

Polar Coordinates

* Cartesian coordinates (x, y)

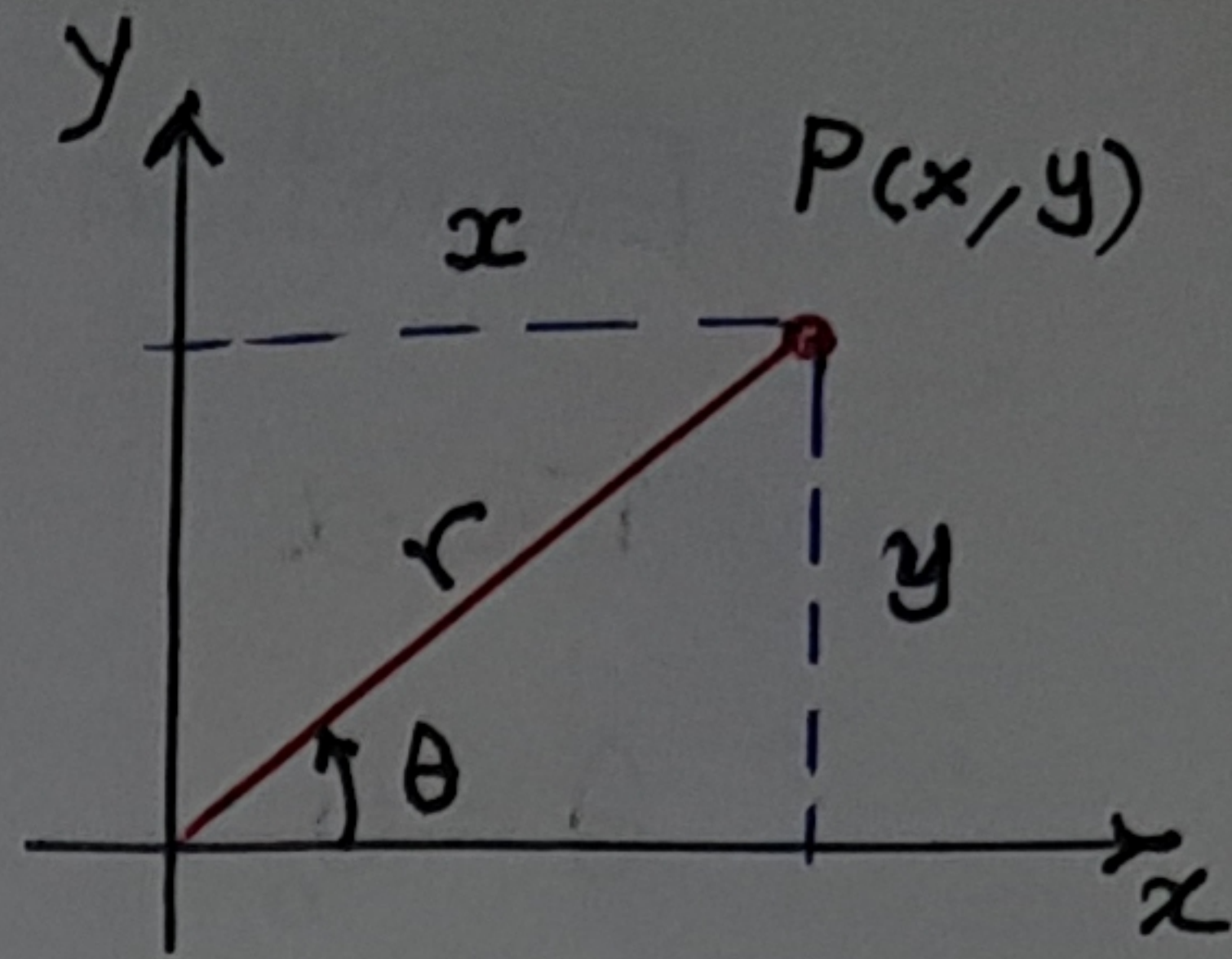
* Polar coordinates (r, θ)

