Mechanics of Materials (MEA214)

Lecture One : Introduction & Motivation

Prepared by
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Syllabus for 1st Semester

Chapter One : Simple Stress and Strain.

Chapter Two : Shearing Force and Bending Moment Diagrams.

Chapter Three : Bending.

Chapter Four : Slope and Deflection of Beams.

Chapter Five : Built-in Beams.

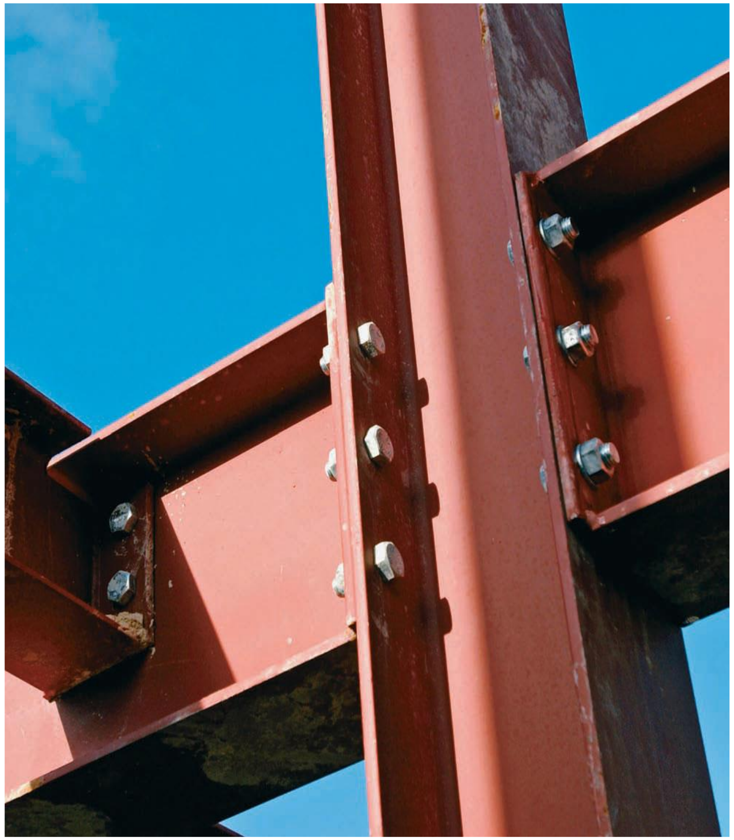
Chapter Six : Shear Stress Distribution.

References

- E. J. HEARN Mechanics of Materials, 1st Edition, Pergamon Press Ltd.,(1977).

What is Mechanics of Materials?

- Is a branch of applied mechanics.
- Deals with the behavior of solid bodies subjected to various types of loading.
- Other names for this field of study are Strength of Materials and Mechanics of Deformable Bodies.



Why Study Mechanics of Materials?

- The main objective of the study of the mechanics of materials is to provide the future engineer with the means of analyzing and designing various machines and load-bearing structures. Both the analysis and the design of a given structure involve the determination of stresses and deformations.

Why Study Mechanics of Materials?

- The Deflections resulting and the stresses and strains set up within bodies , are all considered in an attempt to provide sufficient knowledge to enable any component to be designed such that it will not fail within its service.

Mechanics of Materials (MAE214)

Lecture Two : Simple Stress and Strain

By

Mr. Mohammed Y. Yousif

Load

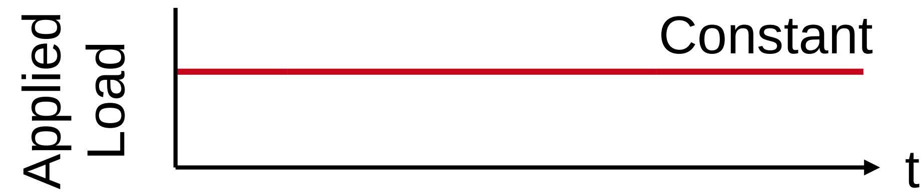
External forces that affect engineering structures and arising from the service conditions or environment in which the component works.

In the SI system of units load is measured in newtons, although a single newton, in engineering terms, is a very small load. In most engineering applications, therefore, loads appear in SI multiples, i.e. kilonewtons (kN) or meganewtons (MN).

Types of Load

(a) Static or dead loads

Non-fluctuating loads , generally caused by gravity effects.



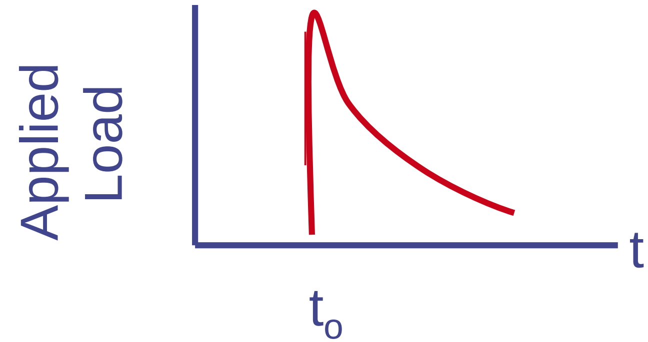
(b) Live Loads

As produced by , for example, lorries crossing a bridge, and characterized by :

1. Slowly applied.
2. Sustained for a period of time.
3. Slowly removed.

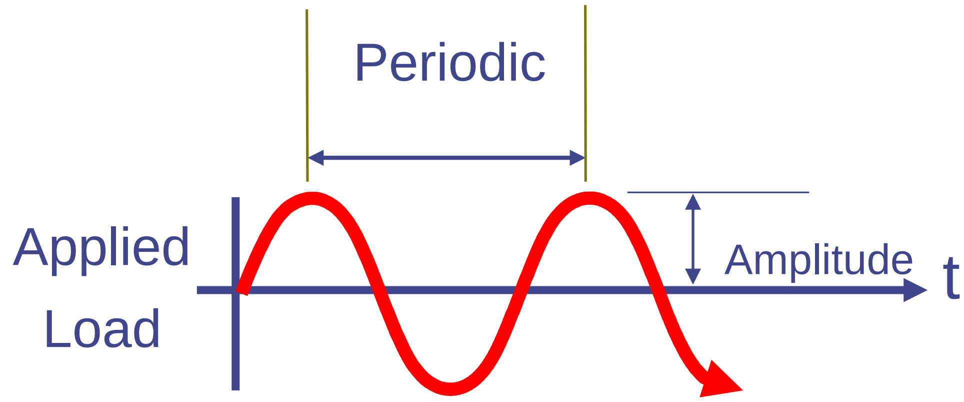
(c) Impact or Shock Loads

Loads caused by sudden blows.



(d) Fatigue, fluctuating or alternating Loads

The magnitude and sign of the load changing with time.



(e) Environmental Loads

loads that act on a structure as a result of environmental conditions (e.g. thermal loads arising from temperature variations etc.).

Direct or Normal Stress (σ)

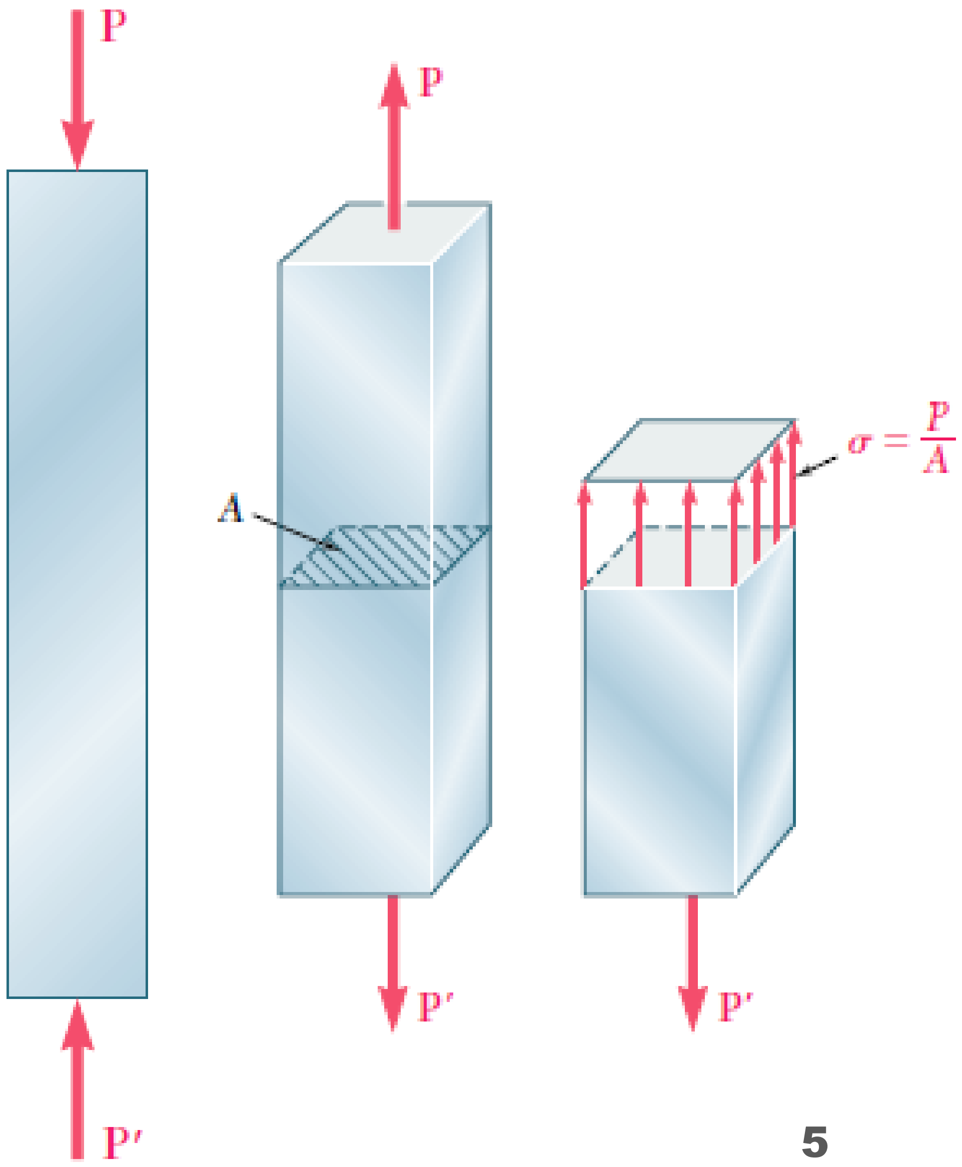
When a bar subjected to a uniform tension or compression, i.e. a direct force which is uniformly applied across the cross section, the internal forces set up are also distributed uniformly and the bar is said to be subjected to a uniform direct or normal stress.

$$\sigma = \frac{P}{A}$$

σ : Stress

P : Applied Load

A : Cross-section Area



Direct or Normal Stress (σ) Unit

In SI units, Stress will be expressed in N/m^2. This unit is called a Pascal (Pa) .

$$1\ \text{kPa} = 10^3\ \text{Pa} = 10^3\ \text{N/m}^2$$

$$1\ \text{MPa} = 10^6\ \text{Pa} = 10^6\ \text{N/m}^2$$

$$1\ \text{GPa} = 10^9\ \text{Pa} = 10^9\ \text{N/m}^2$$

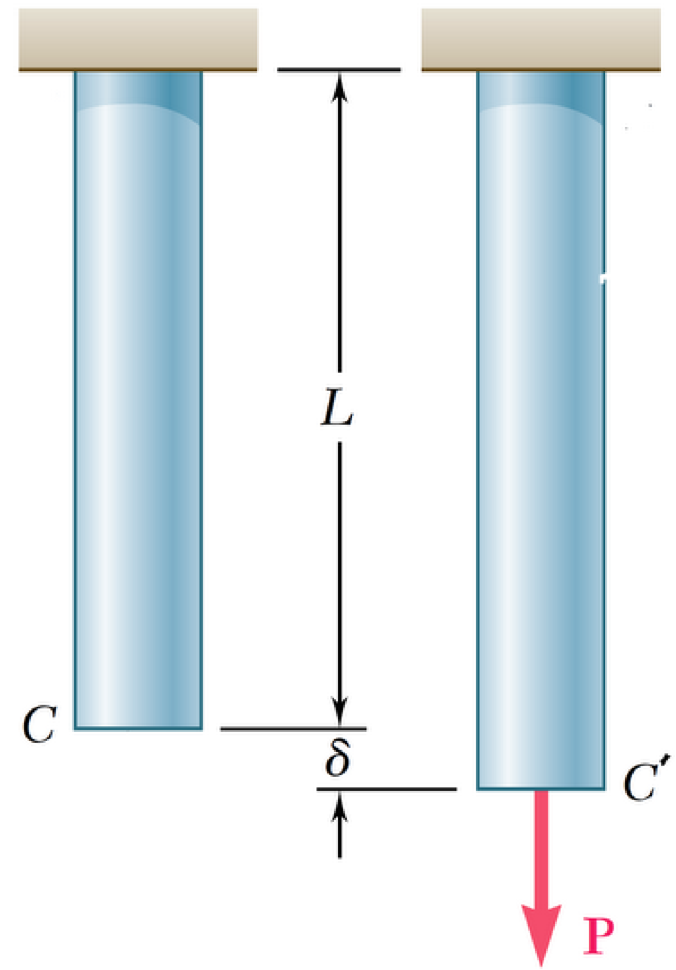
When British units are used, the force P is usually expressed in pounds (lb) or kilopounds (kip), and the cross-sectional area A in square inches (in^2). The stress s will then be expressed in pounds per square inch (psi) or kilopounds per square inch (ksi).

