

Important relations:

1. $\sin^2 x = \frac{1 - \cos 2x}{2}$
2. $\cos^2 x = \frac{1 + \cos 2x}{2}$
3. $\sin 2x = 2 \sin x \cos x$
4. $\cos 2x = \cos^2 x - \sin^2 x$

Integration of trigonometric function

1. $\int \sin u \, du = -\cos u + c$
2. $\int \cos u \, du = \sin u + c$
3. $\int \sec^2 u \, du = \tan u + c$
4. $\int \sec u \tan u \, du = \sec u + c$
5. $\int \csc^2 u \, du = -\cot u + c$
6. $\int \csc u \cot u \, du = -\csc u + c$

Examples:-

$$1. \int \sec^2(2x) dx = \frac{1}{2} \tan(2x) + c.$$

$$2. \int \sin(8x) dx = \frac{-1}{8} \cos(8x) + c.$$

$$\begin{aligned} 3. \int x \sec^2(5x^2 - 3) dx &= \frac{1}{10} \int 10x \sec^2(5x^2 - 3) dx \\ &= \frac{1}{10} \tan(5x^2 - 3) + c \end{aligned}$$

$$\begin{aligned} 4. \int \sin^2(x) dx &= \int \frac{1 - \cos 2x}{2} dx \\ &= \frac{1}{2} \int (1 - \cos 2x) dx \\ &= \frac{1}{2} \int dx - \frac{1}{2} \int \cos 2x dx \\ &= \frac{1}{2} x - \frac{1}{2} \cdot \frac{1}{2} \sin(2x) + c \\ &= \frac{1}{2} x - \frac{1}{4} \sin(2x) + c. \end{aligned}$$

$$\begin{aligned} 5. \int (\sin x + \cos x)^2 dx &= \int (\sin^2 x + 2 \sin x \cos x + \cos^2 x) dx \\ &= \int (1 + 2 \sin x \cos x) dx \\ &= \int (1 + \sin 2x) dx = x - \frac{1}{2} \cos(2x) + c. \end{aligned}$$

Note: $2 \sin x \cos x = \sin 2x$

$$\begin{aligned} 6. \int \cos^2 x dx &= \int \frac{1 + \cos 2x}{2} dx \\ &= \frac{1}{2} \int (1 + \cos 2x) dx \\ &= \frac{1}{2} x + \frac{1}{2} \cdot \frac{1}{2} \sin 2x + c \\ &= \frac{1}{2} x + \frac{1}{4} \sin 2x + c. \end{aligned}$$

$$7. \int \cos\left(\frac{x}{2}\right) dx = \frac{2}{2} \int \cos\left(\frac{x}{2}\right) dx \\ = 2 \sin\left(\frac{x}{2}\right) + c.$$

$$8. \int \frac{\sin 3x}{\cos^3 3x} dx = \int \cos^{-3}(3x) \sin(3x) dx \\ = \frac{-3}{-3} \int \cos^{-3}(3x) \sin(3x) dx \\ = \frac{1}{-3} \int \cos^{-3}(3x) (-3 \sin 3x) dx \\ = \frac{1}{-3} \frac{\cos^{-2}(3x)}{-2} + c \\ = \frac{1}{6 \cos^2 3x} + c.$$

ملاحظة 

إذا كان لدينا قوس (اي داله مرفوعة الى اس مشتقتها موجودة) نستخدم القاعدة الخامسة للتكامل كما موضح بالأمثلة الآتية.

Examples:

$$9. \int \tan^4 x \sec^2 x dx = \frac{\tan^5 x}{5} + c.$$

$$10. \int \sin^5 2x \cos 2x dx = \frac{1}{2} \int \sin^5 2x (2 \cos 2x) dx \\ = \frac{1}{2} \frac{\sin^6 2x}{6} + c = \frac{1}{12} \sin^6 2x + c.$$

$$11. \int \sec^3(7x) \tan(7x) dx = \int \sec^2(7x) \sec(7x) \tan(7x) dx \\ = \frac{1}{7} \int \sec^2(7x) [7 \sec(7x) \tan(7x)] dx \\ = \frac{1}{7} \frac{1}{3} \sec^3(7x) + c \\ = \frac{1}{21} \sec^3(7x) + c.$$

$$12. \int \cos 3x \sqrt[3]{3 - 2 \sin 3x} dx = \int (3 - 2 \sin 3x)^{\frac{1}{3}} \cos 3x dx$$

$$\begin{aligned}
&= \frac{1}{-6} \int (3 - 2 \sin 3x)^{\frac{1}{3}} (-6 \cos 3x) dx \\
&= \frac{-1}{6} \frac{3}{4} (3 - 2 \sin 3x)^{\frac{4}{3}} + c \\
&= \frac{-1}{8} \sqrt[3]{(3 - 2 \sin 3x)^4} + c.
\end{aligned}$$

 **Note**

ملاحظة في حال تعذر توفير مشتقة الدالة نستخدم القوانين التي ذكرناها سابقا كما موضح بالأمثلة الآتية

Examples:

$$\begin{aligned}
13. \int \frac{1 + \tan^2 x}{\tan^3 x} dx &= \int \frac{\sec^2 x}{\tan^3 x} dx = \int \tan^{-3} x \sec^2 x dx \\
&= \frac{\tan^{-2} x}{-2} + c = \frac{-1}{2 \tan^2 x} + c
\end{aligned}$$

$$\begin{aligned}
14. \int \frac{\tan x}{\cos^2 x} dx &= \int \tan x \frac{1}{\cos^2 x} dx \\
&= \int \tan x \sec^2 x dx = \frac{\tan^2 x}{2} + c.
\end{aligned}$$

$$\begin{aligned}
15. \int \frac{\cot x}{\sin^2 x} dx &= \int \cot x \frac{1}{\sin^2 x} dx \\
&= \frac{-1}{-1} \int \cot x \csc^2 x dx = . \\
&= \frac{1}{-1} \int \cot x (-\csc^2 x) dx = \frac{-\cot^2 x}{2} + c.
\end{aligned}$$

$$16. \int \frac{\sin^2 x}{\sec x} dx = \int \sin^2 x \frac{1}{\sec x} dx = \int \sin^2 x \cos x dx = \frac{\sin^3 x}{3} + c.$$

$$\begin{aligned}
17. \int (\cos^4 5x - \sin^4 5x) dx \\
&= \int (\cos^2 5x - \sin^2 5x)(\cos^2 5x + \sin^2 5x) dx \\
&= \int (\cos^2 5x - \sin^2 5x) dx \\
&= \int \cos(10x) dx = \frac{1}{10} \sin 10x + c.
\end{aligned}$$

Note: $\cos 2x = \cos^2 x - \sin^2 x$
 $\cos^2 x + \sin^2 x = 1.$

Note: اذا جاء احد المقادير التالية

$$\begin{cases} 1 \mp \sin x \\ \sin x \mp 1 \\ 1 \mp \cos x \\ \cos x \mp 1 \end{cases}$$

في المقام لغرض حل التكامل يجب الضرب بالمرافق كما موضح في المثال التالي

18. $\int \frac{1}{1+\sin x} dx = \int \frac{1}{1+\sin x} * \frac{1-\sin x}{1-\sin x} dx$

$$\int \frac{1 - \sin x}{1 - \sin^2 x} dx = \int \frac{1 - \sin x}{\cos^2 x} dx$$

$$= \int \left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} \right) dx$$

$$= \int \left(\sec^2 x - \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} \right) dx$$

$$= \int (\sec^2 x - \tan x \cdot \sec x) dx$$

$$= \tan x - \sec x + c.$$