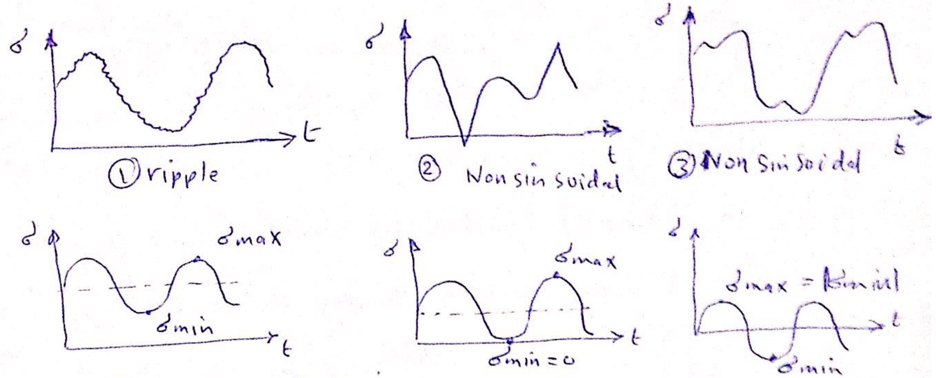
Characterizing Fluctuating stress:

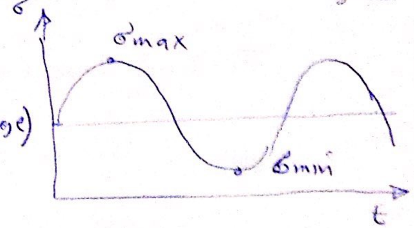
- Various dynamic stresses (stress-time relationships) may occure. (see Fig. below)



① ripple ② Non sinsoidal ③ Non sinsoidal

④ ~~Six~~ General fluctuating stress (could be sinsoidal) ⑤ repeated stress ⑥ Completely reversed stress ~~Cycles~~

our Concern: ④, ⑤ & ⑥



Define:

* σ_m : Mean or average stress (Midrange)

$$\sigma_m = \frac{\sigma_{min} + \sigma_{max}}{2}$$

* σ_a : Stress amplitude (~~Midrange~~)

$$\sigma_a = \left| \frac{\sigma_{max} - \sigma_{min}}{2} \right|$$

Notes

① In rotating beam test (Fatigue test) the specimen is subjected to completely reversed stress [type ⑥]

$\sigma_a = \sigma_{max} = |\sigma_{min}|$; $\sigma_m = 0$

② In repeated stress (type ⑤) ; $\sigma_{min} = 0$

$$\sigma_a = \sigma_m = \frac{\sigma_{max}}{2}$$

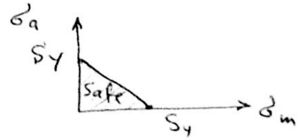
Fatigue Failure Criteria :
(Design against Fatigue loads):

* When structure is subjected to completely reversed stress ; use endurance limit obtained from fatigue test (after applying the necessary modifying factors). However, If the structure is subjected to general fluctuating stress ($\sigma_m \neq 0$), The situation is different and a fatigue failure criteria is needed.

Theories:

① Langer line (Yield line):

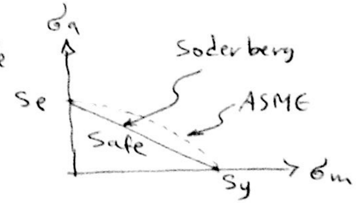
Not realistic since ($S_y > S_e$)



② Soderberg line : Most conservative

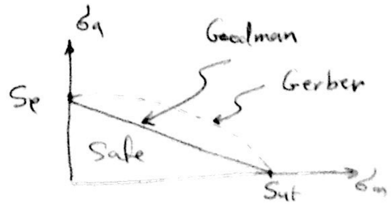
$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}$$

S_e: modified endurance limit; S_y: yield strength; n: safety factor



③ ASME elliptic line : fits experimental data better

$$\left(\frac{n\sigma_a}{S'_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1$$



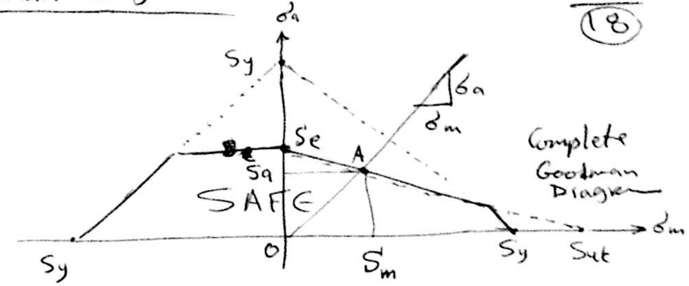
④ Goodman line

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

⑤ Gerber line : parabola line

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

⑥ Modified Goodman diagram:



steps

① Draw diagram

② Draw line with

Slope $\frac{\sigma_a}{\sigma_m}$ till

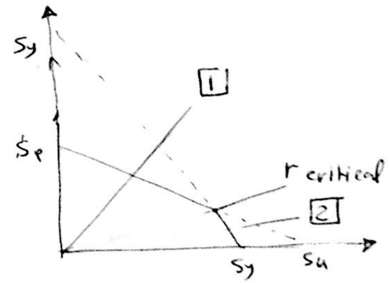
intersect the Goodman diagram @ A ; locate (S_a, S_m)

