Thermodynamics

2^{nd} Semester, Chapter 4 -PART 2

Reciprocating Compressor Including Clearance
Isothermal Efficiency & Volumetric Efficiency

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Example 3

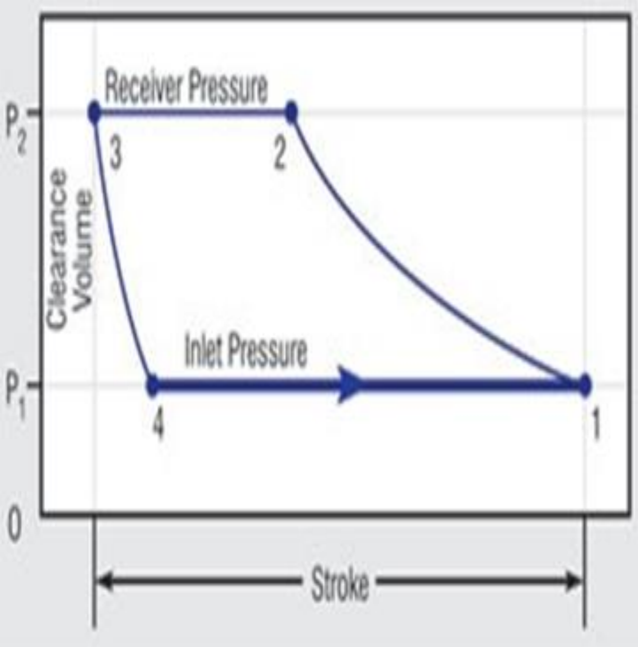
Using the data of the compressor of example 1 . Calculate the isothermal efficiency of the compressor .

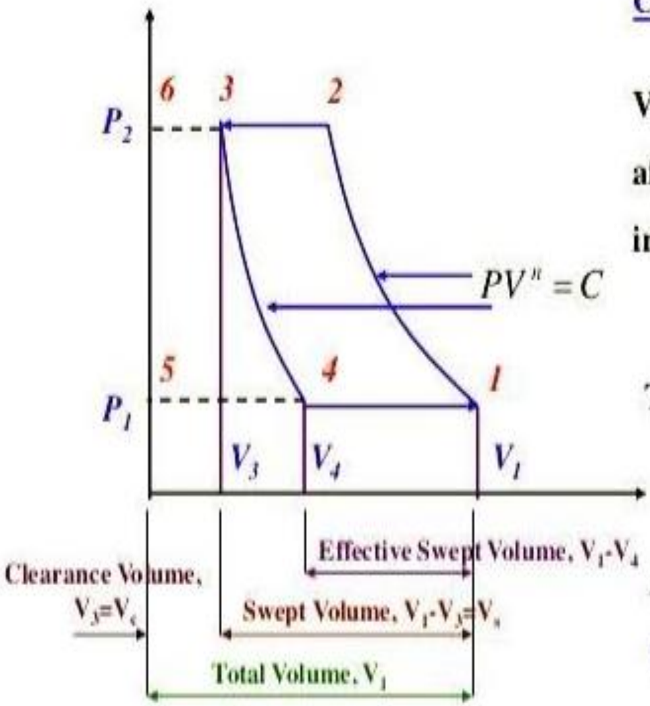
$$W_{isothermal} = m\,R\,T\,Ln\left(\frac{P_2}{P_1}\right) = \left(\frac{1.226}{60}\right) x\ 0.287\ x\ 288\ Ln\left(\frac{7}{1.013}\right) = 3.2666\ kW$$

From example 1, indicated work = 4.23 kW

$$\eta_{isothermal} = \frac{W_{isothermal}}{W_{net\ (\ indicated)}} = \frac{3.2666}{4.23} = 0.7722 = 77.22\ \%$$

Reciprocating Compressor Including Clearance





Clearance Volume :

Volume that remains inside the cylinder after the piston reaches the end of its inward stroke.

