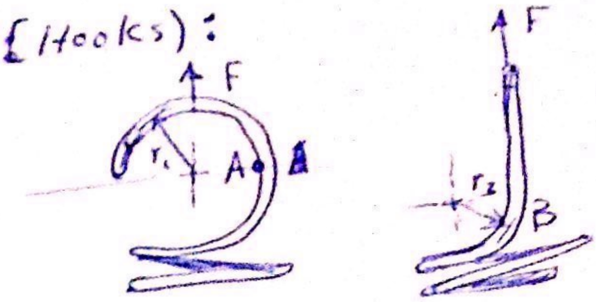
(6) Extension springs:

* load is tensile F  F , spring extends

If load is removed ⇒ shorten back to original length

* Hooks , threaded plugs.... are types of spring ends to transfer load to body of spring of spring.
* Stresses in the body is the same as Compression springs.

* stresses at spring ends [Hooks):



(i) Maximum tensile stress @ A
(inner fibre of ring)

$$\sigma_A = K_A \frac{16FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

K_A: bending stress correction factor for curvature.

$$K_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} \quad \& \quad C_1 = \frac{2r_1}{d} \quad , \quad r_1 = \frac{D_{mean}}{2}$$

(ii) Maximum shear stress @ B

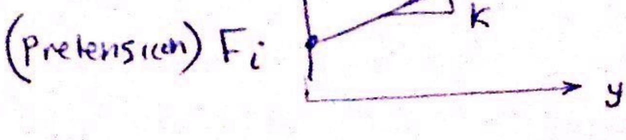
$$\tau_B = K_B \frac{8FD}{\pi d^3}$$

K_B: shear correction factor for curvature.

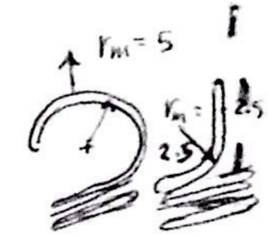
$$K_B = \frac{4C_2 - 1}{4C_2 - 4} \quad \& \quad C_2 = \frac{2r_2}{d} \quad , \quad r_2 : \text{Curvature radius}$$

* Extension springs usually have "initial tension" (pretension)

$$\Rightarrow \quad F = F_i + ky$$



Example: the following specifications were taken from tension spring removed from junked washing machine:



$D = 10$ mm , $d = 1.8$ mm , $N = 122$ active Coils

distance between hook = 244 mm , Preload = 25 N

and material is hard drawn (HD) wire $\left\{ \begin{array}{l} G = 79.3 \text{ Gpa} \\ S_{ut} = 1560 \text{ Mpa} \end{array} \right\}$

(a) Compute initial stress in the spring?

(b) What force to cause spring body stressed to yield strength?

(c) What is spring stiffness?

(d) What force to cause hook's ends torsionally stressed to yield strength?

(e) " " " : normal stress @ hook's end to reach tensile yield strength?

Sol.

(a) $\tau = K_s \dfrac{8FD}{\pi d^3}$, $K_s = \dfrac{2C+1}{2C}$, $C = \dfrac{D}{d}$

$\Rightarrow C = \dfrac{10}{1.8} = 5.56 \Rightarrow K_s = 1.09 \Rightarrow \tau = 1.09 \dfrac{8 * 25 * 10 * 10^{-3}}{\pi (1.8)^3 * 10^{-3}}$

$= 119$ Mpa

(b) $S_y = 0.75\, S_{ut} \Rightarrow S_{y\,tensile} = 1170$ Mpa

$S_{y\,shear} = .58\, S_{y\,tensile} = 679$ Mpa

$S_{y\,shear} = K_s \dfrac{8FD}{\pi d^3} \Rightarrow 679 = 1.09 * \dfrac{8F * 10}{\pi (1.8)^3} \Rightarrow F = 142$ N

(c) $k = \dfrac{d^4 G}{8 D^3 N} = \dfrac{(1.8)^4 * 79.3 * 10^6}{8 (10)^3 * 122} = 853$ N/m

(d) $r_2 = 2.5 \Rightarrow C_2 = \dfrac{2 r_2}{d} = 2.78 \Rightarrow K_B = \dfrac{4C_2 - 1}{4C_2 - 4} = 1.42$

$S_{y\,shear} = K_B \dfrac{8FD}{\pi d^3} \Rightarrow 679 = 1.42 * \dfrac{8F * 10}{\pi (1.8)^3} \Rightarrow F = 109.5$ N

(e) $r_1 = 5 \Rightarrow C_1 = \dfrac{2 r_1}{d} = 5.55 \Rightarrow K_A = \dfrac{4C_1^2 - C_1 - 1}{4C_1 * (C_1 - 1)} = 1.16$