③ If point (σ_m, σ_a) inside diagram $\Rightarrow$ Safe

$$n = \frac{S_a}{\sigma_a} = \frac{S_m}{\sigma_m}$$

④ Goodman Diagram best fits with exp. data.

IMPORTANT Notes:



(*) slope of loading line passing through the intersection point of the two lines is called "Critical slope"

$$r_{critical} = \frac{S_a}{S_m} \quad , \text{ where}$$

$$S_m = \frac{(S_y - S_e)\, S_{ut}}{S_{ut} - S_e}$$

$$S_a = S_y - S_m$$

(*) According to slope of load line, it could intersect any of the two lines: $\left(r = \frac{\sigma_a}{\sigma_m}\right)$

→ $r > r_{critical}$ $\boxed{1}$ $\quad S_a = \frac{r\, S_e\, S_{ut}}{r\, S_{ut} + S_e} \quad , \quad S_m = \frac{S_a}{r}$

$\Rightarrow$ fatigue $\Rightarrow n_f = \frac{S_a}{\sigma_a} = \frac{S_m}{\sigma_m} = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}}$

→ $r < r_{critical}$ $\boxed{2}$ $\quad S_a = \frac{r\, S_y}{1+r} \quad , \quad S_m = \frac{S_y}{1+r}$

$\Rightarrow$ static yield $\Rightarrow n_s = \frac{S_a}{\sigma_a} = \frac{S_m}{\sigma_m} = \frac{S_y}{\sigma_a + \sigma_m}$

[SOLVE EXAMPLE ONE]

Tursional Fatigue loading:

- If structural elements are "shafts" and subjected to fluctuating shear stress, a fatigue criterion (ASME elliptic ; Gerber) needs to be used.

(**) endurance limit is corrected by $k_c = .59$ (already taken care off).

(**) Another Correction:
Use $(S_y)_{shear}$ and $(S_u)_{shear}$

where
$$\begin{cases} (S_y)_{shear} = 0.577\, S_y \\ (S_u)_{shear} = 0.67\, S_{ut} \end{cases}$$

Combination of Loading Modes:

If structural element is subjected to general fluctuating stress under combination of loading modes (axial, bending & torsional), then;

use Von-Mises as follow:

① Construct an element for mean stress: $(\sigma_x, \sigma_y \,\&\, \tau_{xy})_{mean}$

check Note② $\Rightarrow$ $\Rightarrow \sigma_{xm}, \sigma_{ym} \,\&\, \tau_{xy\,m}$

② Find principal stresses : $(\sigma_1 \,\&\, \sigma_2)_{mean} \Rightarrow \sigma_{1m}, \sigma_{2m}$

③ Use Equivalent mean stress: σ'_m
using Von-mises ; $\sigma'_m = \sqrt{\sigma_{1m}^2 - \sigma_{1m}\sigma_{2m} + \sigma_{2m}^2}$

④ Similarly, $\sigma'_a = \sqrt{\sigma_{1a}^2 - \sigma_{1a}\sigma_{2a} + \sigma_{2a}^2}$

⑤ Select fatigue failure criterion and apply σ'_m & σ'_a as usual

ex: Goodman : $\dfrac{\sigma'_a}{S_e} + \dfrac{\sigma'_m}{S_{ut}} = \dfrac{1}{n}$

Notes

① A simplification, if there exist τ_{xy} and one normal stress (σ_x or σ_y), use

$$\sigma_m' = \sqrt{\sigma_{xm}^2 + 3\tau_{xym}^2}$$

$$\sigma_a' = \sqrt{\sigma_{xa}^2 + 3\tau_{xya}^2}$$

(No need for principal stresses to be calculated)

② Do not use k_f to reduce endurance limit

k_f "modifying factor" is Not working since there is

$\left\{\begin{array}{ll} K_f & \text{for axial} \\ K_f & \text{for bending} \\ K_f & \text{for torsion} \end{array}\right\}$ which one to use ?

