Al Basrah university
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Mathematics and Statistics\ first stage

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Differentiation Rules

1. If $y = k$, where k is constant, $y' = 0$
2. If $y = k.f(x)$, then $y' = k.f'(x)$.
3. If $f(x) = x^n$, (n positive integer) then for every real value of x, $f'(x) = n\,x^{n-1}$.
4. If $y = f(x) \mp g(x)$, then $y' = f'(x) \mp g'(x)$.
5. If $y = f(x)\,.\,g(x)$, then $y' = f(x) * g'(x) + g(x) * f'(x)$.
6. If $y = \frac{f(x)}{g(x)}$, where $(x) \neq 0$ for every x, then $y' = \frac{g(x)*f'(x) - f(x)*g'(x)}{(g(x))^2}$.
7. If $y = (f(x))^n$, (Any real number n) then $y' = n\,(f(x))^{n-1} * f'(x)$.

Derivative of higher orders:-

Frist order derivative of $y = f(x) \rightarrow y', \frac{dy}{dx}, f'(x)$.

Second order derivative of $y = f(x) \rightarrow y'', \frac{d^2y}{dx^2}, f''(x)$.

Third order derivative of $y = f(x) \rightarrow y''', \frac{d^3y}{dx^3}, f'''(x)$.

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The n_{th} order derivative of $y = f(x) \rightarrow y^{(n)}, \frac{d^ny}{dx^n}, f^{(n)}(x)$

Examples:- Find derivative of following functions:

1. $y = x + \frac{1}{x^2} + 3 \longrightarrow y' = 1 - \frac{2}{x^3}$
2. $y = \sqrt{x^3 - 2} + \frac{1}{\sqrt{x+1}} \longrightarrow y' = \frac{1}{2}(x^3-2)^{-\frac{1}{2}}.3x^2 - \frac{1}{2}(x+1)^{-\frac{3}{2}}.1$

$$= \frac{3}{2}\frac{x^2}{\sqrt{x^{3-2}}} - \frac{1}{2}\frac{1}{\sqrt{(x+1)^3}}$$

3. $y = (x^2 + x + 1)^3$

$\longrightarrow y' = 3(x^2 + x + 1)^2 * (2x + 1)$

4. $y = \left(\frac{x+1}{x-1}\right)^{2/3}$

$\longrightarrow y' = \frac{2}{3} . \left(\frac{x+1}{x-1}\right)^{-1/3} . \frac{(x-1)-(x+1).(1)}{(x-1)^2} = \frac{4(x+1)^{-1/3}}{3(x-1)^{5/3}}$

5. $y = \frac{x^2}{2x-1}$

$\longrightarrow y' = \frac{(2x-1)*(2x)-x^2*2}{(2x-1)^2}$

$= \frac{4x^2-2x-2x^2}{(2x-1)^2} = \frac{2x^2-2x}{(2x-1)^2} = \frac{2x(x-1)}{(2x-1)^2}$

6. $y = \sqrt{x+3} \rightarrow y = (x+3)^{\frac{1}{2}}$

$\longrightarrow y' = \frac{1}{2}(x+3)^{\frac{-1}{2}} * 1 = \frac{1}{2\sqrt{x+3}}$

7. $y = \frac{1}{(x^2+3)^2} \rightarrow y = (x^2 + 3)^{-2}$

$\longrightarrow y' = -2(x^2 + 3)^{-3} * (2x)$

$= \frac{-4x}{(x^2+3)^3}.$

8. *Find* y', y'', y''' *of* $y = 6\sqrt[3]{x^2}$

$y = 6(x)^{\frac{2}{3}} \rightarrow y = 6(x)^{\frac{2}{3}}$

$\longrightarrow y' = 6 * \frac{2}{3} (x)^{\frac{-1}{3}} = \frac{4}{\sqrt[3]{x}}$

$\longrightarrow y'' = 4 * \left(\frac{-1}{3}\right) * (x)^{\frac{-4}{3}} = \frac{-4}{3\sqrt[3]{x^4}}$

$y''' = \frac{-4}{3} * \left(\frac{-4}{3}\right) * (x)^{\frac{-7}{3}} = \frac{16}{9\sqrt[3]{x^7}}.$

9. *let* $f(x) = x^2(x-4)^3$, *find* $f'(x)$?

$\longrightarrow f'(x) = x^2 * 3(x-4)^2 + (x-4)^3 * (2x)$

$= x(x-4)^2\big(3x + 2(x-4)\big)$

$= x(x-4)^2(3x + 2x - 8).$

$= x(x-4)^2(5x - 8)$

10. $f(x) = x^3 * \sqrt[3]{(7x^2 + x - 1)^2} = x^3\ (7x^2 + x - 1)^{2/3}$

$$f'(x) = x^3 * \frac{2}{3}(7x^2 + x - 1)^{\frac{-1}{3}} * (14x + 1) + (7x^2 + x - 1)^{2/3} * (3x^2)$$

Trigonomtric Functions

$sinx$, $cosx$, $tanx$, $cotx$, $secx$, and $cscx$.