Thus, *Effective Stroke Volume* $= V_1 - V_4$ (Induced Volume)

Actual Work = W_{act} = Area 1-2-3-4

W_{act} = Area (5-1-2-6) – Area (5-4-3-6)

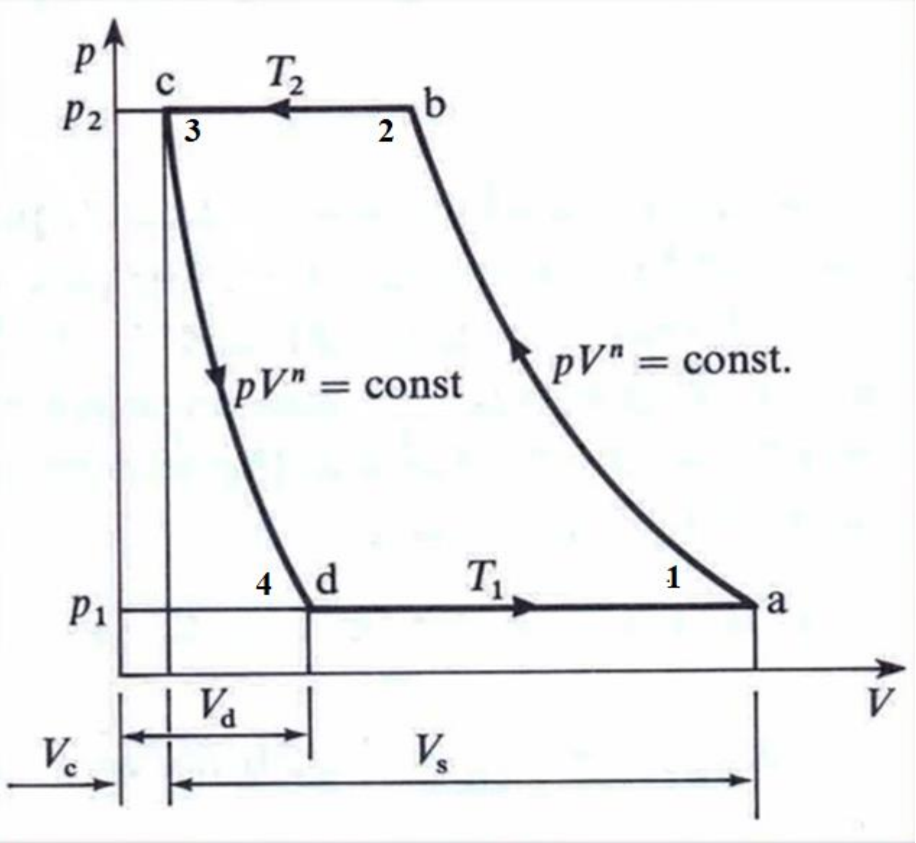
In engineering analysis, the performance was achieved under idealized circumstances for the cycle. The net work of the cycle equal the sum work of all processes. i. e.

$$W_{net\ indicated} = \Sigma W = W_{ab} + W_{bc} + W_{cd} + W_{da}$$

Process ab is polytropic compression, thus perfect gas (air) compress according to the law :

$$PV^n = \text{Constant}, \qquad \text{then } P = \frac{C}{V^n}$$

Then the work found by :



$$W_{ab} = \int_a^b P\, dV$$

$$W_{ab} = \int_a^b \frac{C}{V^n}\, dV = C \int_a^b \frac{dV}{V^n}$$

$$W_{ab} = C \left[\frac{V^{1-n}}{1-n} \right]_a^2 = PV^n \left[\frac{V^{1-n}}{1-n} \right]_a^2$$

$$W_{ab} = \frac{P_2V_b - P_1V_a}{1-n} = -\frac{P_2V_b - P_1V_a}{n-1}$$

By equations PV^n = Constant, and $\frac{PV}{T} = C$

Can found $T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{n-1}$

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{(n-1)/n}$$

Process da is isobaric expansion, the perfect gas (air) expand according to the law of work :

$$\mathbf{Wda = P1\ (\ Va - Vd\ )}$$

$$\mathbf{Wnet\ indicated = \Sigma W = Wab + Wbc + Wcd + Wda}$$

$$\mathbf{Wnet\ (\ indicated\ ) = -\frac{P2Vb - P1Va}{n-1} + P2\ (\ Vc - Vb\ ) + \frac{P2Vc - P1Vd}{n-1} + P1\ (\ Va - Vd\ )}$$