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1} \frac{y}{x}$$

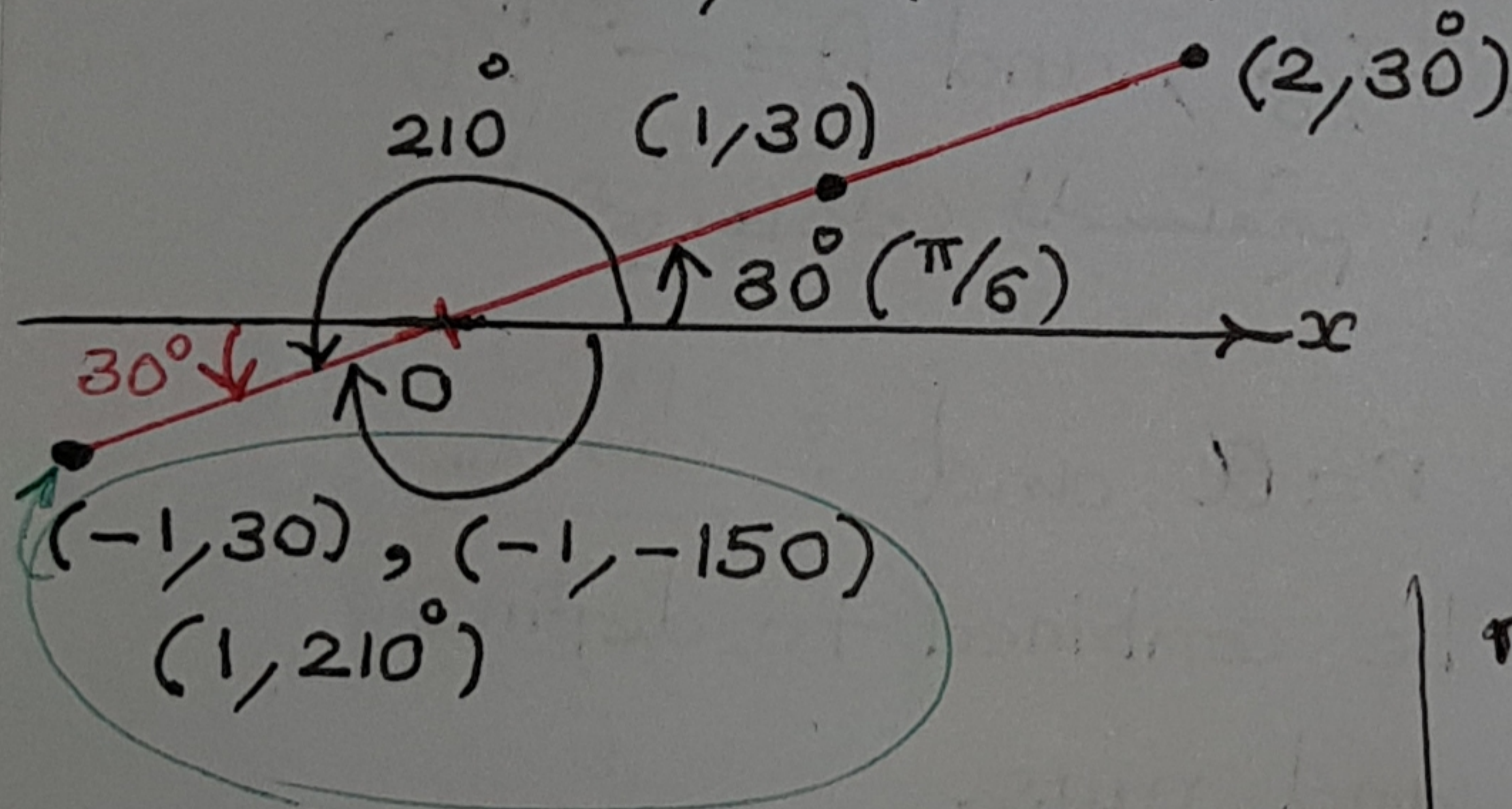
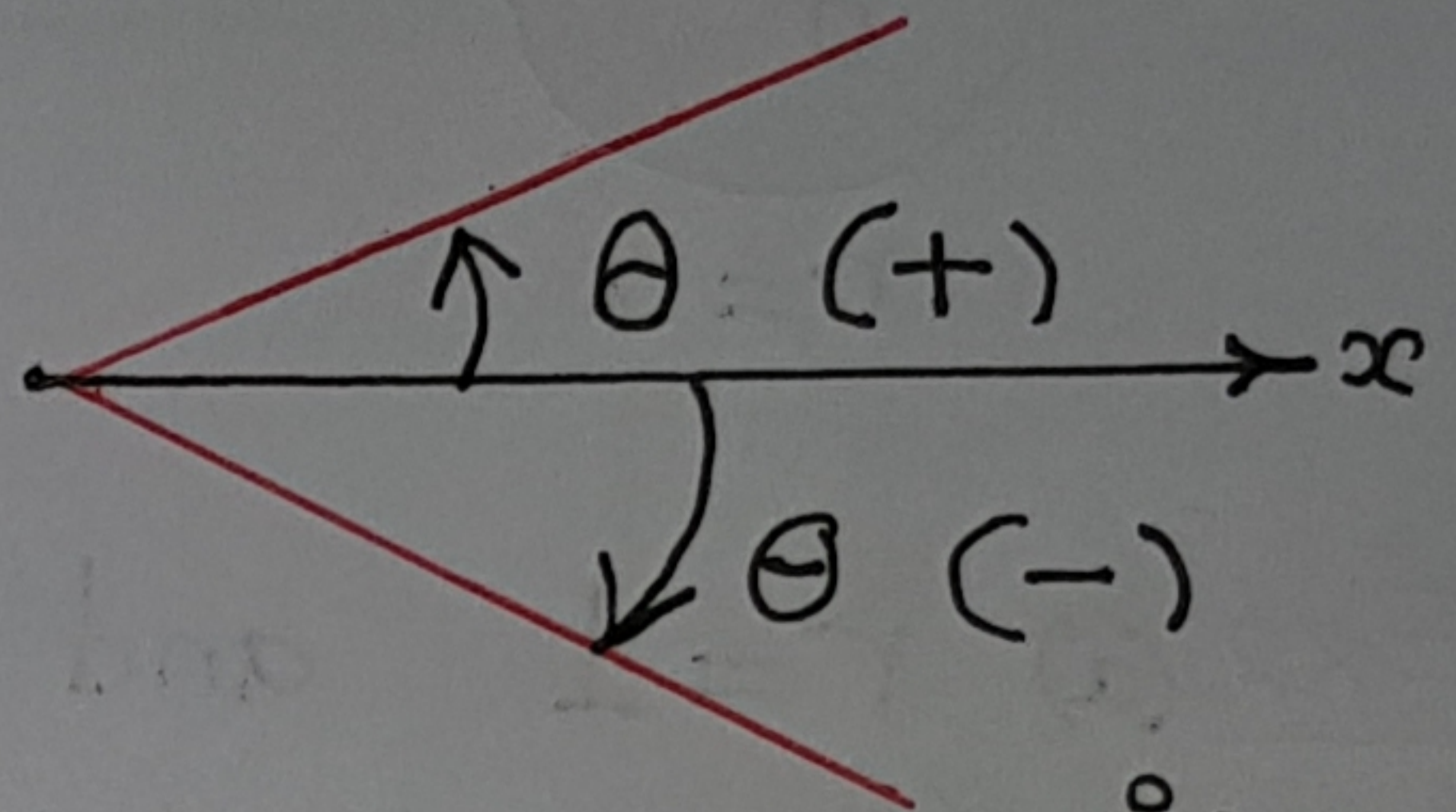


