Part Three Turbulent Flow

1. Introduction

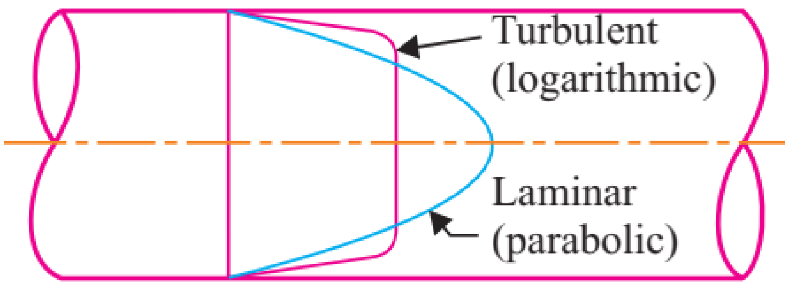
In a pipe, a laminar flow occurs when Reynolds number (Re) is less than 2000 and a turbulent flow occurs **when Re> 4000**. In a turbulent flow, the fluid motion is irregular and chaotic and there is complete mixing of fluid due to collision of fluid masses with one another.

The fluid masses are interchanged between adjacent layers. As the fluid masses in adjacent layers have different velocities, interchange of fluid masses between the adjacent layers is accompanied by a transfer of momentum which causes additional shear stresses of high magnitude between adjacent layers. **The shear in turbulent flow is mainly due to momentum transfer.**

The contribution of fluid viscosity to total shear is small and is usually neglected. In case of laminar flow, because of definite functional relationship 'between shear stress due to viscosity and velocity' it has been possible to derive a mathematical relationship for evaluation of energy dissipation or frictional head but such a simple relationship **does not exist** for turbulent flow. However, to solve some of the practical problems, efforts have been made to evolve semi-empirical theories of turbulence.

Following points are worth noting about turbulent flow:

(i)The velocity distribution is more uniform than in laminar flow.



(ii)The velocity gradients near the boundary shall be quite large resulting in more shear.

(iii)The flatness of velocity distribution curve in the core region away from the wall is because of the mixing of fluid layers and exchange of momentum between them.

(iv)The velocity distribution which is paraboloid in laminar flow, tends to follow power law and logarithmic law in turbulent flow.

(v)Random orientation of fluid particles in a turbulent flow gives rise to additional stresses, called the Reynolds stresses.

(vi)Formation of eddies, mixing and curving of path lines in a turbulent flow results in much greater frictional losses for the same rate of discharge, viscosity and pipe size.

The turbulent motion can be classified as follows:

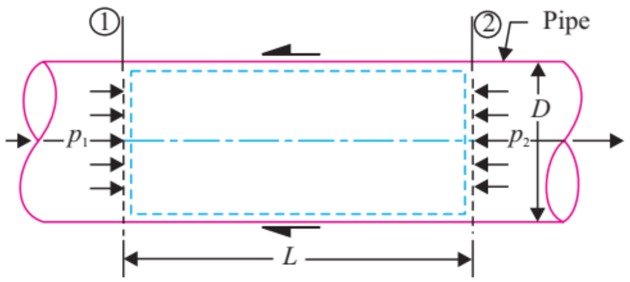
1) Wall turbulence. It occurs in immediate vicinity of solid surfaces and in the boundary layer flows where the fluid has a negligible mean acceleration.
2) Free turbulence. It occurs in jets, wakes, mixing layers etc.
3) Convective turbulence. It takes place where there is conversion of P.E into K.E. by the process of mixing (e.g. the turbulent flow in the annular space between the concentric rotating cylinder, conventional flow between parallel horizontal plates etc.).

2. Loss of Head Due to Friction in Pipe Flow–Darcy Equation

In case of turbulent flow through pipes it has been observed through experiments that the **viscous friction effects** associated with fluid are **proportional** to:

(i)The length of the pipe, L, (ii)The wetted perimeter, P, and(iii)V^n, where V is the average velocity of flow and n is an index varying from 1·5 to 2.

(depending on the material and nature of the pipe surface); for commercial pipes =2 (with turbulent flow).



Forces on a control volume in a pipe flow.