Direct Strain (ε)

If a bar subjected to a direct load, and hence stress, the bar will change in length, as a result, the strain produced as follows:

$$\epsilon = \frac{\delta}{L}$$

ε : Strain
δ or δL : Change in Length
L : Original Length



Note : Strain has no unit

Sign Convention for Direct Stress and Strain

Tensile stresses and strains are considered POSITIVE (+) in sense

Compressive stresses and strains are considered NEGATIVE (-) in sense

Hooke's Law & Young's Modulus

In elastic materials, the deformation produced by any load will completely recovered when the load is removed. According that, stress is proportional to strain , Hooke's Law therefore states that:

$$\sigma \ \alpha \ \varepsilon$$

i.e.

$$\frac{\text{stress}}{\text{strain}} = \text{constant}$$

The constant is given the symbol E and termed the Modulus of Elasticity or Young's Modulus

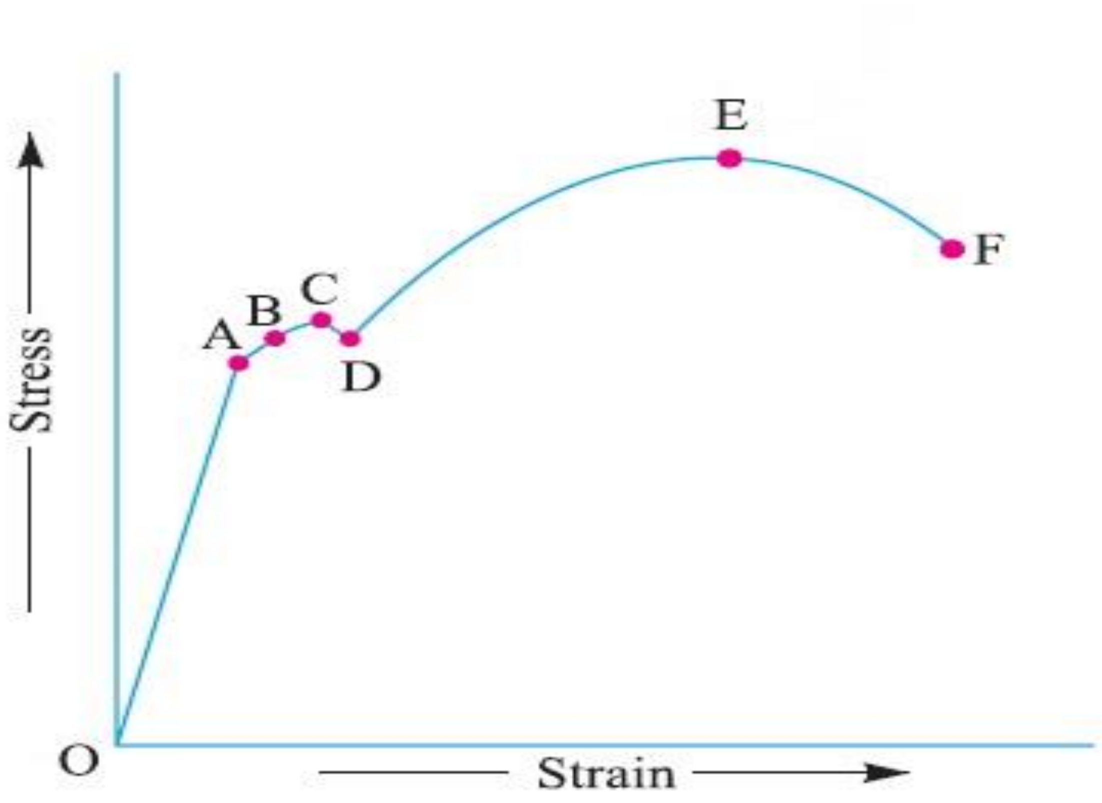
$$E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\varepsilon}$$

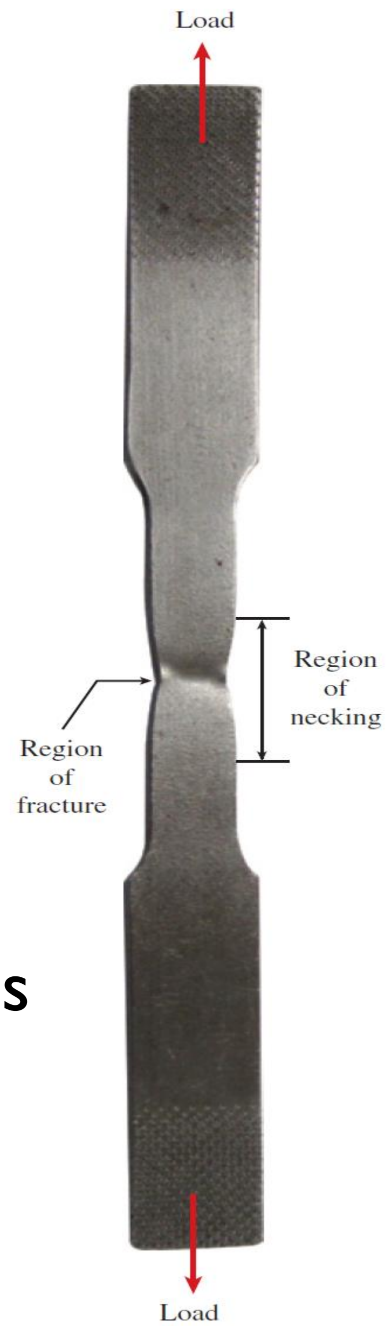
Tensile Test

It is important test to determine the strengths of various materials. During this test a bar of uniform cross-section subjected to a gradually increasing tensile load until failure occurs.

Measurements of the change in length of selected gauge length of the bar are recorded by means of extensometers. The graph of load against extension or stress against strain.

Stress-Strain Diagram





A: limit of proportionally

B : Elastic limit

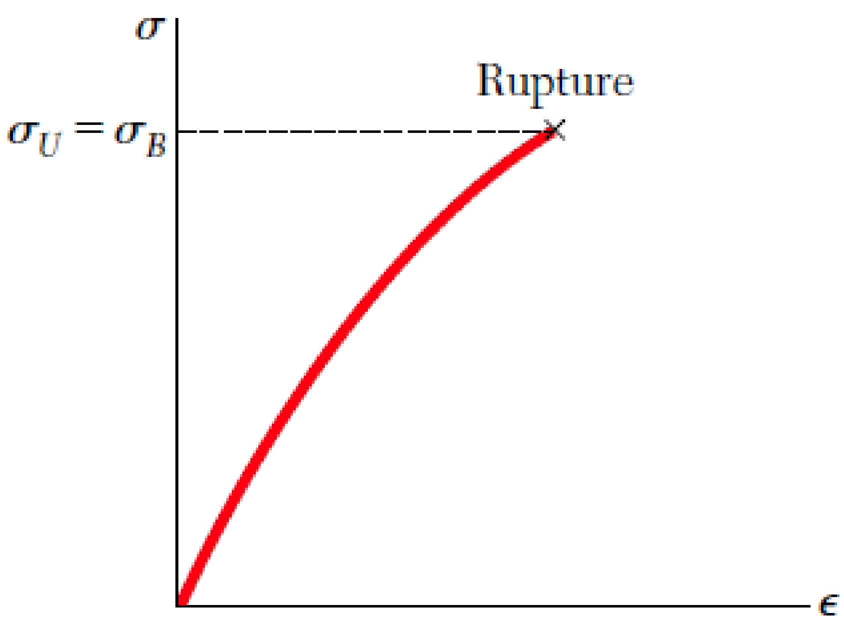
C : Upper Yield Point

D : Lower Yield Point

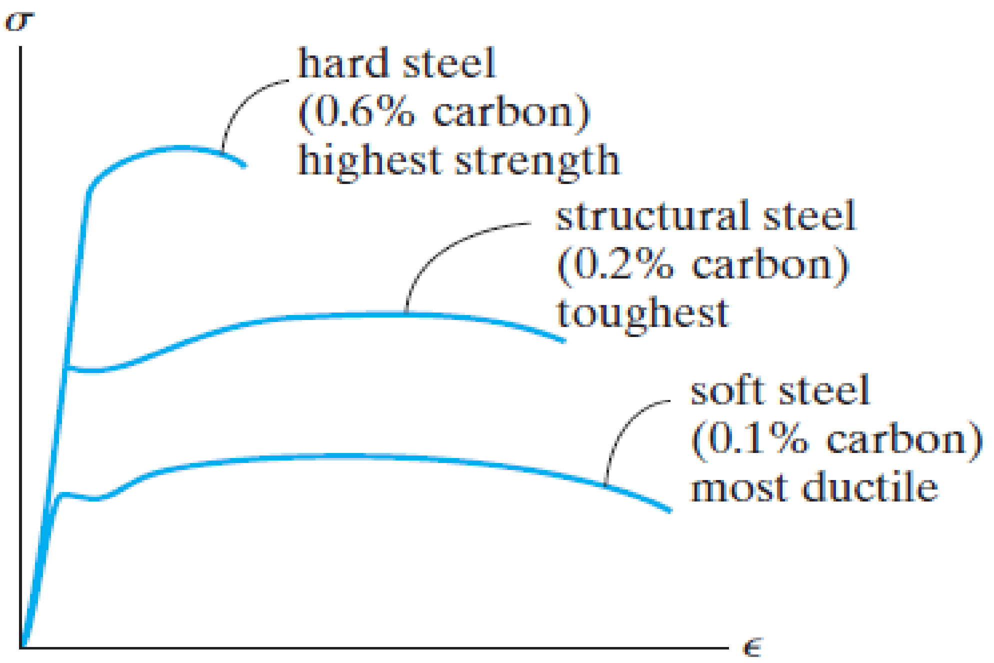
E: Ultimate Tensile Stress

F: Fracture Point

Note: The previous stress-strain diagram is a typical result for a test on a mild (low carbon) steel bar, other materials will exhibit different graphs but of a similar general form.



