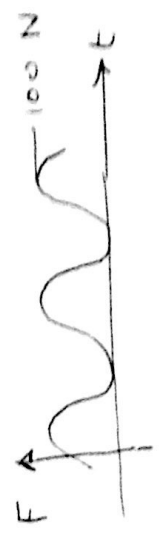
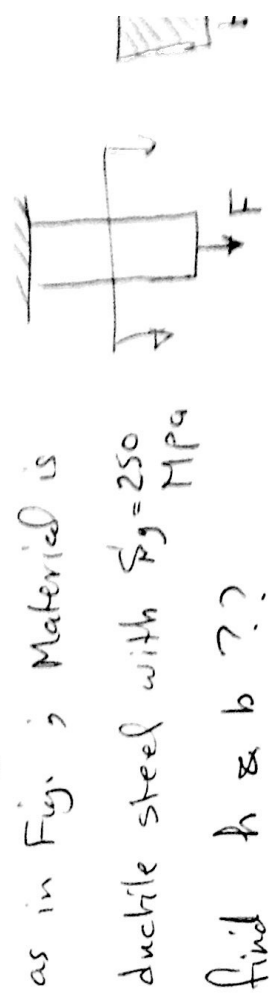


Chapter #3 Design against Fatigue

Problem: let force dynamic say



fluctuate between zero to 100 kN
Sinusoidally, this force is applied on m



as in Fig; Material is
ductile steel with $S_y = 250\text{ MPa}$
find h & b ??

Sol. Attempt

Design based on static load with $F = F_{\text{max}} = 10$

$$\frac{F}{A} = \frac{S_y}{N} \iff \frac{100 \times 10^3}{bh} = \frac{250 \times 10^6}{1}$$

$$2b^2 = \frac{100 \times 10^3}{250 \times 10^6} \iff b = 1414\text{ mm}$$

$$h = 28.3\text{ mm}$$

Is it ok ?? NO

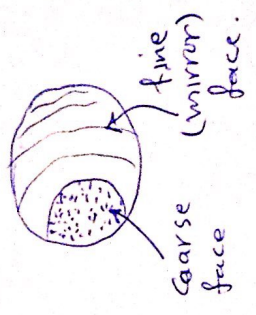
what happens is failure after (1 wt say)

Fact 13

CH3
②

fracture appearance :

- ① mirror face region
- ② rough and coarse region



This appearance is not like ductile materials (static load) where

large deformation (plastic strains) appear and detected

It is ^{also} not like brittle material where all face is rough and coarse, but it is sudden < fatigue } undetectable } brittle and dangerous.

Fact 14

Failure could occur at $\text{stress} < S_{ut}$
it is even @ $\text{stress} < S_y$

If change S_y 250 \rightarrow 200 MPa
(actually you increase dimensions) still failure after 2 wts, S_y

Thus we are here dealing with new phenomenon called \Rightarrow fatigue failure.

\Rightarrow do what? \Rightarrow build fatigue machine

- ① Failure starts with small crack (very small that can not be detected by eye and difficult to locate by X-ray)
- ② It starts at points of discontinuity, holes, change in x-section
Irregularities caused by machining
- ③ It will grow with N (No. of cycles), area of x-section will decrease \Rightarrow stresses rises $\uparrow \Rightarrow$ Sudden failure

CH3
⑧
Fatigue Test (S-N diagram)

Apparatus:

Moore Rotating beam machine;



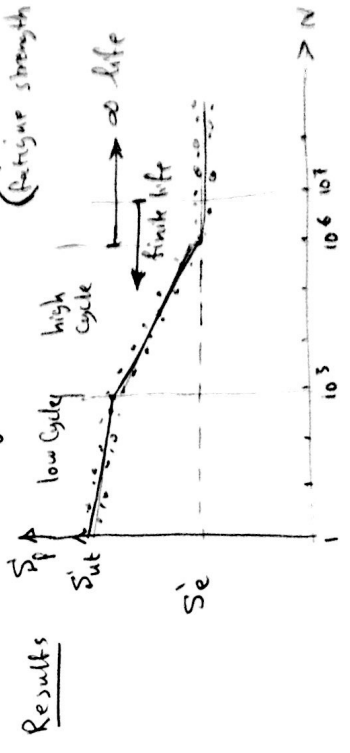
- * check rotating specimen under action of bending loads from motor fiber A (on surface) undergoes tension and compression as shaft rotate, if rpm = 1725, fiber is stressed 1725 tension & compression for each minute
- * specimen is carefully machined and polished with specific dimensions.