Propelling force on the flowing fluid between the two sections is

$$= (p_1 - p_2)\,A$$

(where, A = area of cross-section of the pipe)

Frictional resistance force $= f'\,PLV^2$

where, P = Wetted perimeter, and

V = Average flow velocity.

f' = Non-dimensional factor (whose value depends upon the material and nature of the pipe surface), and

h_f = Loss of head due to friction.

Under equilibrium conditions:

Propelling force=Frictional resistance force

i.e. $(p_1 - p_2)\, A = f'\, PLV^2$

Dividing both sides by weight density *w*, we have:

$$\left(\frac{p_1 - p_2}{w}\right) A = \frac{f'}{w} PLV^2$$

$$h_f = \frac{f'}{w}\left(\frac{P}{A}\right) LV^2$$

$$h_f = \frac{2gf'}{w}\left(\frac{P}{A}\right)\frac{LV^2}{2g} = \frac{2gf'}{w} \times \frac{L}{m} \times \frac{V^2}{2g} \quad ...(1)$$

and after simplify (In case of a circular pipe), we have:

$$h_f = f \times \frac{L}{D/4} \times \frac{V^2}{2g} = \frac{4fLV^2}{D \times 2g} \quad ...(2)$$

(The factor f is known as Darcy coefficient of friction.)

Eqn. (2) is known as Darcy-Weisbach equation and *it holds good for all types of flows provided a proper value of f is chosen.*

Sometimes eqn. (2) is written as:

$$h_f = \frac{f_1 LV^2}{D \times 2g}$$

where, f_1 is *known as* **friction factor** ($f_1 = 4f$)

Expression for co-efficient of friction in terms of shear stress:

Refer to Fig. ,

$$(p_1 - p_2)\, A = \text{Force due to shear stress, } \tau_0$$

$$(\text{where, } \tau_0 = \text{shear stress at the pipe wall})$$

$$= \text{Shear stress } (\tau_0) \times \text{surface area}$$

$$= \tau_0 \times \pi DL$$

or, $$(p_1 - p_2)\frac{\pi}{4} D^2 = \tau_0 \times \pi DL$$

or, $$(p_1 - p_2)\frac{D}{4} = \tau_0 L$$

or, $$(p_1 - p_2) = \frac{4\tau_0 \times L}{D} \quad ...(3)$$

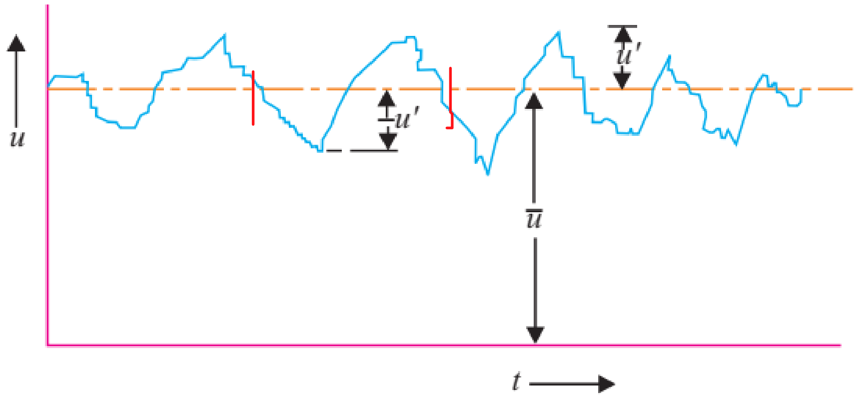
and after simplify, we have:

$$\tau_0 = \frac{fV^2 \times w}{2g} = \frac{fV^2 \times \rho g}{2g} = \frac{f\rho V^2}{2} \quad ...[5\,(a)]$$

$$f = \frac{2\tau_0}{\rho V^2} \quad ...[5\,(b)]$$

3. CHARACTERISTICS OF TURBULENT FLOW

The turbulent flow is characterized by random, Irregular and haphazard movement of fluid particles. It has been observed during experimentation that at any fixed point in turbulent field, the velocity and consequently the pressure fluctuates with time about a mean value.



Variation of *u* with time *t* at a point in turbulent flow.

