Ex

A rectangular section beam ($b = 22$ mm , $h = 80$ mm) subjected to pure bending moment $M = 2200$ N.m so as to produce compression of inner fiber. Find stress if :

1. beam is straight.
2. Centroidal axis has radius of curvature 400 mm
3. $\approx$ $\leq$ $\nsim$, $\approx$ 80 mm.

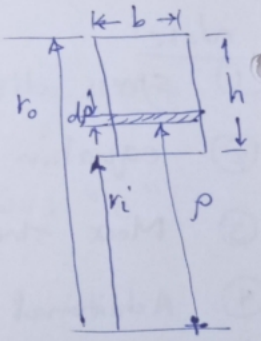
Solution

(1.) $$\sigma_{max} = \frac{My}{I} = \frac{2200 * 40 * 10^{-3}}{\frac{1}{12}(22 * 80^3) * 10^{-12}} = 93.7 \text{ MPa}$$

(2.) Location of neutral axis $$r = \frac{A}{\int \frac{dA}{\rho}}$$

$$A = bh$$

$$\int \frac{dA}{\rho} = \int_{r_i}^{r_o} \frac{1}{\rho}(b\, d\rho) = b \int_{r_i}^{r_o} \frac{d\rho}{\rho} = b \ln\left(\frac{r_o}{r_i}\right)$$

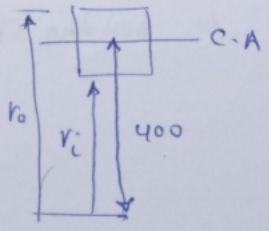


$$r = \frac{bh}{b \ln\left(\frac{r_o}{r_i}\right)} = \frac{h}{\ln\left(\frac{r_o}{r_i}\right)}$$

Now

$r_o = 400 + 40$
$= 440$

$r_i = 400 - 40$
$= 360$



$$r_n = \frac{80}{\ln\left(\frac{440}{360}\right)} = 398.66 \text{ mm}$$

$e = \bar{r} - r_n = 400 - 398.66 = 1.33 \text{ mm}$

$$\sigma_i = \frac{-Mc_i}{Ae r_i} = \frac{-2200\,(38.665)\times10^{-3}}{(22\times80)\times10^{-6}\times1.33\times10^{-3}\times360\times10^{-3}}$$

$$= -100.4 \text{ Mpa}$$

$$\sigma_o = \frac{Mc_o}{Ae r_o} = \frac{2200\times41.33\times10^{-3}}{(22\times80)\times10^{-6}\times1.33\times10^{-3}\times440\times10^{-3}}$$

$$= 87.8 \text{ Mpa.}$$

③ for beam # 3

$r_o = 80+40 = 120$ mm

$r_i = 80-40 = 40$ mm

$$r = \frac{80}{\ln\left(\frac{120}{40}\right)} = 72.8 \text{ mm}$$

$e = \bar{r} - r = 80 - 72.8 = 7.18$ mm

$c_o = 47.181$ mm

$c_i = 32.819$ mm

$\sigma_i = -142.8$ Mpa

$\sigma_o = 68.4$ Mpa

Notes

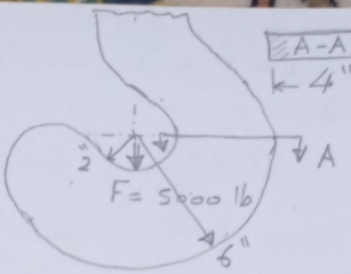
① Case 1 → straight beam
Case 2 → slight Curvature r = big
Case 3 → strong Curvature r = small

as curvature become severe ⇒ stresses change dramatically

Error (bet 1 & 2) = 7% acceptable
Error (bet 1 & 3) ≃ 55% Not acceptable

② If Cross sections Not rectangle → see table p. 77

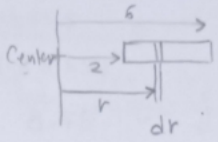
Ex #2 A hook of crane shown has rectangular x-section $\{b = 3/4'', h = 4''\}$ and carry 5000 lbf, find max σ



Sol.

$A = bh = \frac{3}{4} * 4 = 3\ in^2$

$dA = b\,dr$



$$r_n = \frac{A_1}{\int \frac{dA}{r}} = \frac{bh}{\int_{r_i}^{r_o} \frac{b\,dr}{r}} = \frac{h}{\ln \frac{r_o}{r_i}} = \frac{4}{\ln \frac{6}{2}} = 3.641'' \Rightarrow e = 4 - 3.641 = 0.359''$$

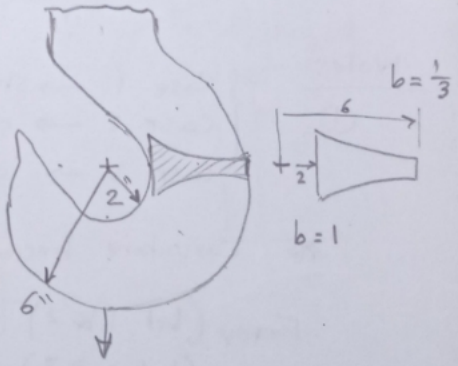
$$M = F\bar{r} = 5000 * 4 = 20\,000\ lb.in$$

$$\sigma = \frac{F}{A} + \frac{My}{Ae(r_n - y)} = \frac{5000}{3} + \frac{20000 * (3.641 - r)}{3 * 0.359 * r} \Rightarrow \begin{cases} \sigma_i = 16.9\ kpsi \\ \sigma_o = -5.63\ kpsi \end{cases}$$

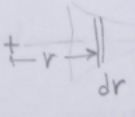
H.W A crane hook carry 20 kip. The x-section uses two concave flanks. width of cross-section $b = \frac{2}{r}$ where r is radius from center, $r_i = 2''$ & $r_o = 6''$. Find stresses @ inner and outer surfaces:

Sol.

$M = F * \bar{r} = 20 * ? = ?\ lb.in$



$$A = \int_2^6 b\,dr = \int_2^6 \frac{2}{r} dr = 2[\ln r]_2^6 = 2.2$$



$$\bar{r}A = \int r\,dA = \int_2^6 r \frac{2}{r} dr = 8$$

$$\bar{r} = \frac{8}{2.2} = 3.636 \Rightarrow M = 72 * 10^3\ lb.in$$

$$r_n = \frac{A}{\int \frac{dA}{r}} = \frac{2.2}{\int_2^6 \frac{b\,dr}{r}} = \frac{2.2}{2\int_2^6 \frac{dr}{r^2}} = \frac{1.1}{\left[\frac{-1}{r}\right]_2^6} = 3.3$$

$e = 3.636 - 3.3 = .336$ $\Rightarrow$ get stresses as above example.