Important relations:

1. $cos(x \pm y) = cos\, x\, cos\, y \mp sin\, x\, sin\, y$
2. $sin(x \pm y) = sin\, x\, cos\, y \pm cos\, x\, sin\, y$
3. $sin^2 x + cos^2 x = 1$
4. $sec^2 x - tan^2 x = 1$
5. $csc^2 x - cot^2 x = 1$
6. $cos^2 x = \frac{1}{2}(1 + cos\, 2x)$
7. $sin^2 x = \frac{1}{2}(1 - cos\, 2x)$
8. $lim_{a \to 0} \frac{sin\, a}{a} = 1$
9. $lim_{a \to 0} \frac{cos\, a - 1}{a} = 0$
10. $tanx = \frac{sin\, x}{cos\, x}$
11. $cot\, x = \frac{cos\, x}{sin\, x}$
12. $sec\, x = \frac{1}{cos\, x}$
13. $csc\, x = \frac{1}{sinx}$

Derivatives of Trigonomtric Functions

Let $u = f(x)$ be a differentiable function, then :−

1. $\frac{d}{dx} sin\, u = cos\, u\, .\, \frac{du}{dx}$
2. $\frac{d}{dx} cos\, u = -\, sin\, u\, .\, \frac{du}{dx}$
3. $\frac{d}{dx} tan\, u = sec^2\, u\, .\, \frac{du}{dx}$
4. $\frac{d}{dx} cot\, u = -csc^2 u\, .\, \frac{du}{dx}$
5. $\frac{d}{dx} sec\, u = sec\, u\, .\, tan\, u * \frac{du}{dx}$
6. $\frac{d}{dx} csc\, u = -\, csc\, u\, .\, cot\, u * \frac{du}{dx}$

Examples:- Find derivative of the following functions:-

1. $y = \sin(2x) + \sec(2x)$

→ $y' = 2\cos(2x) + 3\sec(3x).\tan(3x)$

2. $y = \tan^4 x$

→ $y' = 4\tan^3 x\,.\sec^2 x\,.$

3. $y = \sec^4(x^2+1)$

→ $y' = 4\sec^3(x^2+1) * \sec(x^2+1) * \tan(x^2+1) * (2x)$

$= 8x\sec^4(x^2+1) * \tan(x^2+1).$

4. $y = \cos(3x-2)$

→ $y' = -\sin(3x-2) * (3) = -3\sin(3x-2).$

5. Find y' and y'' if $y = \frac{\sin x}{1+\cos x}$?

→ $y' = \frac{(1+\cos x)*\cos x - \sin x*(-\sin x)}{(1+\cos x)^2}$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \frac{1+\cos x}{(1+\cos x)^2} = \frac{1}{1+\cos x}$$

→ $y'' = \frac{(1+\cos x)*(0)-(-\sin x)}{(1+\cos x)^2} = \frac{\sin x}{(1+\cos x)^2}\,.$

6. $f(x) = (7\sin 5x - \cos\sqrt{x})^3$

→ $f'(x) = 3(7\sin 5x - \cos\sqrt{x})^2 * (35\cos 5x + \frac{1}{2\sqrt{x}}\sin\sqrt{x}).$

7. $f(x) = \sin^3(5x)$

→ $f'(x) = 3(\sin 5x)^2 * \cos 5x * (5) = 15\sin^2 5x * \cos 5x\,.$

8. $y = \cot x * \csc x$

→ $y' = \cot x * \left(-\csc x * \cot x\right) + \csc x * (-\csc^2 x)$

$= -\csc x * \cot^2 x - \csc^3 x.$

9. $y = \sqrt{x^3 - \sin x} = (x^3 - \sin x)^{\frac{1}{2}}$

→ $y' = \frac{1}{2}(x^3 - \sin x)^{\frac{-1}{2}} * (3x^2 - \cos x)$

$= \frac{3x^2 - \cos x}{2\sqrt{x^3 - \sin x}}.$

10. $y = \sin(\sqrt[3]{x})$

$\longrightarrow y' = \cos\sqrt[3]{x} * \left(\frac{1}{3\sqrt[3]{x^2}}\right)$

11. $f(x) = \cos^3 7x = (\cos 7x)^3$

$\longrightarrow f'(x) = 3\,(\cos 7x)^2 * (-\sin 7x) * 7$

$= -21\cos^2 7x * \sin 7x\,.$

12. $f(x) = \cos 3x - \tan 5x + \sec 4x$

$\longrightarrow f'(x) = -\sin 3x * (3) - \sec^2 5x * (5) + \sec 4x * \tan 4x * 4$

$= -3\sin 3x - 5\sec^2 5x + 4\sec 4x * \tan 4x\,.$