m\dot{\omega} \times r'$

Coriolis force $\quad F'_{cor} = -2m(\omega \times v')$

Centrifugal force $\quad F'_{centr} = -m\omega \times (\omega \times r')$

$$F' = F_{phy} - mA_0 + F'_{trans} + F'_{cor} + F'_{centr}$$

$$F' = ma'$$

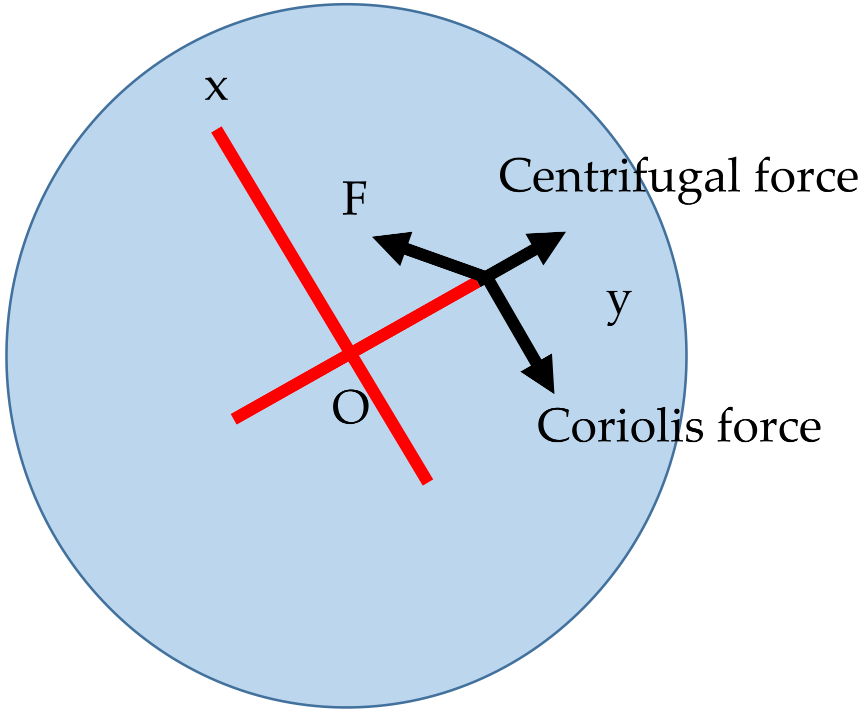


Example 5

A bug crawls outward with a constant speed v' along the spoke of a wheel that is rotating with constant angular velocity ω about a vertical axis. Find all the apparent forces acting on the bug.

$$F' = F_{phy} - mA_0 + F'_{trans} + F'_{cor} + F'_{centr}$$

Let the bug crawls along $x' - direction$



$$r' = ix = iv't$$

$$\dot{r}' = i\dot{x} = iv' \qquad \ddot{r}' = 0$$

$Let\ choose\ z' - direction\ to\ be\ vertical$ $\Rightarrow$ $\omega = k'\omega$

$$m\ddot{r}' = F - mA_0 - 2\,m\omega \times \dot{r}' - m\dot{\omega} \times r' - m\omega \times (\omega \times r')$$

Coriolis force

$$-2\,m\omega \times \dot{r}' = -2m\omega v'(k' \times i') \quad = -2m\omega v' j'$$

$\boldsymbol{Transverse\ force}$

$$-m\dot{\omega} \times r' \quad = 0 \qquad\qquad \omega \text{ is constant}$$

$\boldsymbol{centrifugal\ force}$

$$-m\omega \times (\omega \times r') \quad = -m\omega^2\,[k' \times (k' \times i'x')]$$

$$= -m\omega^2\,[k' \times j'x']$$

$$= m\omega^2 x' i'$$

$$m\ddot{r}' = F - mA_0 - 2\,m\omega \times \dot{r}' - m\dot{\omega} \times r' - m\omega \times (\omega \times r')$$

$$0 = F - 0 - 2m\omega v' j' - 0 + m\omega^2 x' i'$$

$$0 = F - 2m\omega v' j' + m\omega^2 x' i'$$

H.W

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In Example 5, Find how far the bug crawl before it starts to slip, given the coefficient of static friction μ_s between the bug and the spoke.