Thermodynamics

Chapter One

The heat engine Cycles

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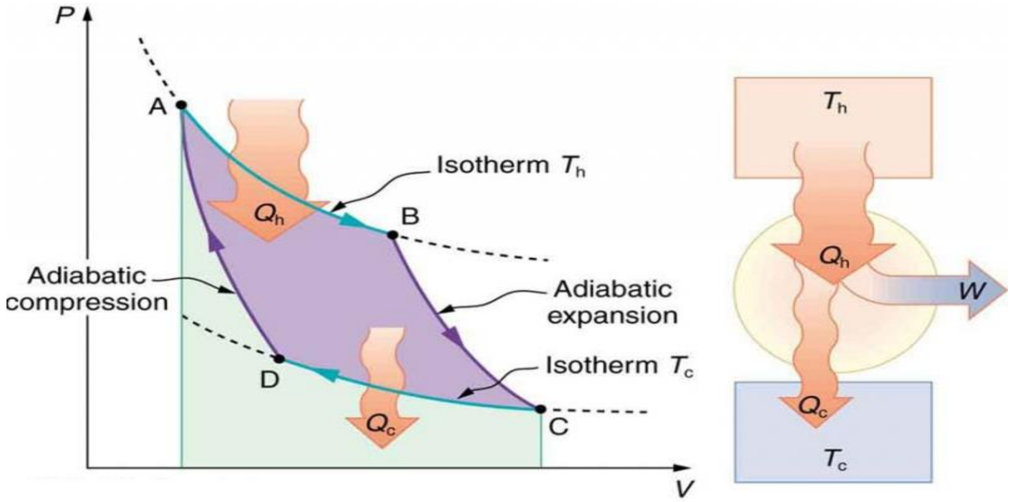
Power (Heat Engine) cycles

Thermodynamic power cycles are the basis for the operation of heat engines, which supply most of the world's electric power and run the vast majority of motor vehicles. Power cycles can be organized into two categories: real cycles and ideal cycles. Cycles encountered in real world devices (real cycles) are difficult to analyze because of the presence of complicating effects (friction), and the absence of sufficient time for the establishment of equilibrium conditions. For the purpose of analysis and design, idealized models (ideal cycles) are created; these ideal models allow engineers to study the effects of major parameters that dominate the cycle without having to spend significant time working out intricate details present in the real cycle model.

Power cycles can also be divided according to the type of heat engine they seek to model. The most common cycles used to model internal combustion engines are the Otto cycle, which models gasoline engines, and the Diesel cycle, which models diesel engines. Cycles that model external combustion engines include the Brayton cycle, which models gas turbines, the Rankine cycle, which models steam turbines, the Stirling cycle, which models hot air engines, and the Ericsson cycle, which also

models hot air engines.

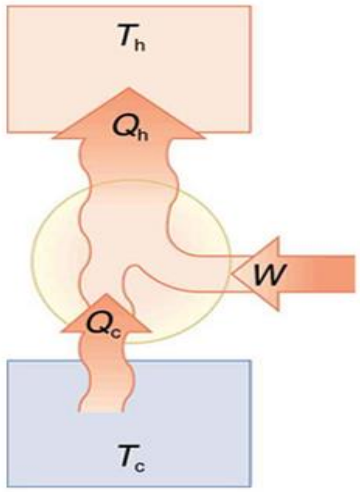
The clockwise thermodynamic cycle indicated by the arrows shows that the cycle represents a heat engine. The cycle consists of four states (the point shown by crosses and four thermodynamic processes (lines).

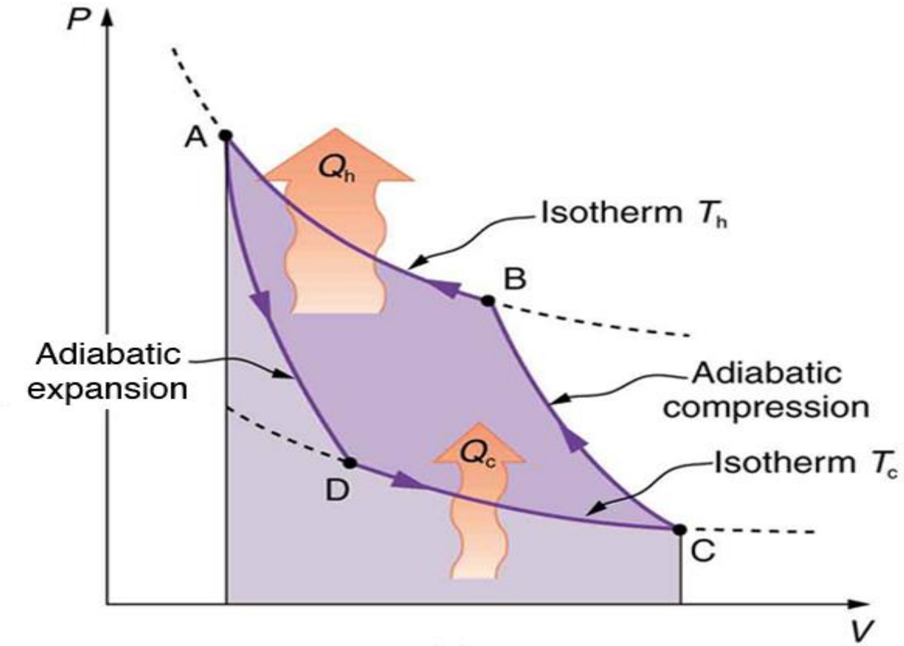


Heat pump cycles

Thermodynamic heat pump cycles are the models for household heat pumps and refrigerators. There is no difference between the two except the purpose of the refrigerator is to cool a very small space while the household heat pump is intended to warm a house. Both work by moving heat from a cold space to a warm space. The most common refrigeration cycle is the vapor compression cycle, which models systems using refrigerants that change phase.

The absorption refrigeration cycle is an alternative that absorbs the refrigerant in a liquid solution rather than evaporating it. Gas refrigeration cycles include the reversed Brayton cycle and the Hampson–Linde cycle. Multiple compression and expansion cycles allow gas refrigeration systems to liquify gases.





