Sheet 3

Q1:- Let A, B, C, D, E be subsets of the Euclidean space $\mathbb{R}^2$. Find their boundary, their interior, and their exterior. Conclude from here whether these sets are open, closed, or neither.

a) $A = \{x \in \mathbb{R}^2 \mid d_2(x, x_0) \leq 2\}$, where $x_0 \in \mathbb{R}^2$.
b) $B = \mathbb{R} \times [a, b)$, where $a, b \in \mathbb{R}$, $a < b$.
c) $C = (a, b)^2 = (a, b) \times (a, b)$, where $a, b \in \mathbb{R}$, $a < b$.
d) $D = \{a\} \times [b, c)$, where $a, b, c \in \mathbb{R}$, $b < c$.
e) $E = \{a\} \times \{b, c\}$, where $a, b, c \in \mathbb{R}$, $b \neq c$.

Remark: In this problem, it is sufficient to sketch the set and to give the correct answers without justification.

Q2:- In the Euclidean space $\mathbb{R}^3$, find the set

$$A = \bigcap_{n=1}^{\infty} \left[-1 - \frac{1}{n}, 2 + \frac{1}{n}\right] \times \left[-\frac{1}{n^2}, \frac{1}{n^2}\right] \times [0, e^{-n}].$$

Is this set open, closed, or neither?

Q3:- Let (X, d) be a discrete metric space and let $x_0 \in X$ be a point. Describe the open ball $B_r(x_0)$ where (a) $0 < r \leq 1$. (b) $r > 1$. (c) $r > 1$ and $r \neq 0$

Q4:- Consider the following sets

1. $A_n = [\frac{1}{n}, \infty)$ closed for each $n = 1,2,\cdots$. However, $\bigcup_{n=1}^{\infty} A_n =$
2. $B_n = [\frac{1}{n}, 1 - \frac{1}{n}]$ closed for each $n = 1,2,\cdots$. However, $\bigcup_{n=1}^{\infty} B_n =$
3. $C_n = [\frac{1}{n}, 1]$ closed for each $n = 1,2,\cdots$. However, $\bigcup_{n=1}^{\infty} C_n =$
4. $D_n = [-n, n]$ closed for each $n = 1,2,\cdots$. However, $\bigcup_{n=1}^{\infty} D_n =$

What can you conclude from the union of the above sets?

Q7:- In the Euclidean space $\mathbb{R}^2$, find the closure of the following sets:
(i) $A = [a, b) \times (a, b]$, where $a, b \in \mathbb{R}$, $a < b$.
(ii) $B = [a, b) \times Q$, where $a, b \in \mathbb{R}$, $a < b$.