Chapter

2

PRESSURE MEASUREMENT

2.1. Pressure of a liquid
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2.1. PRESSURE OF A LIQUID

When a fluid is contained in a vessel, it exerts force at all points on the sides and bottom and top of the container. The *force per unit area is called* **pressure.**

If, P = The force, and

A = Area on which the force acts; then

intensity of pressure, $p = \dfrac{P}{A}$ ...(2.1)

The pressure of a fluid on a surface will *always act normal to the surface.*

2.2. PRESSURE HEAD OF A LIQUID

A liquid is subjected to pressure due to its own weight, this pressure increases as the depth of the liquid increases.

Consider a vessel containing liquid, as shown in Fig. 2.1. The liquid will exert pressure on all sides and bottom of the vessel. Now, let cylinder be made to stand in the liquid, as shown in the figure.

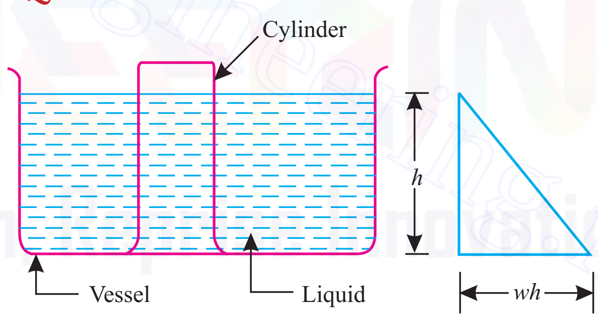


Fig. 2.1. Pressure head.

Let, h = Height of liquid in the cylinder,

A = Area of the cylinder base,

w = Specific weight of the liquid,

and, p = Intensity of pressure.

Now, Total pressure on the base of the cylinder = Weight of liquid in the cylinder

i.e., $p. A. = wAh$

$$p = \frac{wAh}{A} = wh \qquad i.e., \; p = wh \qquad ...(2.2)$$

As $p = wh$, *the intensity of pressure in a liquid due to its depth will vary directly with depth.*

As the pressure at any point in a liquid depends on height of the free surface above that point, it is sometimes convenient to express a liquid pressure by the *height of the free surface* which would cause the pressure, *i.e.*,

$$h = \frac{p}{w} \qquad \text{[from eqn. (2.2)]}$$

The height of the free surface above any point is known as the *static head* at that point. In this case, static head is h.

Hence, the intensity of pressure of a liquid may be expressed in the following two ways:

1. As a force per unit area (*i.e.*, N/mm^2, N/m^2), and
2. As an equivalent static head (*i.e.*, metres, mm or cm of liquid).

Alternatively:

Pressure variation in fluid at rest:

In order to determine the pressure at any point in a fluid at rest "*hydrostatic law*" is used; the law states as follows:

"The rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point."

The proof of the law is as follows.

Refer to Fig. 2.2

Let, p = Intensity of pressure on face LM,

ΔA = Cross-sectional area of the element,

Z = Distance of the fluid element from free surface, and

ΔZ = Height of the fluid element.

The forces acting on the element are:

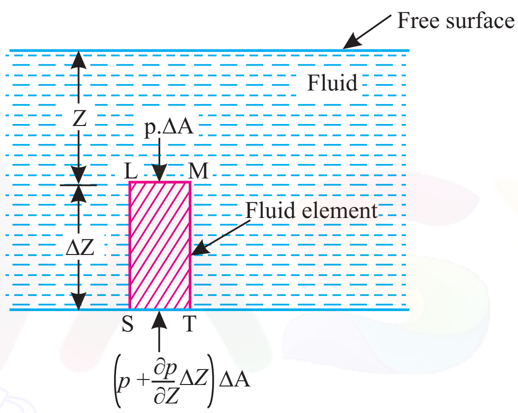


Fig. 2.2. Forces acting on a fluid element.

(*i*) Pressure force on the face $LM = p \times \Delta A$ ...(acting downward)

(*ii*) Pressure force on the face $ST = \left(p + \frac{\partial p}{\partial Z} \times \Delta Z\right) \times \Delta A$ ... (acting upward)

(*iii*) Weight of the fluid element = Weight density × volume

$= w \times (\Delta A \times \Delta Z)$

(*iv*) Pressure forces on surfaces MT and LS ..... are equal and opposite.

For equilibrium of the fluid element, we have:

$$p \times \Delta A - \left[p + \frac{\partial p}{\partial Z} \times \Delta Z\right] \times \Delta A + w \times (\Delta A \times \Delta Z) = 0$$

or, $$p \times \Delta A - p \times \Delta A - \frac{\partial p}{\partial Z} \times \Delta Z \times \Delta A + w \times \Delta A \times \Delta Z = 0$$

or, $$\frac{\partial p}{\partial Z} \Delta Z \times \Delta A + w \times \Delta A \times \Delta Z = 0$$

or, $$\frac{\partial p}{\partial Z} = w \text{ (cancelling } \Delta Z \times \Delta A \text{ from both the sides)}$$

or, $$\frac{\partial p}{\partial Z} = \rho \times g \qquad (\because w = \rho \times g) \qquad \text{...(2.3)}$$

Eqn. (2.3.) states *that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point*. This is "**hydrostatic law**".

On integrating the eqn. (2.3), we get:

$$\int dp = \int \rho g . dZ$$

or, $$p = \rho\, g.\ \ Z\,(= wZ) \qquad ...(2.4)$$

where, p is the pressure above atmospheric pressure.

From eqn. (2.4), we have:

$$Z = \frac{p}{\rho.g}\left(=\frac{p}{w}\right) \qquad ... (2.5)$$

Here Z is known as *pressure head*.

Example 2.1. *Find the pressure at a depth of 15 m below the free surface of water in a reservoir.*

Solution. Depth of water, h = 15 m

Specific weight of water, w = 9.81 kN/m^3

Pressure *p*:

We know that, $p = wh = 9.81 \times 15 = 147.15$ kN/m^2

i.e., p = 147.15 kN/m^2 = **147.15 kPa (Ans.)**

Example 2.2. *Find the height of water column corresponding to a pressure of 54 kN/m^2.*

Solution. Intensity of pressure, p = 54 kN/m^2

Specific weight of water, w = 9.81 kN/m^3

Height of water column, *h*:

Using the relation: $p = wh;\ h = \dfrac{p}{w} = \dfrac{54}{9.81}$ = **5.5 m (Ans.)**

2.3. PASCAL'S LAW

The **Pascal's law** states as follows :

"The intensity of pressure at any point in a liquid at rest, is the same in all directions".

Proof. Let us consider a very small wedge shaped element LMN of a liquid, as shown in Fig. 2.3.

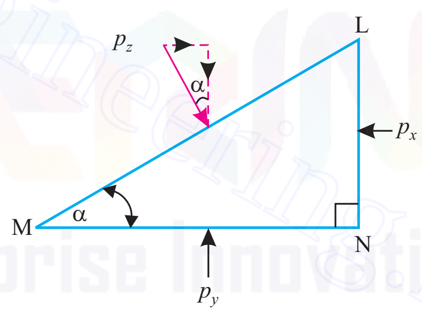


Fig. 2.3. Pressure on a fluid element at rest.

