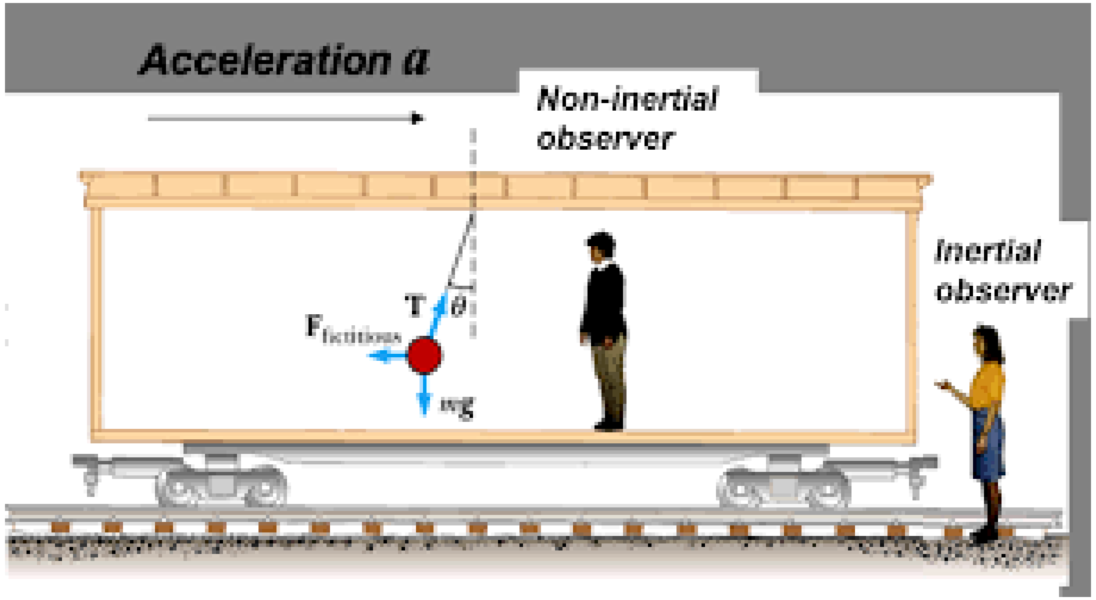
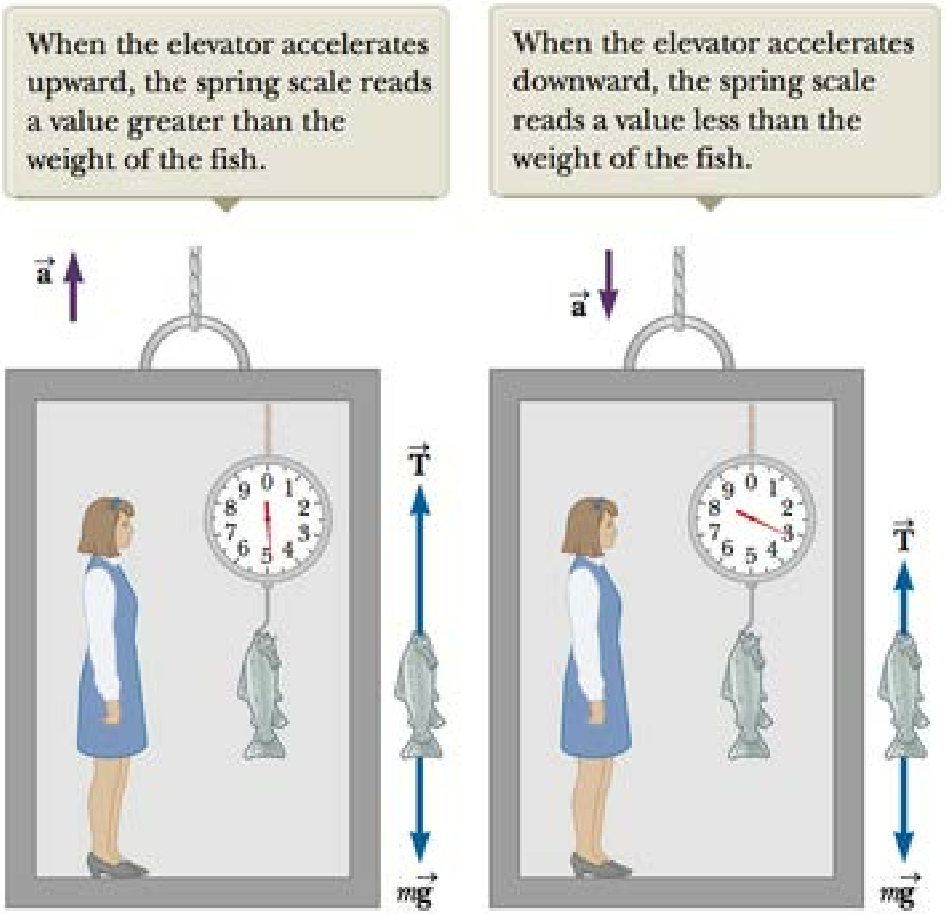
Chapter 5