Instead; use K_f (fatigue stress concentration factor) to each mode of loading ~~factor~~, apply it to (σ_a and σ_m) of that mode. ex:

$$\sigma_a' = \sqrt{\left[K_{f_{bending}}\sigma_{a_{bending}} + K_{f_{axial}}(\sigma_{a_{axial}}/.85)\right]^2 + 3\left[K_{f_{torsion}}\tau_{a_{torsion}}\right]^2}$$

Similarly

$$\sigma_m' = \sqrt{\left[\underbrace{K_f\sigma_m}_{bending} + \underbrace{K_f(\sigma_m/.85)}_{axial}\right]^2 + 3\left[\underbrace{K_f\tau_m}_{torsion}\right]^2}$$

③ K_c (loading modification factor) is taken care of since $K_c = 1$ (bending) and $\left(\frac{\sigma_m}{.85}\text{ and }\frac{\sigma_a}{.85}\text{ for axial}\right)$ and von mises already accounted for torsional loading (No need for $K_c = .59$) $\Rightarrow$ Thus, In Calculation of endurance limit, drop (K_c "modifying factor")

Example # 1

A 40 mm diameter bar has been machined from AISI 1045 CD bar. The bar is subjected to a fluctuating tensile load varying from 0 to 100 KN. Because of the ends fillets radius, $K_f = 1.85$ to be used. Find the critical mean and alternating stress values S_a and S_m and fatigue safety factor n_f according to Modified Goodman fatigue criterion.

Sol.

Data book $\rightarrow \begin{cases} S_{ut} = 630 \text{ Mpa} \\ S_y = 530 \text{ Mpa} \end{cases}$

$\Rightarrow S_e' = \frac{1}{2} S_{ut} = 315 \text{ MPa}.$

Modifying factors:

- Surface factor: $k_a = 4.51 (630)^{-.265} = .817$
- Size factor : $k_b = 1$ (axial load)
- load factor : $k_c = .85$ (axial load)
- other factors : $k_d = k_e$ ~~...~~ $= 1$
- use K_f instead of k_f ; just proceed with sol.

$\rightarrow S_e = k_a k_c S_e' \Rightarrow S_e = .817 * (.85) * 315 = 218.8 \text{ Mpa}$

Stresses : $\sigma = \frac{F}{A}$ (axial load) , $A = \frac{\pi}{4} d^2 = 1.25 \times 10^3 \text{ mm}^2$

$\sigma_{min} = 0$

$\sigma_{max} = \frac{100 * 10^3}{1.257 * 10^3} = 79. \text{ Mpa}$

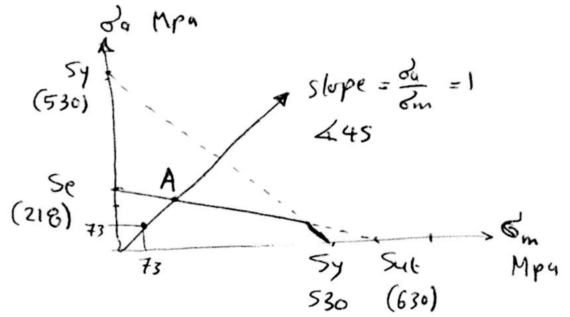
$\Rightarrow \sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 39.8 \text{ Mpa}$

$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 39.8 \text{ Mpa}$

Apply K_f to both components

$\sigma_m = \sigma_a = 1.85 (39.8) = 73.6 \text{ Mpa}$

the plot shows that the load line intersects the modified Goodman line



$$S_a = \frac{r\, S_e\, S_{ut}}{r\, S_{ut} + S_e}$$

$$= \frac{1*(218)\,630}{1*(630) + 218} = 162.4$$

$$S_m = S_a = 162.4$$

$$n = \frac{S_a}{\sigma_a} = \frac{S_m}{\sigma_m} = \frac{162}{73.6} = 2.21$$

or Solve Graphically.
$\Rightarrow S_a = 160$
$n = 160/73.6 = 2.17$

Example #2

It is desired to determine the size of a cold drawn steel bar to withstand a tensile preload of 36 KN and fluctuating tensile load varying from (0 – 72) KN. The bar have theoritical (geometric) stress concentration factor $K_t = 2.02$ corresponding to a fillet radius of 3.5 mm. Determine suitable diameter for infinite life and safety factor of 3.0

use [$S_y = 510$ Mpa , $S_{ut} = 650$ Mpa & S_e' (uncorrected) $= 364$ Mpa

Sol.

$S_e' = 364$

modifiying factors:

$k_a = 4.51(650)^{-.265} = .81$

$k_b = 1$ axial load

$k_c = .85$

$k_d = k_e = 1$

since $K_t = 2.02$ & $q = .85$ (data books, $r = 3.5$ mm)

$K_f = 1 + q(K_t - 1)$

$= 1 + .85(2.02 - 1) = 1.87$

$k_f = \frac{1}{K_f} = .5356$ (see, too much)

$S_e = .81 * .85 * .5356 * 364$

$= 134.2$ Mpa.

