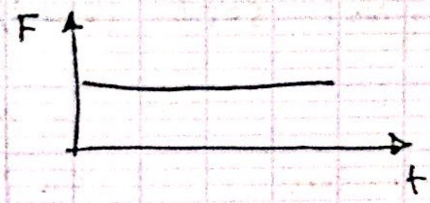
Design against static load

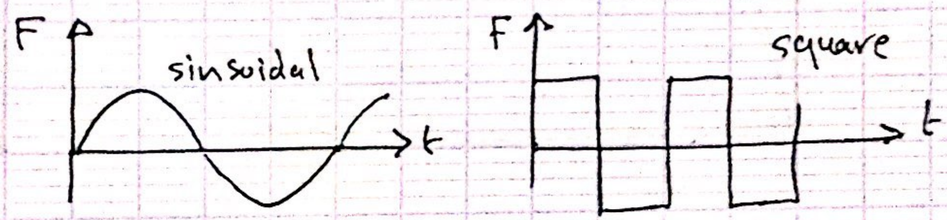
Basics :

$\boxed{1}$ static load vs dynamic load

- If load does not vary with time ⇒ static otherwize it is dynamic load.



⇐ static load



examples of dynamic loads

for this chapter ; load is always $\boxed{\text{Static}}$

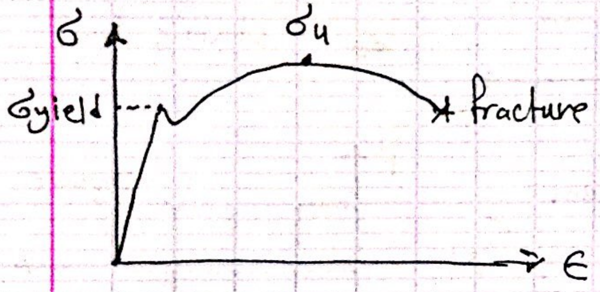
for next chapter ; load is dynamic

2 Ductile vs brittle materials

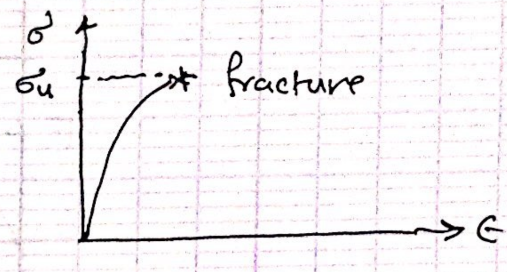
Material is either :

(a) ductile : able to be drawn into wire or elongate (soft material)

(b) brittle : able to rupture with small deformation (Not soft (hard) material)



ductile material
ex mild steel



brittle material
ex. - Cast Iron
- Concrete

<u>Note</u> for ductile material

S_y : yield stress from tensile test

S_u : Ultimate stress from " "

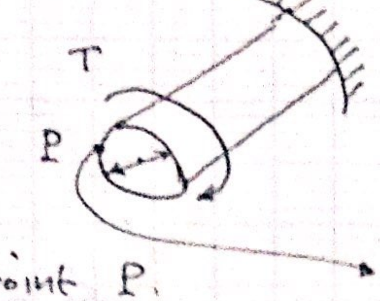
for brittle material

S_u : ultimate stress from tensile test.

(No yield)

3 Principal stresses

Case 1



shaft subjected to torque only.

take point P.

stresses are:

$$\tau = \frac{Tr}{J}$$

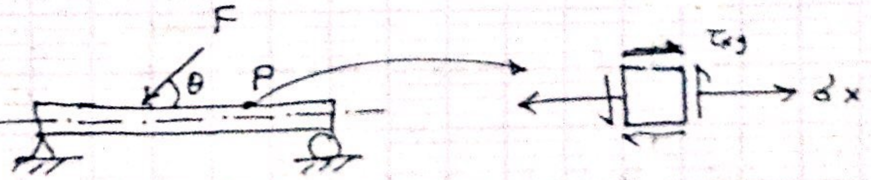
$\tau_{xy} = \frac{Tr}{J}$

ex. say

$\tau = 10$ Mpa

Case 2

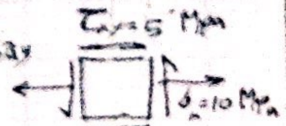
beam under concentrated load



stresses :