Carnot's Cycle

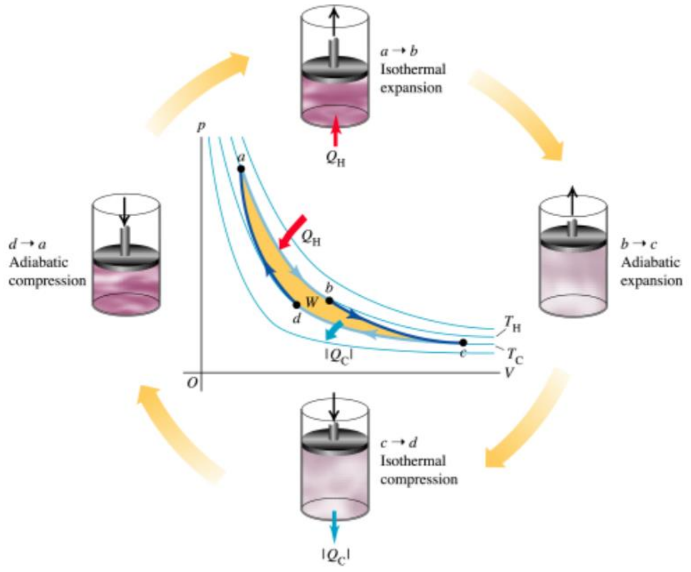
Carnot's cycle consists of taking the working substance in the cylinder through the following four operations that together constitute a reversible, cyclic transformation .

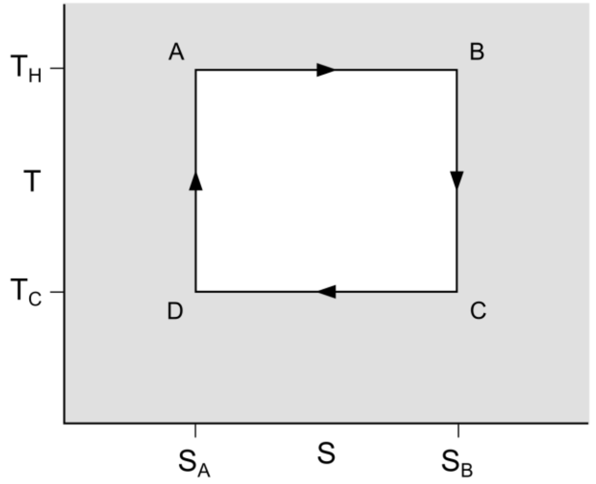
1. The substance starts at point A with temperature T_H. The cylinder is now placed on the warm (Hot) reservoir, from which it extracts a quantity of heat Q_H. The working substance expands isothermally at temperature T_H to point B. During this process the working substance does work.

2. The working substance undergoes an adiabatic expansion from point B to point C and its temperature falls to T_C. Again The working substance does work against the force applied to the piston.

3. The working substance is compressed isothermally from point C to point D. In this transformation the working substance gives up a quantity of heat Q_C to the cold reservoir at temperature T_C.

4. Finally, working substance is compressed adiabatically from point D back to its original state A. Its temperature rises from T_C to T_H. Carnot's cycle is Ideal engine with maximum efficiency that is still consistent with the second law of thermodynamics, and has several consequences regarding the Carnot's cycle.





* A 100% efficient Carnot engine would convert all heat absorbed from a warm reservoir into work, directly Contradictor to the second law. We hence conclude that $\eta < 1$. η is = 1 (100%) if there is no waste heat ($Q_C = 0$). However, we will see that this is impossible due to the 2nd law of thermodynamics.

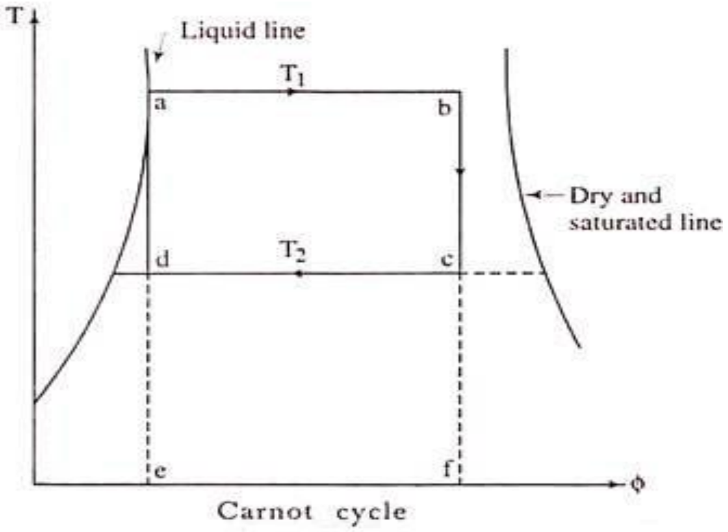
* All reversible heat engines operating between heat bath with temperatures T_H and T_C have the same efficiency.

Generally, the definition of efficiency is

$$\eta = \frac{\textbf{Desired Result}}{\textbf{Required Input}}$$

For a **Heat engine** operate according to **Carnot's Cycle,** the efficiency is :

$$\eta_{\text{ther.Carnot}} = \frac{W_{net}}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 -$$



$$\Delta S = Q / T$$

$$Q = T \ \Delta S$$

$$\eta_{\text{ther.Carnot}} = 1 - \frac{T_C\,(S_C - S_D)}{T_H\,(S_B - S_A)} = 1 - \frac{T_C \ \Delta S}{T_H \ \Delta S}$$

$$\eta_{\text{ther.Carnot}} = 1 - \frac{T_C}{T_H}$$

Coefficient Of Performance (COP)

Just as efficiency was defined for a heat engine, for a heat pump the coefficient of performance (COP) is a measure of how well it is doing the job.

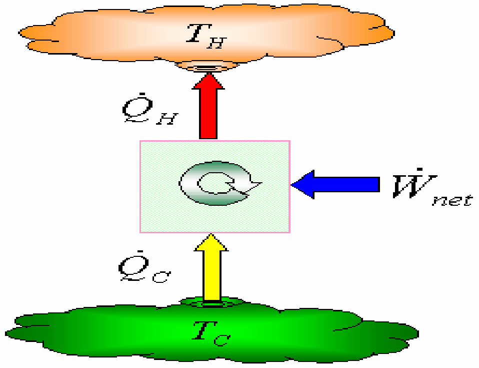


Fig. 1 . Refrigerator/Heat Pump.

