Sheet 1

Q1:- Let $X = \{1,2,3\}$. Let a function $d : X \times X \to [0,\infty)$ be as below. Decide whether d is a metric on X . You must justify your answers.

(i) $d(1,1)=d(2,2)=d(3,3)=0$,
$d(1,2) = d(2,1)=2$,
$d(2,3)=d(3,2)=4$,
$d(1,3)=d(3,1) = 5$.

(ii) $d(1,1)=d(2,2)=d(3,3)=0$,
$d(1,2) = d(2,1)=2$,
$d(2,3)=d(3,2)=4$,
$d(1,3)=d(3,1) = 7$.

Q2:- Let $x = (1,5,-3)$ and $y = (3,8,-9)$ are two vectors in $\mathbb{R}^3$. Find (a) $d_1(x,y)$, (b) $d_2(x,y)$, (c) $d_\infty(x,y)$, where $d_1,\ d_2, d_\infty$ are metrics on $\mathbb{R}^3$.

Q3:- Consider the metric space ($C([a,b]), d_\infty$).

A. Let $f(x) = x^2$ and $g(x) = x^3$. Find

(i) $d_\infty(f,g)$ in $C([0,1])$. (ii) $d_\infty(f,g)$ in $C([-1,1])$.

B. Let $f(x) = x^2$ and $g(x) = x^4$. Find

(i) $d_\infty(f,g)$ in $C([0,1])$. (ii)$d_\infty(f,g)$ in $C([0,2])$

Q4:- Prove that $(\mathbb{R}^N,\ \|\cdot\|_2)$ is a normed space, where $\|\cdot\|_2$ is the Euclidean norm on $\mathbb{R}^N$.

Q5:- In $\mathbb{R}^N$ we define

1. $d_1(x,y) = \sum_{i=1}^N |x_i - y_i|$,
2. $d_2(x,y) = \left(\sum_{i=1}^N |x_i - y_i|^2\right)^{\frac{1}{2}}$,
3. $d_\infty(x,y) = \max\limits_{1\le i\le N}\{|x_i - y_i|\}$.

Prove that $d_1,\ d_2$ and d_∞ are metrics on $\mathbb{R}^N$.

Q6:- Let d be a metric on X. Determine all constants K such that
(1)_kd is a metric on X. (2) d + k is a metric on X

Q7:- Inverse triangle inequality. Let $(X, \|\cdot\|)$ be a normed space. Prove that

$$\|x - y\| \ge \big|\|x\| - \|y\|\big| \qquad \forall\, x,y \in X.$$