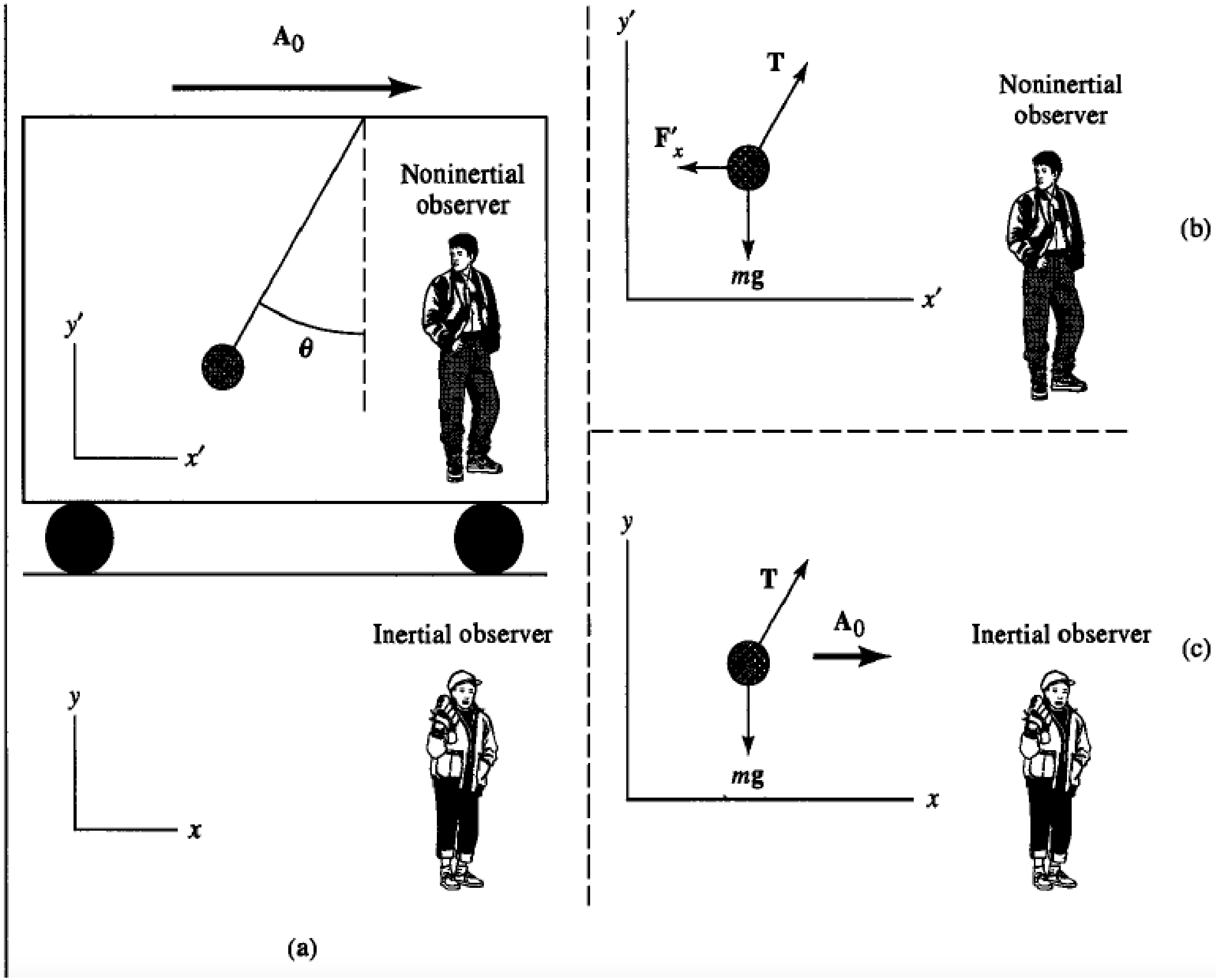
Chapter 5

Noninertial Reference System

Example 2

A pendulum is suspended from the ceiling of a railroad car, as shown in Figure 5.1.2a. Assume that the car is accelerating uniformly toward the right ($+x$ direction). A non-inertial observer, the boy inside the car, sees the pendulum hanging at an angle θ, left of vertical. He believes it hangs this way because of the existence of an inertial force $\mathbf{F}'_x$, which acts on all objects in his accelerated frame of reference (Figure 5.1.2b). An inertial observer, the girl outside the car, sees the same thing. She knows, however, that there is no real force $\mathbf{F}'_x$ acting on the pendulum. She knows that it hangs this way because a net force in the horizontal