Al Basrah university
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Mathematics \ first stage

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Integration of the Exponential and Logarithmic functions :

Let $u = f(x)$,where $f(x)$ is a differentiable function of x.

1. $\int \frac{du}{u} = ln\ |u| + c$.
2. $\int e^u\, du = e^u + c$.
3. $\int a^u\, du = \frac{a^u}{ln\, a} + c$.

Examples:- Evaluate the following integrals

1. $\int \frac{3x^2\, dx}{x^3+5} = ln\left|x^3+5\right| + C$ . .

2. $$\int \frac{sin\, x}{2+cos\, x}dx = \frac{-1}{-1}\int \frac{sin\, x}{2+cos\, x}dx$$
$$= \frac{-1}{-1}\int \frac{sin\, x}{2+cos\, x}dx = -ln\,|2+cos\, x| + C\ .$$

3. $\int e^{3x}\, dx = \frac{3}{3}\int e^{3x}\, dx = \frac{1}{3}\, e^{3x} + C$.

4. $\int \frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}\, dx = \frac{2}{2}\int \frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}\, dx = \frac{1}{2}\, ln\left|e^{2x}-e^{-2x}\right| + C$.

5. $\int 10^{3x}\, dx = \frac{3}{3}\int 10^{3x}\, dx = \frac{1}{3}\,\frac{10^{3x}}{ln\,(10)} + C$.

6. $$\int cot^3\,(10x)\, dx = \int cot^2(10x).cot\,(10x)\, dx$$
$$= \int (csc^2(10x)-1)\,.cot\,(10x)\, dx$$

$$= \int \left(csc^2(10x).cot\,(10x) - cot\,(10x)\right) dx$$
$$= \frac{-10}{-10} \int csc^2(10x).cot\,(10x)\,dx - \frac{10}{10}\int \frac{cos(10x)}{sin(10x)}\,dx$$
$$= \frac{1}{-10}\int (-10csc^2(10x)).cot\,(10x)\,dx - \frac{1}{10}\int \frac{10.cos(10x)}{sin(10x)}\,dx$$
$$= \frac{-1}{10}\,\frac{cot^{\,2}(10x)}{2} - \frac{1}{10}\,ln|sin(10x)| + C\,.$$

7. $\int 3^{\,x}\,dx = \frac{3^{\,x}}{ln\,(3)} + C\,.$

8. $\int \frac{x+1}{x^2+2x+3}\,dx = \frac{2}{2}\int \frac{x+1}{x^2+2x+3}\,dx$

$$= \frac{1}{2}\int \frac{2x+2}{x^2+2x+3}\,dx = \frac{1}{2}\,ln\left|x^2+2x+3\right| + C\,.$$

9. $\int \frac{dx}{x\,ln(x)}\,dx = \int \frac{\frac{1}{x}}{ln(x)}\,dx = ln\,|ln(x)| + C\,.$

10. Prove $\int tan(x)\,dx = ln\,|sec(x)| + c$???

$\rightarrow \int tan(x)\,dx = \int tan(x) * \frac{sec(x)}{sec(x)}\,dx = \int \frac{tan(x).sec(x)}{sec(x)}\,dx$

$$= ln\,|\,sec(x)| + c$$

11. $\int sec^2(3x)\,e^{\,tan(3x)}dx = \frac{1}{3}\int 3sec^2(x)\,e^{\,tan(3x)}dx$

$$= \frac{1}{3}e^{\,tan(3x)} + c.$$

12. $\int \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx = e^{\sqrt{x}} + c.$

13. $\int x.e^{-lnx}\,dx = \int x.e^{\,lnx^{-1}}dx = \int x.\,x^{-1}dx$

$$= \int x.\,\frac{1}{x}\,dx = \int 1\,dx = x + c.$$

H.W

1. Prove $\int cot(x)\,dx = ln\,|sin(x)| + c$??**H.W**.

2.Find the following integral $\int sin^3(x)cos^4(x)dx$??**H.W**