$$\mathbf{Wnet\ (\ indicated\ ) = -\frac{P2Vb - P1Va}{n-1} + P2\ Vc - P2\ Vb + \frac{P2Vc - P1Vd}{n-1} + P1\ Va - P1\ Vd}$$

$$\mathbf{Wnet\ (\ indicated\ ) = -\frac{P2Vb - P1Va}{n-1} - (\ P2\ Vb - P1\ Va\ ) + \frac{P2Vc - P1Vd}{n-1} + (\ P2\ Vc - P1\ Vd\ )}$$

$$\mathbf{Wnet\ (\ indicated\ ) = -\ (\ P2\ Vb - P1\ Va\ )\ \{\frac{1}{n-1} + 1\} + (\ P2\ Vc - P1\ Vd\ )\ \{\frac{1}{n-1} + 1\}}$$

$$\mathbf{Wnet\ (\ indicated\ ) = -\frac{n}{n-1}\ (\ P2\ Vb - P1\ Va\ ) + \frac{n}{n-1}\ (\ P2\ Vc - P1\ Vd\ )}$$

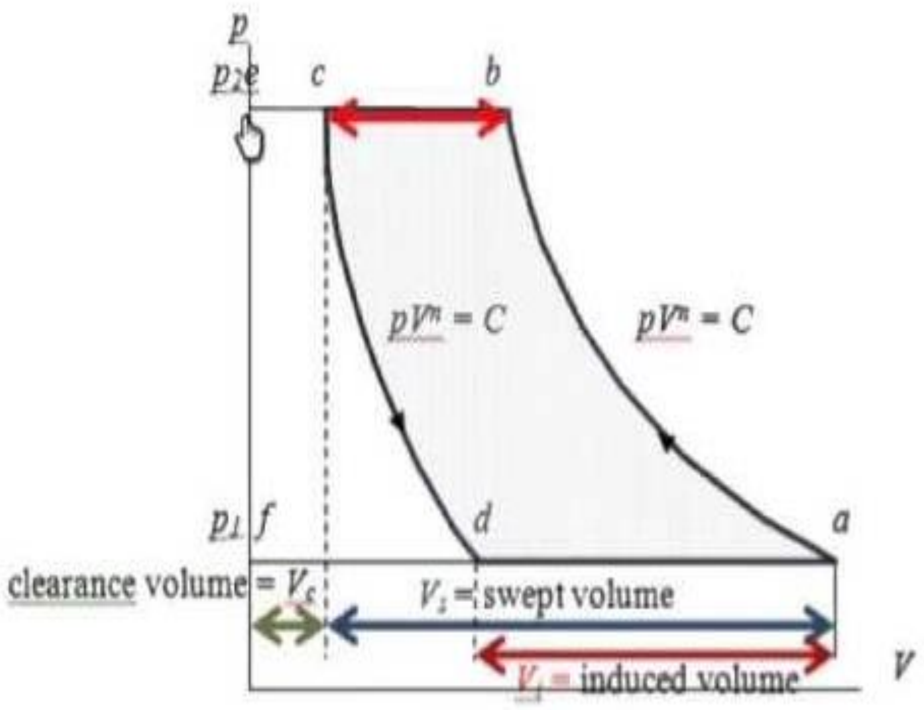
$$\mathbf{Wnet\ (\ indicated\ ) = -\ \{\frac{n}{n-1}\ (\ P2\ Vb - P1\ Va\ ) - \frac{n}{n-1}\ (\ P2\ Vc - P1\ Vd\ )\}}$$

For perfect gas (air)   PV = mRT , and the work of compressor is done one cycle (– ve), thus the absolute value of the compressor work is :

$$\mathbf{Wnet\ (\ indicated\ ) = \frac{n}{n-1}\ ma\ R\ (\ T2 - T1\ ) - \frac{n}{n-1}\ md\ R\ (\ T2 - T1\ )}$$