The instantaneous velocity *i.e.* velocity at any time at the given point can be expressed as:

$$u = \overline{u} + u' \quad \text{...(6)}$$

where, u = Instantaneous velocity,

$\overline{u}$ = Time average or temporal mean velocity, and

u' = Velocity fluctuation (fluctuating component).

Similarly, $v = \overline{v} + v'$,

$w = \overline{w} + w'$,

and, $p = \overline{p} + p'$ ...(7)

From the definition of average-velocities, we have:

$$\left\{\begin{matrix} \frac{1}{T}\int_0^T u\,dt = \overline{u}; & \frac{1}{T}\int_0^T v\,dt = \overline{v}; \\ \frac{1}{T}\int_0^T w\,dt = \overline{w}; & \frac{1}{T}\int_0^T p\,dt = \overline{p} \end{matrix}\right\} \quad \text{...(8)}$$

and,

$$\left\{\begin{matrix} \frac{1}{T}\int_0^T u'dt = \overline{u}' = 0; & \frac{1}{T}\int_0^T v'dt = \overline{v}' = 0; \\ \frac{1}{T}\int_0^T w'dt = \overline{w}' = 0; & \frac{1}{T}\int_0^T p'dt = \overline{p}' = 0 \end{matrix}\right\} \quad \text{...(9)}$$

where, T = Large interval of time.

Magnitude of turbulence = Arithmetic mean of root-mean square value of turbulent fluctuations in the three directions

$$= \sqrt{\left(\frac{\overline{u'^2} + \overline{v'^2} + \overline{w'^2}}{3}\right)} \quad ...(10)$$

Intensity of turbulence

$$= \frac{\sqrt{\dfrac{\overline{u'^2} + \overline{v'^2} + \overline{w'^2}}{3}}}{\overline{V}} \quad ...(11)$$

where, $\overline{V}$ = Line average resultant velocity at the point.

For describing the turbulence fully, besides ***the intensity of turbulence***, ***the average size of the eddy*** is also necessary which can be obtained from the curve of velocity variation with time (as shown in Fig.) by multiplying **the average time interval at which the curve crosses the mean value**, with **the average velocity of flow**.

4. SHEAR STRESSES IN TURBULENT FLOW

In turbulent flow, as stated earlier, velocity fluctuations cause momentum transport which results in developing additional shear stresses of high magnitude between adjacent layers of the fluid. In order to determine the magnitude of the turbulent shear stress a number of semi-empirical theories have been developed some of which are discussed below.

4·1 Boussinesq's Theory

According to this theory, the expression for the shear stress, τ_t for the turbulent flow can be written as:

$$\tau_t = \eta \cdot \frac{d\overline{u}}{dy} \quad ...(12)$$

where η (eta) is called "*eddy*" *viscosity*, and $\overline{u}$ is the temporal mean velocity in the direction of flow at a point at distance y from the solid boundary.

Similar to kinematic viscosity $\nu = \frac{\mu}{\rho}$, the "*eddy*" *kinematic viscosity* ∈ (Greek 'epsilon') is also obtained by dividing eddy viscosity η, by the mass density of the fluid ρ, thus,

$$\in = \frac{\eta}{\rho}$$

When viscous action is also included, the total shear stress may be expressed as :

$$\tau = \tau_\upsilon + \tau_t$$

(where τ_υ = shear stress due to viscosity)

or,
$$\tau = \mu \frac{du}{dy} + \eta \frac{d\overline{u}}{dy} \quad ...(13)$$

The magnitude of η may vary from zero (if the flow is laminar) to several thousand times that of μ. As the values of η and ∈ cannot be predicted, the Boussinesq's equation has a *limited use*.

4·2 Reynolds Theory

According to this theory (1886), the turbulent shear stress between two layers of a fluid at a small distance apart is given as:

$$\tau = \rho u' \upsilon' \qquad ...(14)$$

where u' and υ' are the fluctuating components of velocity in the directions of x and y due to turbulence.