$T_H = T_1$, $T_C = T_2$, $Q_H = Q_1$, $Q_C = Q_2$

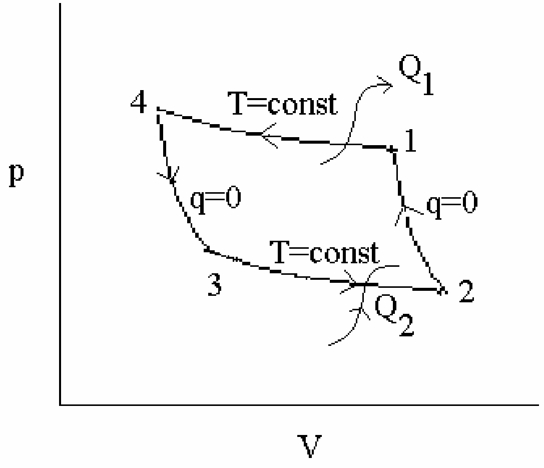
$$Q_C = T_C\,(S_2 - S_3) = T_C\,\Delta S$$
$$Q_H = T_H\,(S_1 - S_4) = T_H\,\Delta S$$

Carnot’s Cycle for a heat Pump

$$COP_{HP} = = \frac{\text{Heat given out}}{\text{Work Input}}$$

$$COP_{HP} = \frac{Q_H}{W_{Input}} = \frac{Q_H}{Q_H - Q_C}$$

$$COP_{HP} = \frac{T_H\,\Delta S}{T_H\,\Delta S - T_C\,\Delta S} = \frac{T_H}{T_H - T_C}$$



Carnot’s Cycle for a Refrigerator

$$COP_{Re} = = \frac{\text{Absorbed Heat}}{\text{Work Input}}$$

$$COP_{Re} = \frac{Q_C}{W_{Input}} = \frac{Q_C}{Q_H - Q_C} = \frac{T_C\,\Delta S}{T_H\,\Delta S - T_C\,\Delta S} = \frac{T_C}{T_H - T_C}$$

Relation between $\eta_{ther.}$ and COP_{HP} of cycle operate according to the Carnot's Cycle

It is not difficult to see that $\eta_{ther.} \;\; COP_{HP} = 1$

Relation between COP_{HP} of Heat Pump and COP_{Re} of Refrigerator which are operating according to the Carnot's Cycle

Apply First law of thermodynamics (Conservation of Energy) to Carnot's cycle as a heat pump/refrigerator :

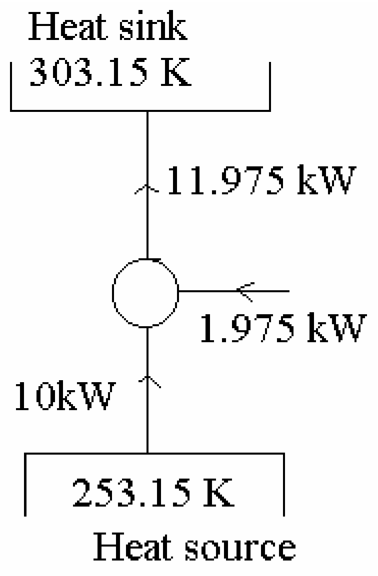
$$Q_H = Q_C + W_{Input}$$

$$\frac{Q_H}{W_{Input}} = \frac{Q_c}{W_{Input}} + \frac{W_{Input}}{W_{Input}}$$

$$COP_{HP} = COP_{Re} + 1$$

Examples

If 10 kw of heat is to be removed from a cold store at - 20$_o$C and rejected to ambient at 30$_o$C. Calculate the input work to the system.



$$COP_{Re} = \frac{T_c \Delta S}{T_H \Delta S - T_C \Delta S} = \frac{T_c}{T_H - T_C} = \frac{253}{303 - 253} = 5.063$$

$$COP_{Re} = \frac{Q_c}{W_{Input}}$$

$$W_{input} = \frac{Q_c}{COP_{Re}} = \frac{10}{5.063} = 1.975 \text{ kW}$$

Air Standard Assumptions

In power engines, energy is provided by burning fuel within the system boundaries, i.e., internal combustion engines. The following assumptions are commonly known as the *air standard* assumptions:

1. The working fluid is air, which continuously circulates in a closed loop (cycle). Air is considered as ideal gas.
2. All the processes in (ideal) power cycles are internally reversible.
3. Combustion process is modeled by a heat-addition process from an external source.
4. The exhaust process is modeled by a heat-rejection process that restores the working fluid (air) at its initial state.

Assuming constant specific heats, at 25°C for air, is called *cold-air-standard* assumption.

Some Definitions for Reciprocation Engines:

The reciprocation engine is one the most common machines that is being used in a wide variety of applications from automobiles to aircrafts to ships, etc.

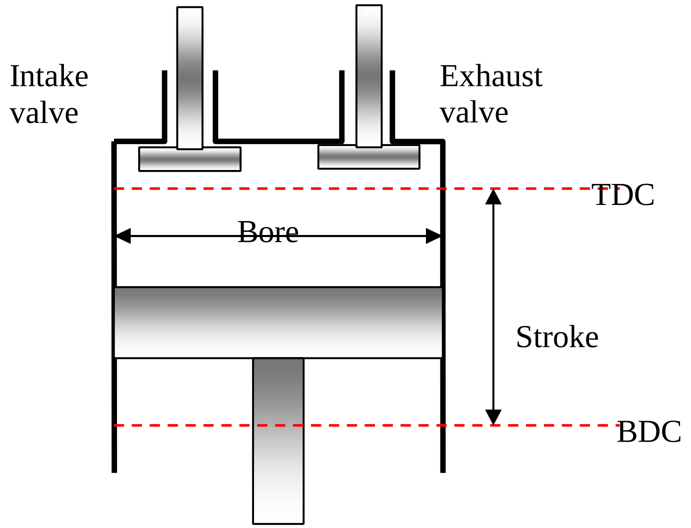


Fig. 1: Reciprocation engine

Top dead center (TDC): The position of the piston when it forms the smallest volume in the cylinder.