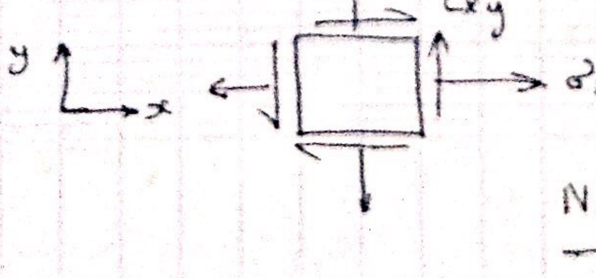
$$\left.\begin{aligned} \sigma_x &= \frac{F\cos\theta}{A} \mp \frac{My}{I} \\ \sigma_y &= 0 \\ \tau_{xy} &= \frac{VA\bar{y}}{It} \end{aligned}\right\}$$

ex. say



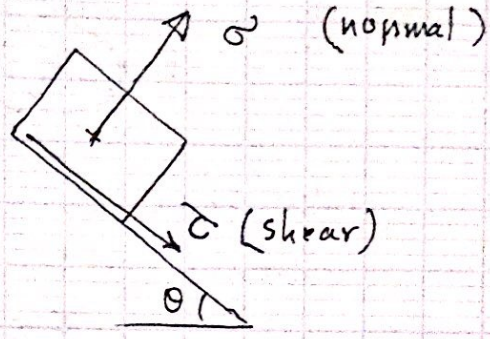
In General : (2-dimensional)

Let a structure (member) subjected to external loads , stresses @ a point (P) in general are:



Not enough

Need to check
stresses (normal and shear) on oblique planes



$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_\theta = - \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy}$$

to find max. normal stress $\Rightarrow \frac{d\sigma_\theta}{d\theta} = 0$

and to find max shear stress $\Rightarrow \frac{d\tau_\theta}{d\theta} = 0$

this leads to principal stresses :

A — Principal Normal stresses :

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

B — Max. shear stress :

$$\tau_{max} = \mp \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Failure Theories:

- too many theories, choose:

A — For ductile materials:

1. Rankine (max. principal stress)
2. Tresca (max. shear stress)
3. Von-mises (max. distortion energy)

B — For brittle materials:

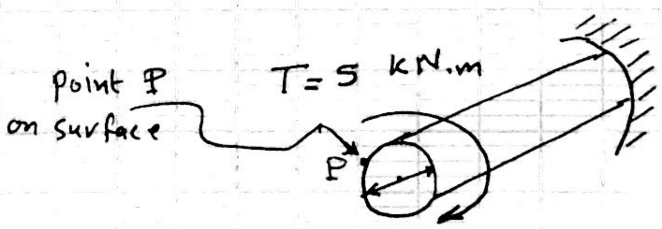
1. Rankine
2. Mohr–Coloumb theory
3. Modified Mohr–Coulomb theory.

— Explaination through following example:

Ex. A shaft is subjected to torque = 5 kN.m

If material has a yield point of 350 Mpa

find diameter of shaft? Use Safety factor 2.5



Solution

① It is a design problem since we are looking for dimentions.

② stresses are only shear @ point P

$$\tau = \frac{T\frac{d}{2}}{\frac{\pi}{32}d^4} = \frac{16\ T}{\pi d^3}$$

Failure Theories

$\boxed{1.}$ Rankine " Failure occure when max. principal stress just exceeds yield point of tensile test"

Now, Since
$$\left\{ \begin{aligned} \sigma_x &= 0 \\ \sigma_y &= 0 \\ \tau_{xy} &= \frac{16T}{\pi d^3} = \frac{16 * 5 * 10^3}{\pi d^3} \end{aligned} \right\} \text{— eq. (1)}$$

and Principal stresses :
$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{— eq. (2)}$$

substitud eq. 1 in eq. 2

$$\sigma_1 = \frac{16 * 5 * 10^3}{\pi d^3}$$

Using failure theory above $\Rightarrow \sigma_1 = \frac{S_y}{\text{Safety factor}}$

$$\rightarrow \frac{16 * 5 * 10^3}{\pi d^3} = \frac{350}{2.5}$$

$$d = 56.6 \text{ mm} \quad \leftarrow \text{Ans.}$$

2. Tresca: "Failure occure when max. shear stress just exceeds yield point of torsional test"

Note that experiments showed that:

$$S \text{ yield of torsional test} = \frac{1}{2} S \text{ yield of tensil test}$$

or $$\underset{\text{shear}}{S}_{s\underset{\text{yield}}{y}} = \frac{1}{2} S'_{\underset{\text{yield}}{y}}$$

Back to example:

$$\sigma_x = \sigma_y = 0 \quad ; \quad \tau_{xy} = \frac{16 * 5 * 10^3}{\pi d^3}$$

Max. shear stress $$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{16 * 5 * 10^3}{\pi d^3}$$

using above theory

$$\frac{16 * 5 * 10^3}{\pi d^3} = \frac{1}{2} * \frac{350}{2.5}$$

$$d = 71 \text{ mm} \leftarrow \text{Ans}$$

$\boxed{3}$ Von-mises : " Failure occure when distortion energy/unit volume for stress state just exceeds distortion energy/unit volume of yield point of tensile test ".