* Other type machine exists -

Procedure:

- * Load specimen @ $\sigma_1 = S_{ut}$, run motor and Count no. of cycles up to failure $\rightarrow N_1 = (\frac{1}{2} \text{ Cycle})$
- * load specimen $P_2 (\sigma_2)$, Count N_2 up to failure
- * ' ' $P_3 (\sigma_3)$, Count N_3 (# of cycles @ failure)

Draw Semi-Log diagram $\leftrightarrow S_f$ vs. N
(Fatigue strength vs. # of cycles)

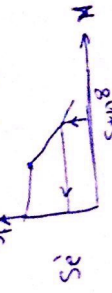


Ferrus ~~Steel~~ alloys

Important Remarks

- 1. $N < 10^3$ } Low cycle fatigue
 $\left\{ \begin{array}{l} N > 10^3 \\ N < 10^6 \end{array} \right\}$ } high cycle fatigue
- 2. $N > 10^6$ } ∞ life } overlap ($10^6 - 10^7$)
 $N < 10^7$ } finite life } period
- 3. For steels and titanium, if we keep reducing stress, we will reach to a stress level for which the specimen will never fail, this value of stress is called [Endurance Limit S_e']
- 4. Endurance limit define boundary between finite life and ∞ life $\Rightarrow 10^6 < N < 10^7$

Ferrous steels & alloys have clear endurance limit but ~~steels & titanium~~ ^{ex AL} other materials do not have S_e' , project back for $N = 15 \times 10^8$

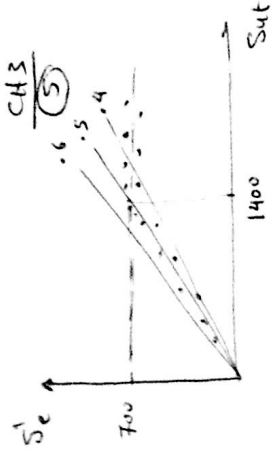


- 6. Take specimen ① test it } different results
 Take specimen ② test it } because of statistical nature.

- An attempt is to relate S_e' to S_{ut} why?
 (fatigue test is time consuming & expensive)

Correlation

Plot S_e' vs S_{ut}
for large no. of specimens
of steel and iron \rightarrow



$$S_e' = \begin{cases} 0.5 S_{ut} & S_{ut} \leq 1400 \text{ MPa} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases}$$

Notes ① For Cast Iron, Cast steel

$$S_e' = \begin{cases} 0.45 S_{ut} & S_{ut} \leq 600 \text{ MPa} \\ 275 \text{ MPa} & S_{ut} > 600 \text{ MPa} \end{cases}$$

② For Al. & Mg. alloys $S_e' : (30-40)\% S_{ut}$
For plastics $S_e' : (18-45)\% S_{ut}$

(Check Tables of manufacturer)

Endurance Limit Modification factors:

- Since Endurance limits are obtained from rotating beam test \rightarrow (specimen is prepared very carefully and tested under controlled conditions)
- But this is not like actual structural member. It is therefore unrealistic to think that both values are equal. Do what??
- Multiply endurance limit S_e' by modifying factors

$$S_e = K_a K_b K_c K_d K_e K_f S_e'$$

where S_e : Corrected endurance limit

- K_c : Load factor
- K_b : gradient (size) factor
- K_a : surface factor
- K_e : reliability factor
- K_d : temp. factor
- K_f : Miscellaneous-effect ~~stress concentration~~ factor
- S_e' : endurance limit from rotating beam test.