Bottom dead center (BDC): The position of the piston when it forms the largest volume in the cylinder.

Stroke: The largest distance that piston travels in one direction.

Bore: The diameter of the piston.

Clearance volume: The minimum volume formed in the cylinder when the piston is at TDC.

Displacement volume: The volume displaced by the piston as it moves between the TDC and BDC, i.e

$$\text{Displacement volume} = (\mathbf{V}_{\mathbf{max}} - \mathbf{V}_{\mathbf{min}})$$

Compression ratio: The ratio of maximum to minimum (clearance) volumes in the cylinder:

$$r = \frac{\mathbf{V}_{\mathbf{max}}}{\mathbf{V}_{\mathbf{min}}}$$

Mean effective pressure (MEP): fictitious (constant throughout the cycle) pressure that if acted on the piston will produce the work.

$$W_{net} = MEP \times A_{Piston} \times Stroke = MEP \times Displacement\ vol$$

$$\mathbf{MEP} = \frac{\mathbf{W}_{\mathbf{net}}}{\mathbf{V}_{\mathbf{max}} - \mathbf{V}_{\mathbf{min}}} = \frac{\mathbf{Q}_{\mathbf{in}} - \mathbf{Q}_{\mathbf{out}}}{\mathbf{V}_{\mathbf{max}} - \mathbf{V}_{\mathbf{min}}}$$

An engine with higher MEP will produce larger net output work.

Internal Combustion Engines

1. spark ignition engines:
    - a mixture of fuel and air is ignited by a spark plug
    - applications requiring power to about 225 kW (300 HP)
    - relatively light and low in cost
2. compression ignition engine:
    - air is compressed to a high enough pressure and temperature that combustion occurs when the fuel is injected
    - applications where fuel economy and relatively large amounts of power are required

Spark-Ignition (Gasoline) Engine

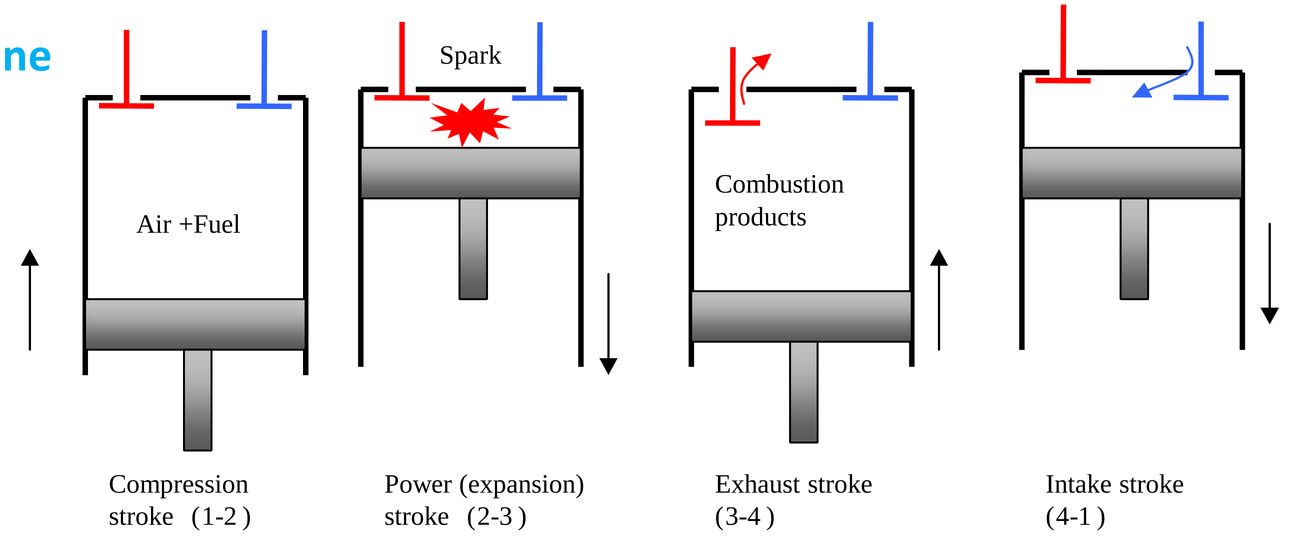


Fig. 2: Actual cycle for spark-ignition engines, four-stroke.

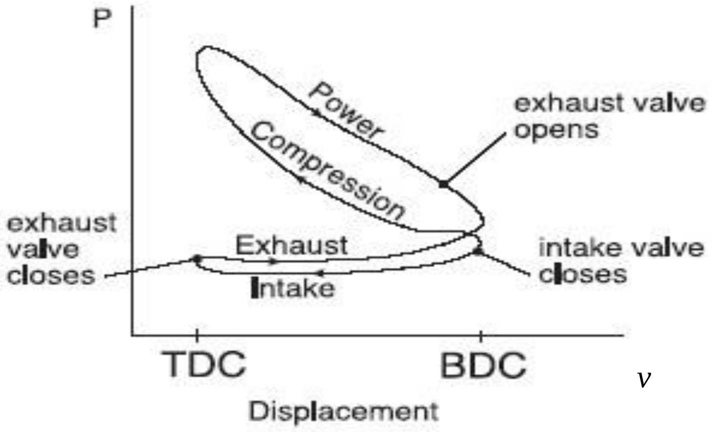


Fig. 3: *P*-*v* diagram for spark-ignition engines

Otto Cycle

The Otto cycle is the ideal cycle for spark-ignition reciprocating engines. It serves as the theoretical model for the gasoline engine:

- Consists of four internally reversible processes
- Heat is transferred to the working fluid at constant volume

The Otto cycle consists of four internally reversible processes in series:

1 – 2 Isentropic compression

2 – 3 Constant-volume heat addition

3 – 4 Isentropic expansion

4 – 1 Constant-volume heat rejection

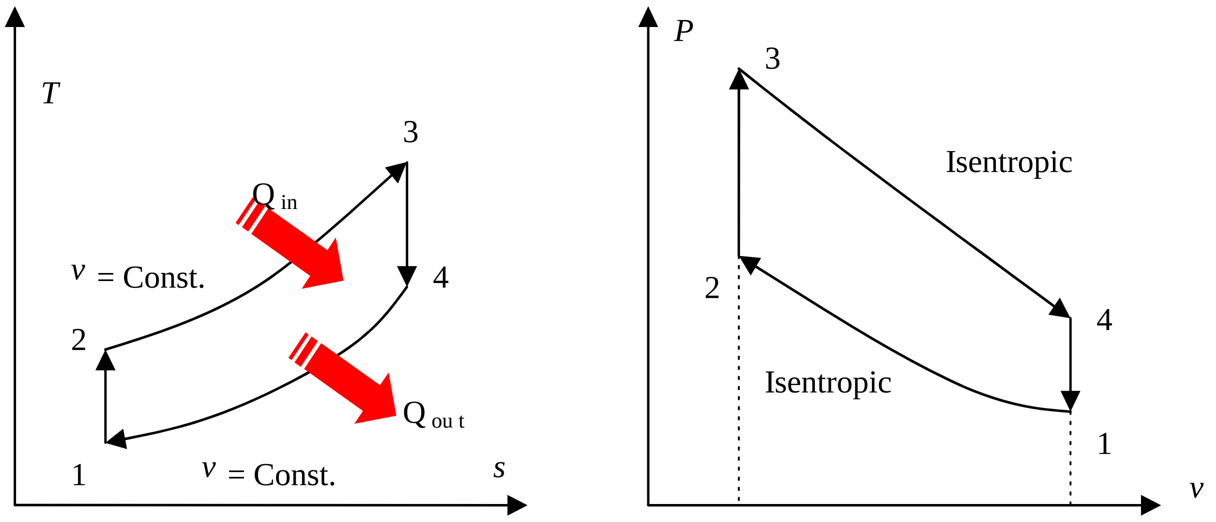


Fig. 4: T-s and P-v diagrams for Otto cycle

The Otto cycle is executed in a closed system. Neglecting the changes in potential and kinetic energies, the 1st law, on a unit mass base, can be written:

$$\mathbf{Q_{in} = u_3 - u_2 = C_v\,(T_3 - T_2)}$$

$$\mathbf{Q_{out} = u_4 - u_1 = C_v\,(T_4 - T_1)}$$

$$\mathbf{W_{net} = Q_{in} - Q_{out}}$$

Thus thermal efficiency can be written :

$$\eta_{\text{ther.otto}} = \frac{\mathbf{W_{net}}}{\mathbf{Q_{in}}} = \frac{\mathbf{Q_{in} - Q_{out}}}{\mathbf{Q_{in}}} = \mathbf{1} - \frac{\mathbf{Q_{out}}}{\mathbf{Q_{in}}} = \mathbf{1} - \frac{\mathbf{C_v\,(T_4 - T_1)}}{\mathbf{C_v\,(T_3 - T_2)}} = \mathbf{1} - \frac{\mathbf{(T_4 - T_1)}}{\mathbf{(T_3 - T_2)}}$$

Processes 1-2 and 3-4 are isentropic, and $V2 = V3$ and $V4 = V1$, then compression ratio is : $\mathbf{r} = \frac{\mathbf{V_1}}{\mathbf{V_2}} = \frac{\mathbf{V_4}}{\mathbf{V_3}}$

Typical compression ratios for spark-ignition engines are between 7 and 10. The thermal efficiency increases as the compression ratio is increased. However, high compression ratios can lead to *auto ignition* or *engine knock*.

Thus :

$$T_2 = T_1 r^{\gamma - 1} \quad \text{and} \quad T_3 = T_4 r^{\gamma - 1}$$

Then :

$$\eta_{ther.otto} = 1 - \frac{(T_4 - T_1)}{(T_4 r^{\gamma - 1} - T_1 r^{\gamma - 1})} = 1 - \frac{(T_4 - T_1)}{r^{\gamma - 1}(T_4 - T_1)}$$

$$\eta_{ther.otto} = 1 - \frac{1}{r^{\gamma - 1}} = 1 - \left(\frac{1}{r}\right)^{\gamma - 1}$$

Mean effective pressure (MEP_{otto}):

MEP_{otto} can be written as :

$$MEP_{otto} = \frac{W_{net}}{V_1 - V_2} = \frac{Q_{in} - Q_{out}}{V_1 - V_2} = \frac{Q_{in} - Q_{out}}{V_1\left(1 - \frac{V_2}{V_1}\right)} = \frac{Q_{in} - Q_{out}}{V_1\left(1 - \frac{1}{r}\right)}$$

$$Q_{in} = m\, C_v\, (T_3 - T_2)\text{, and } Q_{out} = m\, C_v\, (T_4 - T_1)$$

$$V_1 = \frac{m\, R\, T_1}{P_1}\text{, and } R = C_v\, (\gamma - 1)$$

$$V_1 = \frac{m\, C_v\,(\gamma - 1)\; T_1}{P_1}$$

$$MEP_{otto} = \frac{m\, C_v\,(T_3 - T_2) - m\, C_v\,(T_4 - T_1)}{\frac{m\, C_v\,(\gamma - 1)\; T_1}{P_1}\left(1 - \frac{1}{r}\right)}$$

$$MEP_{otto} = \frac{(T_3 - T_2) - (T_4 - T_1)}{\frac{(\gamma - 1)\; T_1}{P_1}\left(\frac{r - 1}{r}\right)}$$

$$MEP_{otto} = \frac{r\, P_1\,\{(T_3 - T_2) - (T_4 - T_1)\}}{T_1\,(\gamma - 1)\,(r - 1)}$$

$T_2 = T_1\, r^{\gamma - 1}$, $T_3 = r_p\, T_2 = r_p\, T_1\, r^{\gamma - 1}$ Where $r_p = \frac{P_3}{P_2}$ is the pressure ratio

$$T_4 = T_3\left(\frac{1}{r}\right)^{\gamma - 1} = r_p\, T_1\, r^{\gamma - 1}\left(\frac{1}{r}\right)^{\gamma - 1} = r_p\, T_1$$

$$MEP_{otto} = \frac{r\, P_1\,\{(r_p\, T_1\, r^{\gamma - 1} - T_1\, r^{\gamma - 1}) - (r_p\, T_1 - T_1)\}}{T_1\,(\gamma - 1)\,(r - 1)}$$

$$MEP_{otto} = \frac{r\, P_1\,\{r^{\gamma - 1}\,(r_p - 1) - (r_p - 1)\}}{(\gamma - 1)\,(r - 1)}$$

$$MEP_{otto} = \frac{r\, P_1\,(r^{\gamma - 1} - 1)\,(r_p - 1)}{(\gamma - 1)\,(r - 1)}$$

Diesel Engine

The Diesel cycle is the ideal cycle for compression ignition engines. It is very similar to spark-ignition, expect the method of ignition. In diesel engine, air compressed to a temperature that is above the ignition temperature of the fuel.