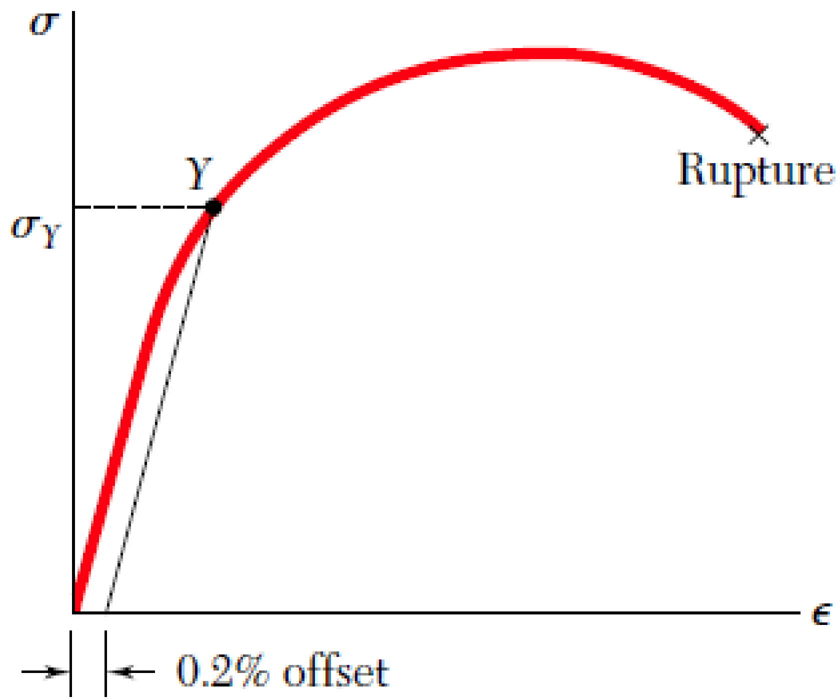
Brittle Material



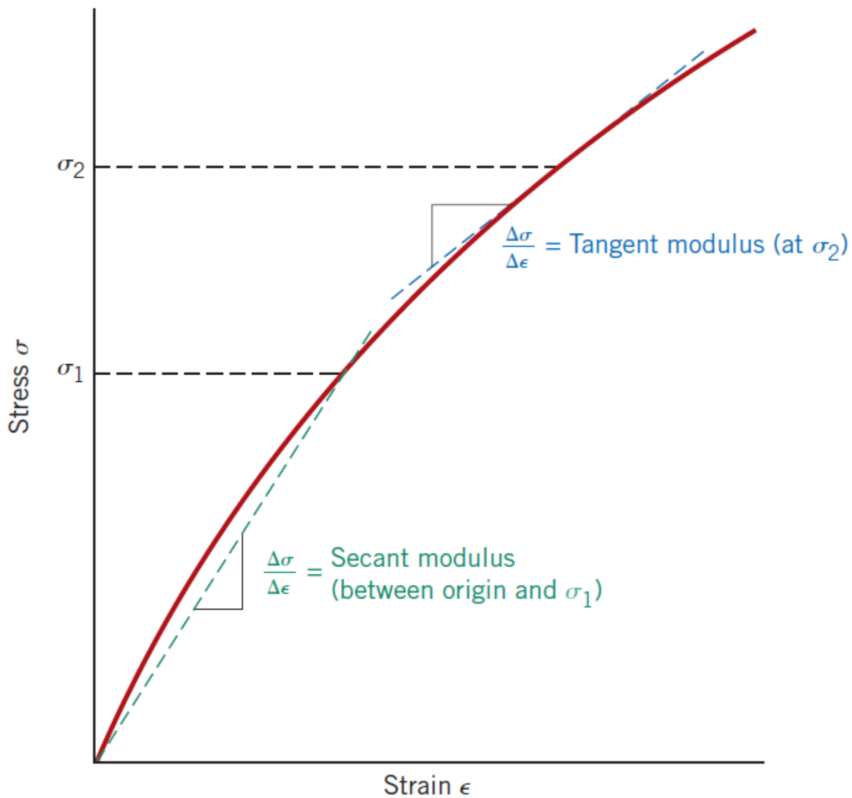
Soft and Hard Materials

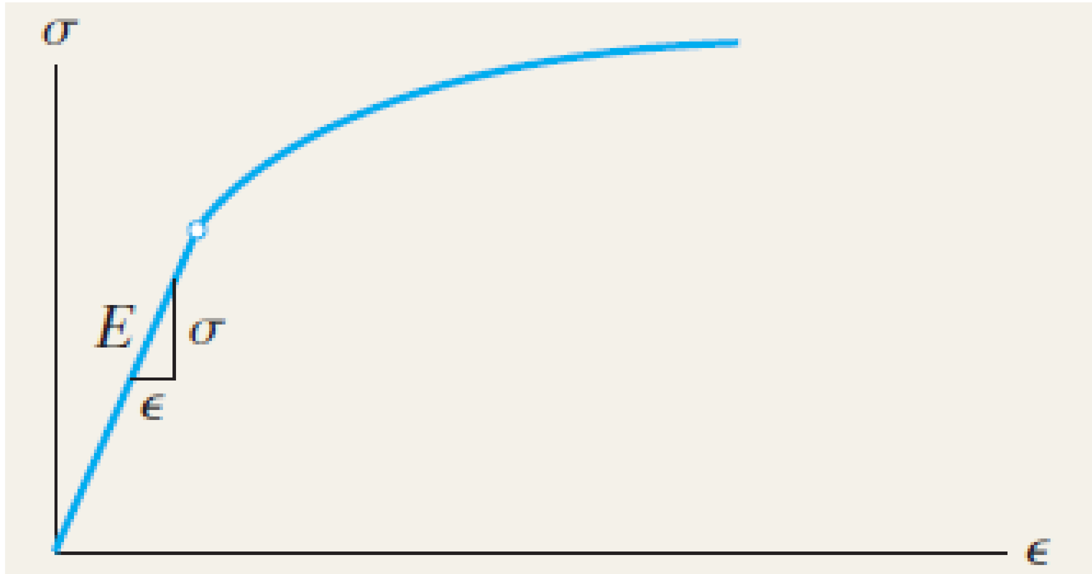
Proof Stress

In the case of aluminum and of many other ductile materials, the onset of yield is not characterized by a horizontal portion of the stress-strain curve. The yield strength is obtained by drawing through the point of the horizontal axis of abscissa ε= 0.2% (or ε= 0.002), a line parallel to the initial straight-line portion of the stress-strain diagram.



Calculation of Young's Modulus





$$E = \frac{\Delta \sigma}{\Delta \varepsilon}$$

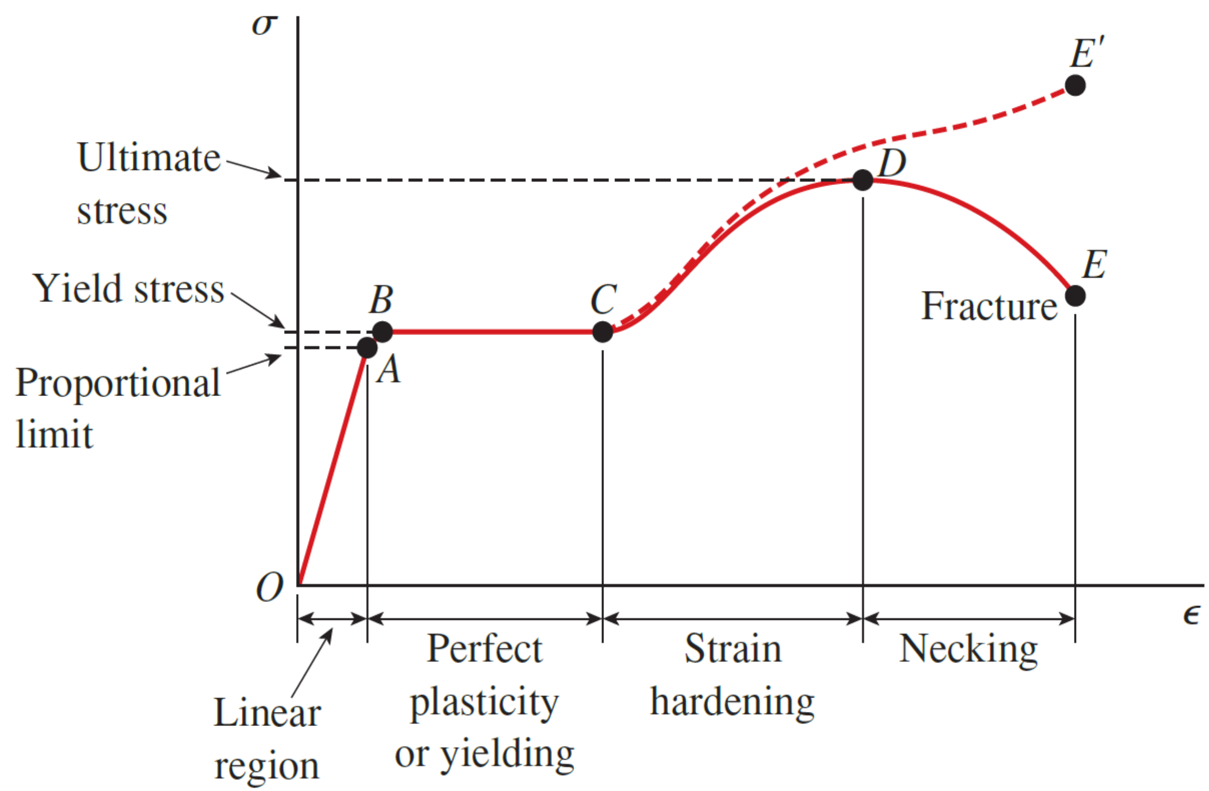
True Stress and True Strain

$$\sigma_t = \frac{F}{A_i}$$

$$\varepsilon_t = ln\frac{l_i}{l_o}$$

$$\varepsilon_t = ln(1 + \varepsilon)$$

$$\sigma_t = \sigma(1 + \varepsilon)$$



σ_t : True Stress

ε_t: True Strain

A_i: Actual (instantaneous) Area

l_i: Actual (instantaneous) Length

Assignment

Proof that:

$$\varepsilon_t = ln(1 + \varepsilon)$$

$$\sigma_t = \sigma(1 + \varepsilon)$$

Ductility

Define as material's ability to exhibit the plastic deformation. The ductility of a material in tension can be characterized by its elongation and by the reduction in area at the cross section where fracture occurs.

$$\text{Percent elongation} = \frac{L_1 - L_0}{L_0}(100)$$

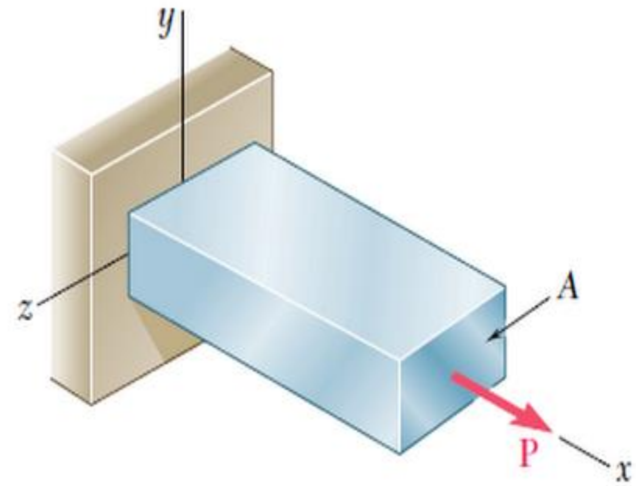
$$\text{Percent reduction in area} = \frac{A_0 - A_1}{A_0}(100)$$

Poisson's ratio

In all engineering materials, the elongation produced by an axial tensile force **P** in the direction of the force is accompanied by a contraction in any transverse direction.