Now, for 3D complex stress state, distortion energ

$$U_d = \frac{1+\mu}{3E} \left\{ \frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2} \right\} \quad \text{— (1)}$$

and distortion energy at yield point of tensile test

$$U_d = \frac{1+\mu}{3E} S_y^2 \quad \text{— (2)}$$

$\left\{ \begin{array}{l} \mu : \text{Poisson's ratio} \\ E : \text{elasticity modulus} \end{array} \right.$

equate eq. (1) and eq. (2)

$$\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2} = S_y^2 \quad \text{— (3)}$$

For 2-dimensional,

$$\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 = S_y^2 \quad \text{— (4)}$$

Now back to example:

$$\sigma_x = 0$$

$$\sigma_y = 0$$

$$\tau_{xy} = \frac{16 * 5 \times 10^3}{\pi d^3}$$

Calculate:

$$\sigma_1 = + \frac{16 * 5 * 10^3}{\pi d^3}$$

$$\sigma_2 = - \frac{16 * 5 * 10^3}{\pi d^3}$$

substitude in eq. (4)

$$3 \left(\frac{16 * 5 * 10^3}{\pi d^3} \right)^2 = S_y'^2$$

or

$$\frac{16 * 5 * 10^3}{\pi d^3} = \frac{S_y}{\sqrt{3}} = \frac{350}{\sqrt{3} * 2.5}$$

safety factor

$$d = 68 \text{ mm} \longleftarrow \text{Ans}$$

Q: What theory to choose ??

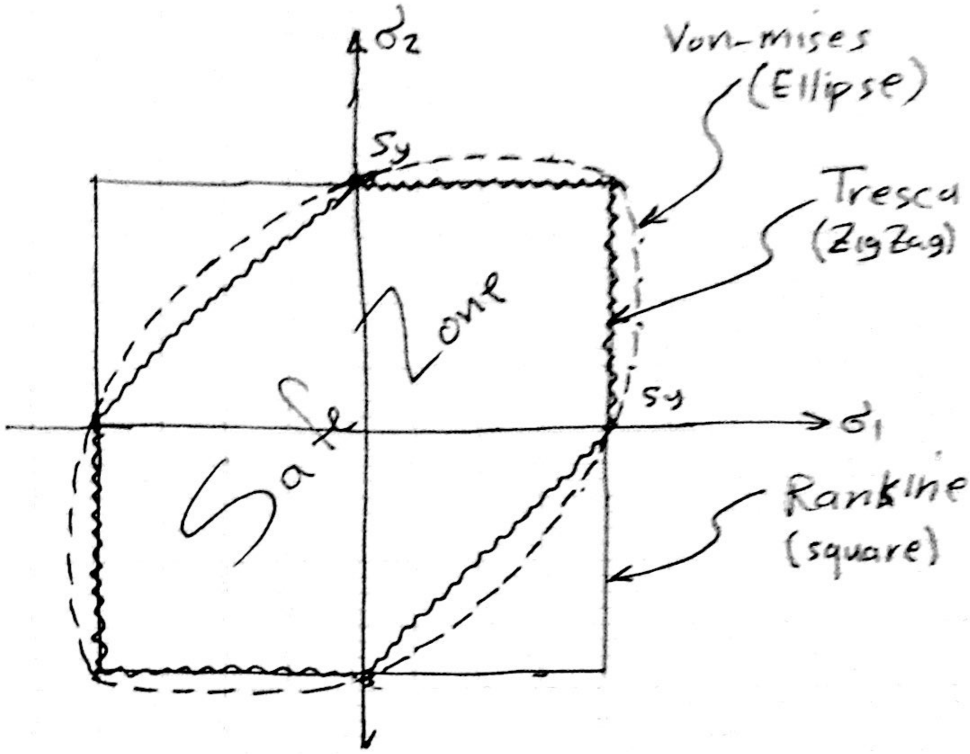
Rankine $\rightarrow$ 56.6 mm

Tresca $\rightarrow$ 71 mm

Von-misés $\rightarrow$ 68 mm

?

Graphs of theories :



① - If your complex stress state lie inside graph of theory ⇒ your design is safe.
other-wize unsafe Zone.