Let, p_x = Intensity of horizontal pressure on the element of liquid,

p_y = Intensity of vertical pressure on the element of liquid,

p_z = Intensity of pressure on the diagonal of the right angled triangular element,

α = Angle of the element of the liquid,

P_x = Total pressure on the vertical side LN of the liquid,

P_y = Total pressure on the horizontal side MN of the liquid, and

P_z = Total pressure on the diagonal LM of the liquid.

Now, $$P_x = p_x \times LN \qquad ...(i)$$

and, $$P_y = p_y \times MN \qquad ...(ii)$$

and, $$P_z = p_z \times LM \qquad ...(iii)$$

As the element of the liquid is at rest, therefore the *sum of horizontal and vertical components of the liquid pressures must be equal to zero*.

Resolving the forces *horizontally*:

$$P_z \sin\alpha = P_x$$

$$p_z \,.\, LM.\, \sin\alpha = p_x \,.\, LN \qquad (\because P_z = p_z.\, LM)$$

But, $LM\,.\,\sin\alpha = LN$ ... From Fig 2.3

$\therefore$ $p_z = p_x$ ...(*iv*)

Resolving the forces *vertically*:

$$P_z\,.\cos\alpha = P_y - W$$

(where, W = weight of the liquid element)

Since the element is very small, neglecting its weight, we have:

$$P_z \cos\alpha = P_y \quad \text{or} \quad p_z\,.\,LM\cos\alpha = p_y\,.MN$$

But, $LM\cos\alpha = MN$ ...From Fig 2.3

$\therefore$ $p_z = p_y$ ...(*v*)

From (*iv*) and (*v*), we get: $p_x = p_y = p_z$,

which is independent of α.

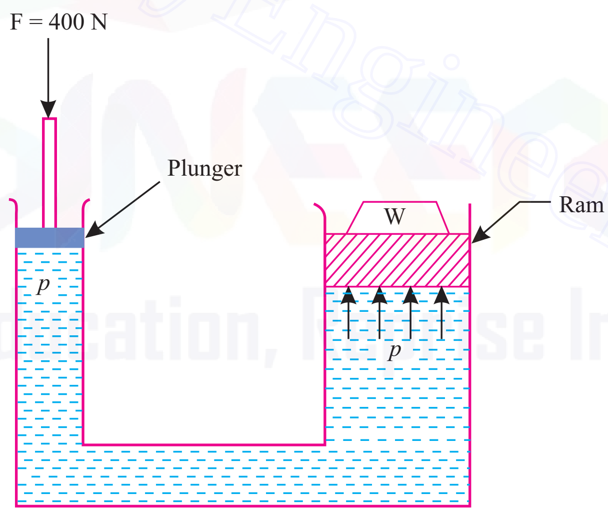
Hence, at *any point in a fluid at rest the intensity of pressure is exerted equally in all directions, which is called* **Pascal's law.**

Example 2.3. *The diameters of ram and plunger of an hydraulic press are 200 mm and 30 mm respectively. Find the weight lifted by the hydraulic press when the force applied at the plunger is 400 N.*

Solution. Diameter of the ram, D = 200 mm = 0.2 m

Diameter of the plunger, d = 30 mm = 0.03 m

Force on the plunger, F = 400 N



Hydraulic press

Fig. 2.4

Load lifted, W:

$$\text{Area of ram, } A = \frac{\pi}{4}D^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$$

$$\text{Area of plunger, } a = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times 0.03^2 = 7.068 \times 10^{-4} \text{ m}^2$$

Intensity of pressure due to plunger,

$$p = \frac{F}{a} = \frac{400}{7.068 \times 10^{-4}} = 5.66 \times 10^5 \text{ N/m}^2$$

Since the intensity of pressure will be equally transmitted (due to Pascal's law), therefore the intensity of pressure at the ram is also

$$= p = 5.66 \times 10^5 \text{ N/m}^2$$

But intensity of pressure at the ram $= \frac{\text{Weight}}{\text{Area of ram}} = \frac{W}{A} = \frac{W}{0.0314} \text{ N/m}^2$

$\therefore \quad \frac{W}{0.0314} = 5.66 \times 10^5$ or $W = 0.0314 \times 5.66 \times 10^5$ N $= 17.77 \times 10^3$ N or **17.77 kN (Ans.)**

Example 2.4. *For the hydraulic jack shown in Fig. 2.5 find the load lifted by the large piston when a force of 400 N is applied on the small piston. Assume the specific weight of the liquid in the jack is 9810 N/m^3.*

Solution. Diameter of small piston, $d = 30$ mm $= 0.03$ m

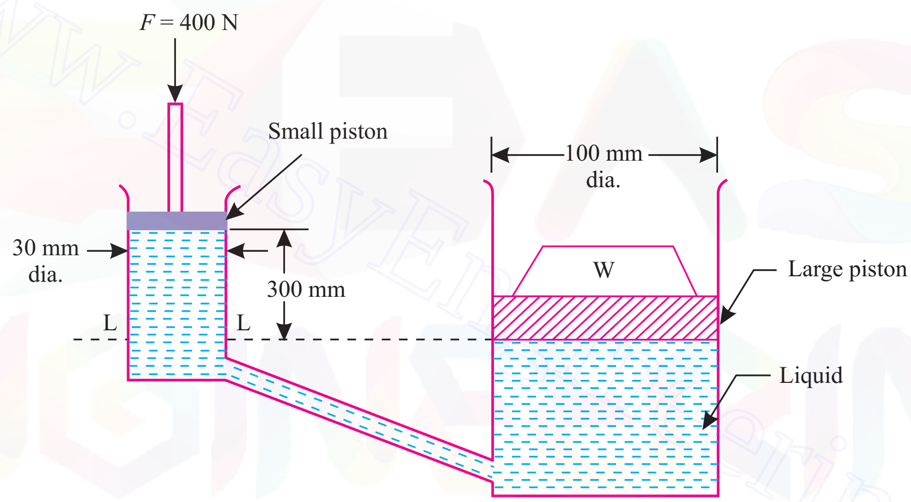


Fig. 2.5

Area of small piston, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.03^2 = 7.068 \times 10^{-4} \text{ m}^2$

Diameter of the large piston, $D = 100$ mm $= 0.1$ m

Area of large piston, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 0.1^2 = 7.854 \times 10^{-3} \text{ m}^2$

Force on small piston, $F = 400$ N

Load lifted, *W*:

Pressure intensity on small piston, $p = \frac{F}{a} = \frac{400}{7.068 \times 10^{-4}} = 5.66 \times 105 \text{ N/m}^2$

Pressure intensity at section LL,

$$p_{LL} = \frac{F}{a} + \text{Pressure intensity due to height of 300 mm of liquid}$$

$$= \frac{F}{a} + wh = 5.66 \times 10^5 + 9810 \times \frac{300}{1000}$$

$$= 5.66 \times 10^5 + 2943 = 5.689 \times 10^5 \text{ N/m}^2$$

Pressure intensity transmitted to the large piston = 5.689×10^5 N/m^2

Force on the large piston = Pressure intensity × area of large piston

$= 5.689 \times 10^5 \times 7.854 \times 10^{-3} = 4468$ N

Hence, *load lifted by the large piston* = **4468 N (Ans.)**

2.4. ABSOLUTE AND GAUGE PRESSURES

Atmospheric pressure:

The atmospheric air exerts a normal pressure upon all surfaces with which it is in contact, and it is known as *atmospheric pressure*. The atmospheric pressure is also known as *'Barometric pressure'*.

