Example (1) (a) Corresponding to relaibility of 99 % estimate endurance limit of a round rotating cold-drawn ~~BS~~ AISI 1018 steel shaft 30 mm in diameter.

(b) What is endurance limit of a non-rotating bar of same material and dimensions?

Sol. * (a) from ~~shigley~~ Data book

$$S_{ut} = 440 \quad Mpa$$

check data book ((Table (17, 19) P. 83 – for steels & heat treated steels))
((Table 18 P. 83 for Cast Iron))

* Relaibility 99% $\Rightarrow K_e = .814$; P. 11

* surface factor ; $K_a = a\, S_{ut}^{\,b}$

P. 9 $\rightarrow$ cold drawn $a = 4.51$; $b = -.265$

$$K_a = 4.51 * 440^{-.265} = .899$$

* load factor, bending $\Rightarrow K_c = 1$

* size factor, $K_b = 1.24\, d^{-.107}$

$$= 1.24\,(30)^{-.107} = 0.862$$

others $K_d = K_f = 1$

$$S_e' = \tfrac{1}{2} S_{ut} = \tfrac{440}{2} = 220 \text{ Mpa}.$$

$$\Rightarrow S_e = .899 * .862 * 1 * 1 * .814 * 1 * 220$$
$$= 138.8 \text{ Mpa} \longleftarrow$$

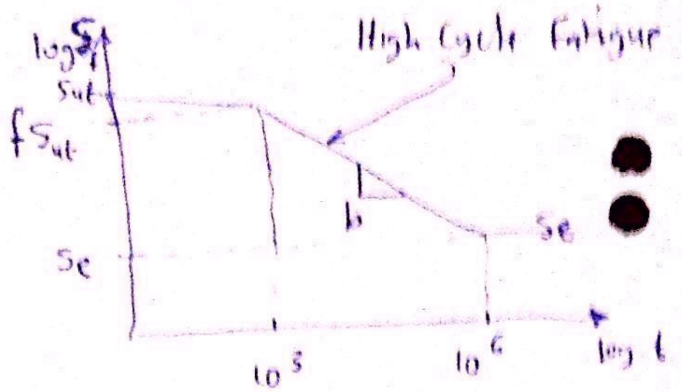
(b) If shaft is not rotating, $d_e = .37\,D$
$= .37 * 30 = 11.1$ mm

$$K_b = 1.24 * (11.1)^{-.107} = 0.958$$

$$S_e = .899 * .958 * .814 * 220$$
$$= 154 \text{ Mpa} \longleftarrow$$

Design for high Cycle Fatigue :
(finite life : $10^3 \le N \le 10^6$)

In some design applications, the structural part is required to have a finite No. of cycles before failure, i.e. only $10^3 \le N \le 10^6$ cycles then the structural part is to be failed. ⇒ No need to design for ∞ life using the endurance limit.



From S-N Curve and for Cycles $10^3 < N < 10^6$ ⇒ line equation:

$$S_f = a N^b$$

where

a : (y-intercept) } Constants

b : slope

$$a = \frac{(f\, S_{ut})^2}{S_e}$$

$$b = -\frac{1}{3} \log\left(\frac{f\, S_{ut}}{S'_e}\right)$$

Notes:

① f: "fraction" is found from chart P. 7 data book

Note: If $S_{ut} \le 490$ MP ⇒ $f = 0.9$ (Conservative)

② $N = \left(\frac{\sigma}{a}\right)^{\frac{1}{b}}$: $N = f(\sigma)$

③ If chart of "f" is not available ⇒ $\sigma_f' = S_{ut} + 345$ MPa, "S_{ut}"

$$b = -\frac{\log \frac{\sigma_f'}{S_e}}{\log(2\times10^6)}$$

$$f = \frac{\sigma_f'}{S_{ut}} (2\times10^3)^b$$

Example (2):

For a rotating beam specimen made of AISI 1045 CD steel find:

(a) endurance limit

(b) fatigue strength corresponding to (5×10^4) cycles to failure

(c) Expected life under completely reversed stress of 400 Mpa.

Sol.