direction is required to accelerate it at the rate $\mathbf{A}_0$ that she observes (Figure 5.1.2c). Calculate the acceleration $\mathbf{A}_0$ of the car from the inertial observer's point of view. Show that, according to the noninertial observer, $\mathbf{F}'_x = -m\mathbf{A}_0$ is the force that causes the pendulum to hang at the angle θ.



A0
θ
y′
x′
Noninertial observer
y
x
Inertial observer
(a)
y′
T
F′x
mg
x′
Noninertial observer
(b)
y
T
A0
mg
x
Inertial observer
(c)

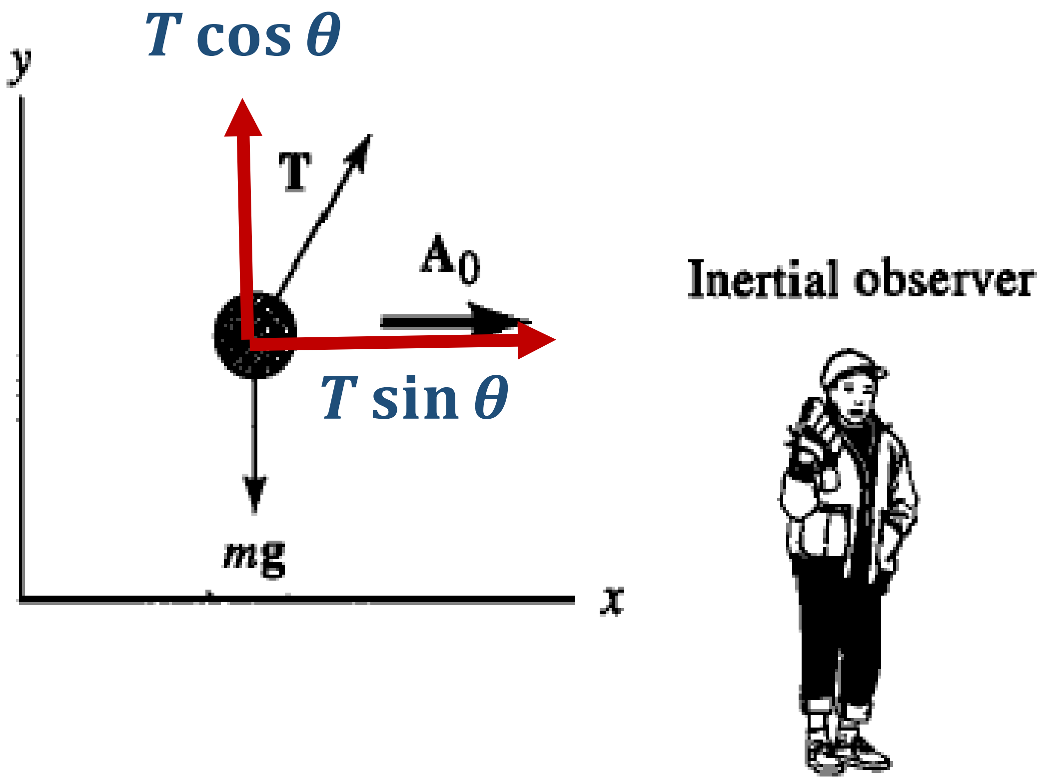
The inertial observer writes down Newton's 2nd law

$$\sum F_i = ma$$

$$T \sin\theta = mA_0$$

$$T \cos\theta = mg$$

$$\frac{\cancel{T} \sin\theta}{\cancel{T} \cos\theta} = \frac{\cancel{m}A_0}{\cancel{m}g} \Rightarrow \boxed{A_0 = g \tan\theta}$$