Ex 1. Sketch the

following points

$(1, 30^\circ)$, $(2, 30^\circ)$,

$(-1, 30^\circ)$, $(1, 210^\circ)$ and $(-1, -150^\circ)$.



$$r \cos \theta = a$$

$$r \sin \theta = a$$

معادلة أفقي

$$x = a$$

(1)

معادلة عمودي

$$y = a$$

Polar Equations and Graphs

Equation

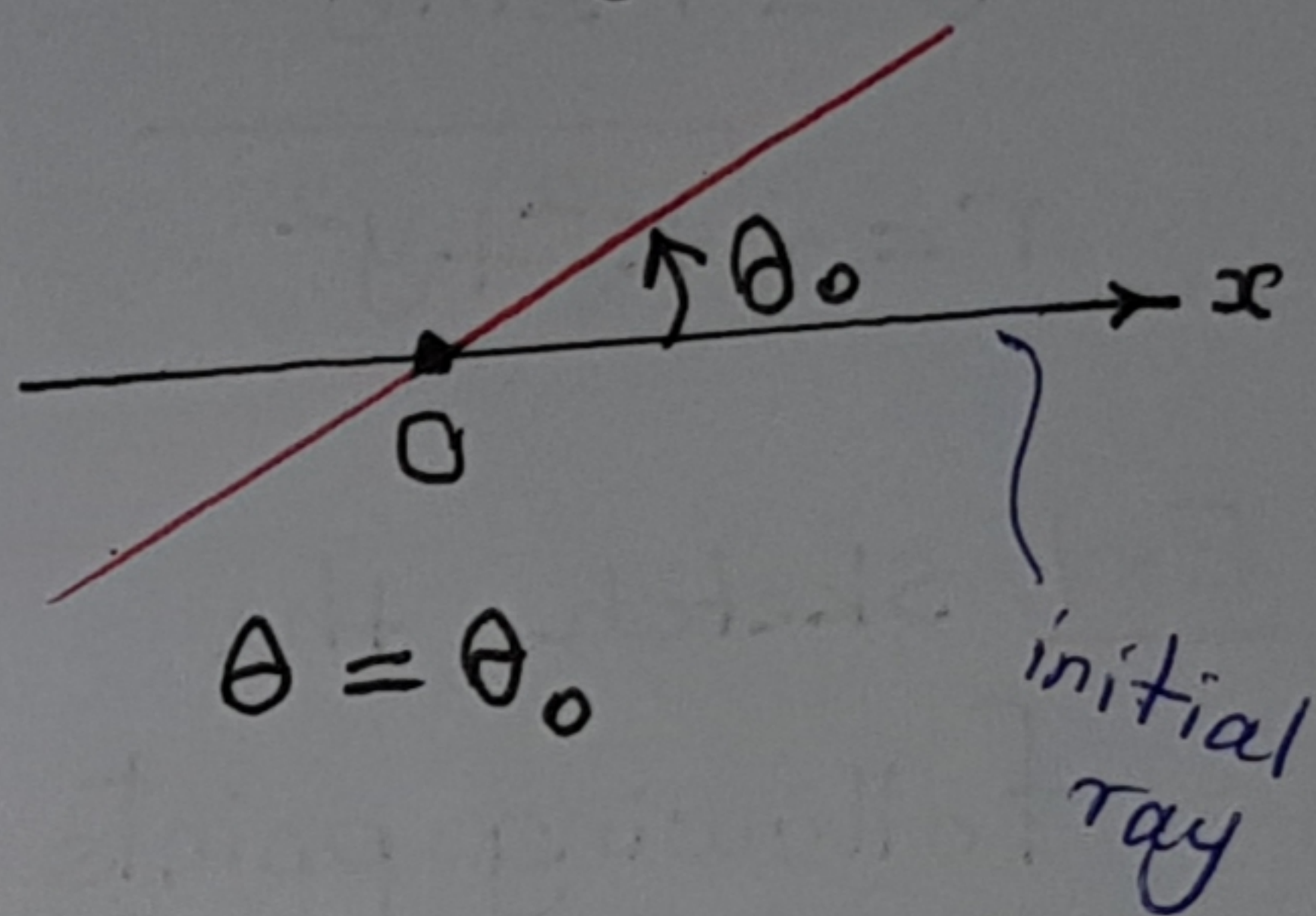
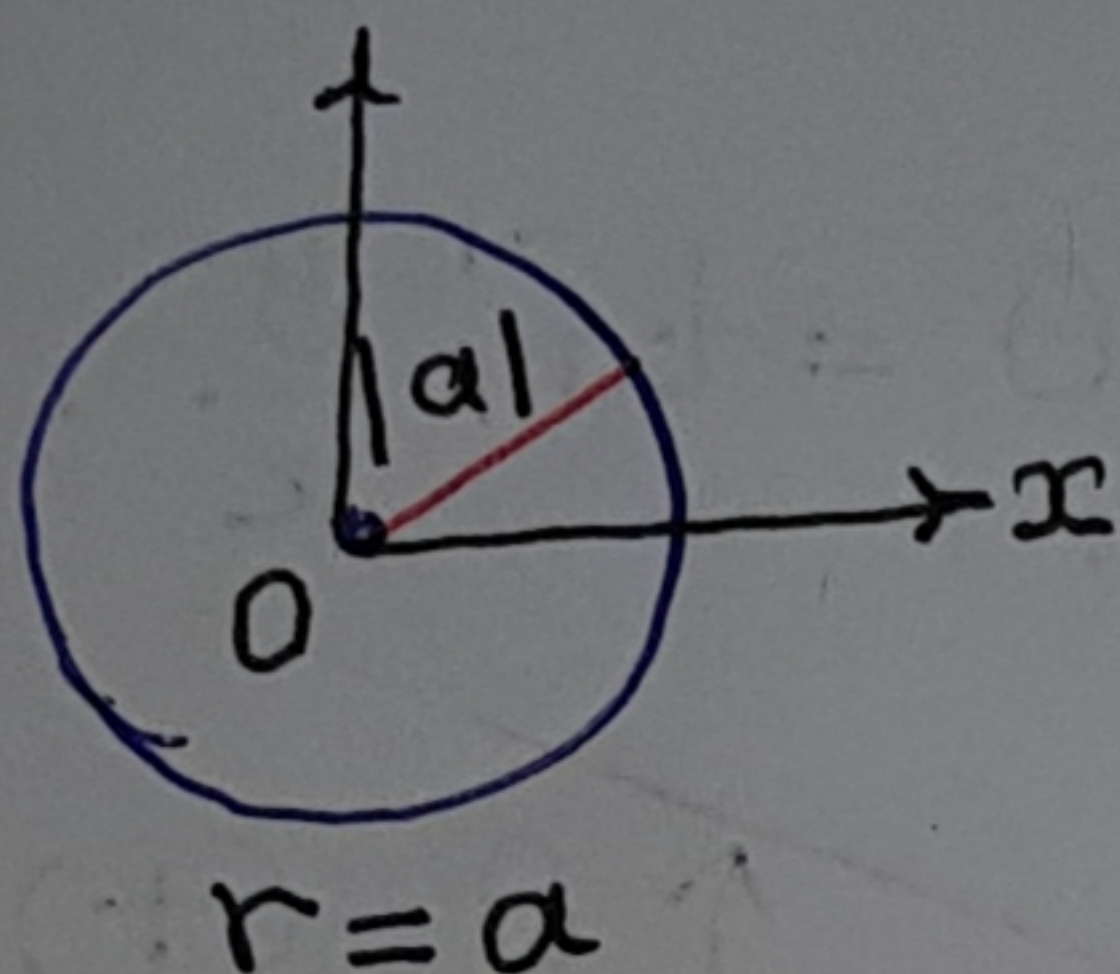
Graph

$$r = a$$

Circle of radius $|a|$ centered at O .

$$\theta = \theta_0$$

Line through O making an angle θ_0 with the initial ray.



Ex 2 (a) $r = 1$ and $r = -1$

معادلة دائرة نصف قطرها $r = 1$ ومركزها O (نصف القطر)