Stresses preload $\Rightarrow \sigma_p = \frac{36 * 10^3}{\frac{\pi}{4} d^2} = \frac{45.8 * 10^3}{d^2}$

fluctuating load (0−72) KN

$\sigma_{min} = 0$

$\sigma_{max} = \frac{72 * 10^3}{\frac{\pi}{4} d^2} = 91.67 * 10^3$ pa

$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{45.8 * 10^3}{d^2}$

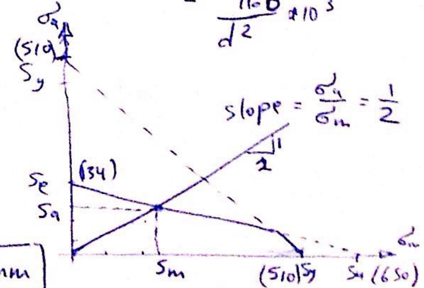
$\sigma_m = \frac{45.8 * 10^3}{d^2}$

now, Preload is added only to σ_m.

$\Rightarrow \sigma_a = \frac{45.8 * 10^3}{d^2}$ & $\sigma_m = \frac{45.8}{d^2} + \frac{45.8}{d^2}$

$= \frac{91.6}{d^2} * 10^3$

Draw modified Goodman Diagram



$S_a' = \frac{r\, S_e\, S_{ut}}{r\, S_{ut} + S_e'} = \frac{0.5 * 134 * 650}{.5(650) + 134}$

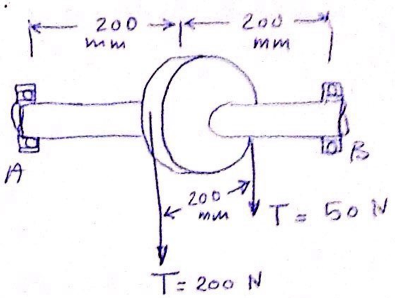
$= 94.9$ Mpa

Now $\sigma_a = \frac{S_a}{n} \Rightarrow \frac{45.8 * 10^3}{d^2} = \frac{94.9 * 10^6}{3}$

$d = 38 * 10^{-3}$ m $\Rightarrow$ $\boxed{d = 38 \text{ mm}}$

Example #3

For the rotating shaft shown in Fig.
Estimate the minimum shaft diameter for a factor of safety of (3). The material is AISI-1050 CD



Sol.

$\boxed{A}$ Stresses

① bending: Reactions @ supports A & B $\Rightarrow R = 125$ N
Moment @ midshaft $M = 125 * 200 = 25000$ N.mm
$= 25$ N.m

Since this type of loading is Completely reversed

$$\sigma_a = \frac{32M}{\pi d^3} = \frac{32 * 25}{\pi d^3} = \frac{254.6}{d^3}$$

$$\sigma_m = 0$$

② Torsion: Actually, torsion is not varying with time since torque is Constant

$$T = (200 - 50) * \frac{100}{1000} = 15 \text{ N.m}$$

$$\Rightarrow \begin{cases} \tau_a = \text{zero} \\ \tau_m = \frac{16T}{\pi d^3} = \frac{16 * 15}{\pi d^3} = \frac{76.4}{d^3} \end{cases}$$

Since Load is Combined:

$$\sigma_a' = \sqrt{\sigma_a^2 + 3\tau_a^2} = \frac{254.6}{d^3}$$

$$\sigma_m' = \sqrt{\sigma_m^2 + 3\tau_m^2} = \frac{132}{d^3}$$

B Material data

AISI-1050 CD $\Rightarrow$ $S_u = 690$ MPa

$S_e' = 345$ MPa

modification factors :

$K_a = 4.51\,(690)^{-.265} = .796$

$K_b = 1$ (assume for now as no diameter is available)

$K_c = 1$ (Just for bending , torque = Constant)

$K_d = K_e = K_f = 1$

$S_e = .796 * 345 = 274.6$ MPa

Use Goodman diagram :

$$\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} = \frac{1}{n}$$

$$\frac{\frac{254.6}{d^3}}{274.6 \times 10^6} + \frac{\frac{132}{d^3}}{690 \times 10^6} = \frac{1}{3}$$

$\Rightarrow$ $d = 14.97$ mm

Now , according to new diameter ,

calculate $K_b = 1.24\, d^{-.107}$

$= 1.24 * (15)^{-.107} = .928$

$S_e = .796 * .928 * 345 = 254.9$ MPa

once again

$$\frac{\frac{254.6}{d^3}}{254.9 \times 10^6} + \frac{\frac{132}{d^3}}{690 \times 10^6} = \frac{1}{3}$$

$d = 15.28$, iterate until Convergence —

$K_b = 1.24\,(15.28)^{-.107} = (.926)$ stop