The atmospheric pressure at sea level (above absolute zero) is called *'Standard atmospheric pressure'*.

Note. The local atmospheric pressure may be a little lower than these values if the place under question is higher than sea level, and higher values if the place is lower than sea level, due to the corresponding decrease or increase of the column of air standing, respectively.

Gauge pressure:

It is the pressure, measured with the help of pressure measuring instrument, in which the atmospheric pressure is taken as datum. The *atmospheric pressure* on the scale is marked as *zero*.

Gauges record pressure above or below the local atmospheric pressure, since they measure the difference in pressure of the liquid to which they are connected and that of surrounding air. If the pressure of the liquid is *below* the local atmospheric pressure, then the gauge is designated as '*vacuum gauge*' and the recorded value indicates the amount by which the pressure of the liquid is *below* local atmospheric pressure, i.e. *negative pressure*.

(*Vacuum pressure* is defined as the pressure *below the atmospheric pressure*).

Absolute pressure:

It is necessary to establish an absolute pressure scale which is independent of the changes in atmospheric pressure. A pressure of absolute zero can exist only in complete vacuum.

Any pressure measured above the absolute zero of pressure is termed as an '**absolute pressure**'.

A schematic diagram showing the gauge pressure, vacuum pressure and the absolute pressure is given in Fig. 2.6.

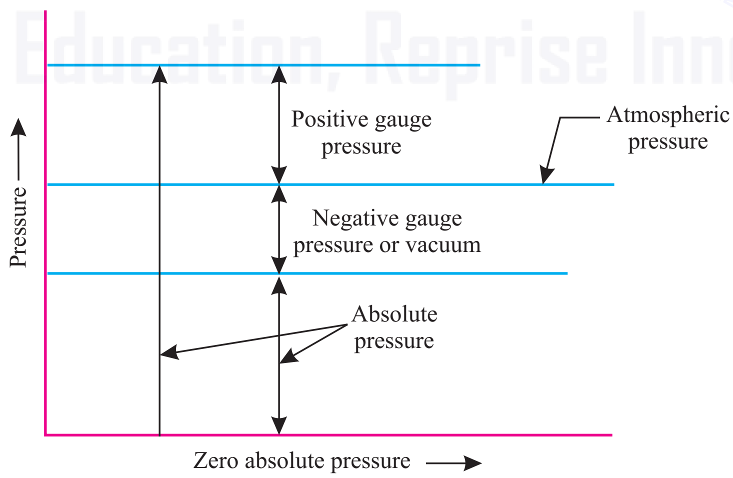


Fig. 2.6. Relationship between pressures.

Mathematically:

1. *Absolute pressure* = Atmospheric pressure + gauge pressure

   i.e., $p_{abs} = p_{atm} + p_{gauge}$

2. *Vacuum pressure* = Atmospheric pressure – absolute pressure

Units for pressure:

The fundamental S.I. unit of pressure is newton per square metre (N/m^2). This is also known as *Pascal*.

Low pressures are often expressed in terms of mm of water or mm of mercury. This is an abbreviated way of saying that the pressure is such that will support a liquid column of stated height.

Note. When the local atmospheric pressure is not given in a problem, it is taken as 100 kN/m^2 or 10 m of water for simplicity of calculations.

Standard atmospheric pressure has the following equivalent values:

101.3 kN/m^2 or 101.3 kPa; 10.3 m of water; 760 mm of mercury; 1013 mb (millibar) ≃ 1 bar ≃100 kPa = 10^5 N/m^2.

Example 2.5. *Given that*:

Barometer reading = 740 mm of mercury;

Specific gravity of mercury = 13.6; Intensity of pressure = 40 kPa.

Express the intensity of pressure in S.I. units, both gauge and absolute.

Solution. Intensity of pressure, $p = 40$ kPa

Gauge pressure:

(i) $p = 40 \text{ kPa} = 40 \text{ kN/m}^2 = 0.4 \times 10^5 \text{ N/m}^2 =$ **0.4 bar (Ans.)**

(1 bar = 10^5 N/m^2)

(ii) $h = \dfrac{p}{w} = \dfrac{0.4 \times 10^5}{9.81 \times 10^3}$ **= 4.077 m of water (Ans.)**

(iii) $h = \dfrac{p}{w} = \dfrac{0.4 \times 10^5}{9.81 \times 10^3 \times 13.6}$ **= 0.299 m of mercury (Ans.)**

$$\left[\begin{array}{l}\text{Where, } w = \text{specific weight;}\\ \text{For water : } w = 9.81 \text{ kN/m}^3\\ \text{For mercury : } w = 9.81 \times 13.6 \text{ kN/m}^3\end{array}\right]$$

Absolute pressure:

Barometer reading (atmospheric pressure)

$= 740$ mm of mercury $= 740 \times 13.6$ mm of water

$= \dfrac{740 \times 13.6}{1000} = 10.6$ m of water

Absolute pressure ($p_{abs.}$) = Atmospheric pressure ($p_{atm.}$) + gauge pressure (p_{gauge}).

∴ p_{abs} = 10.06 + 4.077 = **14.137 m of water (Ans.)**

= 14.137 × (9.81 × 10^3) = **1.38 × 10^5 N/m^2 (Ans.)** ($p = wh$)

= **1.38 bar (Ans.)** (1 bar = 10^5 N/m^2)

$= \dfrac{14.137}{13.6}$ **= 1.039 m of mercury. (Ans.)**

Example 2.6. *Calculate the pressure at a point 5 m below the free water surface in a liquid that has a variable density given by relation:*

$$\rho = (350 + Ay)\ kg/m^3$$

where, $A = 8$ kg/m^4 and y is the distance in metres measured from the free surface.

Solution. As per hydrostatic equation

$$dp = \rho.g.dy = g\,(350 + Ay)dy$$

Integrating both sides, we get:

$$\int dp = \int_0^5 g(350 + Ay)\,dy = g\int_0^5 (350 + 8y)\,dy$$

$$p = g\left|350\,y + 8 \times \frac{y^2}{2}\right|_0^5$$

$$= 9.81\,(350 \times 5 + 8 \times \frac{5^2}{2}) = 18148\ \text{N/m}^2 \simeq \mathbf{18.15\ kN/m^2}$$

(Ans.)

Example 2.7. *On the suction side of a pump a gauge shows a negative pressure of 0.35 bar. Express this pressure in terms of:*

(*i*) *Intensity of pressure, kPa,*
(*ii*) *N/m^2 absolute,*
(*iii*) *Metres of water gauge,*
(*iv*) *Metres of oil (specific gravity 0.82) absolute, and*
(*v*) *Centimetres of mercury gauge,*

Take atmospheric pressure as 76 cm of Hg and relative density of mercury as 13.6.

