Thermodynamics

2^{nd} Semester-Chapter 4

Positive Displacement Compressors
Reciprocating Machines
The Conditions For Minimum Work
Isothermal Efficiency & Volumetric Efficiency
Multi – Stage Compression

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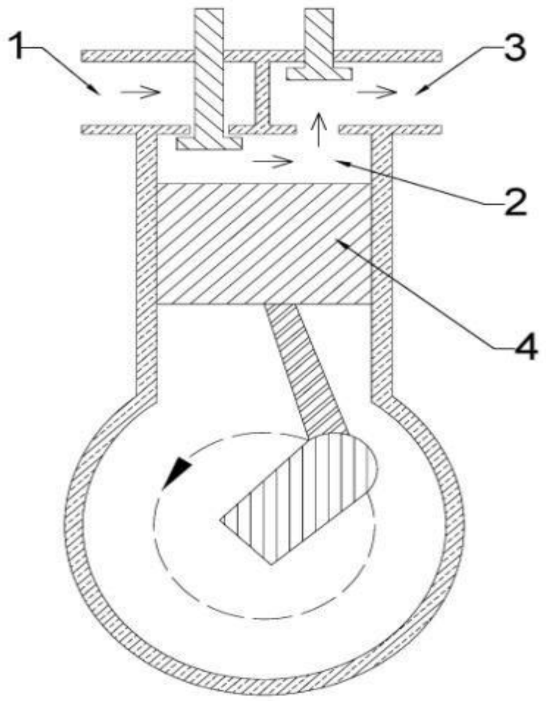
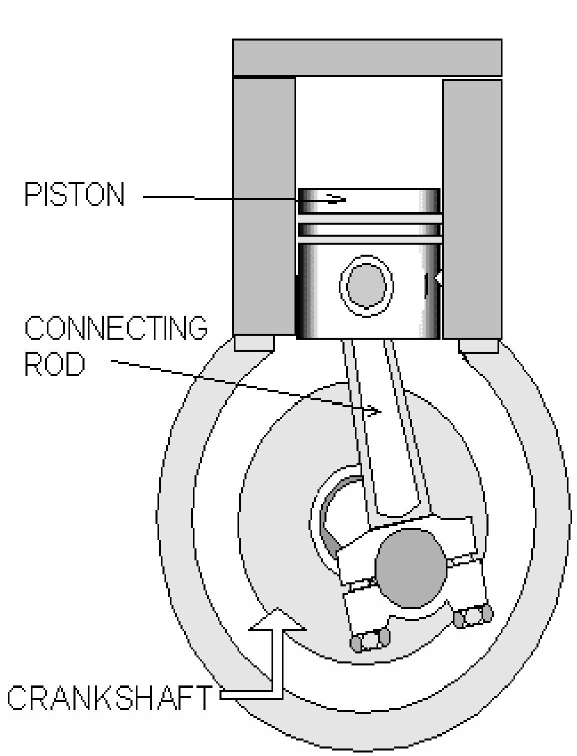
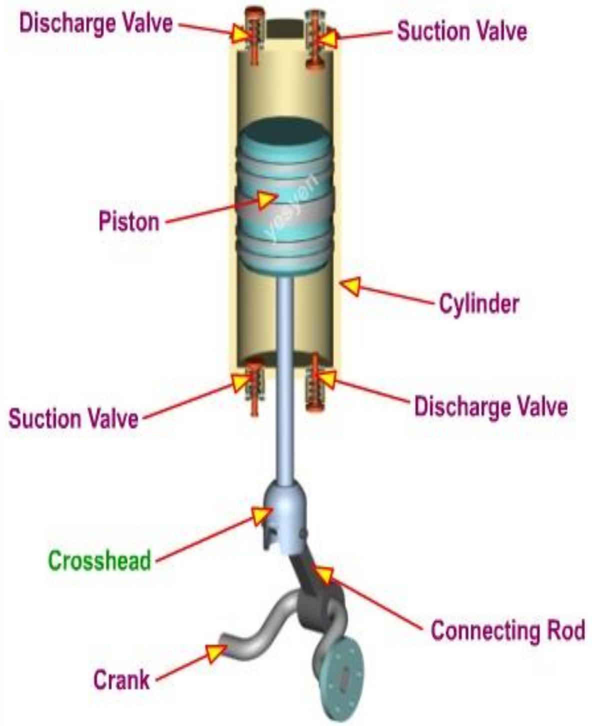
Introduction

A reciprocating compressor or piston compressor is a positive-displacement compressor that uses pistons driven by a crankshaft to deliver gases at high pressure. The intake gas enters the suction manifold, then flows into the compression cylinder where it gets compressed by a piston driven in a reciprocating motion via a crankshaft, and is then discharged. Applications include oil refineries, gas pipelines, chemical plants, natural gas processing plants, air conditioning, and refrigeration plants. One specialty application is the blowing of plastic bottles made of polyethylene terephthalate (PET).