The pendulum hangs at angle θ because the railroad car is accelerating in the horizontal direction

The acceleration of the car is proportional to the tangent of the angle of the defection.

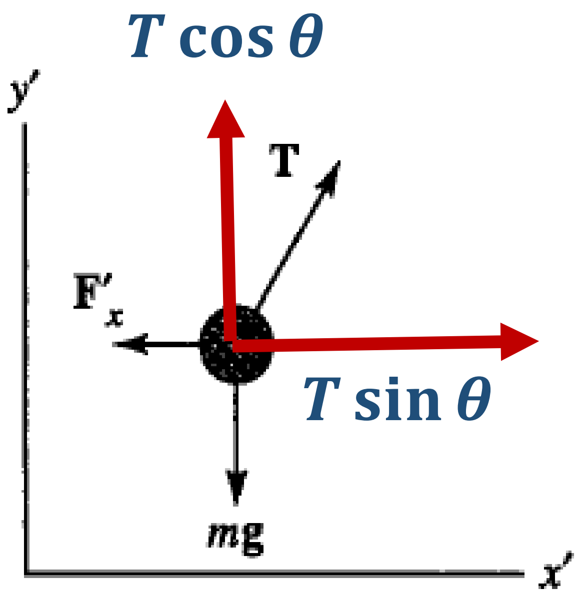
The non-inertial observer writes down Newton's 2nd law

$$\sum F'_i = ma'$$

$$F'_x = T\sin\theta$$

$$mg = T\cos\theta$$

$$\frac{\cancel{T}\sin\theta}{\cancel{T}\cos\theta} = \frac{F'_x}{mg} \Rightarrow \boxed{F'_x = mg\tan\theta}$$



All the forces acting on the pendulum are in balance and the pendulum hangs left the vertical due to the force ($F'_x = mg\tan\theta = -mA_0$)

H.W

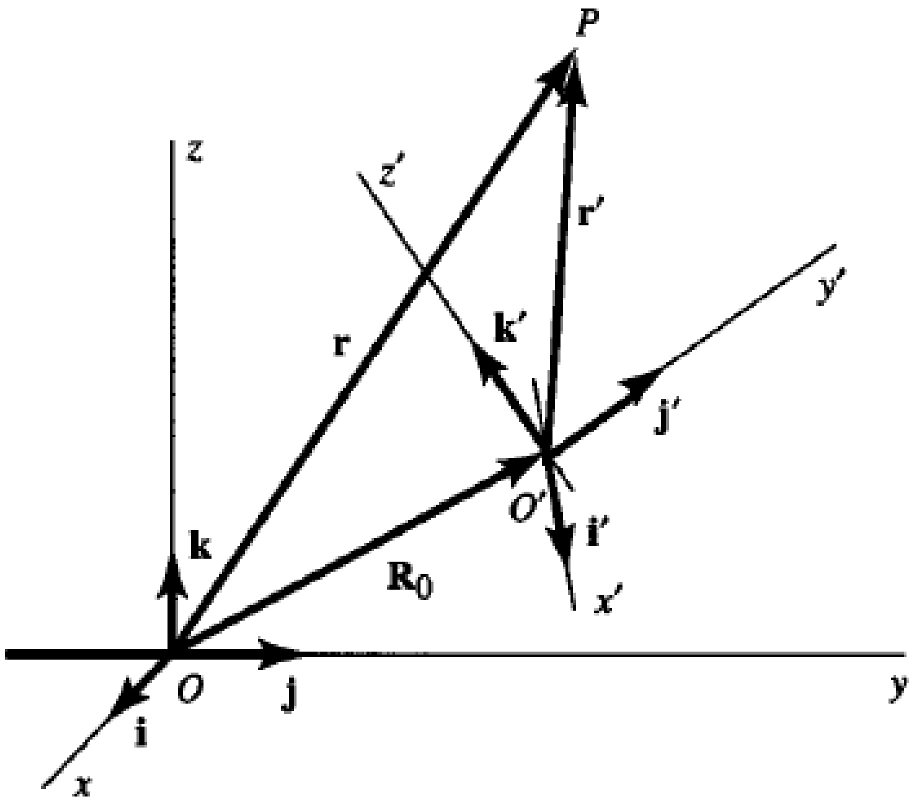
1. A 120-lb person stands on a bathroom spring scale while riding in an elevator. If the elevator has (a) upward and (b) downward acceleration of $g/4$, what is the weight indicated on the scale in each case?