Since both u' and υ' vary and subsequently τ also varies, therefore, to find the shear stress, the time average is taken and eqn. (14) becomes:

$$\bar{\tau} = \overline{\rho u' \upsilon'} \qquad ...(15)$$

4·3 Prandtl's Mixing Length Theory

According to Prandtl (1925), the **mixing length** (l) is *defined as* the *average lateral distance through which a small mass of fluid particles would move from one layer to the other adjacent layers before acquiring the velocity of the new layer*. He assumed that components u' and υ' are of the same order and the velocity fluctuation in X-direction is related to the mixing length as:

$$u' = l \frac{du}{dy}$$

$$\therefore \quad \overline{u' \times \upsilon'} = \overline{u' \upsilon'} = \left(l \frac{du}{dy}\right) \times \left(l \frac{du}{dy}\right) = l^2 \left(\frac{du}{dy}\right)^2 \qquad \left(\because \upsilon' = l \frac{du}{dy}\right)$$

Substituting the value of $\overline{u' \upsilon'}$ in eqn. (15), we get:

$$\bar{\tau} = \rho l^2 \left(\frac{du}{dy}\right)^2 \qquad ...(16)$$

When the viscous action is also included the total shear stress may be expressed as :

$$\bar{\tau} = \mu \frac{du}{dy} + \rho l^2 \left(\frac{du}{dy}\right)^2 \qquad ...(17)$$

Eqn. (17) is used for most of the turbulent flow problems for determining the shear stress (viscous shear stress is negligible except near the boundary).

5. UNIVERSAL VELOCITY DISTRIBUTION EQUATION

Assuming the viscous shear stress to be negligible near the boundary the shear stress in turbulent flow is given by the eqn. (16). From this equation, we can obtain velocity distribution if the relation between l, the mixing length, and y is known.

Also $l \propto y$ (from the pipe wall) ...*Prandtl's hypothesis*

or, $l = \lambda y$

where, λ = a constant of proportionality, known as '***Karman universal constant***' (= 0·4).

Substituting the values of l in eqn. (16), we get:

$$\bar{\tau} \text{ or } \tau = \rho \times (\lambda y)^2 \times \left(\frac{du}{dy}\right)^2 = \rho \lambda^2 y^2 \left(\frac{du}{dy}\right)^2 \qquad ...(i)$$

Assuming that the turbulent shear stress remains constant in the vicinity of wall, we have

$$\tau = \tau_0 \qquad (\tau_0 = \text{the boundary shear stress})$$

The eqn. (*i*) becomes:

$$\tau_0 = \rho \lambda^2 y^2 \left(\frac{du}{dy}\right)^2$$

or, $$\frac{du}{dy} = \frac{1}{\lambda y}\sqrt{\frac{\tau_0}{\rho}} = u_f\left(\frac{1}{\lambda y}\right) \quad ...(ii)$$

$$\left[\text{where, } u_f = \textit{shear friction velocity} \text{ or } \textbf{\textit{shear velocity}} = \sqrt{\frac{\tau_0}{\rho}}\right]$$

or, $$du = u_f\left(\frac{1}{\lambda y}\right) dy \quad ...(iii)$$

(u_f is *constant* for a given case of turbulent flow)

Integrating the other equation, we get:

$$u = \frac{u_f}{\lambda}\ln(y) + C \quad ...(18)$$

(where, C = constant of integration)

Eqn. (18) shows that velocity distribution in turbulent flow is *logarithmic* in nature.

The constant of integration C is determined by the boundary condition.

At $y = R$ (radius of the pipe), $u = u_{max}$

By substituting the above values in eqn. (18), we have:

$$u_{max} = \frac{u_f}{\lambda}\ln(R) + C$$

or $$C = u_{max} - \frac{u_f}{\lambda}\ln(R)$$

Substituting this value of C in eqn. (18), we get:

$$u = \frac{u_f}{\lambda}\ln(y) + u_{max} - \frac{u_f}{\lambda}\ln(R)$$

$$= u_{max} + \frac{u_f}{\lambda}[\ln(y) - \ln(R)]$$

or, $$u = u_{max} + \frac{u_f}{\lambda}\ln\left(\frac{y}{R}\right)$$

Taking $\lambda = 0{\cdot}4$, we get:

$$\boldsymbol{u = u_{max} + 2{\cdot}5\, u_f \ln\left(\frac{y}{R}\right)} \quad ...(19)$$

Eqn. (19) is called **Prandtl's universal distribution equation.** This equation is applicableto *smooth as well as rough boundaries*.