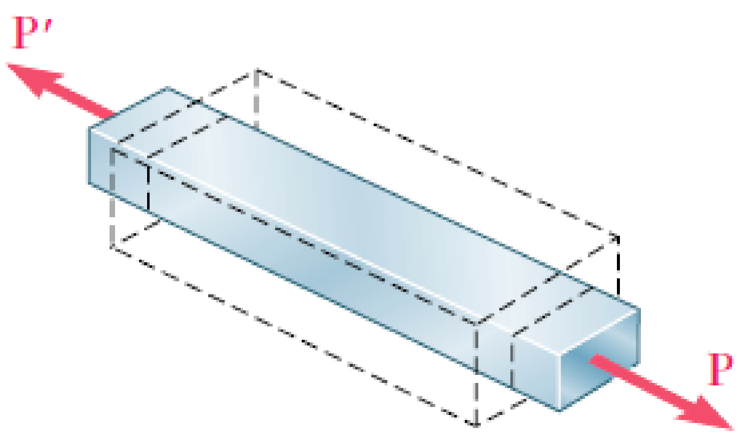
$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}}$$

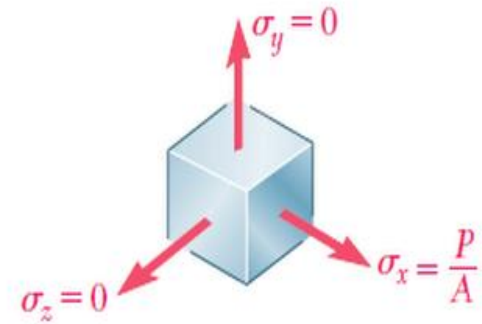
$$\nu = -\frac{\epsilon_y}{\epsilon_x} = -\frac{\epsilon_z}{\epsilon_x}$$



$$\epsilon_x = \frac{\sigma_x}{E}$$

$$\epsilon_y = \epsilon_z = -\frac{\nu\sigma_x}{E}$$

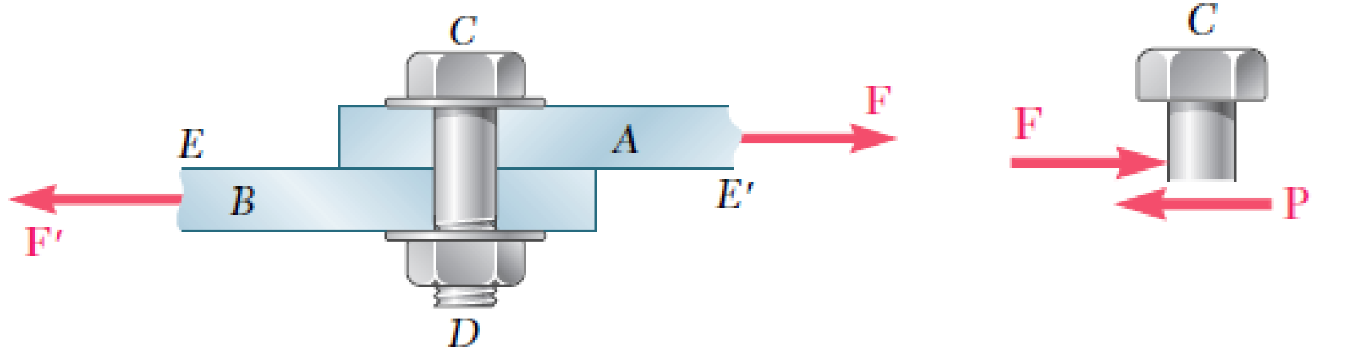




Direct Shear Stress τ

It is also the intensity of internal force but shear stress acts on a surface that is parallel to the surface of material.

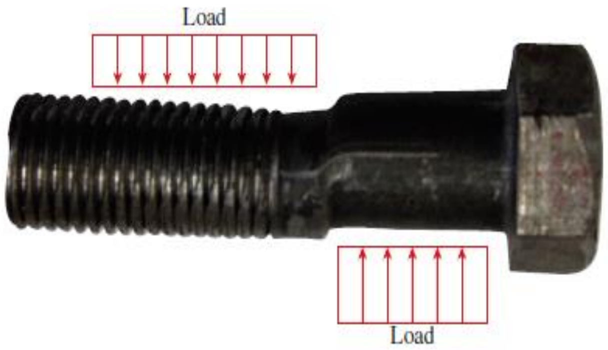
$$\tau_{ave} = \frac{P}{A}$$



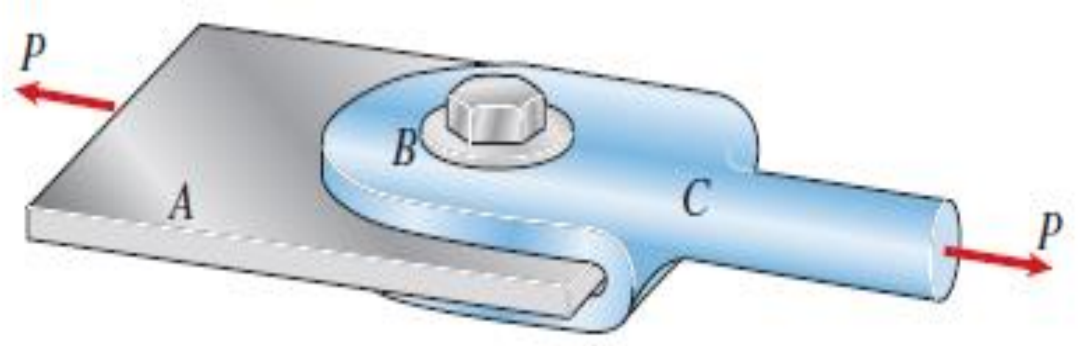
τ_{ave} : Average shear Stress

P : Applied Load

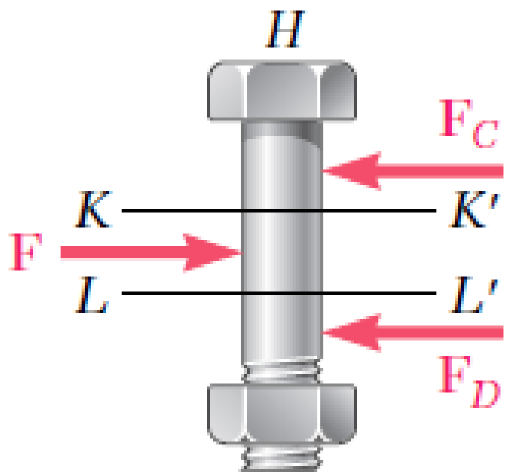
A : Cross-section Area

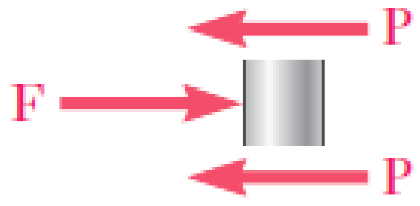


Double Shear



$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F/2}{A} = \frac{F}{2A}$$

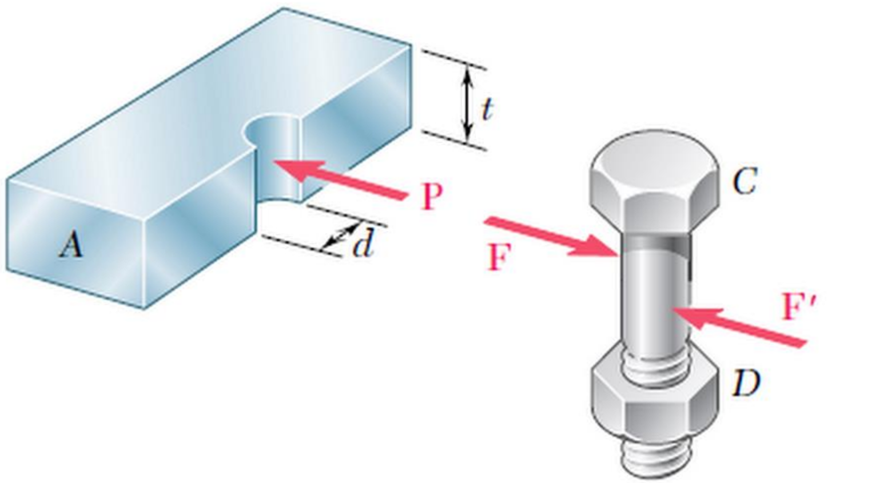




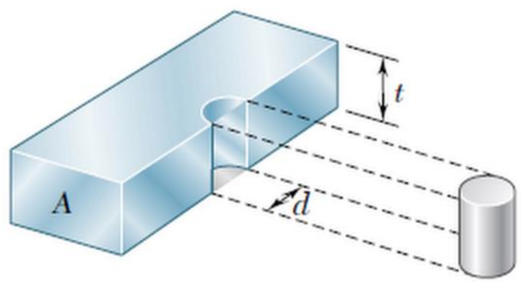
Bearing Stress

Bolts, pins, and rivets create stress in the members they connect, along the bearing surface, or surface of contact. an average nominal value of the stress, called the bearing stress.

$$\sigma_b = \frac{P}{A} = \frac{P}{td}$$

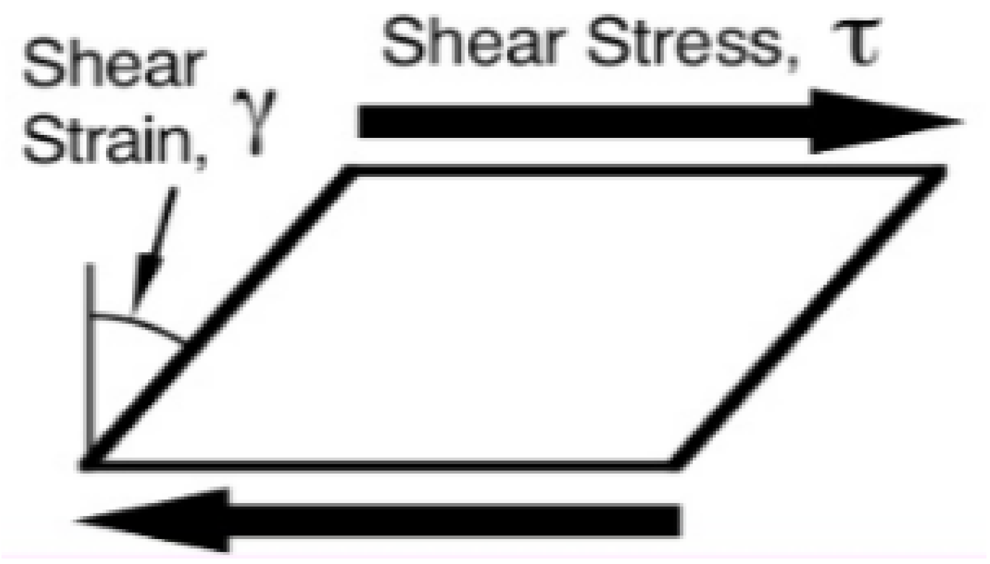


σ_b : Bearing Stress



Shear Strain γ

Change in shape (deformation) of material as a result of shear stress. The angle of deformation is then termed the shear strain.



Shear strain measured in radians and hence is non-dimensional, i.e. it has no units.