2. A plumb line is held steady while being carried along in a moving train. If the mass of the plumb bob is m, find the tension in the cord and the deflection from the local vertical if the train is accelerating forward with constant acceleration $g/10$. (Ignore any effects of Earth's rotation.)

3. A hauling truck is traveling on a level road. The driver suddenly applies the brakes, causing the truck to decelerate by an amount $g/2$. This causes a box in the rear of the truck to slide forward. If the coefficient of sliding friction between the box and the truckbed is $\frac{1}{3}$, find the acceleration of the box relative to (**a**) the truck and (**b**) the road.

2. Rotating Coordinate Systems

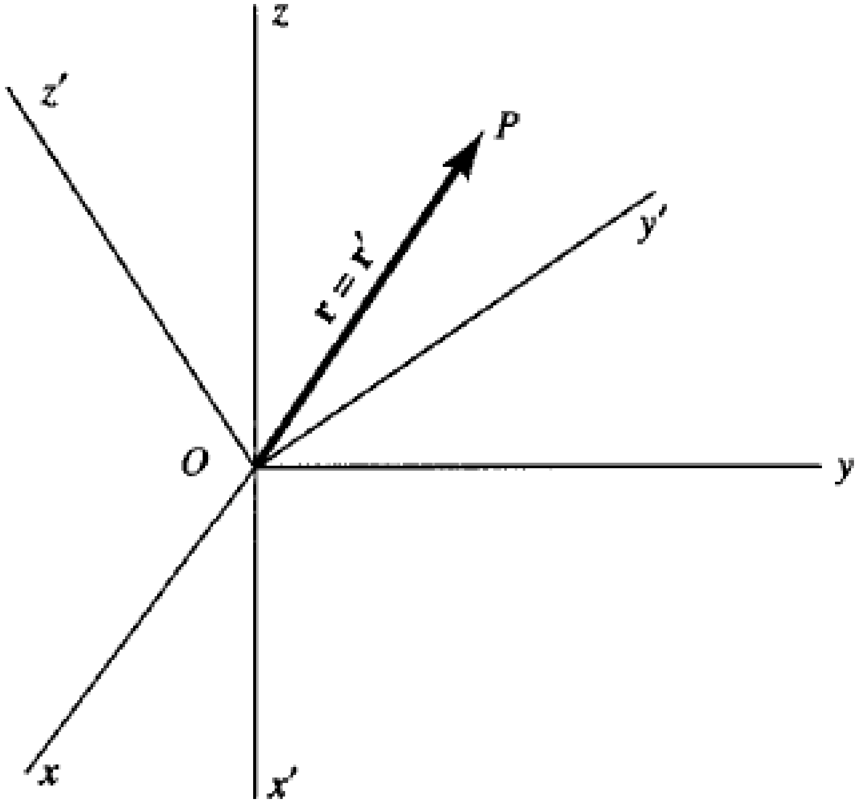
Here we will show that velocity, accelerations, and force transform between an inertial frame of references and non-inertial one, that is rotating as well