(a) Using Data book p. 83, $S_{ut} = 630$ Mpa

$$S'_e = \frac{1}{2} S_{ut} \Rightarrow S'_e = 315 \text{ Mpa}$$

(b) $S'_f = a N^b$

① Use chart p. 7

$$\left.\begin{array}{ll} 100 \text{ kpsi} & \leftarrow 700 \text{ Mpa} \\ x & \leftarrow 630 \end{array}\right\} \quad x = \frac{630 * 100}{700} = 90 \text{ kpsi}$$

$$\Rightarrow f \cong .857$$

② $$a = \frac{(f\, S_{ut})^2}{S_e} = \frac{(.857 * 630)^2}{315} = 925.4 \text{ Mpa}$$

③ $$b = -\frac{1}{3}\log\left(\frac{f\, S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{.857 * 630}{315}\right) = -.078$$

$$\Rightarrow S'_f = 925.4 * (5*10^4)^{-.078} = 398 \text{ Mpa.}$$

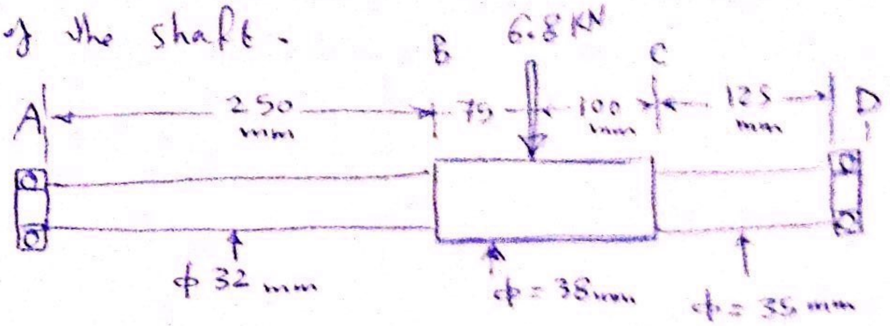
(c) $$N = \left(\frac{\sigma}{a}\right)^{\frac{1}{b}}$$

$$= \left(\frac{400}{925.4}\right)^{\frac{1}{-.078}}$$

$$= 46 * 10^3 \text{ cycle.}$$

Example (3)

A rotating shaft is supported by ball bearings @ A & D and loaded by F = 6.8 KN as shown in fig, Estimate life of the shaft.



Material: AISI 1050 CD

Sol. Draw - Bending Moment diagram

Most critical point is (B) why?

① $M_B > M_C$ & section @ B < x-section @ C & stress concentration @ B > @ C
X-area

② Under load $M > M_B$; but x-section is larger and has no stress concentration like point B

∴ Choose B location.

Now From Data book, S_{ut} = 690 MPa (AISI 1050 CD)

$$S_e' = \frac{690}{2} = 345 \text{ MPa}$$

* Surface factor $K_a = a\, S_{ut}^{\,b}$ P.9 (Data)

$$= 4.51\,(690)^{-.265} = 0.798$$

* size factor $K_b = 1.24\, d^{-.107}$

$$= 1.24\,(32)^{-0.107} = 0.856$$

* load factor $K_c = 1$ (bending)

* reliability factor $K_e = 1$

* temp. factor $K_d = 1$

* stress Concentration factor; Data P. 77 : Fig A-9

Ⓐ $K_t = 1.65$ $\left\{ \begin{array}{l} \frac{D}{d} = \frac{38}{32} = 1.1875 \\ \frac{r}{d} = \frac{3}{32} = .09375 \end{array} \right.$

from Data book p. 12 ; Fig. 3.3 $\Rightarrow q = .83$

$$K_f = 1 + q(K_t - 1)$$
$$= 1 + .83(1.65 - 1)$$
$$= 1.54$$

$$k_f \text{ (modifing factor)} = \frac{1}{K_f} = \frac{1}{1.54} = 0.649$$

Corrected (modified) endurance limit $S_e = k_a k_b k_c k_d k_e k_f S_e'$

$$S_e = 0.798 * 0.856 * .649 * 345$$
$$= 152.9 \text{ Mpa} \leftarrow$$

Now

If stress $\leq S_e\ (152.9) \Rightarrow \infty$ life

otherwize finite life.

$$\sigma = \frac{My}{I} = \frac{32 M_B}{\pi d^3} = \frac{32 * 695}{\pi (32*10^{-3})^3} = 216 \text{ Mpa}$$

Since $\sigma > S_e \Rightarrow$ finite life

Therefore Use : $S_f = a N^b$ or $N = \left(\frac{\sigma}{a}\right)^{\frac{1}{b}}$

Use Data book p. 7 $\rightarrow f \cong .844$

$$\left.\begin{matrix} 100 & 700 \\ x & 690 \end{matrix}\right\} x = \frac{100 * 690}{700} \cong 98.5$$

$$a = \frac{(f\ S_{ut})^2}{S_e} = \frac{(.844 * 690)^2}{152.9} = 2\,218$$

$$b = -\frac{1}{3}\log\left(\frac{f\ S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{.844 * 690}{152.9}\right)$$
$$= -.1936$$

$$N = \left(\frac{216}{2\,218}\right)^{\frac{1}{-.1936}}$$

$$= 165.7 * 10^3 \leftarrow$$