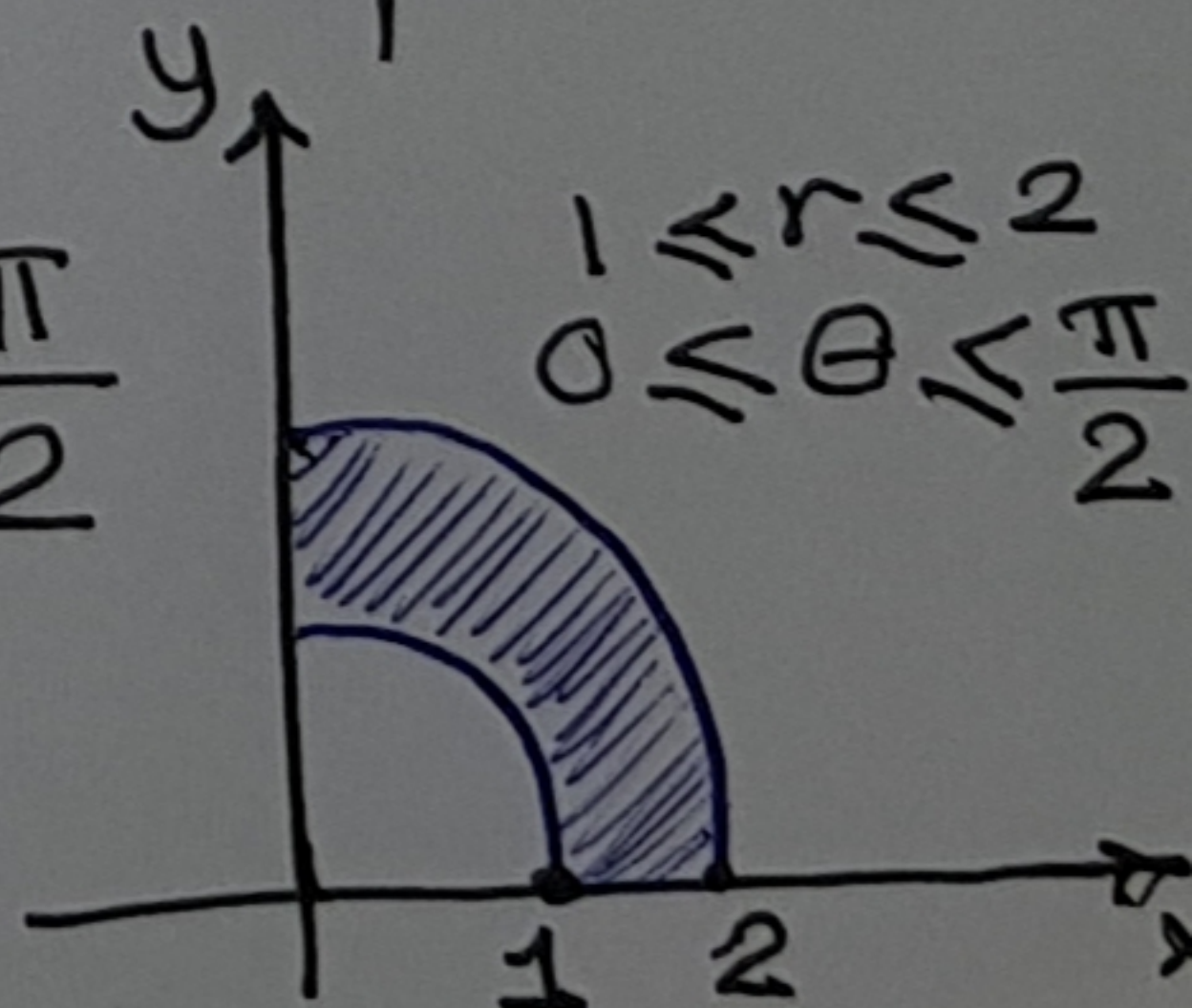
(b) $\theta = \frac{\pi}{6}$, $\theta = \frac{7\pi}{6}$, and $\theta = -\frac{5\pi}{6}$
هي معادلات للمستقيم الموضح في مثال (1).

Note: Equations of $r = a$ and $\theta = \theta_0$ can be combined to define regions, segments and rays.