Modulus of rigidity

For many materials, the initial part of the shear stress-strain diagram is a straight line through the origin, just as it is in tension. For this linearly elastic region, the shear stress and shear strain are proportional, and therefore we have the following equation for Hooke's law in shear.

$$\boldsymbol{\tau = G\gamma}$$

$$\boldsymbol{G = \frac{\tau}{\gamma}}$$

G : Shear modulus or modulus of rigidity

Thermal Stress & Strain

When a material undergoes a change in temperature,

- It either elongates or
- contracts depending upon
- whether temperature is increased or decreased of the material

If the elongation or contraction is not restricted i e free then

- the material does not experience any stress despite the fact that it undergoes a strain
- The strain due to temperature change is called thermal strain and is expressed as

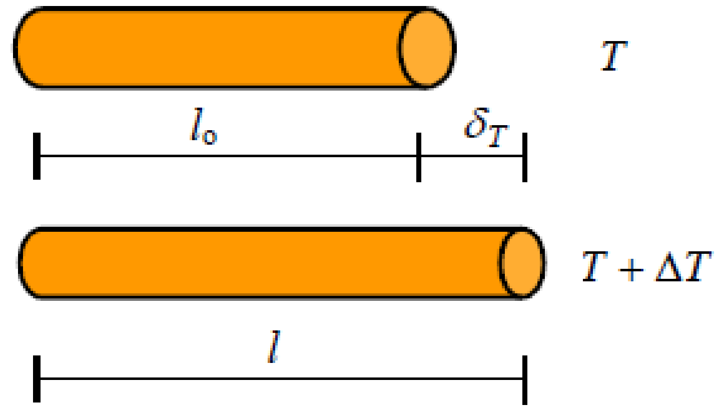
$$\varepsilon_{th} = \Delta T * \alpha$$

ΔT :Temperature Difference

α : linear coefficient of thermal expansion

Increase or decrease in length due to change in temperature is given by;

$$\delta = \boldsymbol{L} * \Delta \boldsymbol{T} * \boldsymbol{\alpha}$$



If the ends of the body are fixed to rigid supports, so that its expansion or contraction is prevented, then stress induced in the body;

$$\sigma_{th} = \varepsilon_{th} * \boldsymbol{E} = \Delta \boldsymbol{T} * \boldsymbol{\alpha} * \boldsymbol{E}$$

Note: Compressive stresses are produced in expansion and tensile in contraction.

Mechanics of Materials (MAE214)

Lecture Three : Shear Force and Bending Moment Diagrams of loaded Beams

By

Mr. Mohammed Y. Yousif

Objective

Draw the shear force (S.F) and Bending Moment (B.M.) diagrams for certain loaded beams.

What is Beam?

A beam is a structural member(horizontal) that is design to support the applied load (vertical). It resists the applied loading by combination of internal transverse shear force and bending moment (i.e. transfers vertical load horizontally).

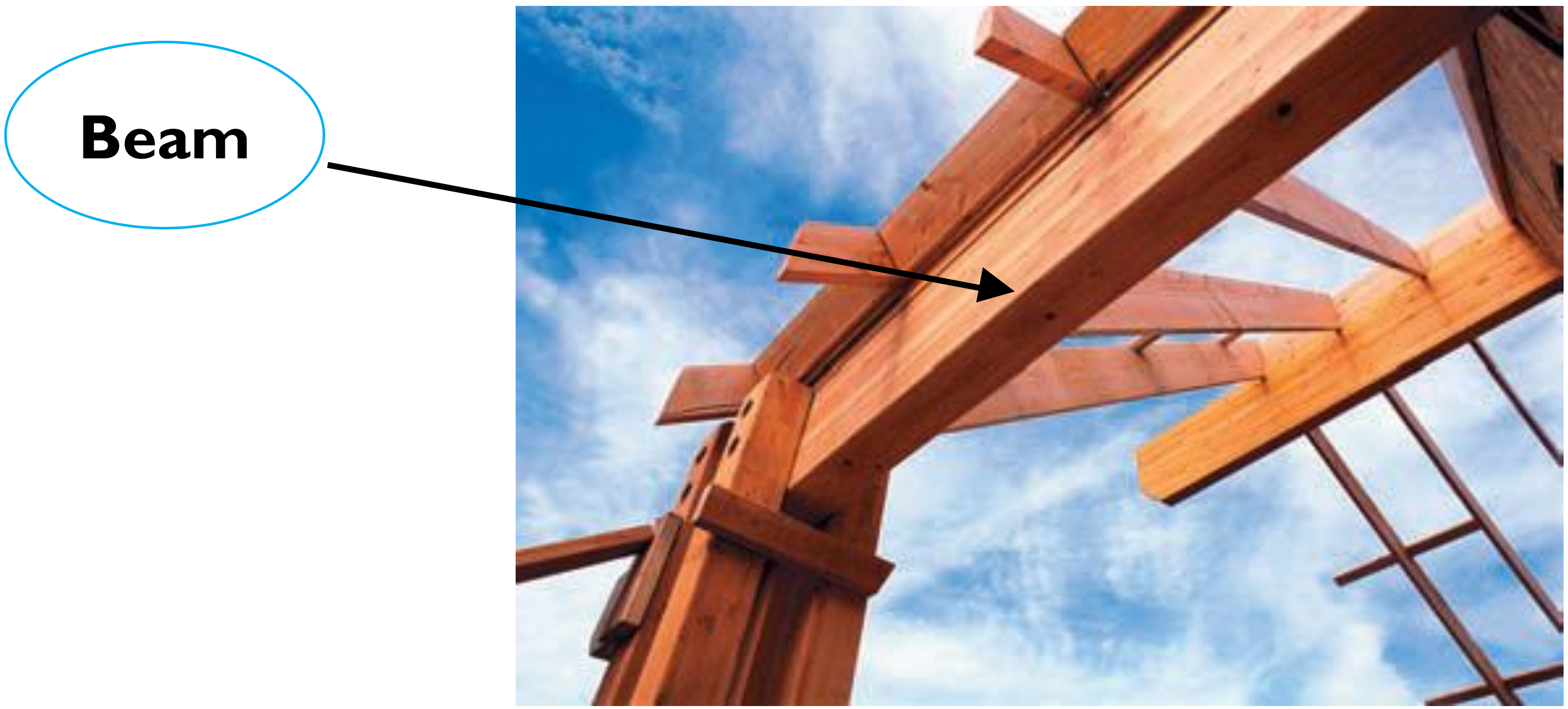
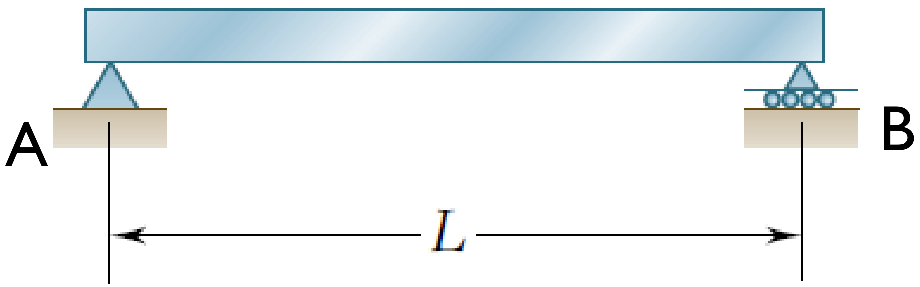


Figure Shows Timber beams used in residential dwelling

Types of Beams

Beams are classified according to the way in which they are supported to :

(a) Simply Supported Beam: A beam resting freely on the supports at its both ends.



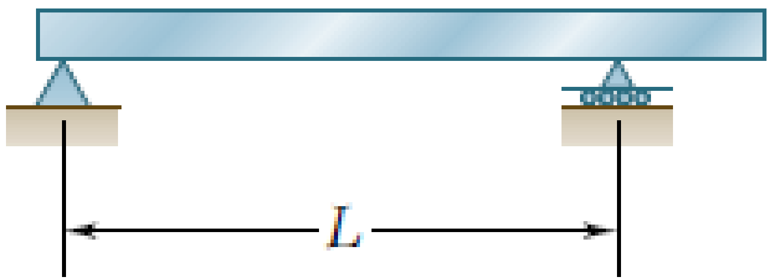
(a) Simply supported beam

At the left (pin support) (A) cannot move horizontally or vertically but the axis of the beam can rotate.

At the right (roller support) (B) can move horizontally and rotate but the vertical movement is restricted.

Types of Beams

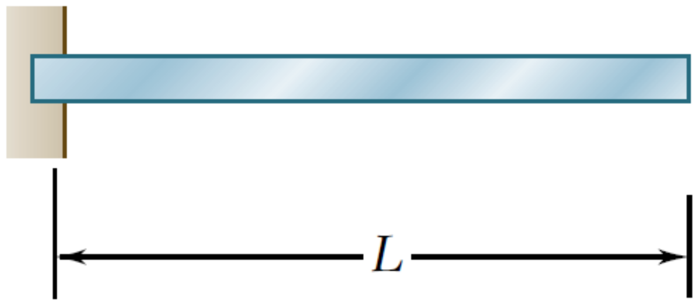
(b)Overhanging Beam : If the end portion of a beam is extended outside the supports.



(b) Overhanging beam

Types of Beams

(c)Cantilever Beam : A beam which is fixed at one end and free at the other end.



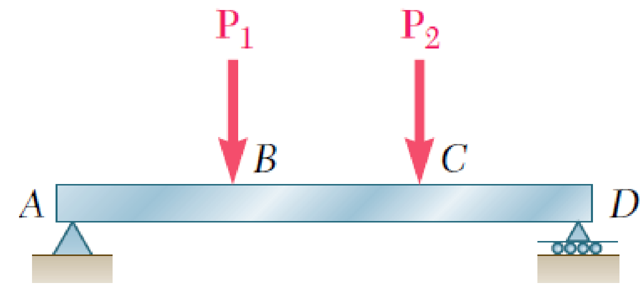
(c) Cantilever beam

At fixing point there is no movement (vertically or horizontally) and also there is no rotation.

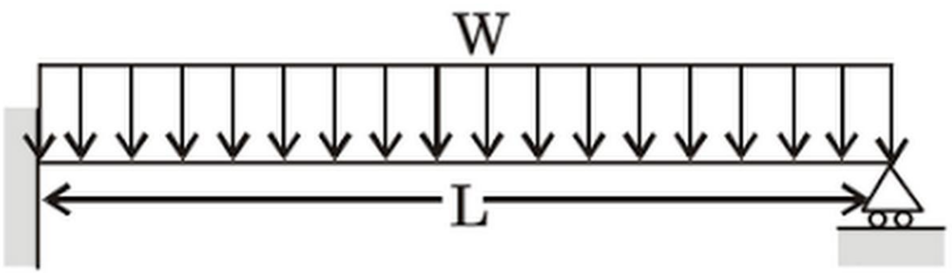
Types of Load

The transverse loading of a beam may consist of:

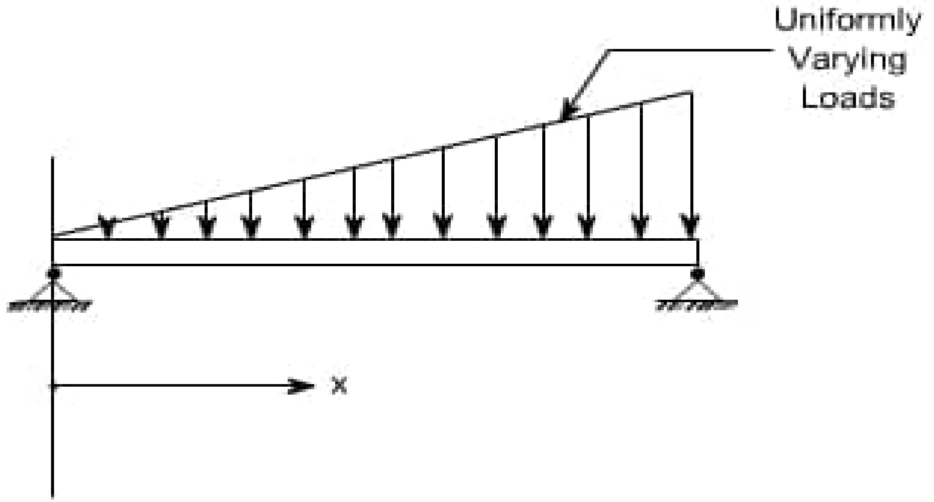
(a) Concentrated Load (Point Load)