① Load factor

Rotating beam test apply pure bending stress, other type of loading will have a different effect, therefore load factor for different type of loading is :

$$K_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ .58 & \text{torsion} \end{cases}$$

② Surface finish factor

The rotating beam test specimens are highly polished, structural elements are not. Then, a surface finish factor is used for to reduce the endurance limit. This factor depends on the surface finish of part (ground, machined, as forged, ... etc) and on the tensile strength of material.

see } Shigley p.244
 } Patn p.9, $K_a = a S_{ut}^b$

③ size factor

The rotating beam specimens have small (7.6mm) diameter, parts (structural elements) of larger size are more likely to contain flaws and more inhomogeneities; use size factor

For axial loads $K_b = 1$ (neglect size effect)

For bending & torsional loading:

$$K_b = \begin{cases} 1.24 d^{-1.07} & 279 \leq d \leq 51 \text{ mm} \\ 1.51 d^{-1.157} & 51 < d \leq 254 \text{ mm} \end{cases}$$

See Data book P. 9

Notes

① If structural element has Non-rotating circular section, then use effective diameter " d_e " instead of actual diameter " d "

$d_e = 0.37 d$; go back to above equations for K_b

② If structural element has rectangular cross-section $h \times b$

$d_e = 0.808 \sqrt{hb}$; go back to above equations for K_b

④ Temperature factors:

As temp. changes, all mechanical properties change, do account for temp. effect. use K_d

$$K_d = \frac{S_T}{S_{RT}} \quad \text{see data book P. 10}$$

⑤ Reliability factor

* A reliability of ~~0.9~~ ($R=0.9$) means that there is 90% chance that the structural member will perform its proper function without failure.

* Failure of 6 parts of every 1000 parts \Rightarrow
 $R = 1 - \frac{6}{1000} = .994$ (99.4% reliability)

* K_e is reliability factor = $1 - .08 Z_a$
 see data book P. 11 (Shigley P. 251)

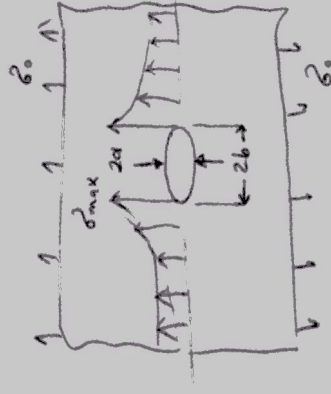
⑥ Miscellaneous-effects factor

other effects (such as residual stresses, corrosion, cyclic frequency, metal spray, ...etc) are not fully characterized and usually not accounted for,

use $K_f = 1$ but effect of stress

Concentration and notch sensitivity is accounted as follows:

A1 - Most machined parts have holes, grooves, notches, or other kind of irregularities or discontinuities which alter stress distribution



- An ∞ plate in tension with elliptical hole see stress distribution

Define "stress concentration factor" (theoretical one) to relate max. stress at discontinuity to nominal stress σ_0

$$K_t = \frac{\sigma_{max}}{\sigma_0} = \begin{cases} K_t = 1 + \frac{2b}{a} & \text{for ellipse} \\ K_t = 3 & \text{for circle} \end{cases}$$

Notes K_t is called theoretical because it depends on geometry of discontinuity and not on material.

② see Data book [P. 73 - P. 80 A1-A15] for practical cases such as [rectangular bar + hole], [bar + notch], [bar + fillets], [shaft + fillets] and [shaft + groove].

③ stress concentration effect is very important in fatigue (dynamic loads) and in static load + brittle materials but very less effect in static load + ductile materials

to notch

Some materials are sensitive & others are not, so define "notch sensitivity q "

Such that:

$q = 0 \Rightarrow$ material has no sensitivity to notch or discontinuity.

$q = 1 \Rightarrow$ Full sensitivity.

To get a value of q see data book P.12

Knowing K_t and q , find:

$$K_f = 1 + q (K_t - 1)$$

↑
fatigue - stress concentration factor

Then, miscellaneous-effects factor

$$K_f = \frac{1}{K_f}$$