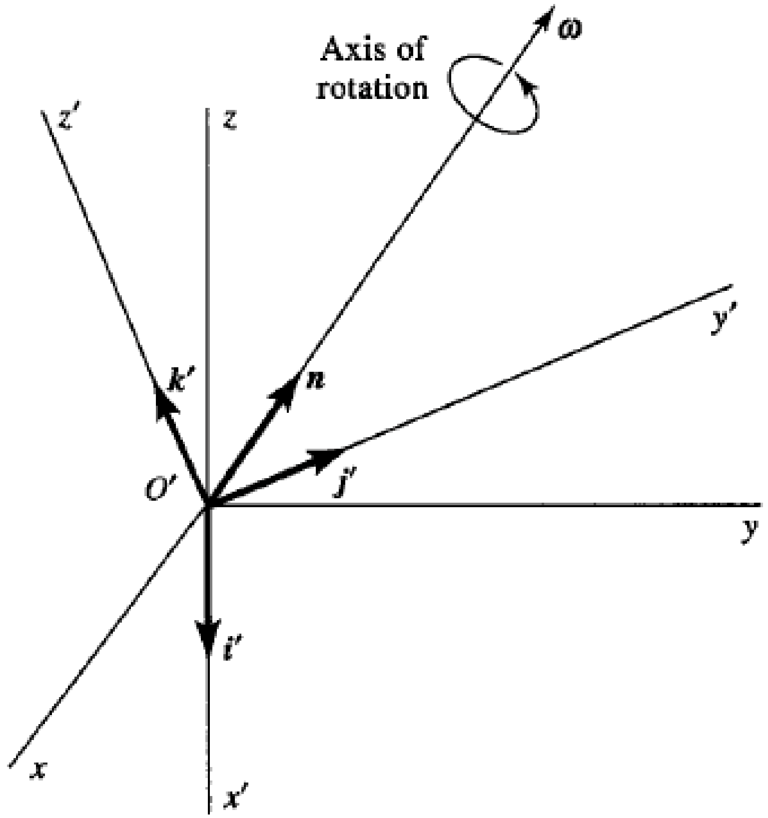


(1) **Fixed** and **rotating** systems have the same origin

The angular velocity $\boldsymbol{\omega}$

$$\boldsymbol{\omega} = \omega\, n$$





$$\boxed{r = r'}$$

$$r = r'$$

$$ix + jy + kz = i'x' + j'y' + k'z'$$

$$\underbrace{i\frac{dx}{dt} + j\frac{dy}{dt} + k\frac{dz}{dt}}_{\text{Velocity } (\boldsymbol{v}) \text{ in the fixed system}} = \underbrace{i'\frac{dx'}{dt} + j'\frac{dy'}{dt} + k'\frac{dz'}{dt}}_{\text{Velocity } (\boldsymbol{v}') \text{ in the rotating system}} + \underbrace{x'\frac{di'}{dt} + y'\frac{dj'}{dt} + z'\frac{dk'}{dt}}_{\text{Velocity due to the rotating of the primed system.}}$$

$$\frac{di'}{dt} \qquad \frac{dj'}{dt} \qquad \frac{dk'}{dt} \qquad ???$$

H.W

$$\boxed{\frac{di'}{dt} = \omega \times i'} \qquad \boxed{\frac{dj'}{dt} = \omega \times j'} \qquad \boxed{\frac{dk'}{dt} = \omega \times k'}$$

$$x'\frac{di'}{dt} + y'\frac{dj'}{dt} + z'\frac{dk'}{dt} = x'(\omega \times i') + y'(\omega \times j') + z'(\omega \times k')$$