Noninertial Reference System

Introduction



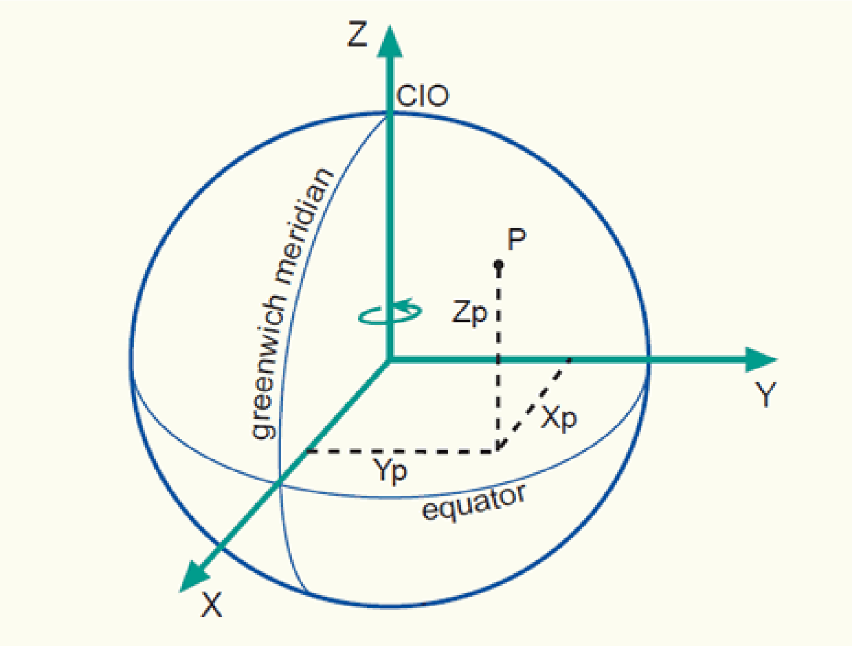


1. Accelerated Coordinate Systems and Inertial Forces

In describing the motion of a particle

The coordinate system is **non-inertial**

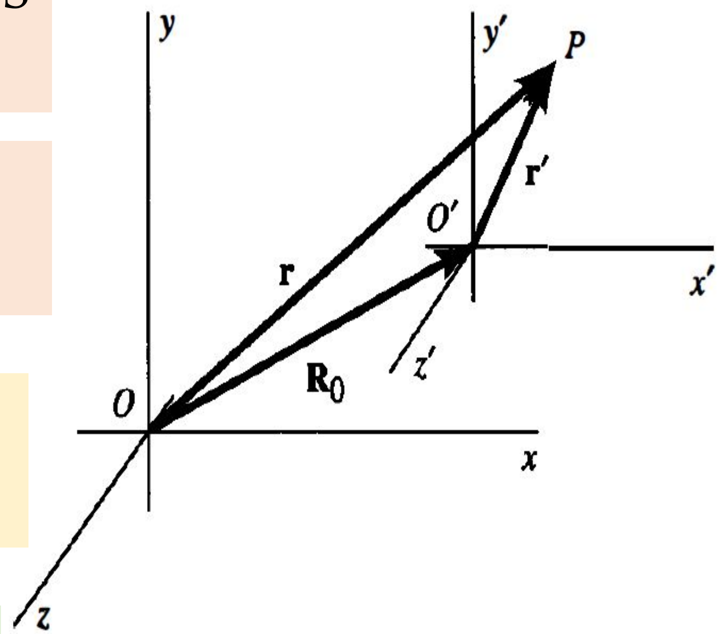
<u>For example</u>, a coordinate system **fixed to the Earth** is the most convenient one to describe the motion of a projectile, even though the Earth is **accelerating and rotating.**



- Consider the case of a coordinate system that undergoes pure translation

- Oxyz are the primary coordinate axes (assumed fixed), and O'x'y'z' are the moving axes

- The position vector of a particle P is denoted by r in the fixed system and by r' in the moving system.