Where the mass flow rate at $\mathbf{m_a = m_b}$ and

$$\mathbf{m_c = m_d}$$



$$\mathbf{W_{net\ (indicated)} = \frac{n}{n-1} R\ (T_2 - T_1)\ (m_a - m_d)}$$

$$\mathbf{W_{net\ (indicated)} = \frac{n}{n-1} m_{induced}\ R\ T_1 \left(\frac{T_2}{T_1} - 1\right)}$$

Where the mass induced per unit time to the compressor is

$$\mathbf{m_{induced} = (m_a - m_d)}$$

$$\mathbf{W_{net\ (indicated)} = \frac{n}{n-1} P_1\ V_{induced} \left(\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1 \right)}$$

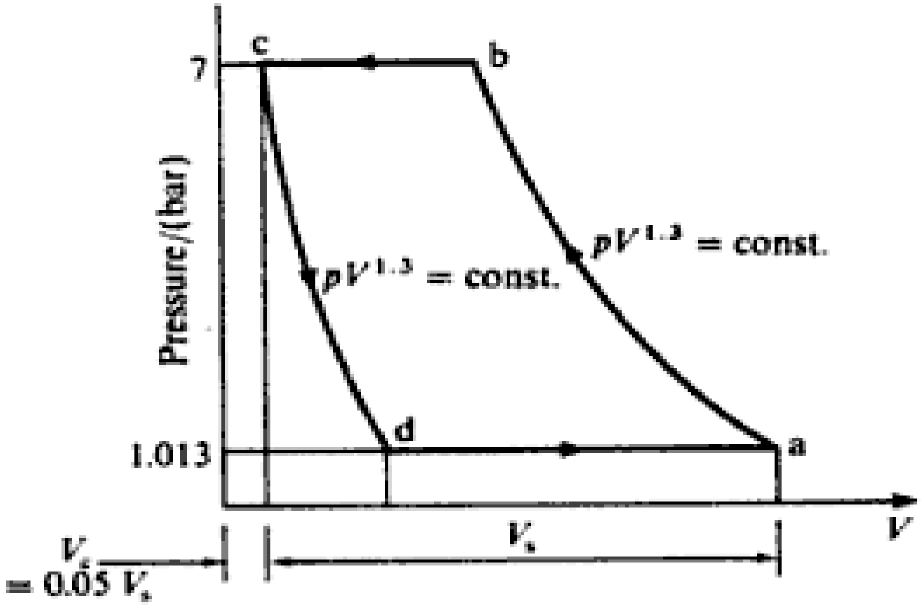
$$\mathbf{W_{net\ (indicated)} = \frac{n}{n-1} P_1\ (V_a - V_d) \left(\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1 \right)}$$

Where the volume induced per unit to the compressor is $\mathbf{V_{induced} = (V_a - V_d)}$

Example 4

A single-stage, double-acting air compressor is required to deliver 14 m^3 of air per minute measured at 1.013 bar and 15 °C. The delivery pressure is 7 bar and the speed 300 rev/min. Take the clearance volume as 5% of the swept volume with a compression and re-expansion index of $n = 1.3$. Calculate the swept volume of the cylinder, the delivery temperature, and the indicated power.

Referring to Fig.



$$\text{Swept volume} = (V_a - V_c) = V_s$$

and $\quad$ Clearance volume, $V_c = 0.05V_s$

i.e. $\quad V_a = 1.05V_s$

Using equation $\qquad$ for a double-acting machine

$$\text{Volume induced per cycle, } (V_a - V_d) = \frac{14}{300 \times 2}$$

$$= 0.0233 \text{ m}^3/\text{cycle}$$

(cycles per minute = revolutions per minute × cycles per revolution).