This equation (19) may be written in *non-dimensional form*:

$$\frac{u_{max} - u}{u_f} = 2{\cdot}5 \ln\left(\frac{R}{y}\right)$$

$$= 5{\cdot}75 \log_{10}\left(\frac{R}{y}\right)$$

The difference *(u_{max}–u)* is known as the **velocity defect.**

Example 1. *In a pipe of 360 mm diameter having turbulent flow, the centre-line velocity is 7 m/s and that at 60 mm from the pipe wall is 6 m/s. Calculate the shear friction velocity.*

Solution. Radius of the pipe $= \frac{360}{2} = 180 \text{ mm} = 0{\cdot}18 \text{ m}$

Centre-line velocity, $u_{max} = 7$ m/s

Velocity at 60 mm (*i.e.* distance *y*), $u = 6$ m/s

Shear velocity, u_f:

We know, $$\frac{u_{max} - u}{u_f} = 5{\cdot}75 \log_{10}\left(\frac{R}{y}\right) \quad \text{...[Eqn. (20)]}$$

$$\therefore \quad \frac{7-6}{u_f} = 5{\cdot}75 \log_{10}\left(\frac{0{\cdot}18}{0{\cdot}06}\right) = 2{\cdot}743$$

$$\therefore \quad u_f = 0{\cdot}36 \text{ m/s } \textbf{(Ans.)}$$

Example 2. *A pipe of 100 mm diameter is carrying water. If the velocities at the pipe centre and 30 mm from the pipe centre are 2.0 m/s and 1.5 m/s respectively and flow in the pipe is turbulent, calculate the wall shearing stress.*

Solution. *Given :* $R = \frac{100}{2} = 50 \text{ mm} = 0.05 \text{ m}; u_{max} = 2.0 \text{ m/s};$

Velocity at $r = 30$ mm or $y = R - r = 50 - 30 = 20$ mm, $u = 1.5$ m/s.

Wall shearing stress, τ_0:

$$\frac{u_{max} - u}{u_f} = 5.75 \log_{10}\left(\frac{R}{y}\right) \quad \text{...[Eqn. (20)]}$$

(where, u_f = shear velocity)

Substituting the values, we get: $$\frac{2.0 - 1.5}{u_f} = 5.75 \log_{10}\left(\frac{0.05}{0.02}\right) = 2.288$$

$$u_f = \frac{(2.0 - 1.5)}{2.288} = 0.218 \text{ m/s}$$

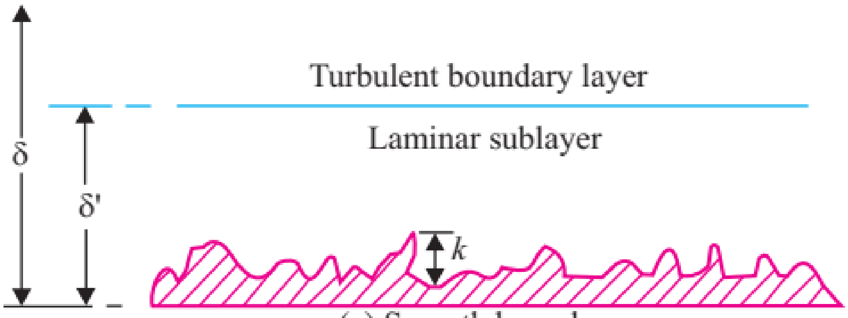
Using the relation: $$u_f = \sqrt{\frac{\tau}{\rho}}$$

or, $$0.218 = \sqrt{\frac{\tau_0}{1000}} \quad (\because \rho \text{ for water} = 1000 \text{ kg/m}^3)$$

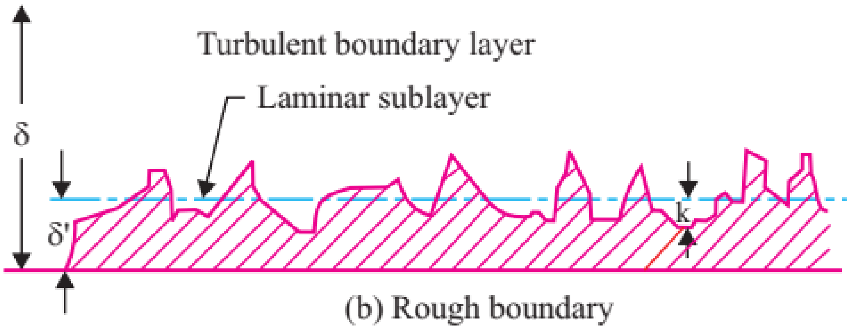
or, $$\tau_0 = 47.524 \text{ N/m}^2 \textbf{ (Ans.)}$$

