Chapter Five: Mechanical Springs:

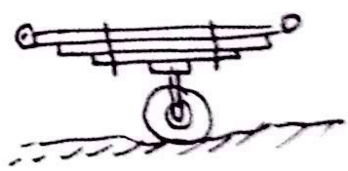
Def. * Spring is a mechanical device designed to store energy when deflected and to return an equivalent amount of energy when released.

Use * Springs are used in wide variety of applications since they allow controlled application of force or torque plus their ability for storing and releasing energy.

Classification

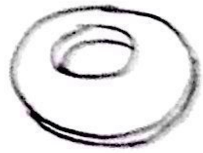
* (i) Helical wire springs (compression, extension and torsional)

(ii) flat springs (leaf) + car springs



(iii) Special shaped springs

- Belleville
  (slightly tapered disc)



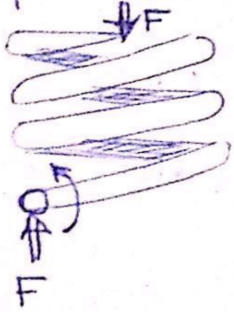
- Volute Springs
  thin stip of material wound such that Coils fit inside one another

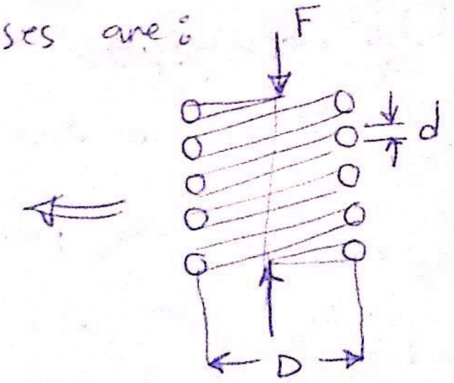
ملوية سامراء !!

① Helical springs (stress)

For a helical compression spring, round section:

let $\left\{ \begin{array}{l} d: \text{ wire diameter} \\ D: \text{ mean Coil diameter} \end{array} \right\}$ spring is loaded by axial force F then stresses are:





① direct shear $\tau = \frac{F}{A}$

② torsional shear $\tau = \mp \frac{Tr}{J}$

Upon Superposition ⇒

$$\tau_{total} = \frac{F}{A} \mp \frac{Tr}{J}$$

$$\tau_{max} = \frac{F}{A} + \frac{Tr}{J}$$ @ inner surface of wire τ_{max}

but $T = \frac{FD}{2}$, $r = \frac{d}{2}$ & $J = \frac{\pi}{32} d^4$

$$\Rightarrow \tau_{max} = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

Define spring index (measure of Coil Curvature)

$$C = \frac{D}{d}$$ (C : 6 ⇒ 12) most springs

$$\tau_{max} = \left(\frac{2C+1}{2C}\right) * \frac{8FD}{\pi d^3}$$

$$= K_s \frac{8FD}{\pi d^3}$$, K_s: shear stress multiplication factor

But

shear stress is effected by wire Curvature (spring wire is not straight). This curvature increases shear stress and this is accounted for by another factor K_w : Wahl correction factor

$$\tau_{max} = K_w \frac{8FD}{\pi d^3}$$

where

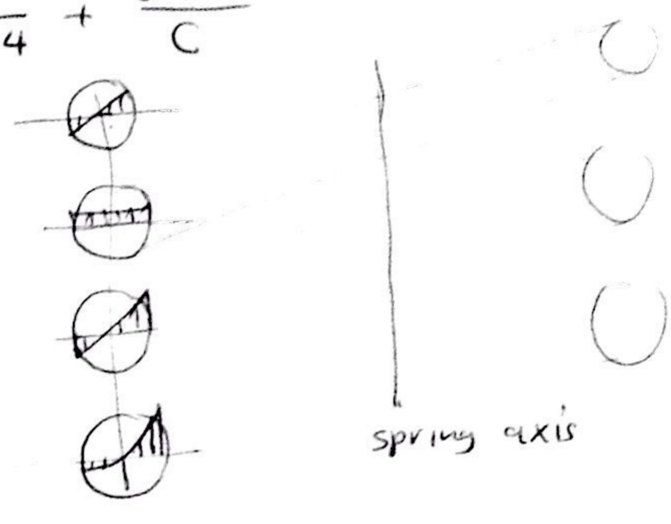
$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

(I) $\frac{Tr}{J}$ torsion

(II) $\frac{F}{A}$ direct

(I) + (II) $\frac{F}{A} + \frac{Tr}{J}$

(I) + (II) + Curvature affect



Notes Experiments shows:

① For static load, Curvature effect is neglected; Use K_s
For dynamic load, Use K_w (curvature is important)

② Square / rectangle x-section wires is Not recommended Unless necessary (above eqs Not valid)

X ③ New editions; Use $K_B = \frac{4C+2}{4C-3}$ for $(K_s K_{curvature})$
(Note: error K_w & K_B is small)

(2) Helical springs. (Deflection):

Using principles of strain energy (strength of material)