Now

$$V_d = V_c\left(\frac{p_2}{p_1}\right)^{1/n} = 0.05V_s\left(\frac{7}{1.013}\right)^{1/1.3}$$

i.e. $\quad V_d = 0.221V_s$

therefore

$$(V_a - V_d) = 1.05V_s - 0.221V_s = 0.0233 \text{ m}^3/\text{cycle}$$

therefore

$$V_s = \frac{0.0233}{0.829} = 0.0281 \text{ m}^3/\text{cycle}$$

i.e. Swept volume of compressor $= 0.0281 \text{ m}^3$

$$\text{Delivery temp., } T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(n-1)/n}$$

and $T_1 = 15 + 273 = 288 \text{ K}$

i.e.

$$T_2 = 288\left(\frac{7}{1.013}\right)^{(1.3-1)/1.3} = 450 \text{ K}$$

therefore

Delivery temp. $= 177\,°\text{C}$

Using equation

Indicated power

$$= \frac{n}{n-1} p_1 \dot{V}\left\{\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1\right\}$$

$$= \frac{1.3}{0.3} \times \frac{1.013 \times 10^5 \times 14}{10^3 \times 60}\left\{\left(\frac{7}{1.013}\right)^{(1.3-1)/1.3} - 1\right\} \text{kW}$$

i.e. Indicated power $= 57.6$ kW

Or, can indicated power can be calculated :

$$\dot{m} = \frac{1.013 \times 14 \times 10^5}{0.287 \times 288 \times 10^3} = 17.16 \text{ kg/min}$$

Then, using equation

$$\text{Indicated power} = \frac{n}{n-1}\dot{m}R(T_2 - T_1)$$

$$= \frac{1.3 \times 17.16 \times 0.287(450 - 288)}{0.3 \times 60}$$

$$= 57.6 \text{ kW} \quad \text{(as before)}$$

Volumetric efficiency, η_v

It has been shown that one of the effects of clearance is to reduce the induced volume to a value less than that of the swept volume. This means that for a required induction the cylinder size must be increased over that calculated on the assumption of zero clearance. The volumetric efficiency is defined as follows:

η_v = the mass of gas delivered, divided by the mass of gas which would fill the swept volume at the free air conditions of pressure and temperature

$$\eta_{volumetric} = \frac{m_{delivered}}{m_{sweptet}}$$

or

η_v = the volume of gas delivered measured at the free air pressure and temperature, divided by the swept volume of the cylinder

$$\eta_{volumetric} = \frac{V_{delivered\ (\ induced\ )}}{V_{sweptet}} = \frac{V_a - V_d}{V_a - V_c}$$

The volume of air dealt with per unit time by an air compressor is quoted as the free air delivery (FAD), and is the rate of volume flow delivered, measured at the pressure and temperature of the atmosphere in which the machine is situated.

Equations above can be shown to be identical, i.e. if the FAD per cycle is V_d at p and T, then the mass delivered per cycle is

$$m_{de} = \frac{pV_{de}}{RT}$$

$$m_{deliverd\ (induced)} = \frac{P\ V_{deliverd\ (induced)}}{R\ T}$$

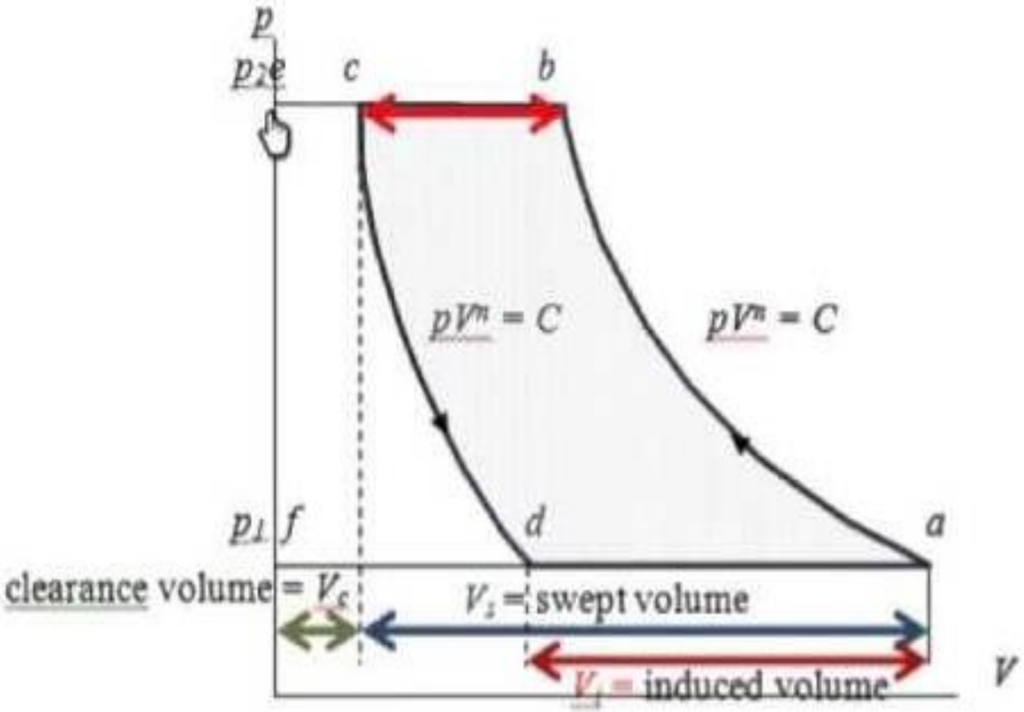
The mass required to fill the swept volume, V_s, at p and T is given by