$$= \omega \times (i'x' + j'y' + k'z')$$

$$= \omega \times r'$$

$$\underbrace{i\frac{dx}{dt} + j\frac{dy}{dt} + k\frac{dz}{dt}}_{\text{Velocity } (\boldsymbol{v}) \text{ in the fixed system}} = \underbrace{i'\frac{dx'}{dt} + j'\frac{dy'}{dt} + k'\frac{dz'}{dt}}_{\text{Velocity } (\boldsymbol{v}') \text{ in the rotating system}} + \underbrace{\overbrace{x'\frac{di'}{dt} + y'\frac{dj'}{dt} + z'\frac{dk'}{dt}}^{\boldsymbol{\omega} \times \boldsymbol{r}'}}_{\text{Velocity due to the rotating of the primed system.}}$$

$$\boxed{v = v' + \omega \times r'}$$

$$\boxed{\left(\frac{dr}{dt}\right)_{fixed} = \left(\frac{dr'}{dt}\right)_{rot} + \omega \times r'}$$

$$\boxed{\left(\frac{dQ}{dt}\right)_{fixed} = \left(\frac{dQ}{dt}\right)_{rot} + \omega \times Q}$$

General form $\boxed{Q = r}$

$$\boxed{\left(\frac{dv}{dt}\right)_{fixed} = \left(\frac{dv}{dt}\right)_{rot} + \omega \times v}$$

Acceleration $\boxed{Q = v}$

$$\boxed{\left(\frac{dv}{dt}\right)_{fixed} = \left(\frac{dv}{dt}\right)_{rot} + \omega \times v}$$

$$v = v' + \omega \times r'$$

$$\left(\frac{dv}{dt}\right)_{fixed} = \left(\frac{d}{dt}\right)_{rot} (v' + \omega \times r') + \omega \times (v' + \omega \times r')$$

$$\left(\frac{dv}{dt}\right)_{fixed} = \left(\frac{dv'}{dt}\right)_{rot} + \left(\frac{d(\omega \times r')}{dt}\right)_{rot} + \omega \times v' + \omega \times (\omega \times r')$$

$$\left(\frac{d\omega}{dt}\right)_{rot} ???$$

$$\left(\frac{dv}{dt}\right)_{fixed} = \left(\frac{dv'}{dt}\right)_{rot} + \left(\frac{d\omega}{dt}\right)_{rot} \times r' + \omega \times \left(\frac{dr'}{dt}\right)_{rot} + \omega \times v' + \omega \times (\omega \times r')$$

$$\boxed{\left(\frac{dQ}{dt}\right)_{fixed} = \left(\frac{dQ}{dt}\right)_{rot} + \omega \times Q}$$

$$\boxed{Q = \omega}$$

$$\left(\frac{d\omega}{dt}\right)_{fixed} = \left(\frac{d\omega}{dt}\right)_{rot} + \omega \times \omega$$

$$\boxed{\omega \times \omega = 0}$$

$$\left(\frac{d\omega}{dt}\right)_{fixed} = \left(\frac{d\omega}{dt}\right)_{rot} = \dot{\omega}$$