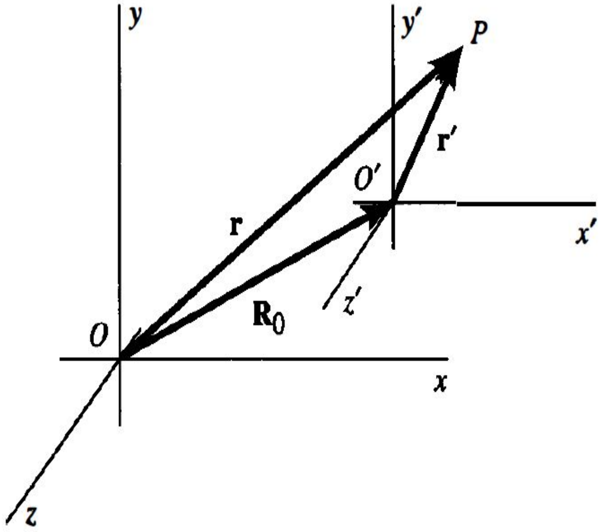
The displacement OO' of the moving origin is denoted by R_o- Thus, from the triangle OO'P, we have

$$r = R_o + r'$$

Taking the first and second time derivatives gives

$$v = V_0 + v' \qquad a = A_0 + a'$$

$r = R_o + r'$ $v = V_0 + v'$ $a = A_0 + a'$



- Where V_0 and A_0 are the velocity and acceleration of the moving system, respectively.

- v' and a' are the velocity and acceleration of the particle with respect to the moving system.

- v and a are the velocity and acceleration of the particle in the with respect to the non-moving system.

In particular, if the moving system is not accelerating, so that $A_0 = 0$, then

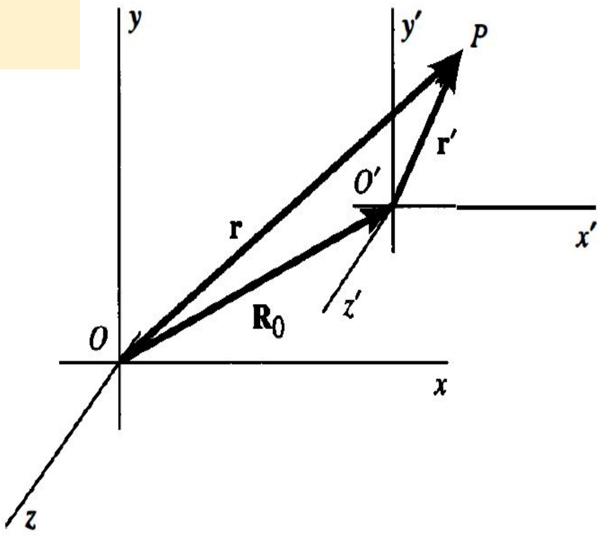
$a = a'$

$$\boxed{\textbf{\textit{Newton's 2nd Low}} \quad F = ma = ma'}$$

- On the other hand if the moving system is accelerating, then Newton's second law becomes