Ex 3. Graph the sets of points whose polar coordinates are given by

(a) $1 \leq r \leq 2$ and

$$0 \leq \theta \leq \frac{\pi}{2}$$

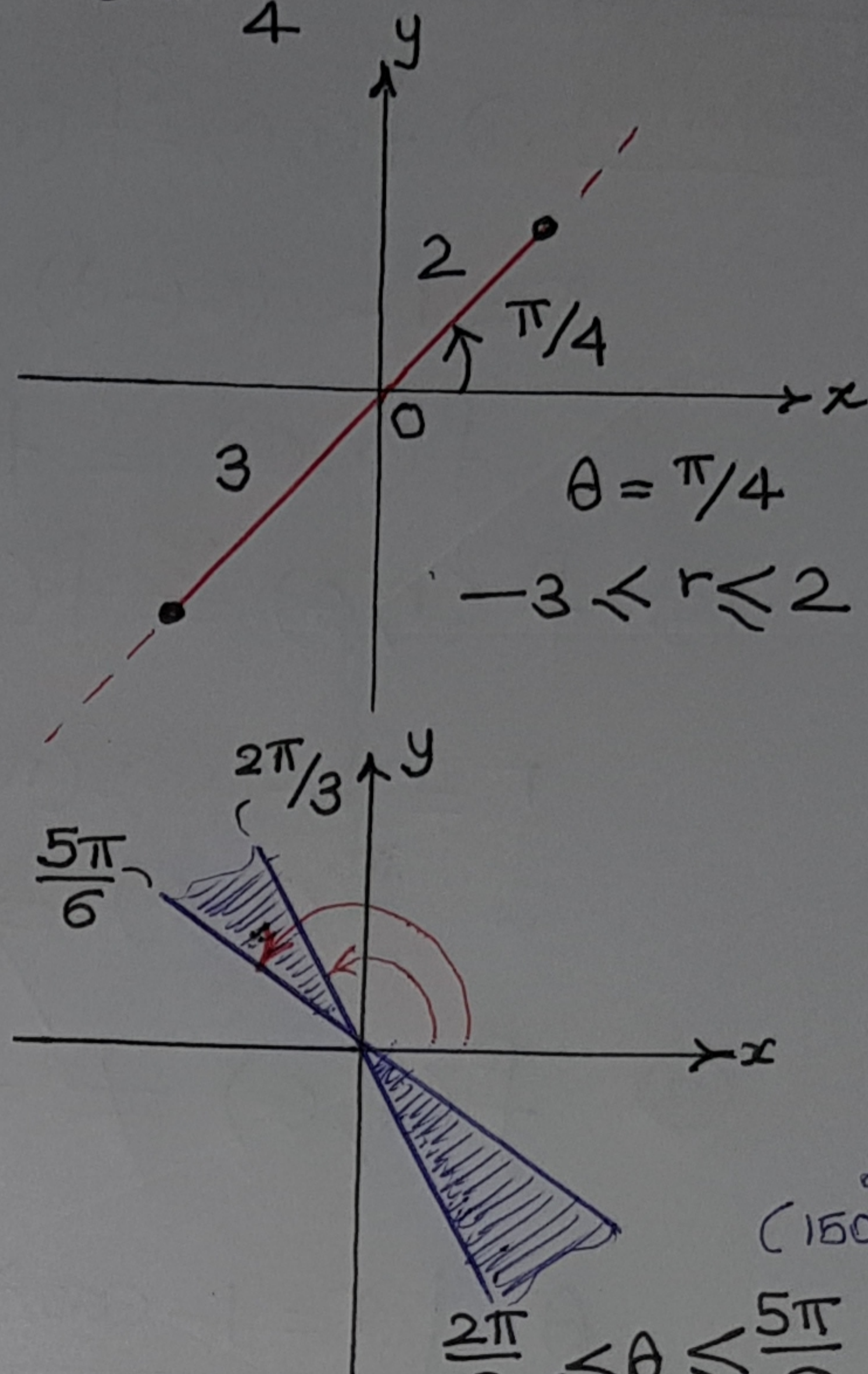


(2)

(b) $-3 \leq r \leq 2$ and $\theta = \frac{\pi}{4}$

(c) $\frac{2\pi}{3} \leq \theta \leq \frac{5\pi}{6}$

(no restriction on r)
 غير مقيد r



Graphing in Polar Coordinates

Symmetry

1. Symmetry about the x-axis:

$$f(r, \theta) = f(r, -\theta)$$

2. Symmetry about the y-axis:

$$f(r, \theta) = f(r, \pi - \theta)$$

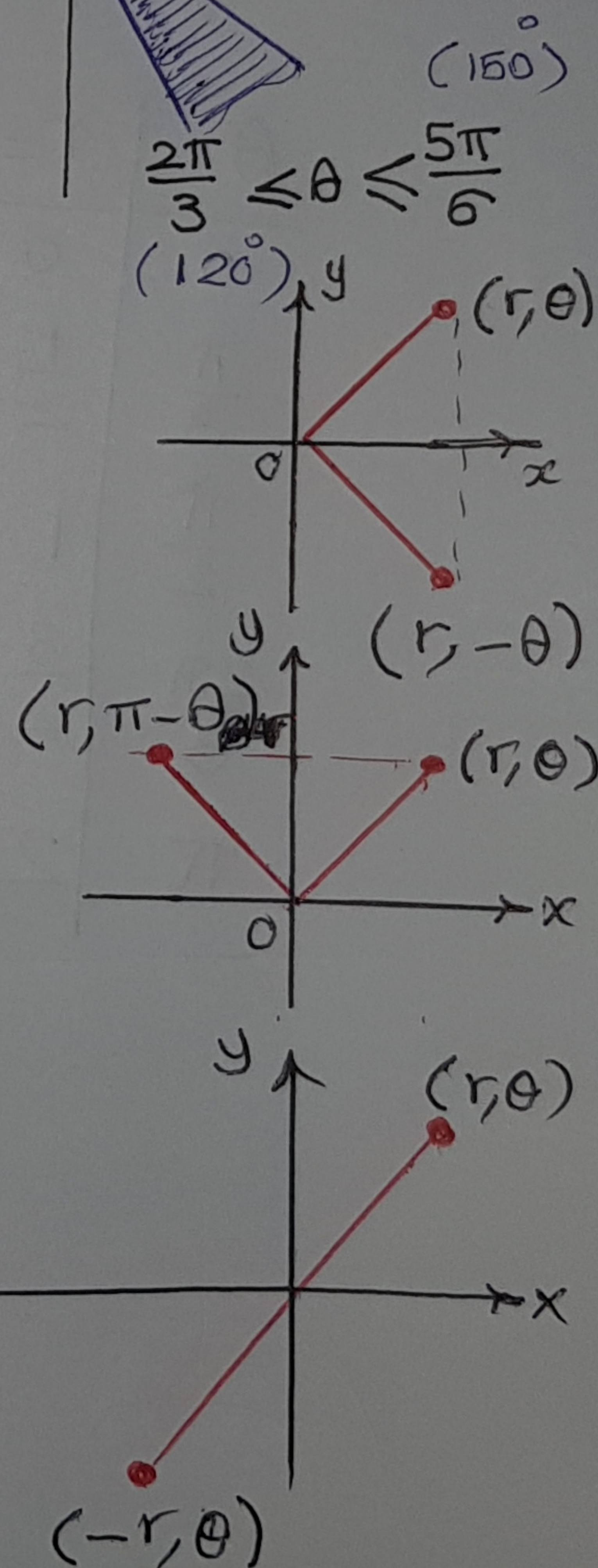
3. Symmetry about the origin:

$$f(r, \theta) = f(-r, \theta)$$

Note

$$\sin(\pi - x) = \sin x$$

$$\cos(\pi - x) = -\cos x$$



Ex. Sketch the curve $r = 1 - \cos \theta$.

Solution ① $f(r, \theta) \stackrel{?}{=} f(r, -\theta)$ symmetry المتماثل ①

$$r = 1 - \cos(-\theta) = 1 - \cos \theta$$

$$\therefore f(r, \theta) = f(r, -\theta)$$

تتماثل حول محور x

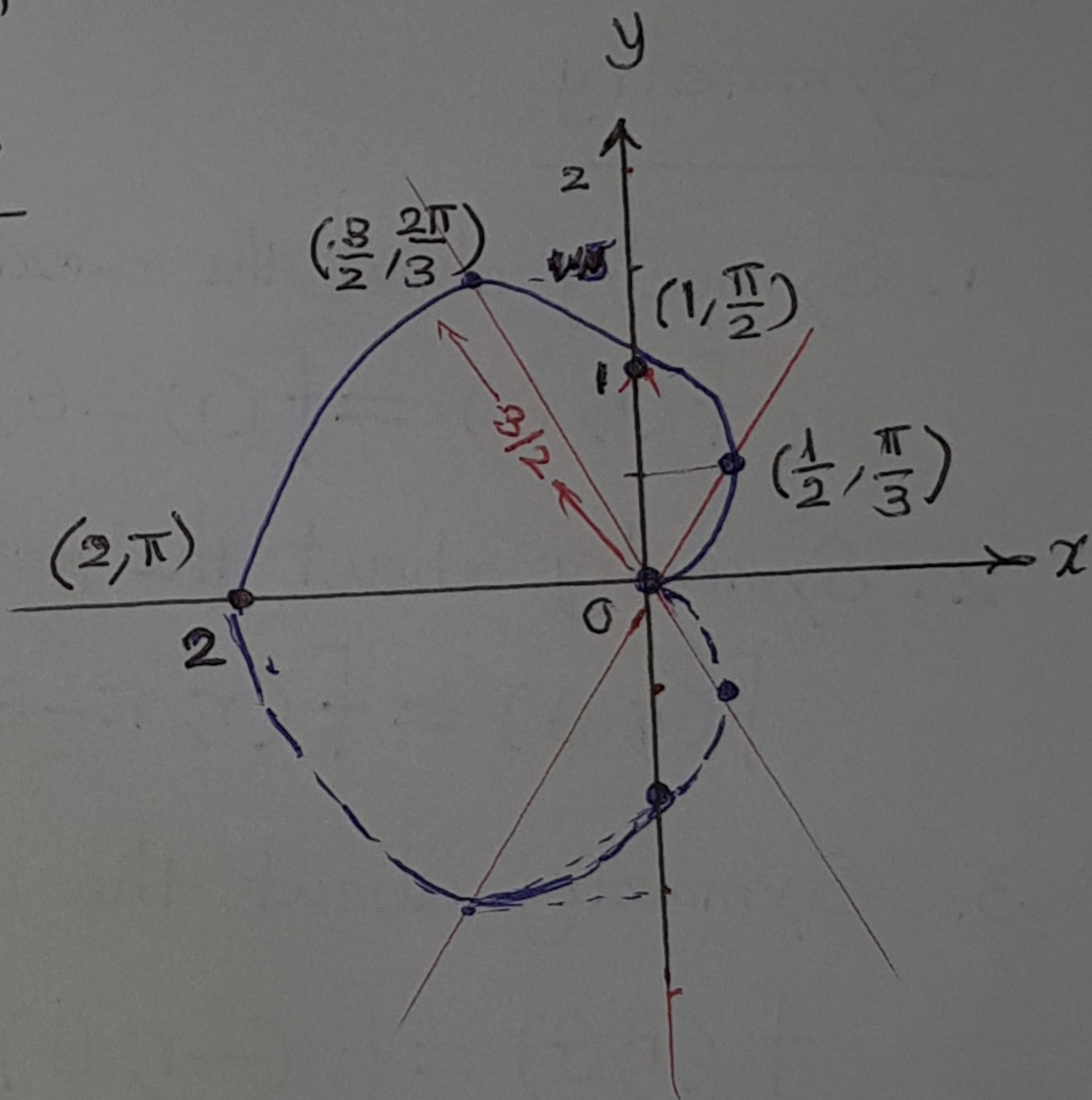
$$\textcircled{2} f(r, \theta) \stackrel{?}{=} f(r, \pi - \theta)$$

$$r = 1 - \cos(\pi - \theta) = 1 + \cos \theta$$

$$\therefore f(r, \theta) \neq f(r, \pi - \theta)$$

$$\textcircled{3} f(r, \theta) \neq f(-r, \theta)$$

θ	$r = 1 - \cos \theta$
0	0
$\frac{\pi}{3}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	$\frac{3}{2}$
π	2