6. HYDRODYNAMICALLY SMOOTH AND ROUGH BOUNDARIES

Refer to Fig. If 'k' is the average height of the irregularities of the surface of a boundary, then in general, the boundary is said to the rough if the value of 'k' compared to the thickness of the laminar sublayer δ' is high and smooth if 'k' is low (in comparison with δ′).



(a) Smooth boundary



(b) Rough boundary

Through experiments Nikuradse found that the boundary behaves as:

(*i*) Hydrodynamically smooth boundary ...when $\left(\frac{k}{\delta'}\right) < 0\cdot25$,

(*ii*) Hydrodynamically rough boundary ...when $\left(\frac{k}{\delta'}\right) > 6\cdot0$, and

(*iii*) Boundary in transition ...when $0\cdot25 < \left(\frac{k}{\delta'}\right) < 6\cdot0$.

In terms of roughness Reynolds number $\frac{u_f k}{\nu}$:

(*i*) For smooth boundary ... $\frac{u_f k}{\nu} < 4$,

(*ii*) For rough boundary ... $\frac{u_f k}{\nu} > 100$, and

(*iii*) For boundary in transition stage ... $\frac{u_f k}{\nu}$ lies between 4 and 100.

6·1. Velocity Distribution for Turbulent Flow in Smooth Pipes

The velocity distribution for turbulent flow in pipes in given by Eqn. 18 as :

$$u = \frac{u_f}{\lambda} \ln(y) + C$$

The peculiarity for this velocity distribution is that at the boundary, that is for $y = 0$, it gives velocity u equal to $-\infty$ (minus infinity). Thus it is only at a certain finite distance above the boundary say $y = y'$, that the velocity will be zero, hence the above equation becomes:

$$0 = \frac{u_f}{\lambda} \ln(y') + C$$

or,

$$C = -\frac{u_f}{\lambda} \ln(y')$$

Substituting the value of C in the above equation and simplified the result, we get:

$$\frac{u}{u_f} = 5\cdot 75 \log_{10}\left(\frac{y}{y'}\right) \qquad ...(21)$$

It has been observed from Nikuradse's experimental studies of turbulent flow in smooth pipes that for turbulent flow in smooth pipes of any size the value of the parameter $\left(\frac{u_f y}{v}\right)$ for $y = \delta'$ is approximately 11·6 and for $y = y'$ it is approximately 0·108.

i.e.

$$\frac{u_f \delta'}{v} = 11\cdot 6 \quad \text{or} \quad \delta' = \frac{11\cdot 6 v}{u_f} \qquad ...(22)$$

and,

$$\frac{u_f y'}{v} = 0.108$$

or,

$$y' = \frac{0\cdot 108 v}{u_f} \left(= \frac{\delta'}{107}\right) \qquad ...(23)$$

Substituting the value of $y'\left(= \frac{0\cdot 108\, v}{u_f}\right)$ in eqn. 21, we get:

$$\frac{u}{u_f} = 5\cdot 75 \log_{10}\left(\frac{y}{\frac{0.108 v}{u_f}}\right)$$

$$= 5\cdot 75 \log_{10}\left(\frac{u_f \cdot y}{0\cdot 108 v}\right) = 5\cdot 75 \log_{10}\left(\frac{u_f \cdot y}{v}\right) = 5.75 \log_{10}(0.108)$$

or,

$$\boldsymbol{\frac{u}{u_f} = 5\cdot 75 \log_{10}\left(\frac{u_f \cdot y}{v}\right) + 5\cdot 5} \qquad ...(24)$$

The eqn. (24) is known as **Karman-Prandtl equation** *for the velocity distribution near hydrodynamically* ***smooth boundaries.***