Reciprocating compressors are positive displacement machines in which the compressing and displacing element is a piston having a reciprocating motion within a cylinder.

Types of reciprocating compressors

There are two types of reciprocating compressors according to the driven way:

- High speed (separable)
- Low speed (integral)

The high-speed category also is referred to as “separable,” and the low-speed category also is known as “integral.”

Separable compressors

The term “separable” is used because this category of reciprocating compressors is separate from its driver. Either an engine or an electric motor usually drives a separable compressor. Often a gearbox is required in the compression train. Operating speed is typically between 900 and 1,800 rpm.

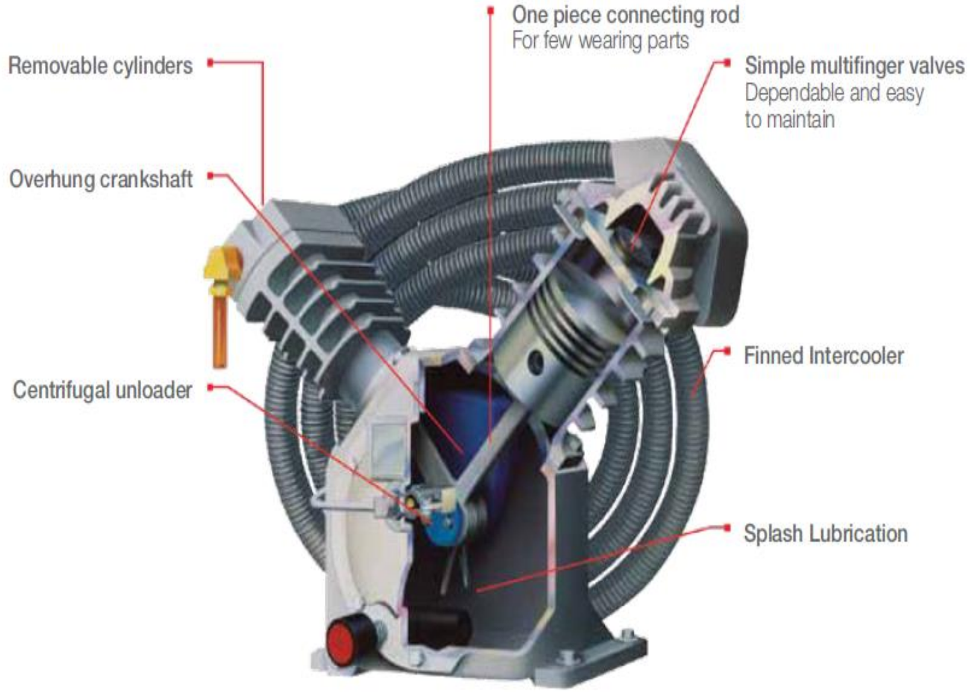
Separable units are skid mounted and self-contained. They are easy to install, offer a relatively small initial cost, are easily moved to different sites, and are available in sizes appropriate for field gathering—both onshore and offshore. However, separable compressors have higher maintenance costs than integral compressors.

Components of a Reciprocating Compressor

A reciprocating compressor uses pistons to compress air. The compressor has a similar design to an internal combustion engine; it even looks similar. There is a central crankshaft that drives anywhere from two to six pistons inside cylinders. The crankshaft is generally driven by an external motor. This motor can be electric or internal combustion. However, it determines the total horsepower of the compressor.

Compressing the Gas

As the pistons draw back, gas is injected from an intake valve in the compressor. This gas is injected into the cylinders of the pistons, and is then compressed by the reciprocating action of the pistons. The gas is then discharged either to be used immediately by a pneumatic machine, or stored in compressed air tanks. However, the gas must be stored or used directly from the compressor to prevent it from losing its pressurization.

