Transformer

Introduction

The transformer is probably one of the most useful electrical devices ever invented. It can change the magnitude of alternating voltage or current from one value to another. This useful property of transformer is mainly responsible for the widespread use of alternating currents rather than direct currents i.e., electric power is generated, transmitted and distributed in the form of alternating current. Transformers have no moving parts, rugged and durable in construction, thus requiring very little attention. They also have a very high efficiency as high as 99%.

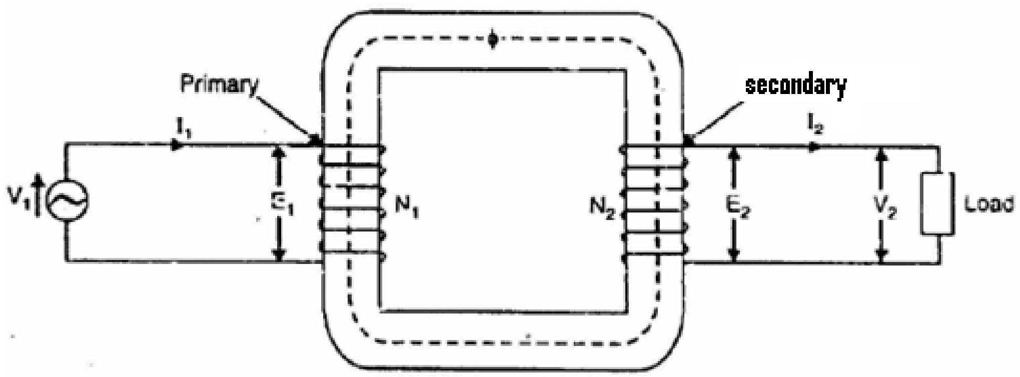
Transformer

A transformer is a static device of equipment used either for raising or lowering the voltage of an a.c. supply with a corresponding decrease or increase in current. It essentially consists of two windings, the primary and secondary, wound on a common laminated magnetic core as shown in Fig below. The winding connected to the a.c. source is called primary winding (or primary) and the one connected to load is called secondary winding (or secondary). The alternating voltage V_1 whose magnitude is to be changed is applied to the primary.

Depending upon the number of turns of the primary (N_1) and secondary (N_2), an alternating $e.m.f. E_2$ is induced in the secondary. This induced $e.m.f. E_2$ in the secondary causes a secondary current I_2. Consequently, terminal voltage V_2 will appear across the load.

If $V_2 > V_1$, it is called a step up-transformer.

If $V_2 < V_1$, it is called a step-down transformer.



Working

When an alternating voltage V_1 is applied to the primary, an alternating flux $\emptyset$ is set up in the core. This alternating flux links both the windings and induces $e.m.f.s.\ E_1$ and E_2 in them according to Faraday's laws of electromagnetic induction. The $e.m.f.\ E_1$ is termed as primary $e.m.f$ and $e.m.f.\ E_2$ is termed as secondary $e.m.f.$

Clearly

$$E_1 = -N_1 \frac{d\emptyset}{dt}$$

And

$$E_2 = -N_2 \frac{d\emptyset}{dt}$$

:-

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Note that magnitudes of E_2 and E_1 depend upon the number of turns on the secondary and primary respectively

If $N_2 > N_1$, then $E_2 > E_1$ (or $V_2 > V_1$) and we get a step-up transformer.

If $N_2 < N_1$, then $E_2 < E_1$ (or $V_2 < V_1$) and we get a step-down transformer.

The following points may be noted carefully:

1- The transformer action is based on the laws of electromagnetic induction.

2- There is no electrical connection between the primary and secondary.

The a.c. power is transferred from primary to secondary through magnetic flux.

3- There is no change in frequency i.e., output power has the same frequency as the input power.

4- the losses that occur in a transformer are:

- a- core losses—eddy current and hysteresis losses
- b- copper losses—in the resistance of the windings

In practice, these losses are very small so that output power is nearly equal to the input primary power. In other words, a transformer has very high efficiency.

Transformer Iron: - The transformer core can be made from

1- low Silicon Iron

1% Silicon , 99% Steel

Specific loss= 1.7 Watt/Kg at 1 Tesla & 50 Hz

1- High Silicon Iron

4 – 5 % Silicon , 95% - 96% Steel

Specific loss= 1.2 Watt/Kg at 1 Tesla & 50 Hz

Note If the silicon percentage is increase above 5% the steel will be hard and it will be difficult to make laminations from it

Transformers classification according to use the application

1- Power transformer

a- Large transformer:

15 – 300 MVA

10 – 500 KV

Used in high voltage transmission lines

b- Medium transformers

10 – 1000 MVA

3 – 30 KV

Used in distribution networks (distribution transformers)