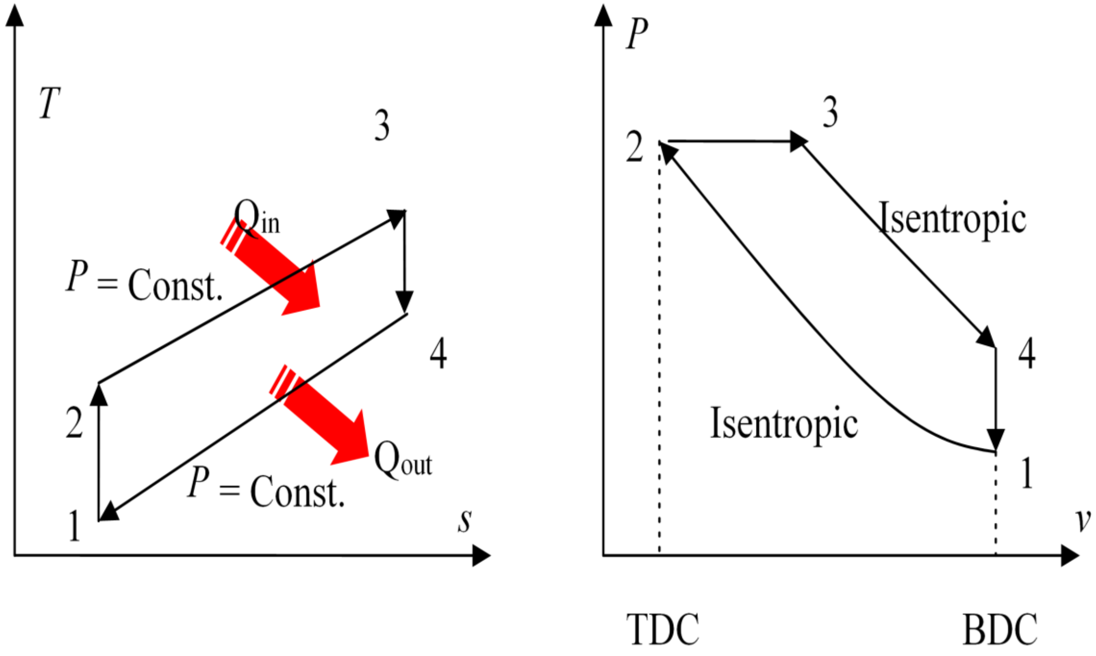


Fig. 5: *T*-*s* and *P*-*v* diagram of Diesel engine

Diesel engine consists of four internally reversible processes Heat is transferred to the working fluid at constant pressure. The Diesel cycle consists of four internally reversible processes in series:

1 – 2 Isentropic compression

2 – 3 Constant-pressure heat addition

3 – 4 Isentropic expansion

4 – 1 Constant-volume heat rejection

The Diesel cycle is executed in a closed system. Neglecting the changes in potential and kinetic energies, the 1st law, on a unit mass base, can be written:

$$Q_{in} = C_p \, (T_3 - T_2)$$

$$Q_{out} = C_v \, (T_4 - T_1)$$

$$W_{net} = Q_{in} - Q_{out}$$

Thus thermal efficiency can be written :

$$\eta_{ther.diesel} = \frac{W_{net}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{C_v\,(T_4 - T_1)}{C_p\,(T_3 - T_2)}$$

Processes 1-2 and 3-4 are isentropic, and $\mathbf{V_1 = V_4}$ and $\mathbf{P_2 = P_3}$,

The compression ratio is

$$r = \frac{V_1}{V_2}$$

C*utoff ratio* $\mathbf{r_c}$, is defined as the ratio of cylinder volumes after and before the combustion process (ignition period)

$$r_c = \frac{V_3}{V_2}$$

Then

$$T_2 = T_1\, r^{\gamma - 1} , \qquad T_3 = r_c\, T_2 = r_c\, T_1\, r^{\gamma - 1}$$

and

$$T_4 = T_3 \left(\frac{V_3}{V_4} \right)^{\gamma - 1} = T_3 \left(\frac{V_3}{V_4} \frac{V_2}{V_2} \right)^{\gamma - 1} = T_3 \left(\frac{V_3}{V_2} \frac{V_2}{V_4} \right)^{\gamma - 1}$$

$$T_4 = T_3 \left(r_c \frac{1}{r} \right)^{\gamma - 1} = T_3\, r_c^{\gamma - 1} \left(\frac{1}{r} \right)^{\gamma - 1} = T_1\, r_c\, r^{\gamma - 1}\, r_c^{\gamma - 1} \left(\frac{1}{r} \right)^{\gamma - 1}$$

$$T_4 = T_1\, r_c^{\gamma}$$

$$\eta_{\text{ther.diesel}} = \frac{W_{net}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{(r_c^{\gamma}\, T_1 - T_1)}{\frac{C_p}{C_v}(r_c\, T_1\, r^{\gamma - 1} - T_1\, r^{\gamma - 1})}$$

$$\eta_{\text{ther.diesel}} = 1 - \left(\frac{1}{r^{\gamma - 1}} \right) \frac{(r_c^{\gamma} - 1)}{\gamma\,(r_c - 1)} = 1 - \left(\frac{1}{r} \right)^{\gamma - 1} \frac{(r_c^{\gamma} - 1)}{\gamma\,(r_c - 1)}$$