$a = A_0 + a'$ ⟹ $ma = m(A_0 + a') = F$

$F = mA_0 + ma'$ ⟹ $F - mA_0 = ma'$



$F' = F + (-mA_0)$

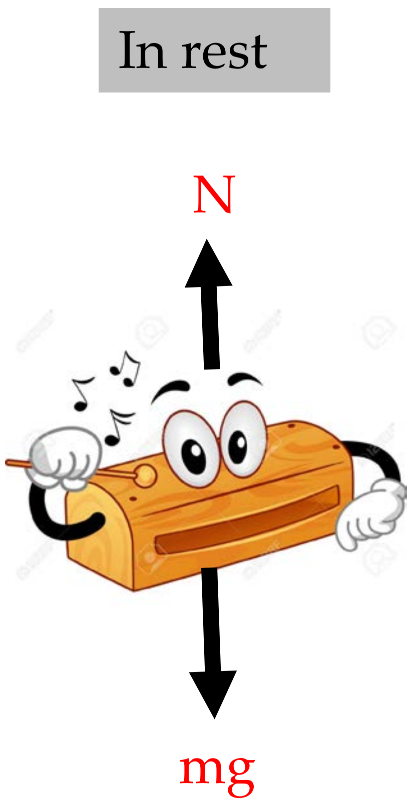
Equation of Motion

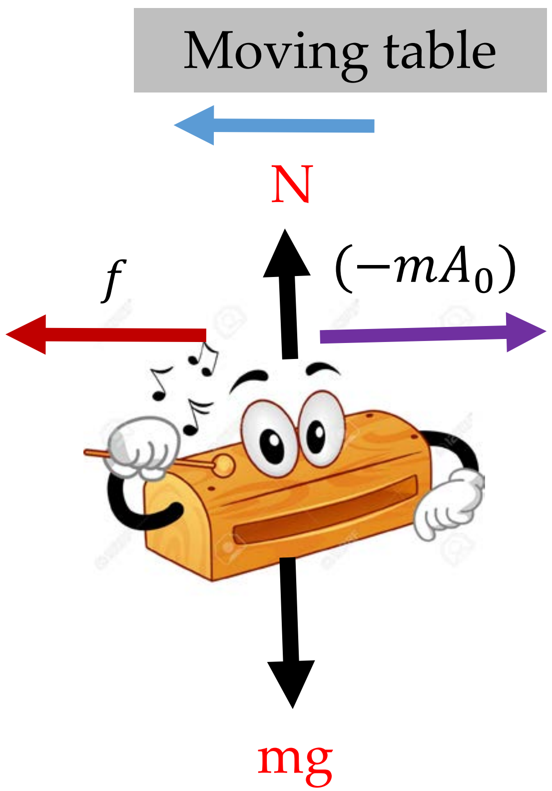
That is, **an acceleration A_0 of the reference system** can be taken into account by adding an **inertial term** **or** **fictitious force** **($-mA_0$) to the force F** and equating the result to the product of mass and acceleration in the moving system.

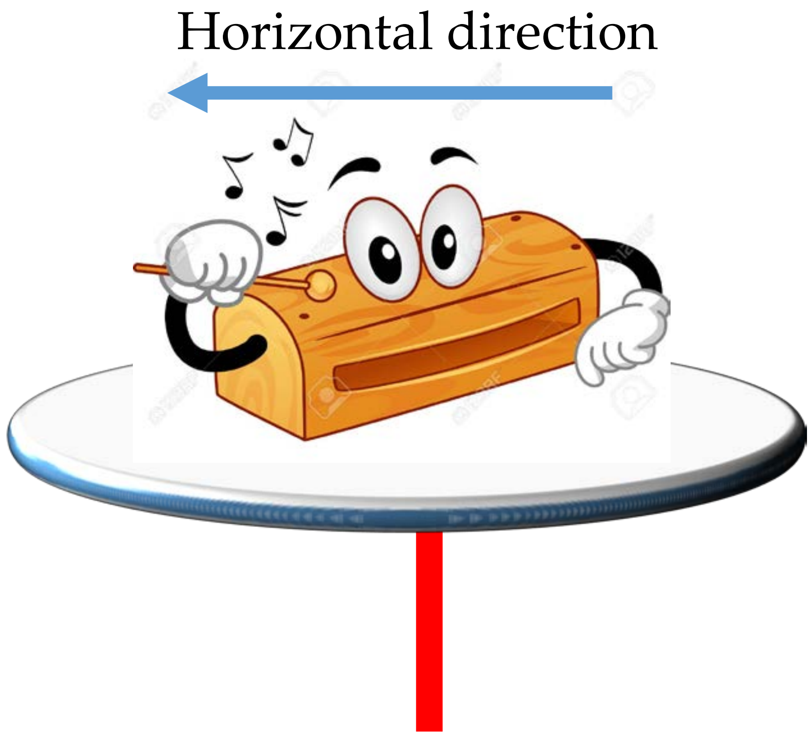
Example 1

A block of wood rests on a rough horizontal table. If the table is accelerated in a horizontal direction, under what condition will the block slip?

Let μ_s be the coefficient of static friction between the block and the table top.







The condition of slipping is that the inertial force ($-mA_0$) exceeds the frictional force, where A_0 is the acceleration of the table.

Thus $$|-mA_0| > \mu_s mg$$

$$A_0 > \mu_s g$$