Solution. *Given*: Reading of the *vacuum* gauge = 0.35 bar

(*i*) Intensity of pressure, kPa:

$$\text{Gauge reading} = 0.35\ \text{bar} = 0.35 \times 10^5\ \text{N/m}^2$$
$$= 0.35 \times 10^5\ \text{Pa} = \mathbf{35\ kPa}\ \textbf{(Ans.)}$$

(*ii*) N/m^2 absolute:

$$\text{Atmospheric pressure, } p_{atm.} = 76\ \text{cm of Hg}$$
$$= (13.6 \times 9810) \times \frac{76}{100} = 101396\ \text{N/m}^2$$

$$\text{Absolute pressure} = \text{Atmospheric pressure} - \textit{Vacuum}\ \text{pressure}$$
$$p_{abs.} = p_{atm} - p_{vac.}$$
$$= 101396 - 35000 = \mathbf{66396\ N/m^2\ absolute}\ \textbf{(Ans.)}$$

(*iii*) Metres of water gauge:

$$p = \rho gh = wh$$

$$\therefore \quad h_{water}\,(\text{gauge}) = \frac{p}{w} = \frac{0.35 \times 10^5}{9810} = \mathbf{3.567\,m}\ (\textit{gauge})\ \textbf{(Ans.)}$$

(*iv*) Metres of oil (sp. gr. = 0.82) absolute:

$$h_{oil}\,(\text{absolute}) = \frac{66396}{0.82 \times 9810} = \mathbf{8.254\ m\ of\ water}\ (\textit{absolute})\ \textbf{(Ans.)}$$

(*v*) Centimetres of mercury gauge:

$$h_{mercury}(\text{gauge}) = \frac{0.35 \times 10^5}{13.6 \times 9810} = 0.2623\ \text{m of mercury}$$
$$= \mathbf{26.236\ cm\ of\ mercury}\ \textbf{(Ans.)}$$

Example 2.8. *The inlet to pump is 10.5 m above the bottom of sump from which it draws water through a suction pipe. If the pressure at the pump inlet is not to fall below 28 kN/m² absolute, work out the minimum depth of water in the tank.*

Assume atmospheric pressure as 100 kPa.

Solution. *Given*: $p_{atm.} = 100$ kPa $= 100$ kN/m²; $p_{abs.} = 28$ kN/m².

Minimum depth of water in the tank:

Let, $p_{vac.}$ = The vacuum (suction) pressure at the pump inlet.

Then,
$$p_{vac.} = p_{atm.} - p_{abs.}$$
$$= (100 - 28) = 72 \text{ kN/m}^2 \quad \text{or} \quad 72000 \text{ N/m}^2$$

Further, let h be the distance between the pump inlet and free water surface in the sump.

Invoking hydrostatic equation, we have:

$$p = wh$$
$$72000 = 9810 \times h$$

or,
$$h = \frac{72000}{9810} = 7.339 \text{ m}$$

∴ Minimum depth of water in the tank

$$= 10.5 - 7.339 = \mathbf{3.161 \text{ m}} \text{ (Ans.)}$$

Example 2.9. *A cylindrical tank of cross-sectional area 600 mm² and 2.6 m height is filled with water upto a height of 1.5 m and remaining with oil of specific gravity 0.78. The vessel is open to atmospheric pressure. Calculate:*

(*i*) *Intensity of pressure at the interface.*

(*ii*) *Absolute and gauge pressures on the base of the tank in terms of water head, oil head and N/m².*

(*iii*) *The net force experienced by the base of the tank.*

Assume atmospheric pressure as 1.0132 bar.

Solution. *Given:* Area of cross-section of the tank, $A = 600$ mm² $= 600 \times 10^{-6}$; Sp.gr. of oil $= 0.78$; $p_{atm.} = 1.0132$ bar.

(*i*) **Intensity of pressure at the interface:**

The pressure intensity at the interface between the oil and water is due to 1.1 m of oil and is given by:

$$p_{interface} = wh$$
$$= (0.78 \times 9810) \times 1.1$$
$$= \mathbf{8417 \text{ N/m}^2} \text{ (Ans.)}$$

(*ii*) **Absolute and gauge pressures on the base of the tank:**

Pressure at the base of the tank

= Pressure at the interface (due to 1.1 m of oil) + pressure due to 1.5 m of water,

i.e.,
$$p_{base\,(gauge)} = 8417 + (9810 \times 1.5)$$
$$= \mathbf{23132 \text{ N/m}^2 \text{ (gauge)}} \text{ (Ans.)}$$
$$= \frac{23132}{9810} = \mathbf{2.358 \text{ m of water (gauge)}} \text{ (Ans.)}$$
$$= \frac{23132}{0.78 \times 9810} = \mathbf{3.023 \text{ m of oil (gauge)}} \text{ (Ans.)}$$

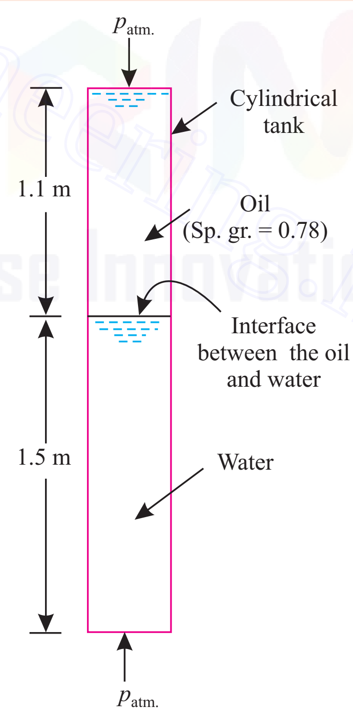


Fig. 2.7

Atmospheric pressure, $p_{atm.}$ $= 1.0132$ bar

$= 1.0132 \times 10^5$ N/m^2

$$= \frac{1.0132 \times 10^5}{9810} = 10.328 \text{ m of water}$$

$$= \frac{1.0132 \times 10^5}{0.78 \times 9810} = 13.241 \text{ m of oil}$$

Absolute pressure = Atmospheric pressure + gauge pressure

p_{base} (absolute) $= 10.328 + 2.358 =$ **12.686 m of water (Ans.)**

$= 13.241 + 3.023 =$ **16.264 m of oil (Ans.)**

$= 101320 + 23132 =$ **124452 N/m^2 (Ans.)**

(*iii*) The net force experienced by the base of the tank:

$F (= P) = p_{base}$(gauge) × cross-sectional area

$= 23132 \times 600 \times 10^{-6} =$ **13.879 N (Ans.)**

Example 2.10. (*a*) *What is hydrostatic paradox?*

(*b*) *A cylinder of 0.25 m diameter and 1.2 m height is fixed centrally on the top of a large cylinder of 0.9 m diameter and 0.8 m height. Both the cylinders are filled with water. Calculate:*

(*i*) *Total pressure at the bottom of the bigger cylinder, and*

(*ii*) *Weight of total volume of water.*

What is hydrostatic paradox between the two results and how this difference can be reconciled?

Solution. (*a*) Hydrostatic paradox:

Fig. 2.8 shows three vessels 1, 2 and 3 having the same area A at the bottom and each filled with a liquid upto the same height h.

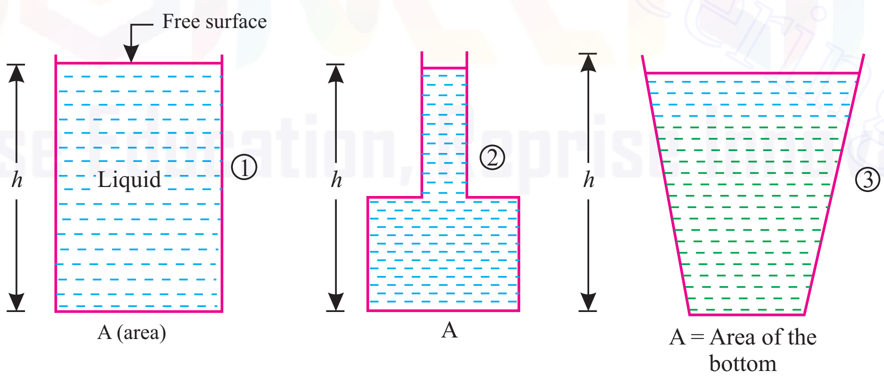


Fig. 2.8. Hydrostatic paradox.