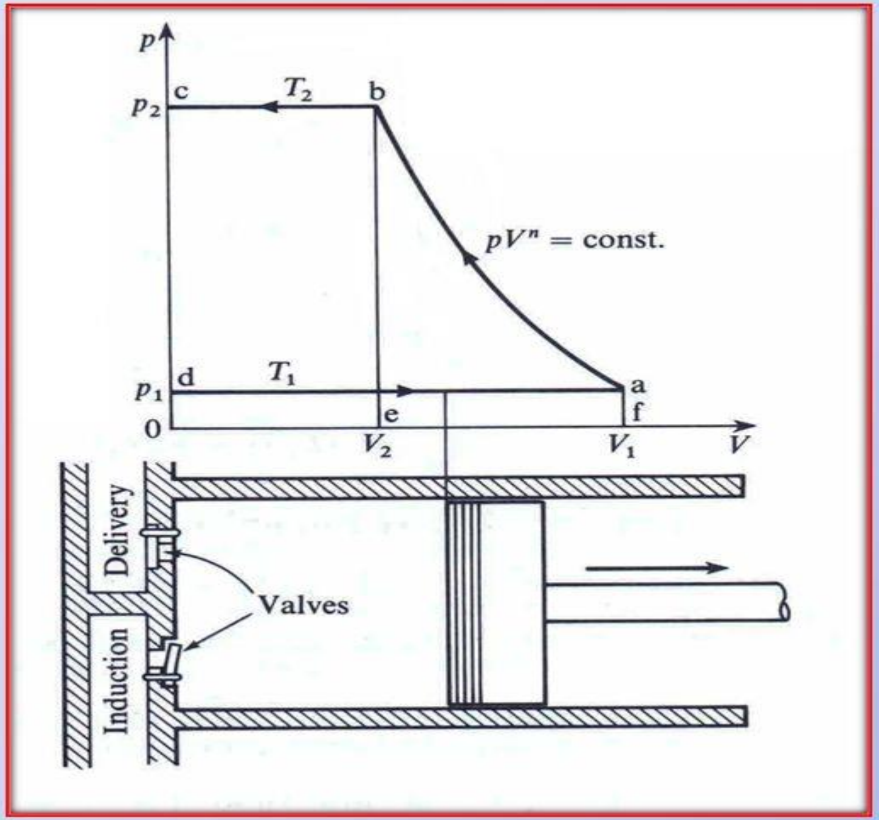


The **Thermodynamic Cycle**. An explanation of a few basic **thermodynamic** principles is necessary to understand the science of **reciprocating compressors**. Compression occurs within the cylinder as a four-part **cycle** that occurs with each advance and retreat of the piston (two strokes per **cycle**).

There are two types of reciprocating compressors according to the cycle:

- Reciprocating Compressor With Clearance Neglected
- Reciprocating Compressor With Clearance

Reciprocating Compressor With Clearance Neglected



Pressure volume diagram for a reciprocating compressor with clearance neglected

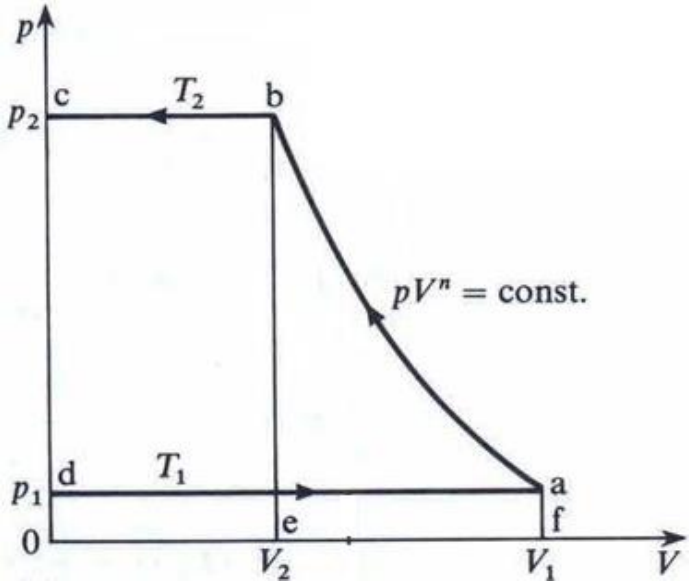
In engineering analysis, the performance was achieved under idealized circumstances for the cycle. The net work of the cycle equal the sum work of all processes. i. e.