Comparison of the Otto and the Diesel Cycle

- $\eta_{Otto} > \eta_{Diesel}$ for the same compression ratio
- Diesel engines burn the fuel more completely since they usually operate at lower rpm and air-fuel ratio is much higher than ignition-spark engines
- Diesel engines compression ratios are typically between 12 and 24, whereas spark-ignition (SI) engines are between 7 and 10. Thus a diesel engine can tolerate a higher ratio since only air is compressed in a diesel cycle and spark knock is not an issue

Mean effective pressure (MEP_{diesel}): can be written as :

$$MEP_{diesel} = \frac{W_{net}}{V_1 - V_2} = \frac{Q_{in} - Q_{out}}{V_1 - V_2} = \frac{Q_{in} - Q_{out}}{V_1\left(1 - \frac{V_2}{V_1}\right)} = \frac{Q_{in} - Q_{out}}{V_1\left(1 - \frac{1}{r}\right)}$$

$$Q_{in} = m\,C_p\,(T_3 - T_2)\text{ , and }Q_{out} = m\,C_v\,(T_4 - T_1)$$

$$V_1 = \frac{m\,R\,T_1}{P_1}\text{ , and }R = C_V\,(\gamma - 1)$$

$$V_1 = \frac{m\,C_V\,(\gamma - 1)\,T_1}{P_1}$$

$$MEP_{diesel} = \frac{m\,C_p\,(T_3 - T_2) - m\,C_v\,(T_4 - T_1)}{\frac{m\,C_v\,(\gamma - 1)\,T_1}{P_1}\left(1 - \frac{1}{r}\right)}$$

$$MEP_{diesel} = \frac{\gamma\,(T_3 - T_2) - (T_4 - T_1)}{\frac{(\gamma - 1)\,T_1}{P_1}\left(\frac{r - 1}{r}\right)}$$

$$MEP_{diesel} = \frac{r\,P_1\,\{\gamma\,(T_3 - T_2) - (T_4 - T_1)\}}{T_1\,(\gamma - 1)\,(r - 1)}$$

$$T_2 = T_1\,r^{\gamma - 1}\text{ , }T_3 = r_c\,T_2 = r_c\,T_1\,r^{\gamma - 1}\text{ and }T_4 = T_1\,r_c^{\gamma}$$

$$MEP_{diesel} = \frac{r\,P_1\,\{\gamma\,(r_c\,T_1\,r^{\gamma - 1} - T_1\,r^{\gamma - 1}) - (T_1\,r_c^{\gamma} - T_1)\}}{T_1\,(\gamma - 1)\,(r - 1)}$$

Constant Pressure (Brayton) Cycle

A constant-pressure cycle is a thermodynamic cycle, which is described by George Brayton when he explains the workings of a heat engine with heat addition and rejection at constant pressure.

The original Brayton engines used a piston compressor and piston expander, but more modern gas turbine engines and airbreathing jet engines also follow the Brayton cycle. Although the cycle is usually run as an open system (and indeed must be run as such if internal combustion is used), it is conventionally assumed for thermodynamic analysis that the exhaust gases are reused in the intake, enabling analysis as a closed system.

The engine cycle is named after George Brayton (1830–1892), the American engineer who developed it originally for use in piston engines, although it was originally proposed and patented by Englishman John Barber in 1791. It is also sometimes known as the Joule cycle. The reversed Joule cycle uses an external heat source and incorporates the use of a regenerator. One type of Brayton cycle is open to the atmosphere and uses an internal combustion chamber, and another type is closed and uses a heat exchanger.

A Brayton-type engine consists of three components: a compressor, a mixing chamber, and an expander.

Modern Brayton engines are almost always a turbine type, although Brayton only made piston engines. In the original 19th-century Brayton engine, ambient air is drawn into a piston compressor, where it is compressed; ideally an isentropic process. The compressed air then runs through a mixing chamber where fuel is added, an isobaric process. The pressurized air and fuel mixture is then ignited in an expansion cylinder and energy is released, causing the heated air and combustion products to expand through a piston/cylinder, another ideally isentropic process.

Some of the work extracted by the piston / cylinder is used to drive the compressor through a crankshaft arrangement.

Gas turbines are also Brayton engines, with three components: a gas compressor, a burner (or combustion chamber), and an expansion turbine.

Ideal Brayton cycle:

- Isentropic process – ambient air is drawn into the compressor, where it is pressurized.
- Isobaric process – the compressed air then runs through a combustion chamber, where fuel is burned, heating that air—a constant-pressure process, since the chamber is open to flow in and out.
- Isentropic process – the heated, pressurized air then gives up its energy, expanding through a turbine (or series of turbines). Some of the work extracted by the turbine is used to drive the compressor.
- Isobaric process – heat rejection (in the atmosphere).

Actual Brayton cycle:

- Adiabatic process – compression
- Isobaric process – heat addition
- Adiabatic process – expansion
- Isobaric process – heat rejection

Brayton cycle For Gas Turbine (Open Cycle)