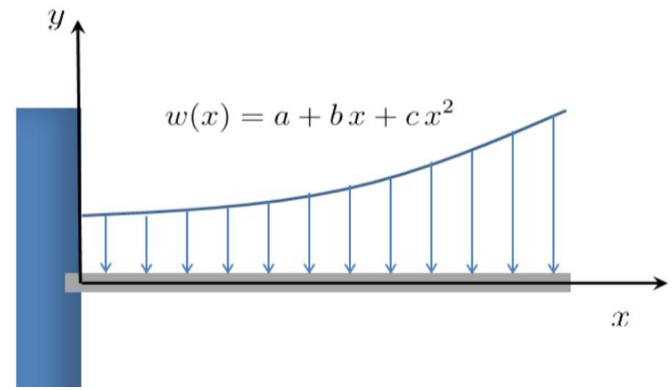
(b) Uniformly Distributed Load



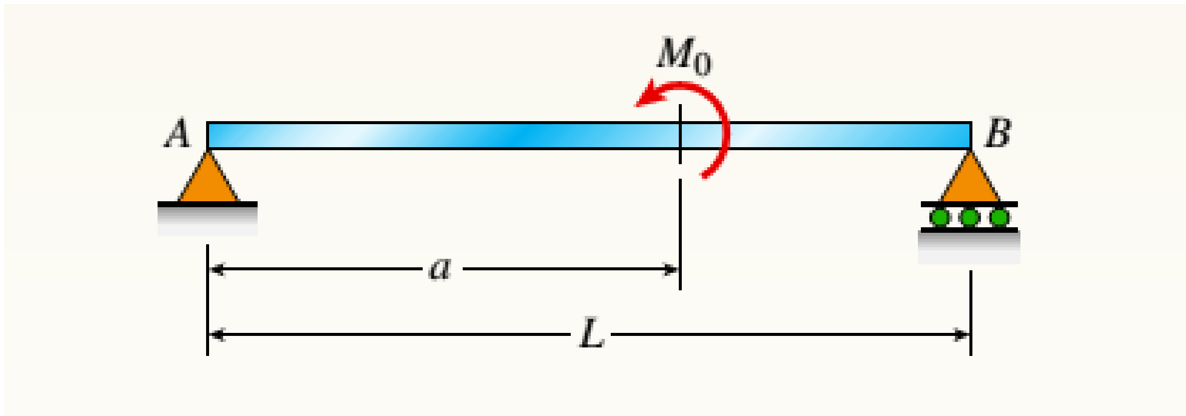
(c) Linearly Distributed Load



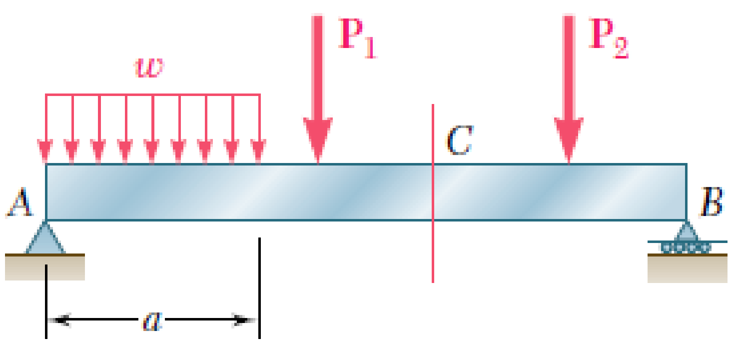
(d) Non-linearly Distributed Load

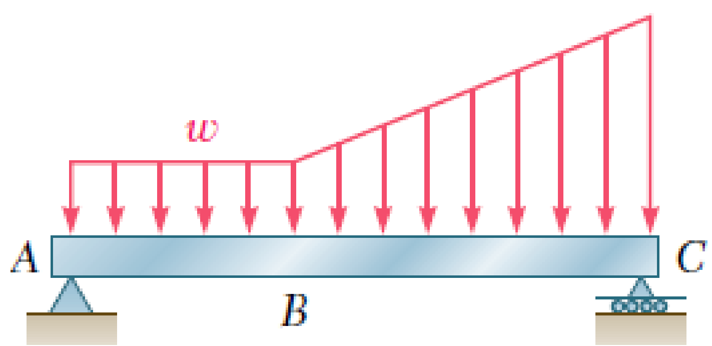


(e) Couple Moment

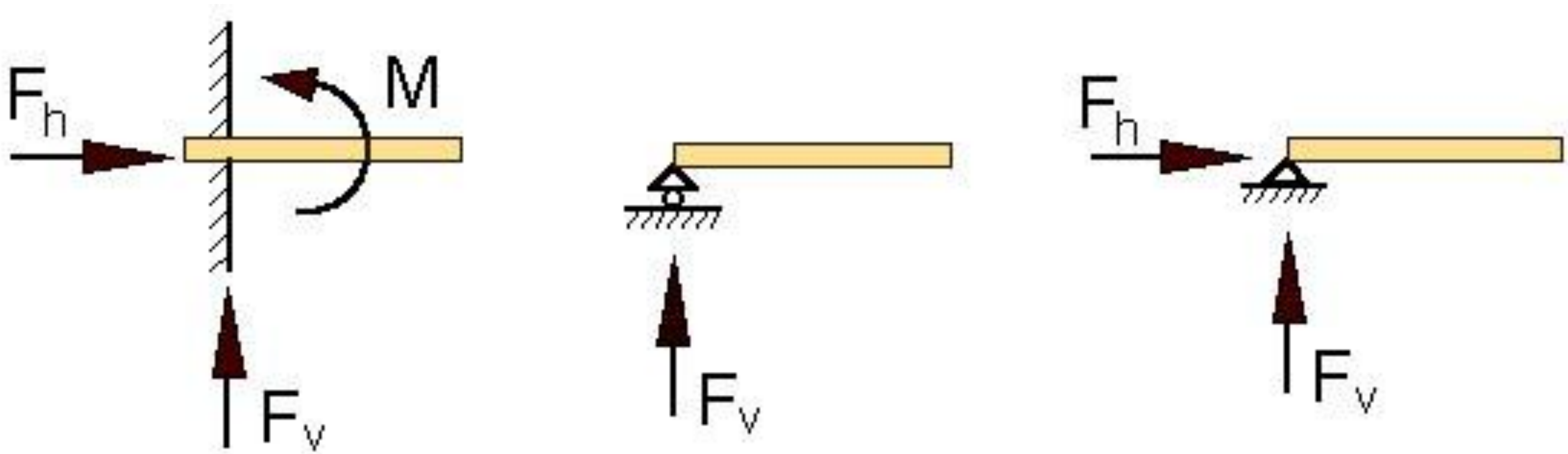


(f) Compound Load





Reactions at Support Points

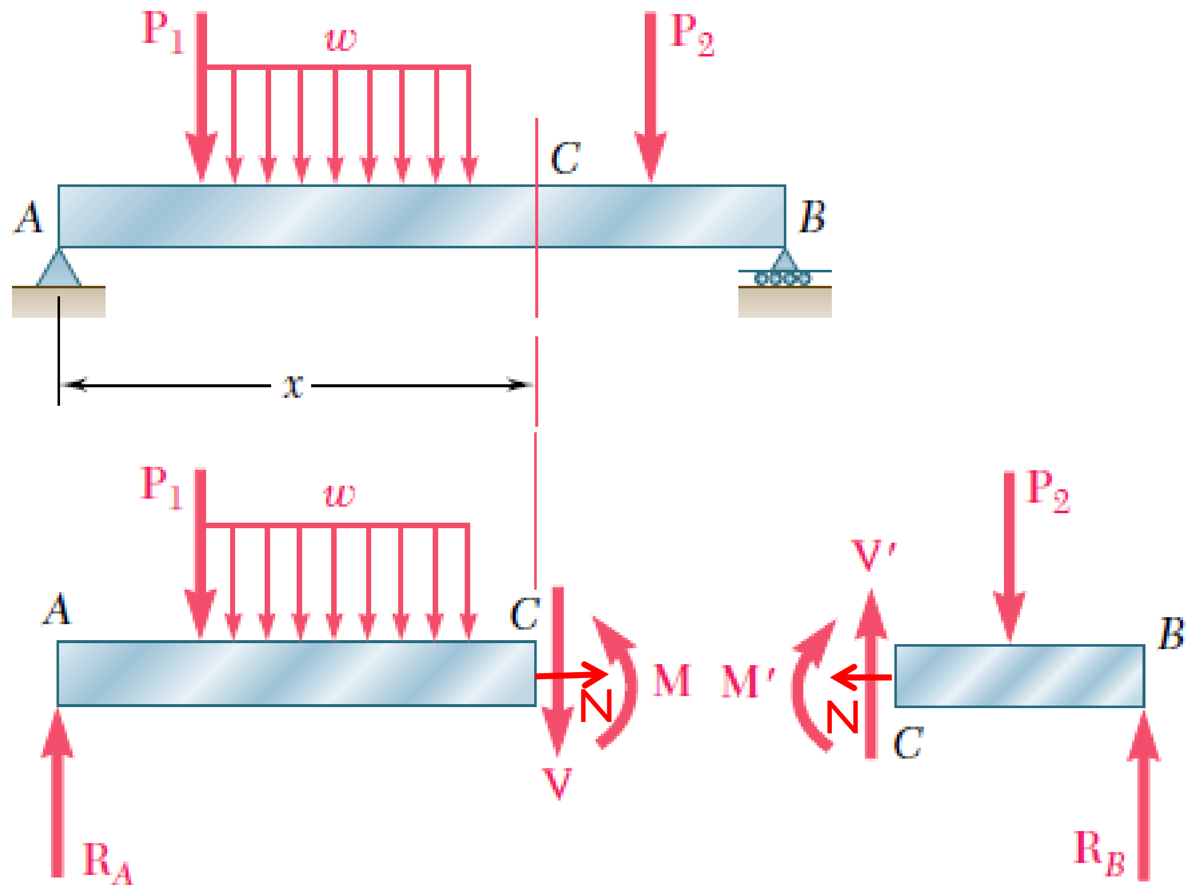


(a) Cantilever Beam (b) Roller support (c) Pin Support

Shear Force and Bending Moment

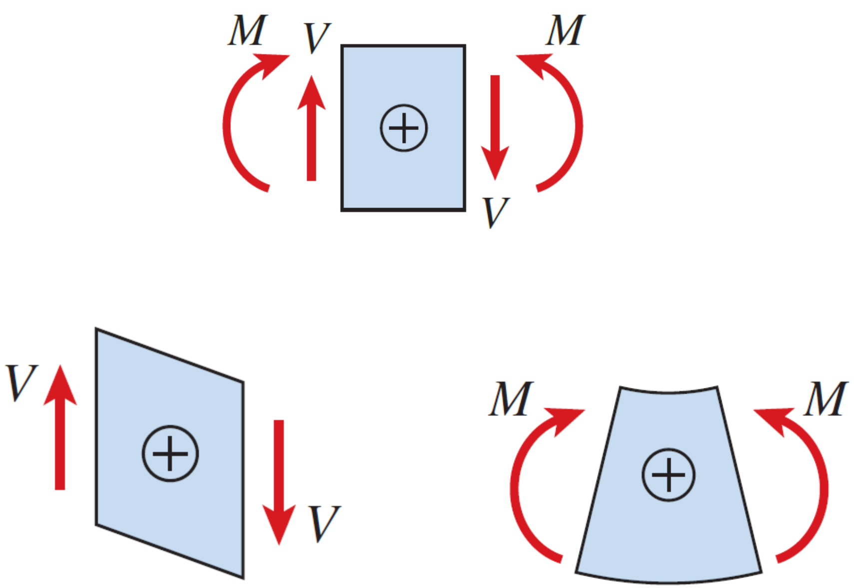
At any cut in a beam, there are 3 possible internal reactions required for equilibrium:

- normal force,
- shear force,
- bending moment

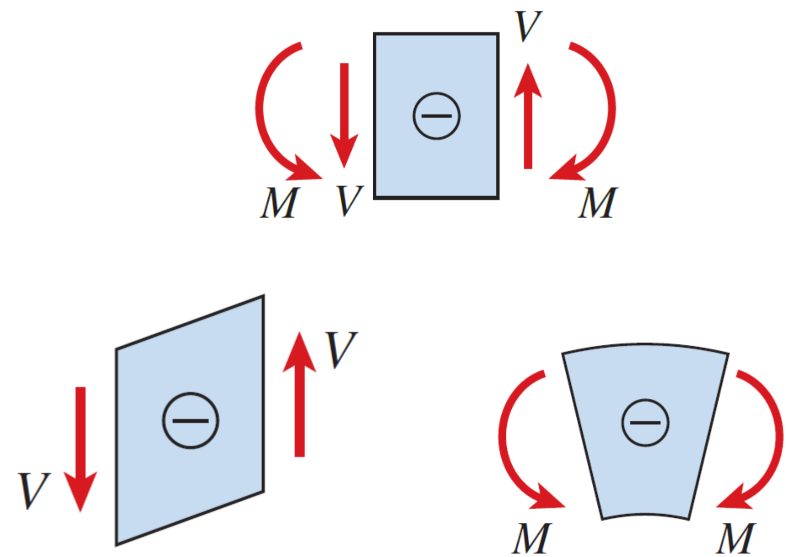


Deformation Sign Conventions

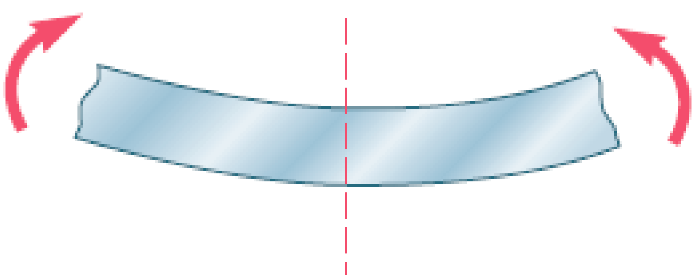
Positive



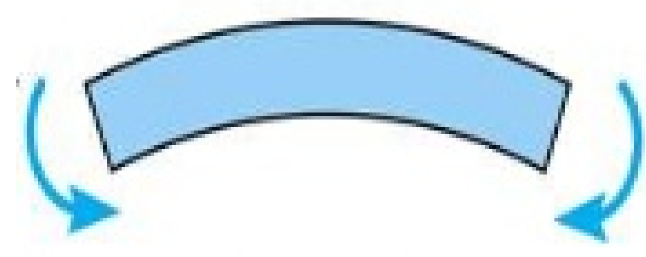
Negative



Sagging

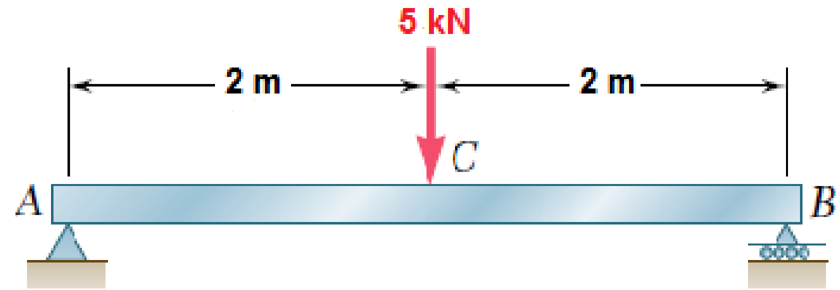


Hogging



Note: deformation sign conventions are based upon how the material is deformed. By contrast, when writing equations of equilibrium we use static sign conventions, in which forces are positive or negative according to their directions along the coordinate axes.

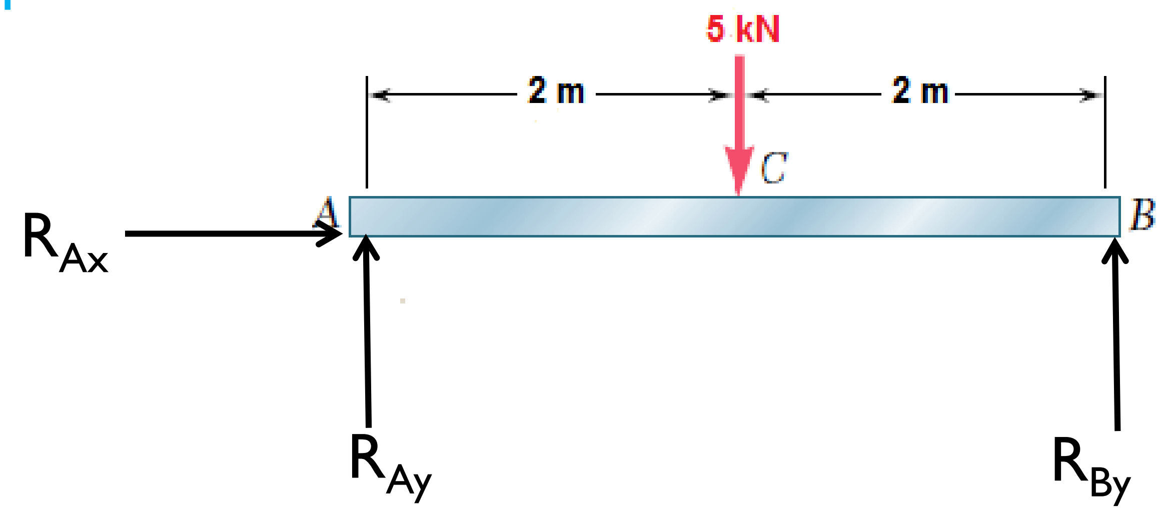
Example No. 1: Draw the shear force and bending moment diagrams for the beam shown.



Solution:

1.Drawing the free body diagram of the loaded beam.

2.Finding the unknown reactions by application the equilibrium equations.



Free Body Diagram

$\sum Fx=0 \Rightarrow R_{AX}=0$

$\sum Fy=0$

$R_{Ay}+R_{By} = 5\ kN$ .... (1)

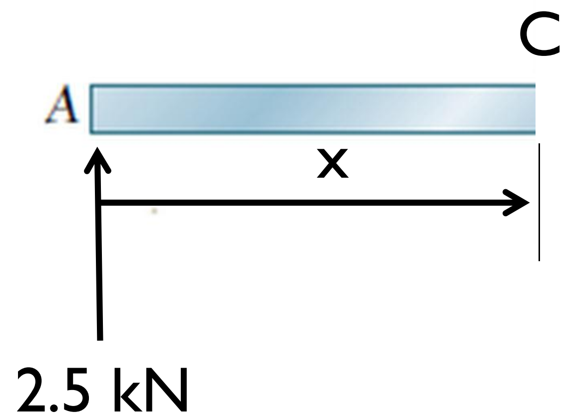
$\sum MB=0$

$R_{Ay} * 4 - 5 * 2=0 \Rightarrow R_{Ay}= 2.5\ kN$

Sub. R_{Ay} value in eq. (1) to find $R_{By.}$

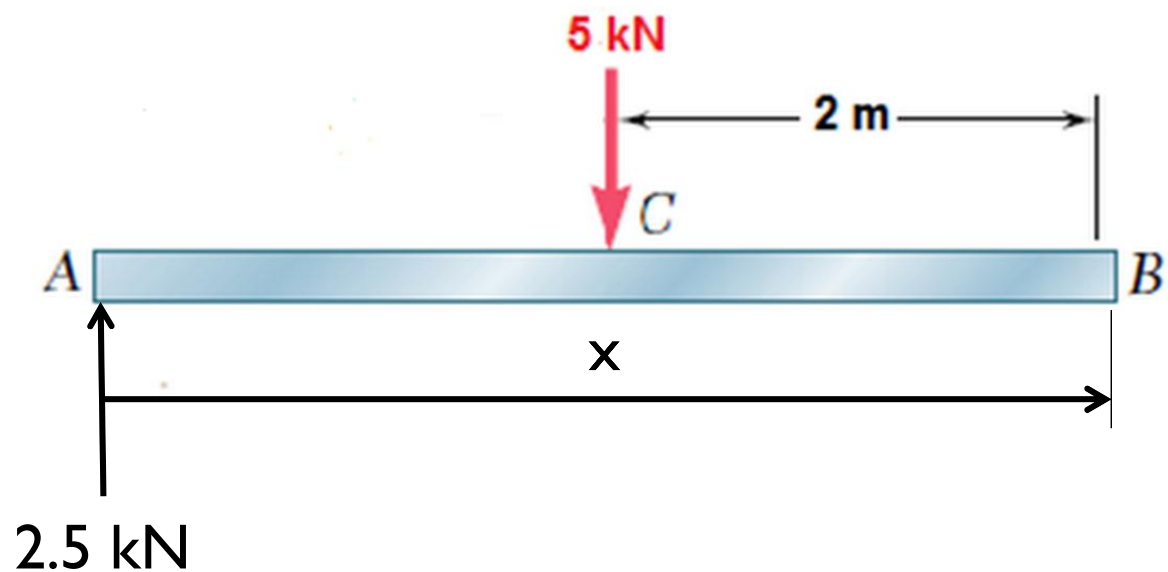
R_{By}= 5 – 2.5= 2.5 kN

A ⟶ C 0 ≤ x ≤ 2



S.F = 2.5 kN

B.M= 2.5 x

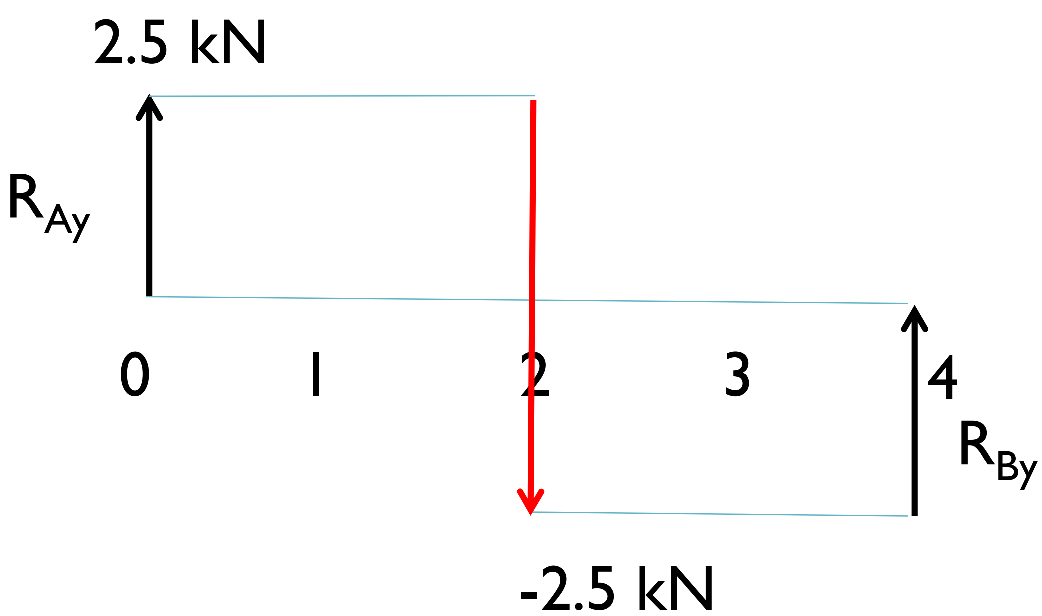


C ⟶ B 2 ≤ x ≤ 4

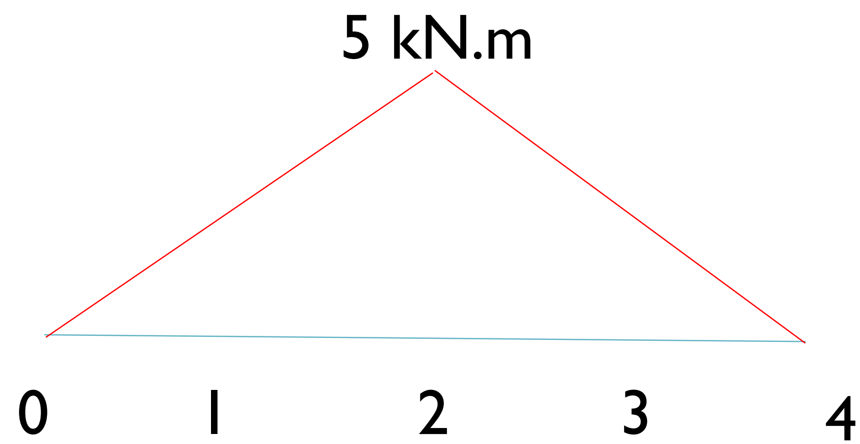
S.F = 2.5 – 5 = -2.5 kN

B.M= 2.5 x – 5 (x-2)