$$W_{net} = \Sigma W = W_{ab} + W_{bc} + W_{cd} + W_{da}$$

Process ab is polytropic, thus perfect gas (air) compress according to the law :

$$PV^n = \text{Constant}, \quad \text{then } P = \frac{C}{V^n}$$

Then the work found by :



$$W_{ab} = {}_a\int^b P\, dV$$

$$W_{ab} = {}_a\int^b \frac{C}{V^n}\, dV = C\ {}_a\int^b \frac{dV}{V^n}$$

$$W_{ab} = C\left[\frac{V^{1-n}}{1-n}\right]_a^2 = PV^n\left[\frac{V^{1-n}}{1-n}\right]_a^2 = \frac{P_2V_b - P_1V_a}{1-n}$$

$$W_{ab} = -\frac{P_2V_b - P_1V_b}{n-1}$$

By equations PV^n **= Constant, and** $\frac{PV}{T}$ **= Constant ,** can found

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{n-1}$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(n-1)/n}$$

Process ab is isobaric, the perfect gas (air) compress according to the law of work :

$$W_{bc} = P_2 (V_c – V_b) = – P_2 (V_b – V_c) = – P_2 (V_b + 0)$$

$$W_{bc} = – P_2 V_b$$

Process ab is isochoric, i. e. ΔV = 0, thus the work = 0 , $W_{cd} = 0$

Process da is isobaric, the perfect gas (air) compress according to the law of work :

$$W_{da} = P_1 (V_a – V_d) = P_1 (V_a – 0)$$

$$W_{da} = P_1 V_a$$

The net work of the cycle is :

$$W_{net\ (Indicated)} = \Sigma W = W_{ab} + W_{bc} + W_{cd} + W_{da}$$

$$W_{net\ (Indicated)} = \Sigma W = -\frac{P_2V_b - P_1V_b}{n-1} - P_2 V_b + 0 + P_1 V_a$$

$$W_{net\ (Indicated)} = \Sigma W = -\frac{P_2V_b - P_1V_b}{n-1} - (P_2 V_b - P_1 V_a)$$

$$W_{net\ (Indicated)} = \Sigma W = -(P_2 V_b - P_1 V_a) \left(\frac{1}{n-1} + 1 \right)$$

$$W_{net\ (Indicated)} = \Sigma W = -\frac{n}{n-1} (P_2 V_b - P_1 V_a)$$

For perfect gas (air) PV = mRT , and the work of compressor is done one cycle (– ve), thus the absolute value of the compressor work is :

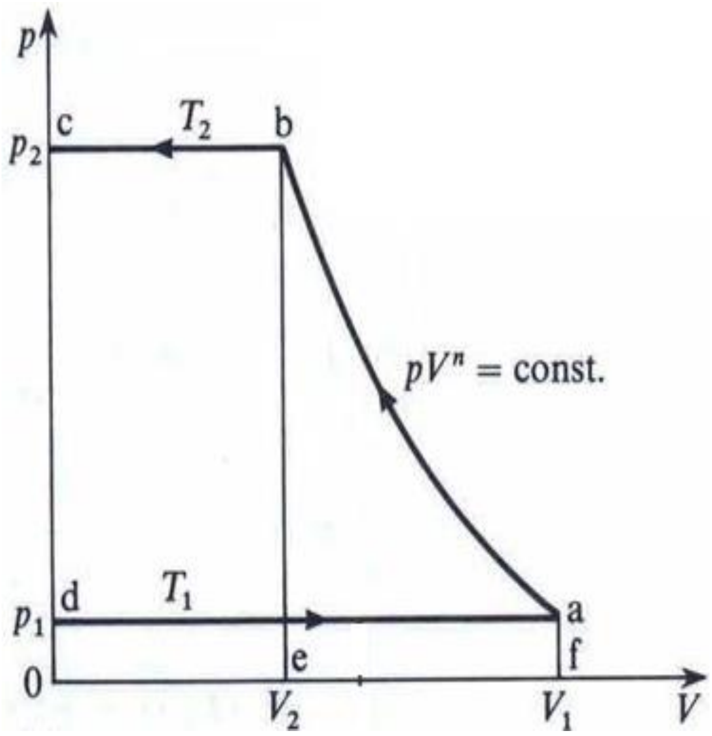
$$W_{net\ (Indicated)} = \frac{n\, m\, R\, (T_2 - T_1)}{n-1}$$

Or

Indicated work done

on the air per cycle = Area (abcd)