$$m_s = \frac{pV_s}{RT}$$

Where $V_s = V_a - V_c$

And V_{de} , at T_a & P_a

$V_{de} = V_b - V_c$

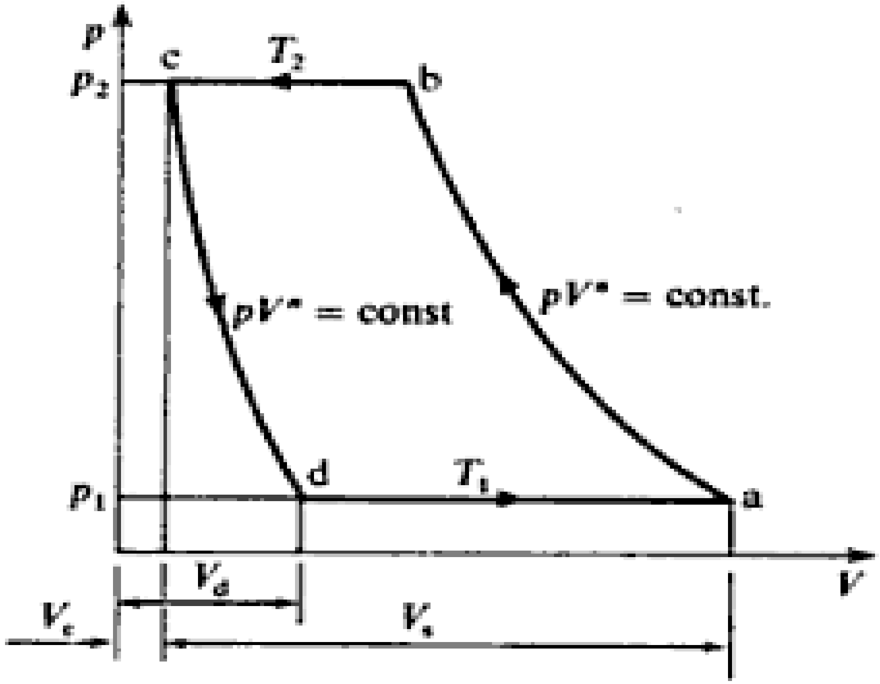


Therefore by equation

$$\eta_v = \frac{m_{de}}{m_s} = \frac{pV_{de}}{RT} \times \frac{RT}{pV_s} = \frac{V_{de}}{V_s}$$

The volumetric efficiency can be obtained from the indicator diagram. Referring to Fig.

