Advanced Mathematical Analysis

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This lectures mainly based on course given by Pro. Vitaly Moroz in Swansea University.

This lecture notes are not intended to cover the entire content of the lectures

Reading books

1. Principles of Mathematical Analysis
   Walter Rudin 3^{rd} ed.

2. A First Course in Real Analysis
   M. H. Protter and C. B. Morry.

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