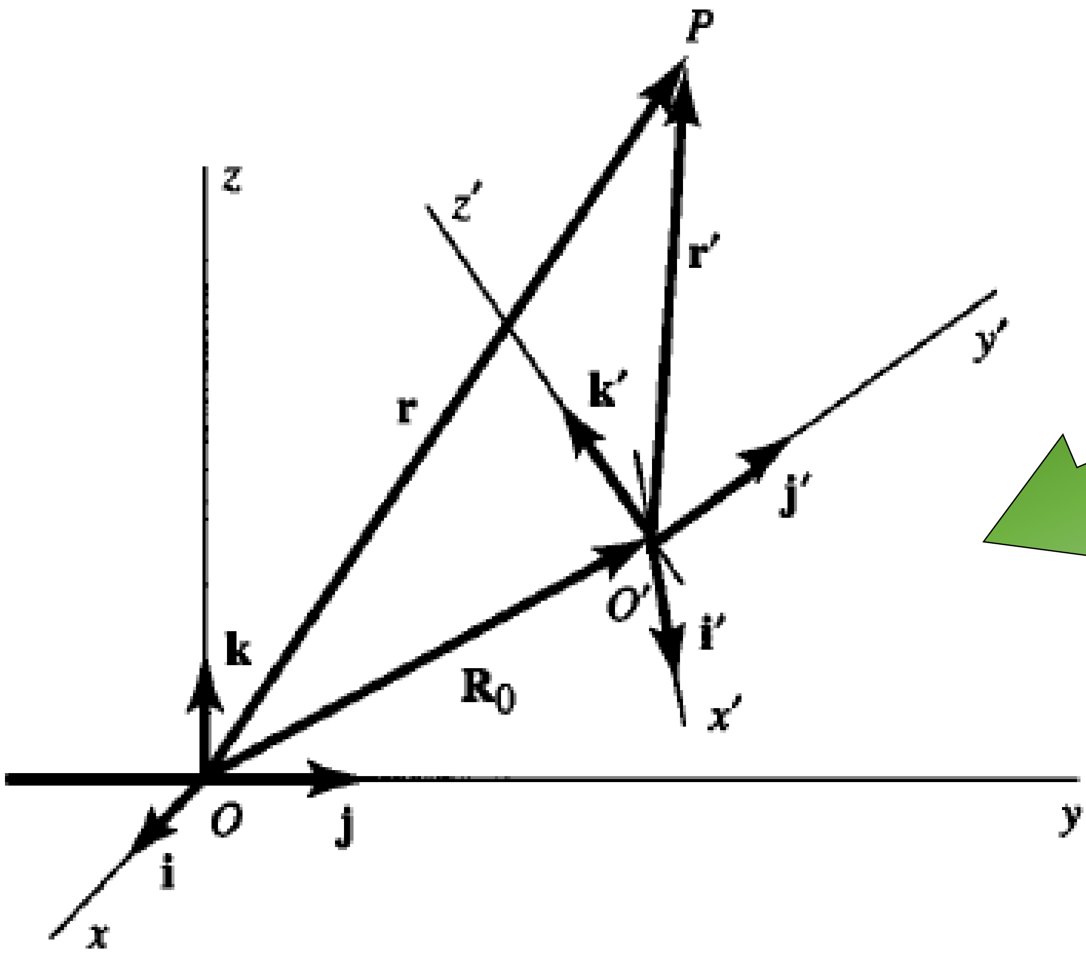
$$\left(\frac{dv}{dt}\right)_{fixed} = \left(\frac{dv'}{dt}\right)_{rot} + \left(\frac{d\omega}{dt}\right)_{rot} \times r' + \omega \times \left(\frac{dr'}{dt}\right)_{rot} + \omega \times v' + \omega \times (\omega \times$$

$$a = \left(\frac{dv}{dt}\right)_{fixed} \qquad a' = \left(\frac{dv'}{dt}\right)_{rot} \qquad \dot{\omega} \qquad v' = \left(\frac{dr'}{dt}\right)_{rot}$$

$$a = a' + \dot{\omega} \times r' + \omega \times v' + \omega \times v' + \omega \times (\omega \times r')$$

$$a = a' + \dot{\omega} \times r' + 2\omega \times v' + \omega \times (\omega \times r')$$

The above equation gives **the acceleration** in the **fixed system** in terms of the **position**, **velocity**, and **acceleration** of the **rotating system**



(2) The general case
Moving and **Rotating** system

$$v = v' + \omega \times r' + V_0$$

$$a = a' + \dot{\omega} \times r' + 2\omega \times v' + \omega \times (\omega \times r') + A_0$$

$2\omega \times v'$ Coriolis acceleration

$\omega \times (\omega \times r')$ Centripetal acceleration

$\dot{\omega} \times r'$ Transverse acceleration