According to the hydrostatic equation, $p = wh$; the intensity of pressure (p) *depends only on the height of the column and not at all upon the size of the column.* Thus, in all these vessels of different shapes and sizes, the same intensity of pressure would be exerted on the bottom of each of these vessels. Since each of the vessels has the same area A at the bottom, the pressure force $P = p \times A$ on the base of each vessel would be same. This is independent of the fact that the weight of liquid in each vessel is different. *This situation is referred to as* **hydrostatic paradox.**

(*b*) Area at the bottom:

$$A = \frac{\pi}{4} \times (0.9)^2 = 0.6362 \text{ m}^2$$

Intensity of pressure at the bottom

$$p = wh = 9810 \times (1.2 + 0.8)$$
$$= 19620 \text{ N/m}^2$$

Total pressure force at the bottom

$$P = p \times A = 19620 \times 0.6362 = 12482 \text{ N}$$

Weight of total volume of water contained in the cylinders,

$$W = w \times \text{volume of water}$$
$$= 9810 \left[\frac{\pi}{4} \times 0.9^2 \times 0.8 + \frac{\pi}{4} \times 0.25^2 \times 1.2 \right]$$
$$= 5571 \text{ N}$$

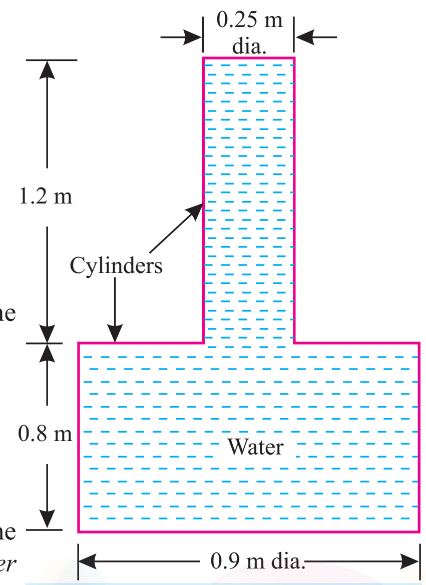


Fig. 2.9

From the above calculations it may be observed that the *total pressure force at the bottom of the cylinder is greater than the weight of total volume of water (W) contained in the cylinders. This is* **hydrostatic paradox.**

The following is the *explanation of the hydrostatic paradox*: Refer to Fig. 2.9.

Total pressure force on the bottom of bigger tank = 12482 N (downward). A reaction at the roof of the lower tank is caused by the upward force which equals,

$$wAh = 9810 \times \frac{\pi}{4}(0.9^2 - 0.25^2) \times 1.2 = 6911 \text{ N (upward)}$$

The distance h corresponding to depth of water in the cylinder fixed centrally on the top of larger cylinder.

Net downward force exerted by water = 12482 – 6911 = 5571 N and it equals the weight of water in the two cylinder.

2.5. MEASUREMENT OF PRESSURE

The pressure of a fluid may be measured by the following devices:

1. Manometers:

Manometers *are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of liquid.* These are classified as follows:

(*a*) Simple manometers:

(*i*) Piezometer, (*ii*) U-tube manometer, and (*iii*) Single column manometer.

(*b*) Differential manometers.

2. Mechanical gauges:

These are the devices in which the pressure is measured by balancing the fluid column by spring (elastic element) or dead weight. Generally these gauges are used for measuring *high pressure* and where *high precision is not required.* Some commonly used mechanical gauges are:

(*i*) Bourdon tube pressure gauge, (*ii*) Diaphragm pressure gauge,

(*iii*) Bellow pressure gauge, and (*iv*) Dead-weight pressure gauge.

2.5.1 Manometers

2.5.1.1. Simple manometers

A "simple manometer" is one which consists of a glass tube whose one end is connected to a point where pressure is to be measured and the other end remains open to atmosphere.

Common types of simple manometers are discussed below:

1. Piezometer:

A piezometer is the simplest form of manometer which can be used for measuring *moderate pressures* of liquids. It consists of a glass tube (Fig 2.10) inserted in the wall of a vessel or of a pipe, containing liquid whose pressure is to be measured. The tube extends vertically upward to such a height that liquid can freely rise in it without overflowing. The pressure at any point in the liquid is indicated by the height of the liquid in the tube above that point, which can be read on the scale attached to it. Thus if w is the specific weight of the liquid, then the pressure at point $A(p)$ is given by:

$$p = wh$$

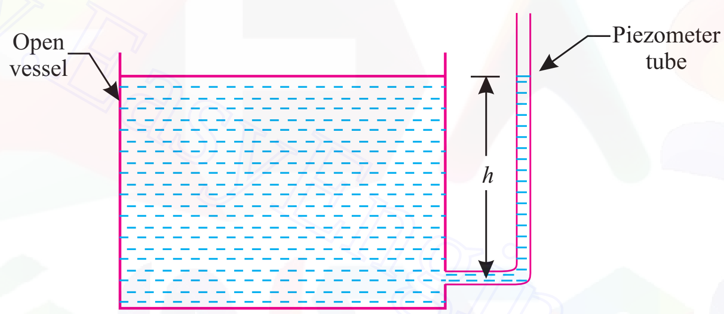


Fig. 2.10. (*a*) Piezometer tube fitted to open vessel.

Piezometers measure *gauge pressure only* (at the surface of the liquid), since the surface of the liquid in the tube is subjected to atmospheric pressure. A piezometer tube is *not suitable* for measuring *negative pressure*; as in such a case the air will enter in pipe through the tube.

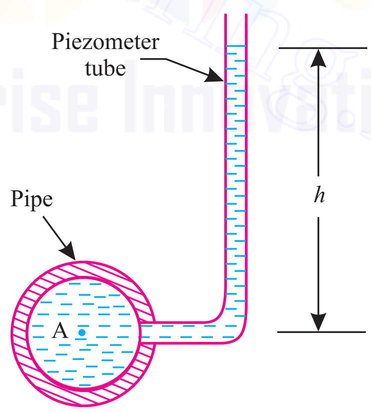


Fig. 2.10. (*b*) Piezometer tube fitted to a closed pipe.

2. U-tube manometer:

Piezometers cannot be employed when large pressures in the *lighter liquids* are to be measured, since this would require *very long tubes*, which cannot be handled conveniently. Furthermore gas pressures cannot be measured by the piezometers because a *gas forms no free atmospheric surface*. These limitations can be overcome by the use of U-tube manometers.

A U-tube manometer consists of a glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in Fig. 2.11.

(*i*) For positive pressure:

Refer to Fig. 2.11 (*a*).