x	S.F (kN)	B.M (kN.m)
0	2.5	0
1	2.5	2.5
2	2.5	5
2	-2.5	5
3	-2.5	2.5
4	-2.5	0

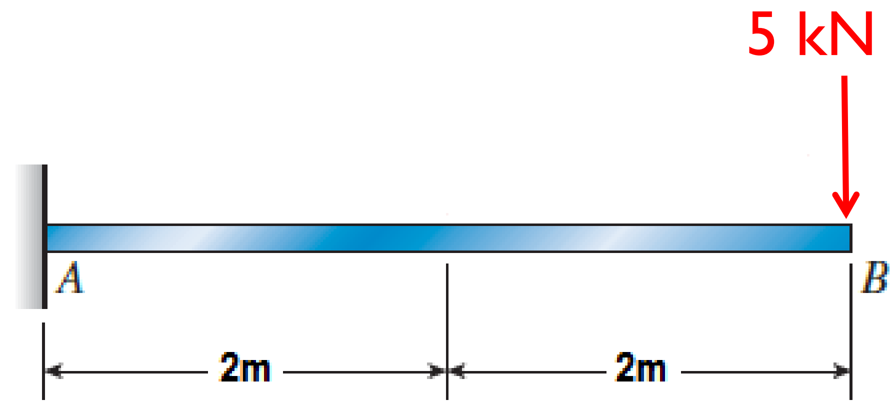


Shear Force Diagram



Bending Moment Diagram

Example No. 2: Draw the shear force and bending moment diagrams for the loaded beam shown.

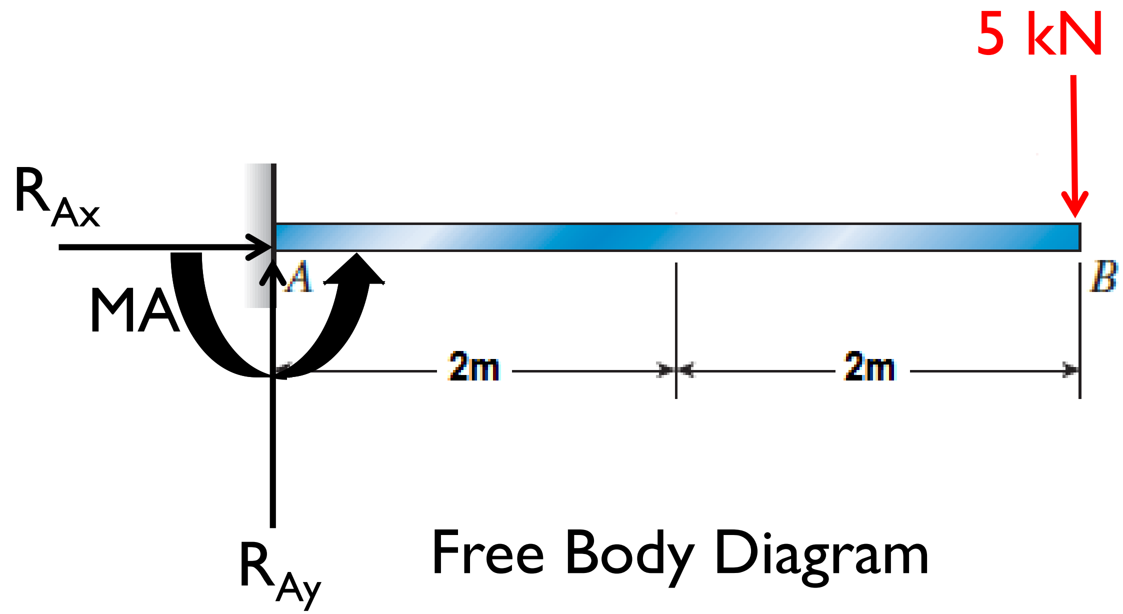


Solution:

$\sum F_x = 0$ ➡ $R_{AX} = 0$

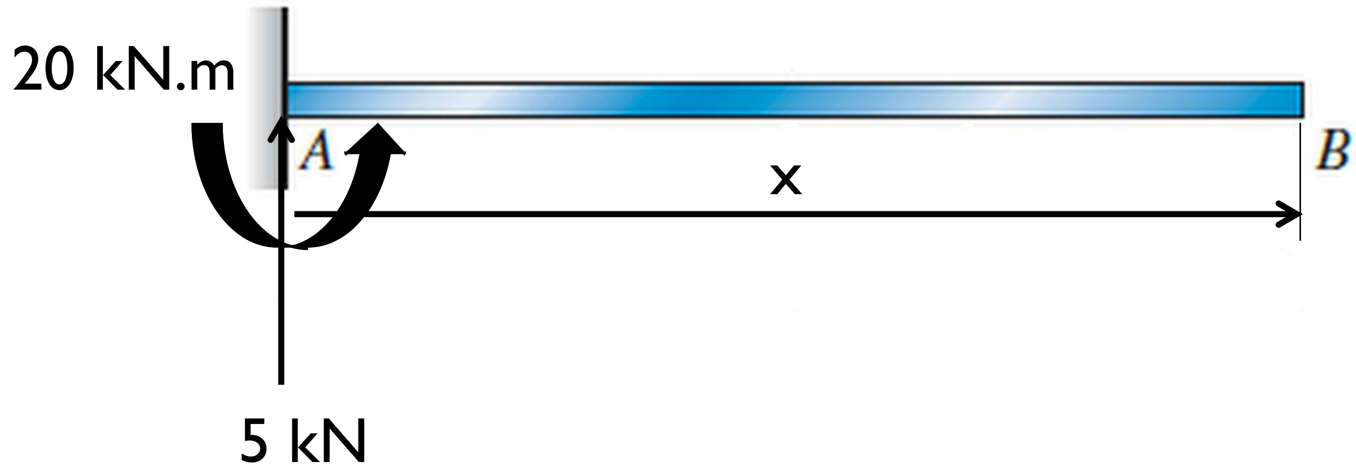
$\sum F_y = 0$
$R_{Ay} = 5$ kN

$\sum M_A = 0$
$MA - 4 * 5 = 0$ ➡ $MA = 20$ kN.m



Free Body Diagram

A ⟶ B $0 \le x \le 4$

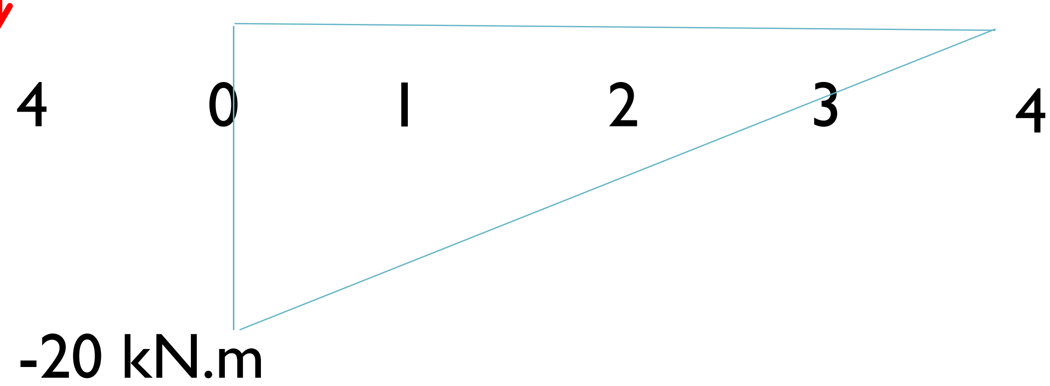


S.F =5 kN

B.M= -20 +5 x

x	S.F (kN)	B.M (kN.m)
0	5	-20
1	5	-15
2	5	-10
3	5	-5
4	5	0

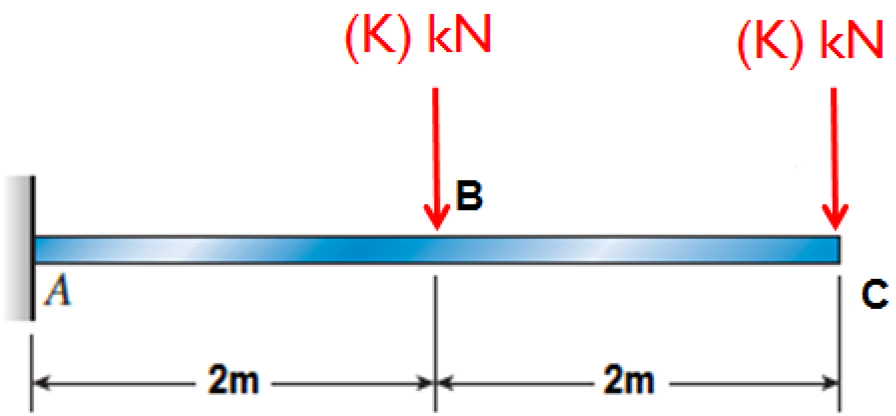
Shear Force Diagram

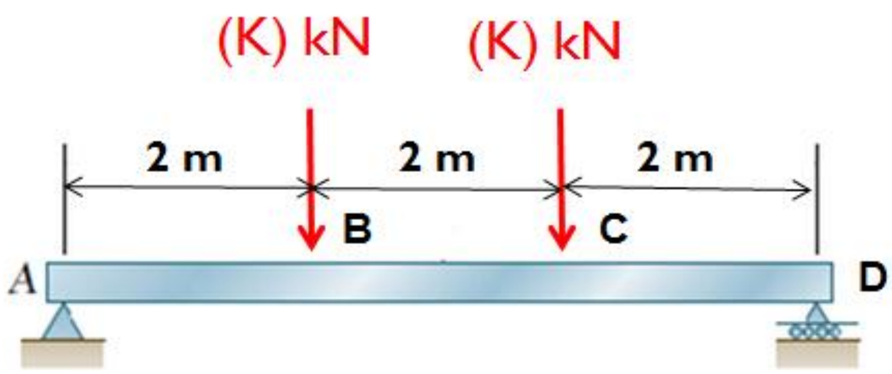


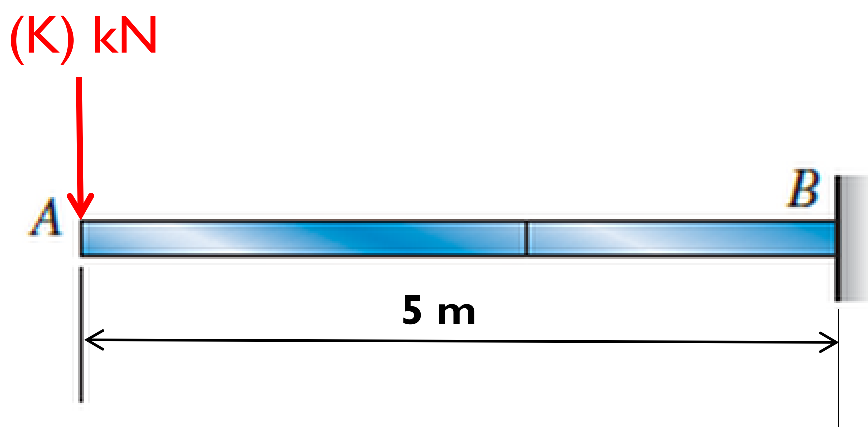
Bending Moment Diagram

Assignment

Draw the shear force and bending moment diagrams for the loaded beam shown below.







Where K, the student number