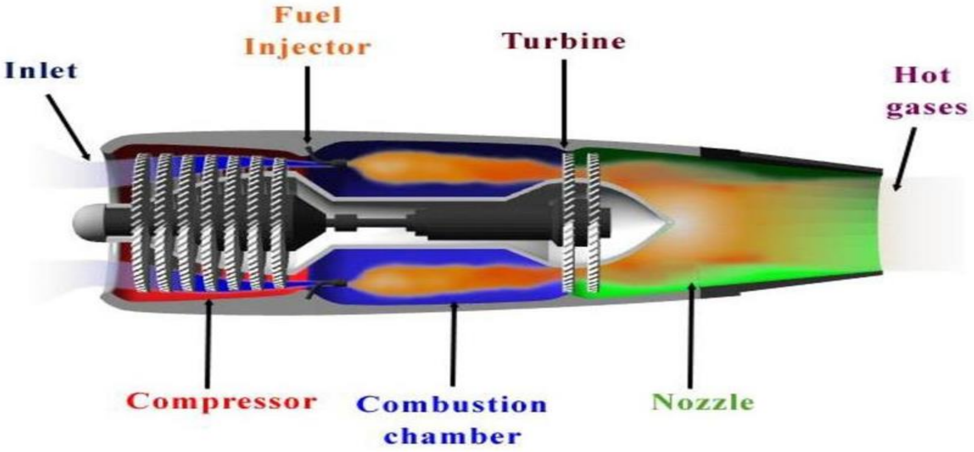
Consist four Processes

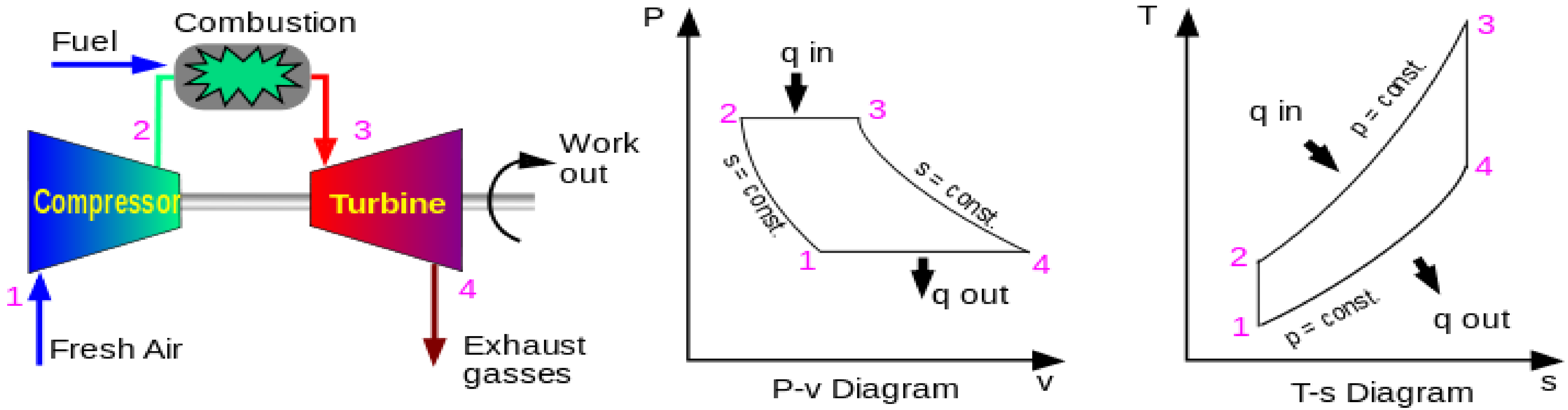
Processes 1 – 2: isentropic compression

Processes 2 – 3: constant pressure energy addition

Processes 3 – 4: isentropic expansion

Processes 4 – 1: constant pressure energy rejection





Heat added is

$$\mathbf{Q_{in} = m\,C_p\,(T_3 - T_2)}$$

Heat rejected is

$$\mathbf{Q_{out} = m\,C_p\,(T_4 - T_1)}$$

Thermal efficiency is

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

$$\eta = 1 - \frac{T_1\left[\left(\frac{T_4}{T_1}\right) - 1\right]}{T_2\left[\left(\frac{T_3}{T_2}\right) - 1\right]}$$

The pressure ratio of the Brayton cycle, r_p is defined as:

$$r_p = \frac{P_2}{P_1}$$

$$\frac{P_3}{P_4} = \frac{P_2}{P_1}$$

Then $r_p =$ $$\frac{P_3}{P_4} = \frac{P_2}{P_1}$$

The processes (1 - 2) and (3 - 4) are isentropic. Hence

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{(\gamma-1)}{\gamma}}$$

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{(\gamma-1)}{\gamma}}$$

We get

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\eta = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}$$

The net work done is

$$\mathbf{W = Q_{in} - Q_{out}}$$

Work delivered by the cycle is given by W = h Q_{in} , Increasing Q_{in} can increase work done by the cycle . Since the Turbine blade material cannot withstand very high temperature T_3 , hence Q_{in} is limited.

Example

Air enters the compressor of an ideal air standard Brayton cycle at 100 kPa, 25 °C , with a flow rate of 9.4712 kg /s. The compressor pressure ratio is 12. The turbine inlet temperature is 1100 °C. Determine (a) the thermal efficiency. (b) net power output .

$$P_2 = (r)^{\gamma} \times P1 = (12)^{1.4} \times 100 = 3242.3041 \text{ kpa}$$

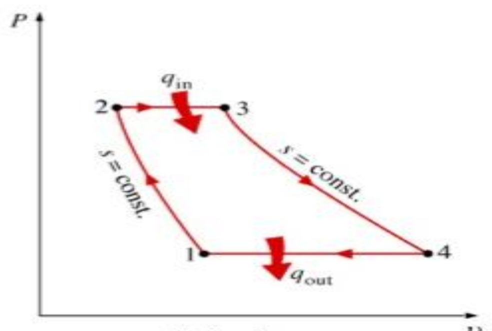
$$r_p = P_2 / P_1 = 3242.3041 / 100 = 32.423041$$

$$\eta = 1 – (1 / r_p)^{(\gamma - 1) / \gamma} = 1 – (1 / 32.423)^{(1.4 - 1) / 1.4} = 62.989 = 63 \%$$

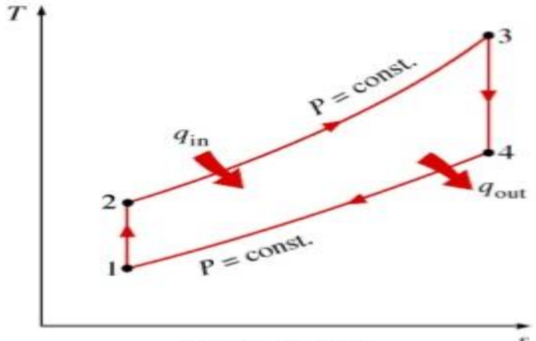
$$Q_{in} = m C_p (T_3 – T_2) = 9.4712 \times 1.005 \times (1373 – 805.172) = 5404.903 \text{ kw}$$

$$Q_{out} = m C_p (T_4 – T_1) = 9.4712 \times 1.005 \times (508.157 – 298) = 2000.391 \text{ kw}$$

$$W = Q_{in} – Q_{out} = 5404.903 – 2000.391 = 3404.511 \text{ kw}$$



(b) P- υdiagram



(a) T-s diagram