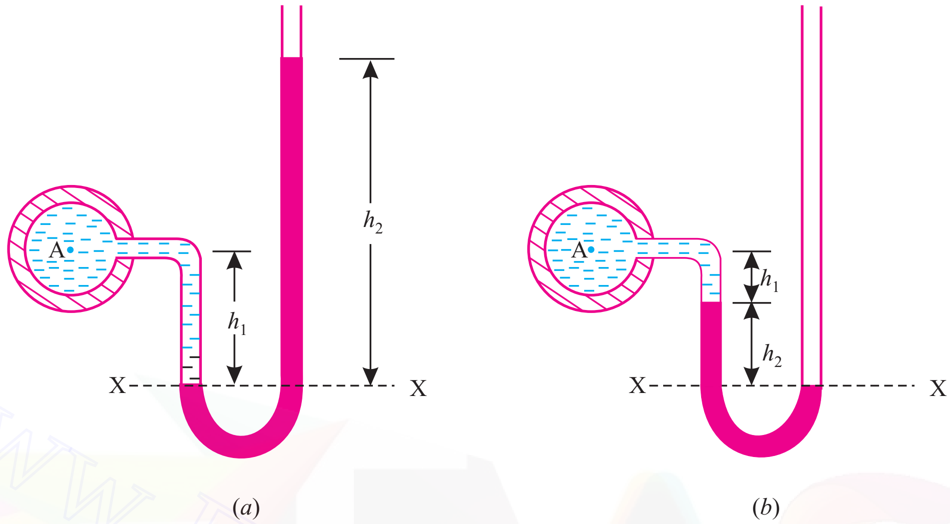


Fig. 2.11. U-tube manometer.

Let, *A* be the point at which pressure is to be measured. *X–X* is the datum line as shown in Fig. 2.11 (*a*).

Let, h_1 = Height of the light liquid in the left limb above the datum line,

h_2 = Height of the heavy liquid in the right limb above the datum line,

h = Pressure in pipe, expressed in terms of head,

S_1 = Specific gravity of the light liquid, and

S_2 = Specific gravity of the heavy liquid.

The pressures in the left limb and right limb above the datum line *X–X* are equal (*as the pressures at two points at the same level in a continuous homogeneous liquid are equal*).

Pressure head above *X–X* in the *left limb* = $h + h_1 S_1$

Pressure head above *X–X* in the *right limb* = $h_2 S_2$

Equating these two pressures, we get:

$$h + h_1 S_1 = h_2 S_2 \quad \text{or} \quad h = h_2 S_2 - h_1 S_1 \qquad ...(2.6)$$

(*ii*) For negative pressure:

Refer to Fig. 2.11 (*b*).

Pressure head above *X–X* in the *left limb* = $h + h_1 S_1 + h_2 S_2$

Pressure head above *X–X* in the *right limb* = 0.

Equating these two pressures, we get:

$$h + h_1 S_1 + h_2 S_2 = 0 \quad \text{or} \quad h = -(h_1 S_1 + h_2 S_2) \qquad ...(2.7)$$

Example 2.11. *In a pipeline water is flowing. A manometer is used to measure the pressure drop for flow through the pipe. The difference in level was found to be 20 cm. If the manometric fluid is CCl_4, find the pressure drop in S.I units (density of CCl_4 = 1.596 g/cm^3). If the manometric fluid is changed to mercury (ρ = 13.6gm/cm^3) what will be the difference in level?*

(UPTU)

Solution. *Given:* h_{CCl_4} = 20 cm = 0.2 m; ρ_{CCl_4} = 1.596 g/cm^3

$$= 1.596 \times 10^3 \text{ kg/m}^3$$

$$\rho_{Hg} = 13.6 \times 10^3 \text{ kg/m}^3$$

$$\text{Pressure drop, } \Delta p = \rho_{CCl_4} g h_{CCl_4}$$

$$= 1.596 \times 10^3 \times 9.81 \times 0.2 \text{ N/m}^2$$

$$= 3131.3 \text{ N/m}^2 \text{ or Pa} = \mathbf{3.131 \text{ kPa}} \text{ (Ans.)}$$

The difference in level with mercury,

$$h_{Hg} = h_{CCl_4} \times \frac{\rho_{CCl_4}}{\rho_{Hg}} = 0.20 \times \frac{1.596 \times 10^3}{13.6 \times 10^3}$$

$$= 0.02347 \text{ m or } \mathbf{2.347 \text{ cm}} \text{ (Ans.)}$$

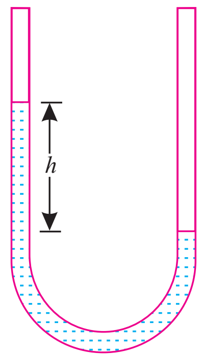


Fig. 2.12

Example 2.12. *A U-tube manometer is used to measure the pressure of oil of specific gravity 0.85 flowing in a pipe line. Its left end is connected to the pipe and the right-limb is open to the atmosphere. The centre of the pipe is 100 mm below the level of mercury (specific gravity = 13.6) in the right limb. If the difference of mercury level in the two limbs is 160 mm, determine the absolute pressure of the oil in the pipe.*

Solution. Specific gravity of oil, $S_1 = 0.85$

Specific gravity of mercury, $S_2 = 13.6$

Height of the oil in the left limb,

$$h_1 = 160 - 100 = 60 \text{ mm} = 0.06 \text{ m}$$

Difference of mercury level,

$$h_2 = 160 \text{ mm} = 0.16 \text{ m.}$$

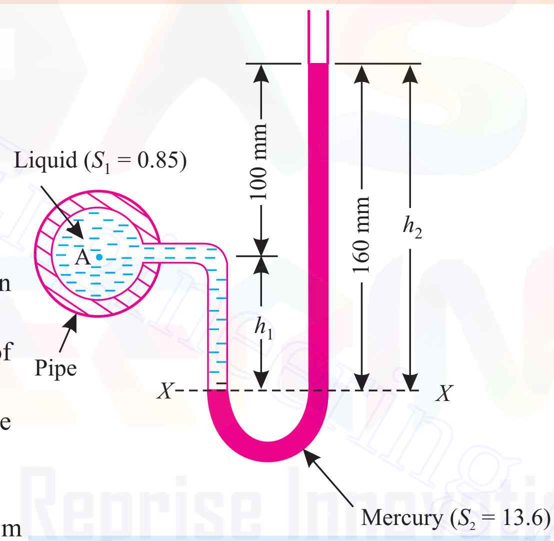


Fig. 2.13

Absolute pressure of oil:

Let, h_1 = Gauge pressure in the pipe in terms of head of water, and

p = Gauge pressure in terms of kN/m^2.

Equating the pressure heads above the datum line X–X, we get:

$$h + h_1 S_1 = h_2 S_2$$

or, $h + 0.06 \times 0.85 = 0.16 \times 13.6 = 2.125$ m

The pressure p is given by:

$$p = wh$$

$$= 9.81 \times 2.125 \text{ kN/m}^2$$

$$= 20.84 \text{ kPa} \quad (\because w = 9.81 \text{ kN/m}^3 \text{ in S.I. units})$$

Absolute pressure of oil in the tube,

$$p_{abs.} = p_{atm.} + p_{gauge}$$

$$= 100 + 20.84 = \mathbf{120.84 \text{ kPa}} \text{ (Ans.)}$$

Example 2.13. *U-tube manometer containing mercury was used to find the negative pressure in the pipe, containing water. The right limb was open to the atmosphere. Find the vacuum pressure in the pipe, if the difference of mercury level in the two limbs was 100 mm and height of water in the left limb from the centre of the pipe was found to be 40 mm below.*

Solution. Specific gravity of water, $S_1 = 1$

Specific gravity of mercury, $S_2 = 13.6$

Height of water in the left limb,

h_1 = 40 mm = 0.04 m

Height of mercury in the left limb,

h_2 = 100 mm = 0.1 m

Let, h = Pressure in the pipe in terms of head of water (*below* the atmosphere).

