Failure of brittle materials :

(A) characteristics of brittle material :

1. Do not have yield point (strength) ; $\sigma-\epsilon$ is smooth curve up to failure by fracture.
2. Comp. strength S_{uc} many times greater tensile S_{ut}
   $\Rightarrow S_{uc} > S_{ut}$
3. Ultimate torsional strength S_{us} (rupture modulus) $= S_{ut}$

(B) Failure theories (brittle materials) :

1. Rankine theory
2. Coulomb-Mohr theory. (internal friction theory).
3. Modified Mohr theory.

1— Rankine @ previous ; only use S_{ut} & S_{uc} (Not yield)

2— Coulomb-Mohr theory : " Failure (fracture) occurs when complex stress state produce a circle tangent to the envelope of ~~that~~ two test circles { tensile test for S_{ut} and Compressive test for S_{uc} }

3.— Modified Mohr theory : see - graphical sol.

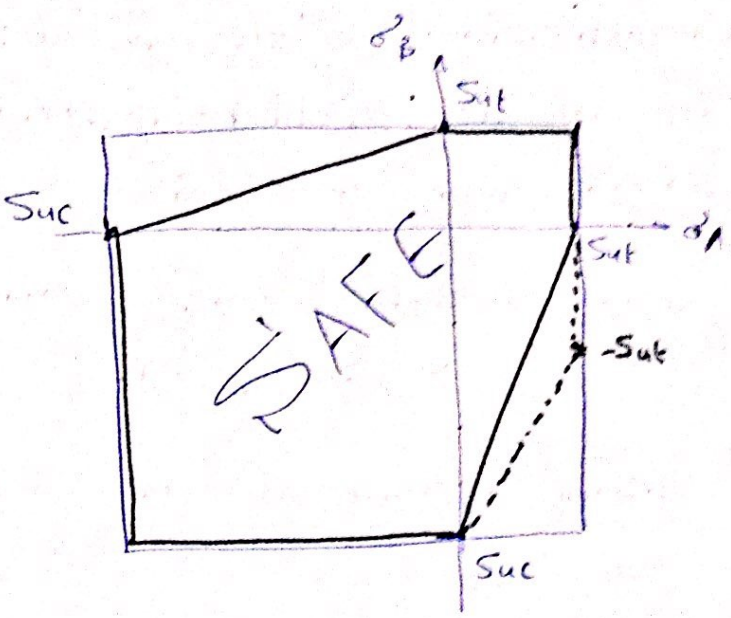
Graphically

- see next page

(1) Inside Zone $\Rightarrow$ safe design ; outside $\Rightarrow$ failure

(2) @ 1st quadrant : Rankine Coincides with Mohr-Coulomb theory

(3) Exp. results Coincides with Modified Mohr theory. espicially @ 4th quadrant -



④ If $\sigma_1 > \sigma_2 > \sigma_3$

and $\left\{ \begin{matrix} \sigma_1 > 0 \\ \sigma_3 < 0 \end{matrix} \right\}$

Use theoritical eq:

$$\frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} = 1$$

blue : Rankine theory
black (solid): Mohr - Coulomb theory
black (dash) : Modified Mohr theory.

Note: Use Suc : + true value.

~~Example~~ Teach students How to use / Get Yield & Ultimate stresses from Tables

Ex A small 6-mm diameter pin was designed of material near BS 300 Cast Iron ($S_{ut} = 293$ Mpa ; $S_{uc} = 965$ Mpa) The pin take an axial compressive load of 3.5 kN combined with a torsional load of 9.8 N.m ; Find safety factor for each of three failure theories ?

Sol. Graphical sol.

step1

- axial comp. stress $\sigma_x = \frac{F}{A} = \frac{4F}{\pi d^2} = \frac{4 * 3.5 * 10^3}{\pi (6)^2} = -124$ Mpa

- torsional shear stress $\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16 * 9.8 * 10^3}{\pi (6)^3} = 231$ Mpa

step2

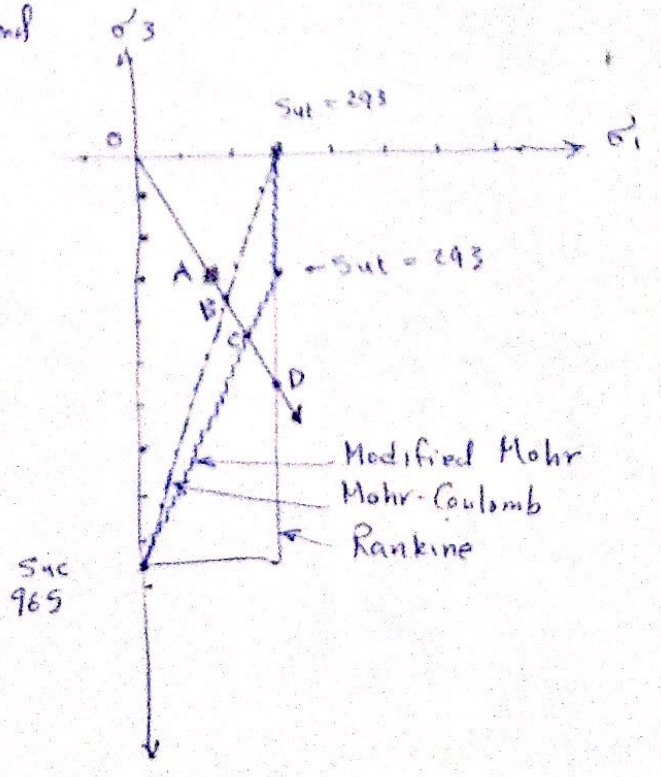
Find principal stresses → equation
→ Mohr Circle

for 2D (σ_1 & σ_2) exist -

$$\begin{cases} \sigma_1 = 177 \text{ Mpa} \\ \sigma_2 = 0 \\ \sigma_3 = -301 \text{ Mpa} \end{cases}$$

step3 use graph paper (scale) , plot 4th quadrent.

1. plot Rankine, Mohr Coulumb and Modified Curves.
2. plot point A : (σ_1 , σ_3)
   A : (177 , -301)
3. Draw line OA through all failure curves $\overrightarrow{OA}$, locate Points : B , C and D
4. $n = \frac{OD}{OA}$ (Rankine)
   $= 1.6$
   $n = \frac{OC}{OA}$ (Modified Mohr)
   $= 1.35$
   $n = \frac{OB}{OA}$ (Mohr-Coulomb)
   $= 1.05$



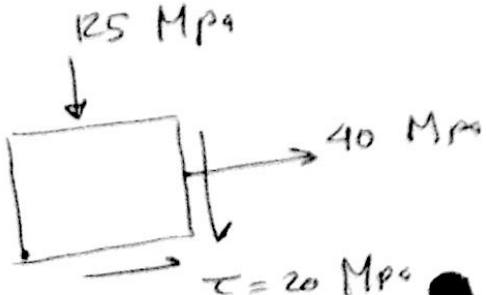
or use Horizental projections

Ex #2

The stress state @ a point is shown in Fig. from a material (same as previous example)

BS 300 Cast Iron (S_{ut} = 293 Mpa , S_{uc} = 965 Mpa)

find safety factor using @ Rankine, (b) Coloumb-Mohr & (c) Modified Mohr theories.



Sol. Using Equations $\begin{cases} \sigma_x = 40 \text{ MPa} \\ \sigma_y = -125 \\ \tau_{xy} = 20 \text{ MPa} \end{cases}$

$\sigma_1 = 42.3$ Mpa

$\sigma_2 = -127.3$ Mpa