6·2Velocity Distribution for Turbulent Flow in Rough Pipes

As shown in Fig. above (b), the thickness of laminar sublayer is very small, the surface irregularities are above the laminar sublayer and hence the laminar sublayer is completely destroyed. From the experiments conducted by Nikuradse and others, using pipes artificially roughened by cemented coatings of sand grains (irregularities/projections) of diameter *k*, it has been found that y' is directly found proportional to *k and y′= k/30.*

Substituting this value of y' in eqn. (21), we get:

$$\frac{u}{u_f} = 5\cdot75 \log_{10}\left(\frac{y}{k/30}\right) = 5\cdot75\,[\log_{10}(y/k)\times 30]$$

$$= 5\cdot75 \log_{10}(y/k) + 5\cdot75 \log_{10} 30$$

$$\boldsymbol{\frac{u}{u_f} = 5\cdot75 \log_{10}(y/k) + 8\cdot5} \quad \text{...(25)}$$

The eqn. (25) is known as **Karman-Prandtl equation** for the velocity distribution near hydrodynamically ***rough boundaries***.

where, u_f = Shear friction velocity $= \sqrt{\frac{\tau_0}{\rho}}$,

ν = Kinematic viscosity of the fluid,

y = Distance from the pipe wall, and

k = Roughness factor.

Read Examples 11·3 – 11.6 in Ref 4.

7. COMMON EQUATION FOR VELOCITY DISTRIBUTION FOR BOTH SMOOTH AND ROUGH PIPES

Refer Fig. 5. Consider an elementary circular ring of radius r and thickness dr as shown in Fig. 5. The distance of the ring from the pipe wall,

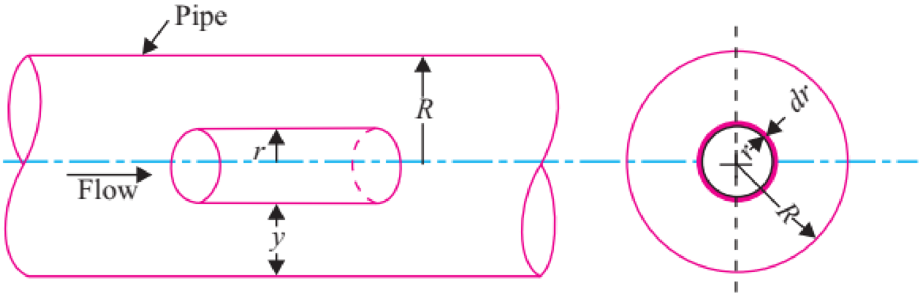


Fig. 5. Average velocity for turbulent flow.

$$y = R - r$$

(where, R = radius of the pipe).

The discharge through the ring is given by:

$$dQ = \text{Area of the ring} \times \text{velocity}$$
$$= 2\pi r \cdot dr \times u$$

$$\therefore \quad \text{Total discharge, } Q = \int dQ = \int_0^R u \times 2\pi r \cdot dr$$

(i)For smooth pipes:

Substituting the value of u in eqn. (24), we get:

$$Q = \int_0^R \left[5.75 \log_{10} \frac{u_f (R - r)}{\nu} + 5.5\right] u_f \times 2\pi r \, . \, dr$$

$$\therefore \quad \text{Average velocity, } \bar{U} = \frac{Q}{A} = \frac{Q}{\pi R^2}$$

$$\bar{U} = \frac{1}{\pi R^2} \int_0^R \left[5.75 \log_{10} \frac{u_f (R - r)}{\nu} + 5.5\right] u_f \times 2\pi r \, . \, dr$$

After integration and simplification, we have:

$$\frac{\bar{U}}{u_f} = 5.75 \log_{10} \frac{u_f R}{\nu} + 1.75 \quad \text{...(27)}$$

(ii)For rough pipes:

Substituting the value of u in eqn. (25), we get:

$$Q = \int_0^R u_f \left[5.75 \log_{10}\left(\frac{R-r}{k}\right) + 8.\right] 2\pi r \cdot dr$$

$$\therefore \quad \text{Average velocity, } \bar{U} = \frac{Q}{\pi R^2} = \frac{1}{\pi R^2}\int_0^R u_f \left[5.75 \log_{10}\left(\frac{R-r}{k}\right) + 8.5\right] 2\pi r \cdot dr$$