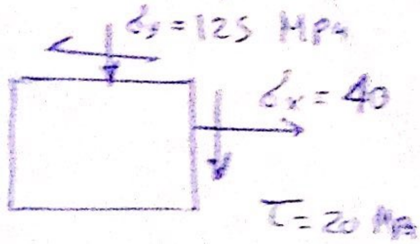
② Most Conservative theory is Tresca.

③ Experimental data mostly coincides with Von-mises

Example 2 :

State of stress @ a point for a material is shown in Fig. Find safety factor using (a) Tresca (b) Von-mises

Use tensile yield strength of material = 400 Mpa.



Sol

Use equation / Mohr's circle

$\sigma_1 = 42.3$ Mpa

$\sigma_2 = -127.38$ Mpa

(a) Tresca $\frac{\sigma_1 - \sigma_2}{2} = \frac{S_y}{2 * n}$ ⇒ safety factor, n = 2.356

(b) Von-mises

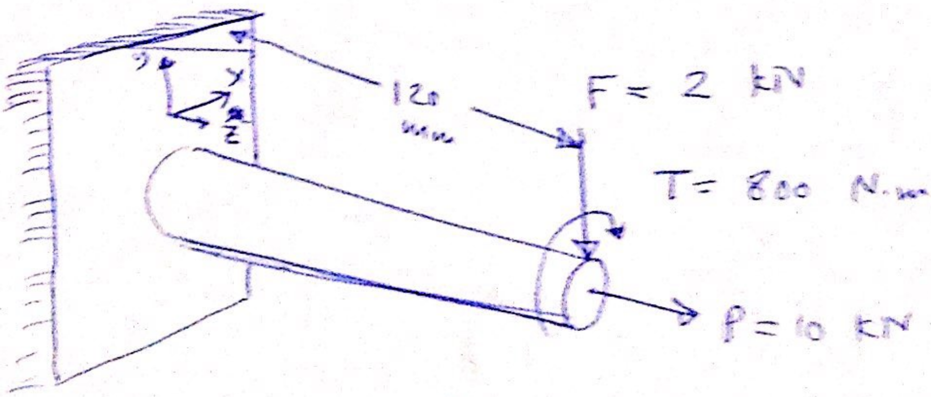
$\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 = \left(\frac{S_y}{n}\right)^2$ ⇒ n = 2.613

Example 3 A cantilever rod is loaded @ shown,

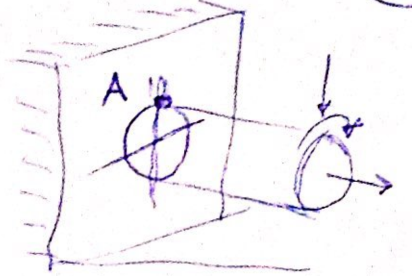
If S_y = 300 Mpa, find diameter of rod using

(a) Rankine (b) Tresca (c) Von Mises

Sol



solution Most Critical point is A

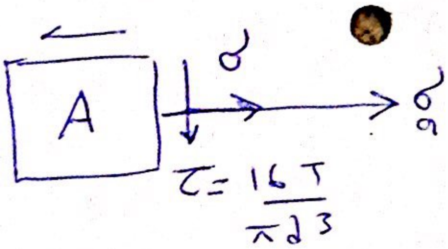


① direct axial stress $\sigma = \dfrac{P}{\frac{\pi}{4} d^2}$ (tensile)

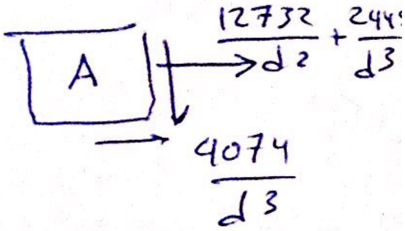
② Bending stress $\sigma_A = \dfrac{32 F \times l}{\pi d^3}$

③ Shear (due torsion) $\tau = \dfrac{16 T}{\pi d^3}$

shear due $\dfrac{VQ\,Ay}{It}$ is neglected (small)



$\sigma_{1,2} = \frac{1}{2}\left(\frac{12732}{d^2} + \frac{2445}{d^3}\right) \mp \sqrt{\frac{1}{4}\left(\frac{12732}{d^2} + \frac{2445}{d^3}\right)^2 + \left(\frac{4074}{d^3}\right)^2}$



(a) Rankine Set $\sigma_1 = \sigma_y \Rightarrow d = 26.67$ mm
(by trail & error)

(b) Tresca set $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{\sigma_y}{2} \Rightarrow d = 30.6$ mm

(c) Von mises set $\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = S_y^2 \Rightarrow d = 29.36$ mm