Ex. Graph the curve $r^2 = \sin 2\theta$

Solution

$\therefore f(-r, \theta) = f(r, \theta)$

نقاط مرسومة باليد

نقاط تقاطع

$(0.93, \pi/3)$

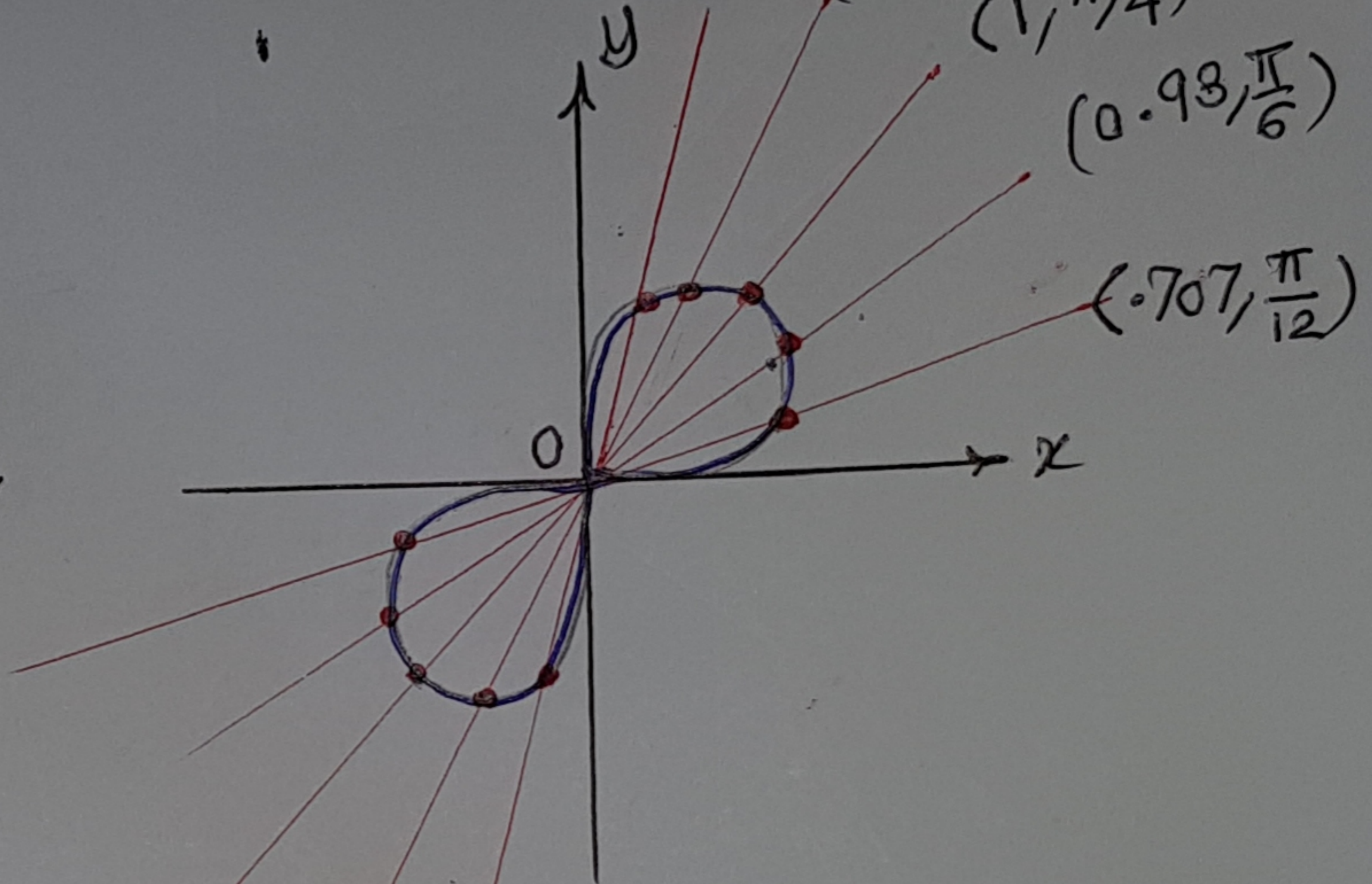
$(1, \pi/4)$

$(0.93, \pi/6)$

$(0.707, \pi/12)$

$0 \leq \theta \leq \frac{\pi}{2}$

θ	$r = \pm \sqrt{\sin 2\theta}$
0	0
$\pi/12$	± 0.707
$\pi/6$	± 0.93
$\pi/4$	± 1
$\pi/3$	± 0.93
$5\pi/12$	± 0.707
$\pi/2$	0



Home work

Graph the following Curves

(a) $r^2 = 4 \cos \theta$

(b) $r = 1 + \cos \theta$

(c) $r = 1 - \sin \theta$

(d) $r = \sin(\theta/2)$

(e) $r^2 = \cos \theta$