After integration and simplication, we have:

$$\frac{\bar{U}}{u_f} = 575 \log_{10}\left(\frac{R}{k}\right) + 4.75 \qquad \text{...(28)}$$

From eqns. (24) and (27) by subtraction, we have:

$$\frac{u}{u_f} - \frac{\bar{U}}{u_f} = \left[5.75 \log_{10}\left(\frac{u_f \cdot y}{\nu}\right) + 5.5\right] - \left[5.75 \log_{10}\frac{u_f R}{\nu} + 1.75\right]$$

And then

$$\frac{u - \bar{U}}{u_f} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75 \qquad \text{...}(i)$$

Similarly, from eqns. (25) and (28), we get:

$$\frac{u}{u_f} - \frac{\bar{U}}{u_f} = \left[5.75 \log_{10}\left(\frac{y}{k}\right) + 8.5\right] - \left[5.75 \log_{10}\left(\frac{R}{k}\right) + 4.75\right]$$

$$\frac{u - \bar{U}}{u_f} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75 \qquad \text{...}(ii)$$

As eqns. (i) and (ii) are identical, the ***velocity distribution in both types of pipes is the same.***

$$\therefore \quad \boxed{\frac{u - \bar{U}}{u_f} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75} \qquad \text{...(29)}$$

The common equation holds good for both types of pipes due to the reason that the *velocity distribution for the turbulent core is identical in both cases.*

Read Examples 11·7 in Ref 4

8. VELOCITY DISTRIBUTION FOR TURBULENT FLOW IN SMOOTH PIPES BY POWER LAW

The eqns. (20), (24) and (25) of velocity distribution for turbulent flow are inconvenient to use, being logarithmic in nature. Nikuradse, through experiments, established the following velocity distribution law (exponential form) for smooth pipes:

$$\boxed{\frac{u}{u_{max}} = \left(\frac{y}{R}\right)^{\frac{1}{n}}} \qquad \text{...(30)}$$

where, exponent $\frac{1}{n}$ depends on Reynolds number (Re) and it *decreases* with the *increasing Re.*

For:
$$Re = 400, \frac{1}{n} = \frac{1}{6}$$
$$Re = 1.1 \times 10^5, \frac{1}{n} = \frac{1}{7}$$
$$Re \geq 2 \times 10^6, \frac{1}{n} = \frac{1}{10}$$

Therefore, for $\frac{1}{n} = \frac{1}{7}$, the velocity distribution law becomes:

$$\frac{u}{u_{max}} = \left(\frac{y}{R}\right)^{1/7} \quad ...(31)$$

Eqn. (31) is known as $\mathbf{\frac{1}{7}}$ **th power law** *of velocity distribution for smooth pipes*.

9. RESISTANCE TO FLOW OF FLUID IN SMOOTH AND ROUGH PIPES

When a fluid flows through a pipe frictional resistance is offered to the motion of the fluid and the loss of head due to friction is expressed by Darcy-Weisbach equation. But the loss of head can be predicted correctly only if the friction coefficient can be evaluated accurately. It can be shown by dimensional analysis that the friction coefficient ***f*** depends upon the Reynolds number and the ratio *k/D* thus,

$$f = \phi\left[\left(\frac{\rho VD}{\mu}\right), \frac{k}{D}\right] \quad ...(32)$$

The co-efficient of friction is given by:

$$f = \frac{16}{Re} \quad \text{...for laminar flow}$$

$$= \frac{0.0791}{(Re)^{1/4}} \quad \text{for turbulent flow in smooth pipes}$$

for $Re \geq 4000$ but $\leq 10^5$

$$= 0.0008 + \frac{0.05525}{(Re)^{0.237}} \quad \text{for } Re \geq 10^5 \text{ but } \leq 4 \times 10^7$$

$$\frac{1}{\sqrt{4f}} = 2\log_{10}(R/k) + 1{\cdot}74 \quad \text{for rough pipes} \quad (\text{where, } Re = \text{Reynolds number})$$

Re/k

Read & resolve examples in ch.11 ref. 04

Solve HW -04