$$\text{Volume induced} = V_a - V_d = V_s + V_c - V_d$$

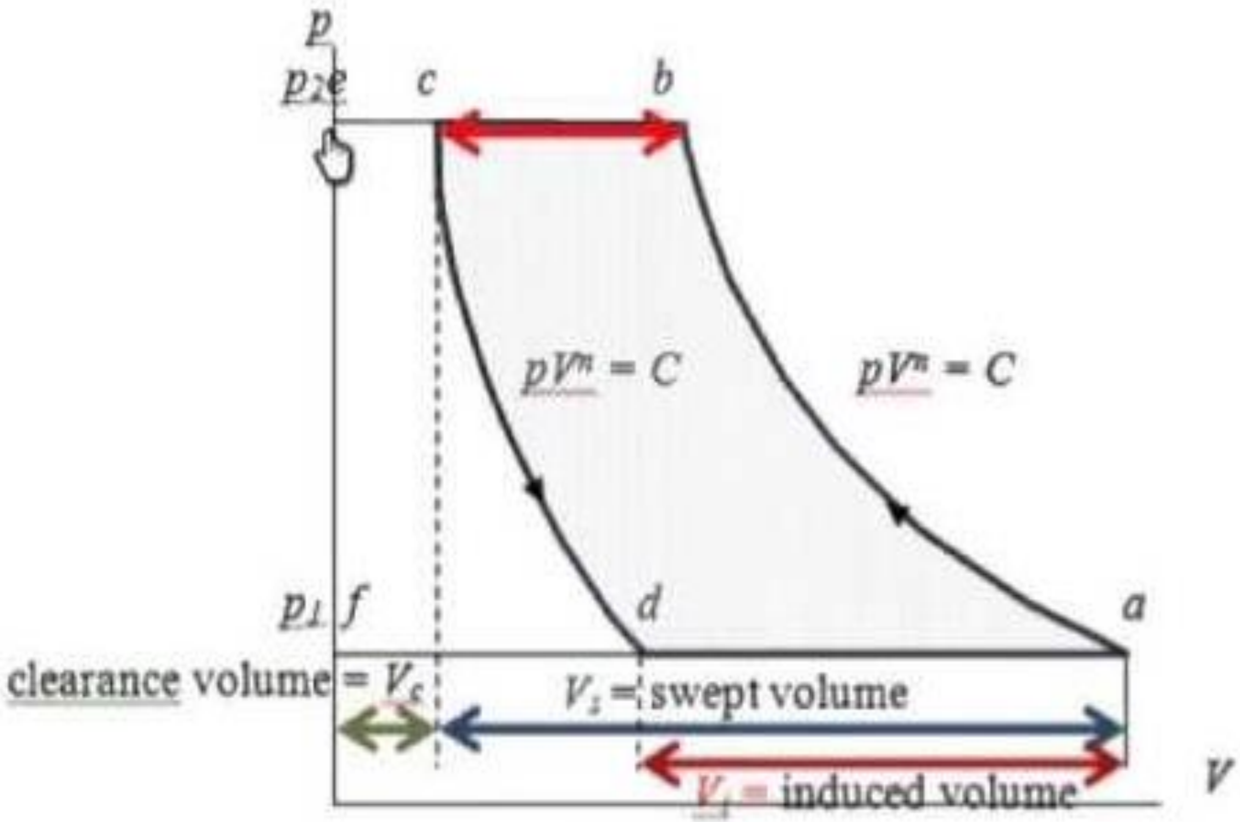


$$Vd = Vc \left(\frac{P2}{P1}\right)^{1/n}$$

Volume induced $= Va - Vd \quad = VS + VC - Vd$

Volume induced $= Va - Vd \quad = VS + VC - VC \left(\frac{P2}{P1}\right)^{1/n}$

Volume induced $= Va - Vd \quad = VS - VC \left\{ \left(\frac{P2}{P1}\right)^{1/n} - 1 \right\}$



$$\eta volumetric = \frac{VS - VC \left\{ \left(\frac{P2}{P1}\right)^{1/n} - 1 \right\}}{VS}$$

$$\eta volumetric = 1 - \frac{VC \left\{ \left(\frac{P2}{P1}\right)^{1/n} - 1 \right\}}{VS}$$

$$\eta volumetric = 1 - \frac{VC}{VS} \left\{ \left(\frac{P2}{P1}\right)^{1/n} - 1 \right\}$$

As before the F.A.D. per cycle is denoted

by **Vdeliverd (induced) at P & T**

$$mdeliverd\ (induced) = \frac{P\ Vdeliverd\ (induced)}{R\ T} = \frac{P1\ (Va - Vd)}{R\ T1}$$

$$\frac{F.A.D.}{cycle},\ Vdeliverd\ (induced) = (Va - Vd)\ \frac{T}{T1}\ \frac{P1}{P}$$