Equating the pressure heads above the datum line *X–X*, we get:

$$h + h_1S_1 + h_2S_2 = 0$$

or,

$$h = -(h_1S_1 + h_2S_2)$$
$$= -(0.04 \times 1 + 0.1 \times 13.6)$$
$$= -1.4 \text{ m of water}$$

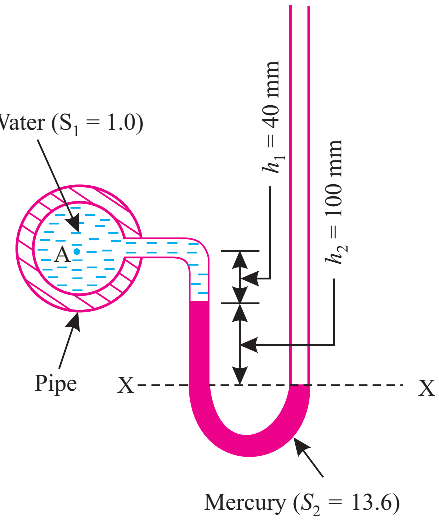


Fig. 2.14

Pressure p is given by:

$$p = wh$$
$$= 9.81 \times (-1.4) \text{ kN/m}^2$$
$$= -13.73 \text{ kPa}$$
$$= \mathbf{13.73 \text{ kPa (vacuum)}} \text{ (Ans.)}$$

Example 2.14. *A simple U-tube manometer is installed across an orificemeter. The manometer is filled with mercury (sp. gravity = 13.6) and the liquid above the mercury is carbon tetrachloride (sp. gravity = 1.6). The manometer reads 200 mm. What is the pressure difference over the manometer in newtons per square metre.*

Solution. Specific gravity of heavier liquid, S_{hl} = 13.6

Specific gravity of lighter liquid, S_{ll} = 1.6

Reading of the manometer, y = 200 mm

Pressure difference over the manometer : p

Differential head,

$$h = y\left[\frac{S_{hl}}{S_{ll}} - 1\right]$$

$$200\left[\frac{13.6}{1.6} - 1\right] = 1500 \text{ mm of carbon tetrachloride}$$

Pressure difference over manometer,

$$p = wh = (1.6 \times 9810) \times \left(\frac{1500}{1000}\right)$$

or $\quad$ $\mathbf{p = 23544 \text{ N/m}^2}$ **(Ans.)**

Example 2.15. *In Fig. 2.15 is shown a conical vessel having its outlet at L to which U-tube manometer is connected. The reading of the manometer given in figure shows when the vessel is empty. Find the reading of the manometer when the vessel is completely filled with water.*

Solution. When vessel is empty: (Refer to Fig. 2.15)

Let, h_1 = Height of water above *X–X*

Specific gravity of water, S_1 = 1.0

Specific gravity of mercury, S_2 = 13.6

Equating the pressure heads about the datum line *X–X*, we get:

Equating the pressure heads above $X–X$, we get:

$$h_L = 0.9\ h_M \qquad ...(i)$$

When the pressure on the surface in bulb M is increased by 20 mm of water, let the separation level fall by an amount equal to y. Then Z–Z is the *new separation level*.

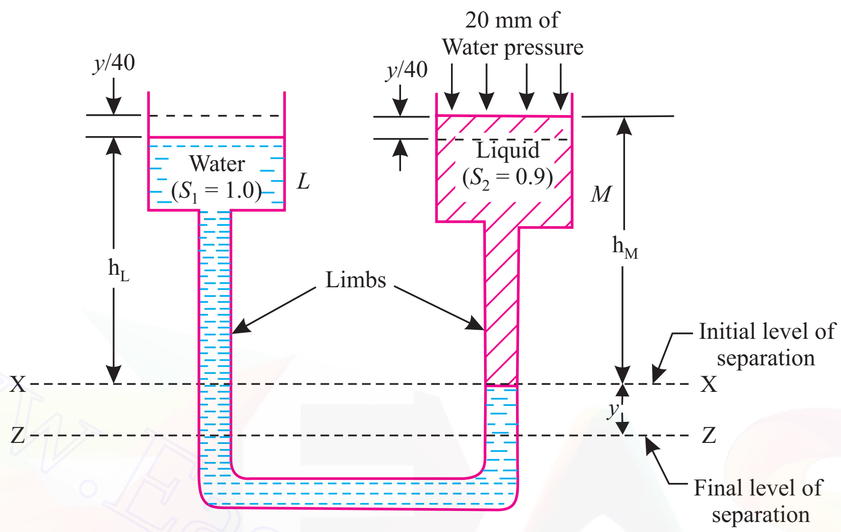


Fig. 2.17

Now, $A \times$ fall in separation level in bulb $M = a \times$ fall in separation level in the limb (y).

Fall in separation level in bulb $M = \dfrac{a \times y}{A} = \dfrac{30 \times y}{1200} = \dfrac{y}{40}$

Also, fall in separation level in bulb M = Rise in surface level of $L = \dfrac{y}{40}$

Considering pressure heads above $Z–Z$, we have:

Pressure head in the left limb $= \left[\dfrac{y}{40} + h_L + y\right]$

Pressure head in the right limb $= \left(h_M + y - \dfrac{y}{40}\right) \times 0.9 + 20$

Equating the pressure heads, we get:

$$\left[\frac{y}{40} + h_L + y\right] = \left[h_M + y - \frac{y}{40}\right] \times 0.9 + 20$$

or, $\dfrac{y}{40} + 0.9\ h_M + y = 0.9\ h_M + \dfrac{39\,y}{40} \times 0.9 + 20 \qquad (\because\ h_L = 0.9\ h_M)$

or, $\dfrac{41y}{40} = \dfrac{39\,y}{40} \times 0.9 + 20$ or $\dfrac{41y}{40} - \dfrac{39\,y}{40} \times 0.9 = 20$

or, $1.025y - 0.877y = 20$ or $y = 135.1$ mm

Hence, *displacement of the surface of separation* **= 135.1 mm (Ans.)**

3. Single column manometer (micro-manometer):

The U-tube manometer described above usually requires reading of fluid levels at two or more points since a change in pressure causes a rise of liquid in one limb of the manometer and a drop in the other. This difficulty is however overcome by using single column manometers. A single column manometer is a modified form of a U-tube manometer in which a shallow reservoir having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one limb of the manometer, as shown in Fig. 2.18. *For any variation in pressure, the change in the liquid level in the reservoir will be so small that it may be neglected, and the pressure is indicated by the height of the liquid in the other limb*. As such only *one reading in the narrow limb of the manometer need be taken for all pressure measurements*. The narrow limb may be vertical or inclined. Thus there are two types of single column manometer as given below:

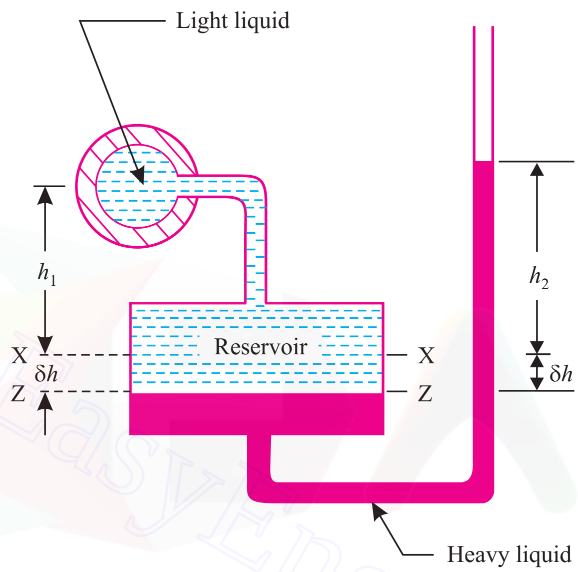


Fig. 2.18. Vertical single column manometer.