- Total strain energy in the spring wire has two Componen : shear and torsional ⇒

$$U = \frac{F^2 L}{2AG} + \frac{T^2 L}{2GJ}$$

substitute: $T = \frac{FD}{2}$; $l = \pi D N$ (spring length)

and $J = \frac{\pi}{32} d^4$

(Note $N = N_a$ is number of active coils)

$$\Rightarrow \quad U = \frac{2F^2 DN}{d^2 G} + \frac{4F^2 D^3 N}{d^4 G}$$

Use Castigliano's theorem to get the deflection "y"

$$y = \frac{\partial U}{\partial F} = \frac{4FDN}{d^2 G} + \frac{8FD^3 N}{d^4 G}$$

Use $C = \frac{D}{d}$

$$y = \frac{8FD^3N}{d^4 G}\left(1 + \underbrace{\frac{1}{2C^2}}_{\text{Very small}}\right)$$

$$y \simeq \frac{8FD^3N}{d^4 G} \quad \longleftarrow \quad *$$

spring stiffness (constant, rate) $K = \frac{F}{y}$

or $$K = \frac{d^4 G}{8D^3 N} \quad \longleftarrow \quad **$$

Notes
1. N is N_a = active number of Coils.
2. Four types of spring ends are used for Compression springs - See data p.31
3. active Coils: $N_a = \underbrace{N_t}_{\text{total}} - \underbrace{N_e}_{\text{end coils (inactive coils)}}$

(4) * free length "l_o" of a spring : the length of spring with No force applied to it.

* Solid length "l_s" of a spring : spring is fully compressed such that Coils are touching each other.

* full range of motion $= l_o - l_s$

(3) Stability of Compression Springs :
(students read this)

Similar to columns, If the length of a compression spring is large relative to its diameter, the spring may buckle while it is being Compressed as shown in Fig. [buckling is a sudden lateral deflection that occures in members subjected to axial Compressive loading]. In general :

* the Use of squared ends is better in stability of spring than using plain ends.

* To prevent buckling, the spring Can be inserted inside a hole or a rod is inserted inside the sp

(i)- Clearance between hole and spring Outside diameter :

$$\text{Hole diameter} = \text{spring OD} + 1.5 \text{ mm}$$

(ii) - Clearance between rod and spring inside diameter :

$$\text{Rod diameter} = \text{Spring ID} - 1 \text{ mm}$$

(4) Spring Materials :

A great variety of spring materials are available includ

- plain Carbon steel
- alloy steel
- corrosion resisting steel
- Nonferrous
  - phosphor bronze
  - beryllium Copper
  - Nickel alloys

See Shigley

Spring Material	description
Music wire (.8 - .95) C	the best, toughest, most widely used for small springs
oil tempered wire (0.6 - 0.7) C	Cheapest, general purpose than music wire but larger in siz
HD (Hard drawn) wire (0.6 - 0.7) C	Cheapest, general purpose
Chrome - Vanadium Wire	steel alloy spring, good fatigue resistance up to 220 °C
Chrome - Silicon wire	Excellent material for highley stressed spring, good shock resistance up to 250 °C. hard spring.

⟨5⟩ Spring Strength

- Tensile strength of wires depends on wire diameter

$$S_{ut} = \frac{A}{d^m}$$ A & m are material Constants

See data book P 31 ; gives material constants A & m

- However, springs are subjected to shear not tension and we need to consider yield strength Not ultimate

- An approximate relation :

$$S_y = 0.75\, S_{ut}$$

- Using Von Mices :

shear yield strength $S_{y\,shear} = 0.58\, S_y$

- Table 6-2 / Data book p.32 gives elastic Constants E & G for different spring materials.

Important Notes for static design :

* Material : HD steel is 1st choice since it has lowest cost. (cheapest).
* Type of ends : squared ends is 1st choice since it gives good stability & has low cost.
* Manufacturing : as-wound is 1st choice since it has lowest Cost
* Safety : Use design safety factor at solid length $n \geq 1.2$

X • Working Range : Avoid closing spring to its solid length under max. load.
(75 % of its possible Compression distance)
when $F : 0 \rightarrow F_s$

- spring Index $4 \leq C \leq 12$
- Number of active turns $3 \leq N_a \leq 15$

Example #1

A helical Compression spring made of music wire ($d = 1.397$ mm). The wire is to be inserted inside a hole with (20 mm) diameter. The free length of the spring is to be (100 mm) and it should have a squared and ground ends. The maximum load applied to spring = 30 N. Find total number of coils and full range of motion. (Under load 30N, spring Compress 60 mm)

Solution

Spring Outer diameter $OD =$ Hole diameter − clearance

$= 20 - 1.5 \quad = 18.5$ mm

mean diameter $D = OD - d$

$= 18.5 - 1.397 \quad = 17.103$ mm

$$K = \frac{F}{y} = \frac{30\ N}{60} = \frac{1}{2}\ N/mm$$

$$K = \frac{d^4 G}{8 D^3 N_a}$$, let $G = 80$ Gpa (check table 6-2)

$$0.5 = \frac{1.397^4 * 80 \times 10^3}{8\,(17.103)^3 N_a} \Rightarrow N_a = 15.23 \text{ turns}$$

For squared & ground ends $\Rightarrow N_e = 2$

$N_t = 15.23 + 2$

$= 17.23$ turn

solid length $= d * N_t = 1.397 * 17.23$

$= 24.07$ mm

Full Range of motion $= l_o - l_s$

$= 100 - 24.07$

$= 75.93$