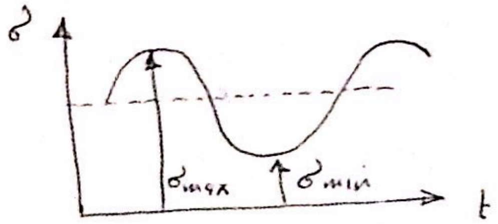
$S_{y\,tensile} = K_A \dfrac{16FD}{\pi d^3} + \dfrac{4F}{\pi d^2}$

$1170 = 1.16 * \dfrac{16F * 10}{\pi (1.8)^3} + \dfrac{4F}{\pi (1.8)^2} \Rightarrow F = 111$ N

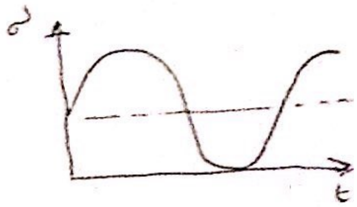
Notes ① $N_a = 122$ too much
② failure @ hook because of shear (weakest) then hook (bending) then body of spring
$109 < 111 < 142$ N

(7) Fatigue Loading

- Springs are almost always subjected to fatigue loading
- Some springs are designed to operate without failure (∞ life).
- Preload is always exerted on helical springs such that usual condition is ask shown below:



Usual Case



Worst Case
(No preload)

- To design against Fatigue:

(1) Find $F_a = \dfrac{F_{max} - F_{min}}{2}$, $\tau_a = K_w \dfrac{8 F_a D}{\pi d^3}$

$F_m = \dfrac{F_{max} + F_{min}}{2}$, $\tau_m = K_s * \dfrac{8 F_m D}{\pi d^3}$

(2) Use Goodman Diagram $\left[\dfrac{\tau_a}{S_{es}} + \dfrac{\tau_m}{S_{us}} = \dfrac{1}{n}\right]$ ($S_u|_{shear} = 0.6\, S_u|_{tensile}$)

Use $\tau_a = \dfrac{S_e}{n}\Big|_{shear}$ ✓ (This equation is valid since load is shear Only)

(3) "Zimmerli" found that endurance limit for different type of springs material and sizes ⟪diameter < 10 mm⟫:

$$S_e'\Big|_{shear} = \begin{cases} 310 \text{ Mpa} & \text{for Unpeened springs} \\ 465 \text{ Mpa} & \text{for peened springs} \end{cases}$$

(4) Above endurance limit should corrected for relaibility & temp cond

Example

A 2.24 mm helical Compression spring (music wire) has an outside Coil diameter of 14.3 mm, free length of 105 mm, 21 active Coils. The spring to be assembeled with a preload of 45 N and will operate to a maximum load of 225 N during Use. Determine the Safety factor gaurding against fatigue failure based on a life of 50×10^3 Cycles and relaibility of 99 percent.

Solution

- mean Coil diameter : $D_m = OD - d = 14.3 - 2.24 = 12.06$ mm
- Sprind index : $C = \frac{D_m}{d} = \frac{12.06}{2.24} = 5.38$
- $K_s = \frac{2C+1}{2C} = 1.092$; $K_w = \frac{4C-1}{4C-4} + \frac{.615}{C} = 1.285$
- $F_a = \frac{F_{max} - F_{min}}{2} = \frac{225-45}{2} = 90$ N

  $F_m = \frac{F_{max} + F_{min}}{2} = \frac{225+45}{2} = 135$ N
- $\tau_a = K_w \frac{8 F_a D}{\pi d^3} = 1.285 \frac{8 * 90 * 12.06}{\pi (2.24)^3} = 316$ Mpa

  $\tau_m = K_s * \frac{8 F_m D}{\pi d^3} = 1.09 * \frac{8 * 135 * 12.06}{\pi (2.24)^3} = 402.8$ MPa
- Material Data

① Unpeened spring $\Rightarrow S_e' = 310$ Mpa

for 99% relaibility $\Rightarrow K = .814$

Corrected Endurance limit $S_e = .814 * 310 = \boxed{252.3}$ MPa

② music wire spring $\Rightarrow m = .145$; $A = 2211$

$S_{ut} = \frac{A}{d^m} = \frac{2211}{(2.24)^{.145}} = 1967$ Mpa

$S_{ut}|_{shear} = .6\, S_{ut}|_{tensile} = \boxed{1180 \text{ Mpa}}$

Now, $S_f = a N^b$, $\left\{ \begin{array}{l} b = -\frac{1}{3} \log \frac{f\, S_{ut}|_{shear}}{S_e} = -.18359 \\ a = \frac{(f\, S_{ut}|_{shear})^2}{S_e'} = 4,194 \end{array} \right.$

$S_f = 4194 (50 \times 10^3)^{-.1835}$

$= 575$ Mpa $\Rightarrow n = \frac{S_f}{\tau_a} = \frac{575}{316} = 1.82$