= Area (abef) + Area (bcoe) – Area (adof)



$$\text{Work input} = \frac{(p_2V_b - p_1V_a)}{n-1} + p_2V_b - p_1V_a$$

$$= (p_2V_b - p_1V_a)\left(\frac{1}{n-1} + 1\right)$$

i.e. $$\text{Work input} = (p_2V_b - p_1V_a)\frac{1+n-1}{n-1}$$

$$= \frac{n}{n-1}(p_2V_b - p_1V_a)$$

From equation of gas, we can write

$$p_1V_a = mRT_1 \quad \text{and} \quad p_2V_b = mRT_2$$

where m is the mass induced and delivered per cycle. Then

$$\text{Work input per cycle} = \frac{n}{n-1}mR(T_2 - T_1)$$

The delivery temperature is given by the equation

i.e. $$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(n-1)/n}$$

$$W_{net\ (indicated)} = \frac{n}{n-1} mRT_1 \left(\frac{T_2}{T_1} - 1 \right)$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{(n-1)/n}$$

$$W_{net\ (indicated)} = \frac{n}{n-1} mRT_1 \left\{ \left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right\}$$

$$W_{net\ (indicated)} = \frac{n}{n-1} P_1 V \left\{ \left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right\}$$

Where **V** is the volume induced per time.

The actual work of the compressor is larger than the net work (Indicated work) done on the air (cycle), due to the work necessary to over come the losses due to friction

Shaft work Wsh = net work (indicated)+ friction work

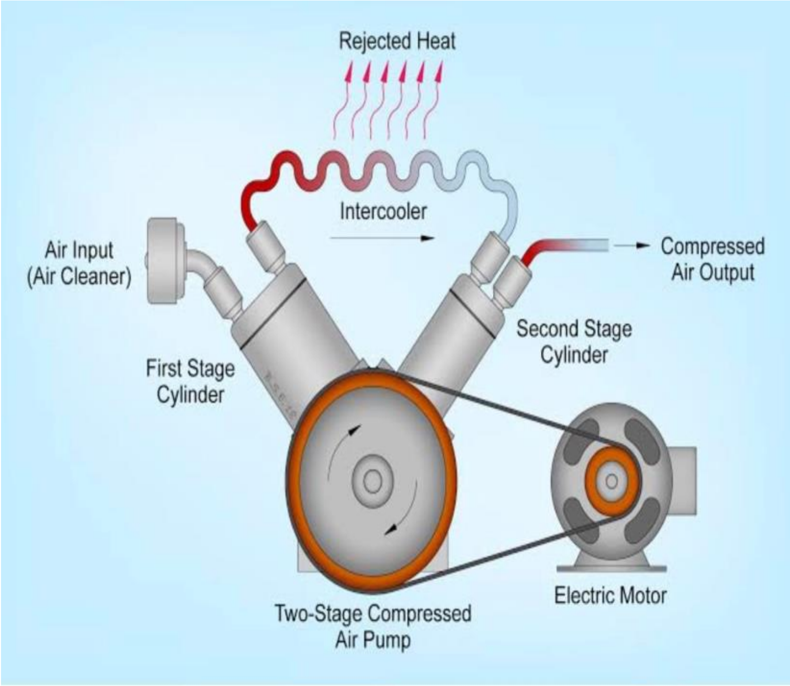
$$W_{sh.} = W_{net\ (\ indicated)} + W_{fri.}$$

The mechanical efficiency of the compressor is

$$\eta_{mech.\ Comp.} = \frac{W_{net\ (\ indicated\ )}}{W_{sh.}}$$

The input required power to the compressor is larger than the shaft work, thus it is necessary to know the efficiency of the driving motor, then

$$\text{Input Power} = \frac{W_{Sh.}}{\eta_{motor\ and\ drive}}$$



Example 1

A single-stage reciprocating compressor takes 1 m^3 of air per minute at 1.013 bar and 15 °C and delivers it at 7 bar. Assuming that the law of compression is $pV^{1.35}$ = constant, and that clearance is negligible, calculate the indicated power.

Solution

$$\text{Mass delivered per min}, \dot{m} = \frac{p_1 V_1}{R T_1}$$

$$= \frac{1.013 \times 1 \times 10^5}{287 \times 288} = 1.226 \text{ kg/min}$$

where $T_1 = 15 + 273 = 288$ K.

$$\text{Delivery temp., } T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(n-1)/n} = 288\left(\frac{7}{1.013}\right)^{(1.35-1)/1.35}$$

$$= 475.4 \text{ K}$$

$$W_{comp.} = \frac{n\, m\, R\,(T_2 - T_1)}{n - 1}$$

$$W_{comp.} = \frac{1.35 \times 1.226 \times 0.287\,(475.4 - 288)}{60\,(1.35 - 1)} = 4.2389 \text{ kw}$$