(*a*) Vertical single column manometer, and
(*b*) Inclined single column manometer.

(*a*) Vertical single column manometer:

Refer to Fig. 2.18

Let *X–X* be the datum line in the reservoir when the single column manometer is not connected to the pipe. Now consider that the manometer is connected to a pipe containing light liquid under a very high pressure. The pressure in the pipe will force the light liquid to push the heavy liquid in the reservoir downwards. As the area of the reservoir is very large, the fall of the heavy liquid level will be very small. This downward movement of the heavy liquid, in the reservoir, will cause a considerable rise of the heavy liquid in the right limb.

Let, h_1 = Height of the centre of the pipe above X–X,
h_2 = Rise of heavy liquid (after experiment) in the right limb,
δh = Fall of heavy liquid level in the reservoir,
h = Pressure in the pipe, expressed in terms of head of water,
A = Cross-sectional area of the reservoir,
a = Cross-sectional area of the tube (right limb),
S_1 = Specific gravity of light liquid in pipe, and
S_2 = Specific gravity of the heavy liquid.

We know that fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

Thus, $A \times \delta h = a \times h_2$ or $\delta h = \dfrac{a \times h_2}{A}$ ...(*i*)

Let us now consider pressure heads above the datum line *Z–Z* as shown in Fig. 2.18.

Pressure head in the left limb $= h + (h_1 + \delta h)S_1$

Pressure head in the right limb $= (h_2 + \delta h)S_2$

Equating the pressure heads, we get:

$$h + (h_1 + \delta h)S_1 = (h_2 + \delta h)S_2 \quad \text{or} \quad h = (h_2 + \delta h)\,S_2 - (h_1 + \delta h)\,S_1$$
$$= \delta h\,(S_2 - S_1) + h_2S_2 - h_1S_1$$

But, $\delta h = \dfrac{a \times h_2}{A}$ ...[Eqn. (*i*)]

$$h = \frac{a \times h_2}{A}(S_2 - S_1) + h_2S_2 - h_1S_1 \qquad \text{...(2.8)}$$

When the area A is very large as compared to *a*, then the ratio $\dfrac{a}{A}$ becomes very small, and thus is neglected. Then the above equation becomes

$$h = h_2S_2 - h_1S_1 \qquad \text{...(2.9)}$$

(*b*) Inclined single column manometer:

This type of manometer is useful for the measurement of *small pressures and is more sensitive than the vertical tube type*. Due to inclination the distance moved by the heavy liquid in the right limb is *more*.

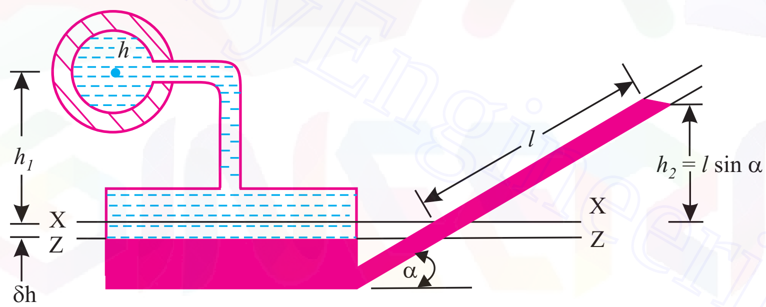


Fig. 2.19. Inclined single column manometer.

Let, l = Length of the heavy liquid moved in right limb,

α = Inclination of right limb horizontal, and

h_2 = Vertical rise of liquid in right limb from X–$X = l \sin \alpha$.

Putting the value of h_2 in eqn. 2.9, we get:

$$h = l \sin \alpha \times S_2 - h_1\,S_1 \qquad \text{...(2.10)}$$

Example. 2.17. *Fig. 2.20 shows a single column manometer connected to a pipe containing liquid of specific gravity 0.8. The ratio of area of the reservoir to that of the limb is 100. Find the pressure in the pipe.*

Take specific gravity of mercury as 13.6.

Solution. Specific gravity of liquid in the pipe, $S_1 = 0.8$.

Specific gravity of mercury, $S_2 = 13.6$

$$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a} = 100$$

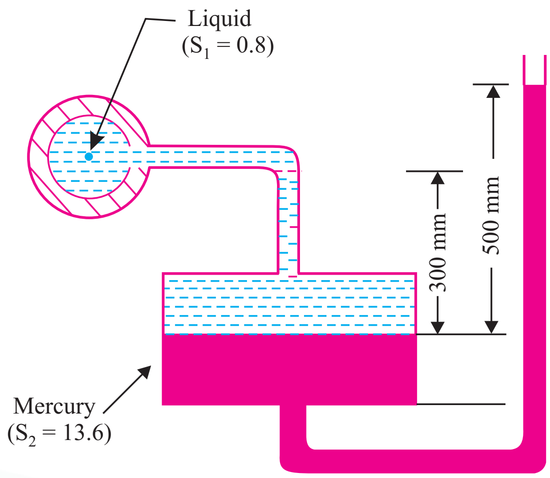


Fig. 2.20

Height of the liquid in the left limb,

$$h_1 = 300 \text{ mm}$$

Height of mercury in the right limb,

$$h_2 = 500 \text{ mm}$$

Let, h = Pressure head in the pipe.

Using the relation:

$$h = \frac{a}{A} h_2 (S_2 - S_1) + h_2 S_2 - h_1 S_1$$

or,
$$h = \frac{1}{100} \times 500\,(13.6 - 0.8) + 500 \times 13.6 - 300 \times 0.8 \text{ mm of water}$$
$$= 6624 \text{ mm of water} \quad \text{or} \quad 6.624 \text{ m of water}$$

Pressure,
$$p = wh = 9.81 \times 6.624$$
$$= 64.98 \text{ kN/m}^2 \text{ or } 64.98 \text{ kPa}$$

i.e., $\quad$ **p = 64.98 kPa (Ans.)**

Example 2.18. *A manometer consists of an inclined glass tube which communicates with a metal cylinder standing upright; liquid fills the apparatus to a fixed zero mark on the tube when both the cylinder and the tube are open to atmosphere. The upper end of the cylinder is then connected to a gas supply at a pressure p and manometric liquid rises through a distance l in the tube. Establish the relation:*

$$h = Sl \left[\sin \alpha + \left(\frac{d}{D}\right)^2 \right]$$

for the pressure head h of water column in terms of inclination α of the tube, specific gravity S of the liquid, and ratio of diameter d of the tube to the diameter D of the cylinder.

Also determine the value of $\left(\frac{D}{d}\right)$ *so that the error due to disregarding the change in level in the cylinder will not exceed 0.1 percent when α = 25°.*

Solution. Vertical rise in the tube = $l \sin \alpha$

$$\text{Fall of liquid level in the cylinder} = l \times \frac{a}{A} = l \times \left(\frac{d}{D}\right)^2$$