Example 2

If the compressor of example 1 is to be driven at 300 $\frac{\text{rev.}}{\text{min.}}$ and is a single-cylinder machine, calculate the cylinder bore required, assuming a stroke to bore ratio is $\frac{1.5}{1}$. Calculate the power of the motor required to drive the compressor if the mechanical efficiency of the compressor is 85 % and the that of motor transmission is 90 %

Volume was drawn in per minute = 1 m^3 , and driven at speed 300 rpm, thus

$$\text{Volume was drawn in per cycle} = \frac{1}{300} = 0.00333 \, \frac{m^3}{\text{cycl}}$$

$$\text{Cylinder volume} = \frac{\Pi}{4} \, d^2 \, L = \text{Volume was drawn in per cycle} = 0.00333 \, \frac{m^3}{\text{cycl}}$$

$$\frac{\Pi}{4} \, d^2 \, L = 0.00333$$

$$\frac{3.14}{4} \, d^2 \, (1.5 \; d) = 0.00333$$

$$d^3 = 0.00283 \, m^3$$

$$\text{Cylinder bore} \quad d = 0.1415 \text{ m} = 141.5 \text{ mm}$$

$$\text{Power input to compressor} = \frac{W_{net}}{\eta_{mech.\ Comp.}} = \frac{4.2389}{0.85} = 4.9869 \text{ kW}$$

$$\text{Then , Motor power} = \frac{W_{sh.}}{\eta_{motor\ and\ drive}} = \frac{4.9869}{0.90} = 5.541 \text{ kW}$$

The condition for minimum work

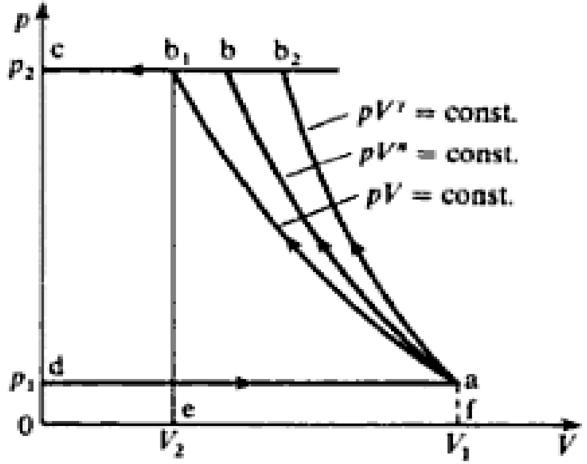
The work done on the gas is given by the area of the indicator diagram, and the work done will be a minimum when the area of the diagram is a minimum. The height of the diagram is fixed by the required pressure ratio (when p_1 is fixed), and the length of the line da is fixed by the cylinder volume, which is itself fixed by the required induction of gas. The only process which can influence the area of the diagram is the line ab. The position taken by this line is decided by the value of the index n; Fig. shows the limits of the possible processes.

Line ab_1 is according to the law pV = constant (i.e. isothermal)

Line ab_2 is according to the law pV^{γ} = constant (i.e. isentropic)

Both processes are reversible.

Isothermal compression is the most desirable process between a and b, giving the minimum work to be done on the gas. This means that in an actual



compressor the gas temperature must be kept as close as possible to its initial value, and a means of cooling the gas is always provided, either by air or by water.

The indicated work done when the gas is compressed isothermally is given by the area ab_1cd.

$$\text{Area } ab_1cd = \text{area } ab_1ef + \text{area } b_1c0e - \text{area } ad0f$$

$$\text{Area } ab_1ef = p_2 V_{b_1} \ln\frac{p_2}{p_1} \quad \text{(Worke at constant temperature)}$$

i.e. $$\text{indicated work per cycle} = p_2 V_{b_1} \ln\frac{p_2}{p_1} + p_1 V_{b_1} - p_1 V_a$$

Also $p_1 V_a = p_2 V_{b_1}$, since the process ab_1 is isothermal, therefore

$$\text{Isothermal work per cycle} = p_2 V_{b_1} \ln\frac{p_2}{p_1}$$

$$W_{isothermal} = p_1 V_a \ln\frac{p_2}{p_1}$$

$$W_{isothermal} = mRT \ln\frac{p_2}{p_1}$$

When **m** and **V_a** in equations are the mass and volume induced per unit time, then these equations give the isothermal power .

Isothermal Efficiency

By definition, based on the indicator diagram

$$\textbf{Isothermal Efficiency} = \frac{\textbf{Isothermal Work}}{\textbf{Indicated Work}}$$

$$\eta_{isothermal} = \frac{W_{isothermal}}{W_{net\ (indicated)}}$$