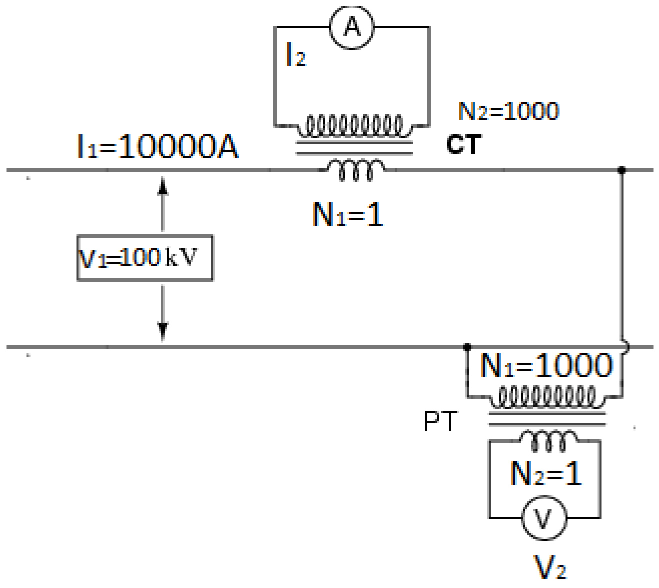
c- Small transformers

They are used industry and general application

2- Instrument transformers

Used for measuring the current ("CT" current transformers) or for measuring the voltage ("PT" potential transformers)

Example



$$I_2N_2 = I_1N_1 \rightarrow \frac{I_2}{I_1} = \frac{N_1}{N_2} \rightarrow I_2 = \frac{1}{1000} \times 10000 = 10A$$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \rightarrow V_2 = \frac{1}{1000} \times 100000 = 100V$$

3- High frequency transformers

Used in electronic circuits high frequencies and low power.

These transformers have air core to reduce the iron loses at high frequencies and for linearity (no saturation).

These transformers are also called air- core transformers

4- Impedance matching transformers

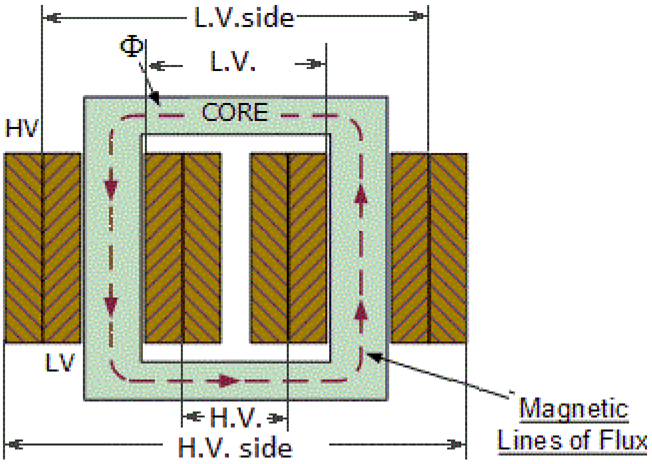
Used for impedance matching in communication circuits.

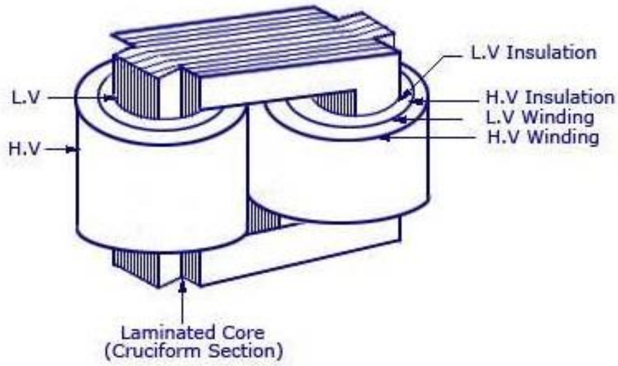
The can be used to match the load impedance with the internal source impedance in order to have maximum power transferred to the load

Iron core type

a- Core type

high voltage & Low current

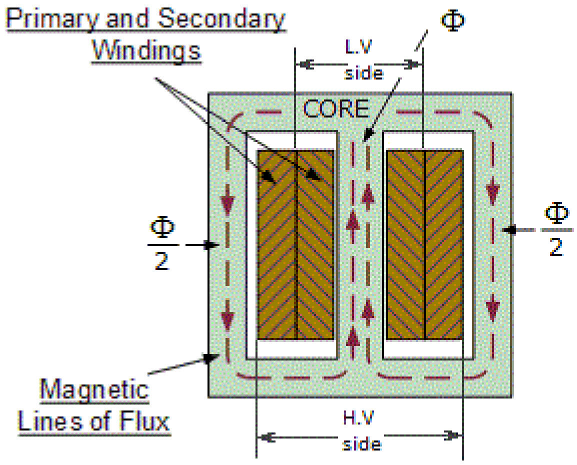


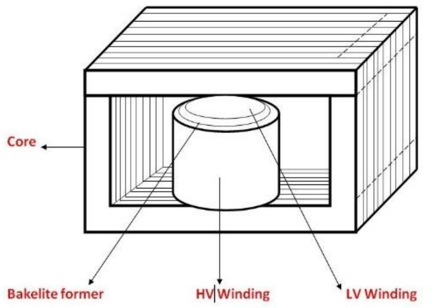


In core type transformers half of primary winding and half of the secondary winding are placed round each limb. this reduced the leakage flux. It is a usual practice to place the low-voltage winding below the high-voltage winding for mechanical considerations and to reduce the size of the insulator used

b- Shell-type transformer

Low voltage & high current

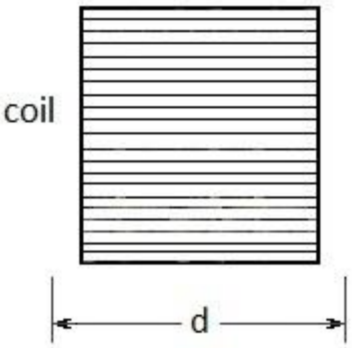




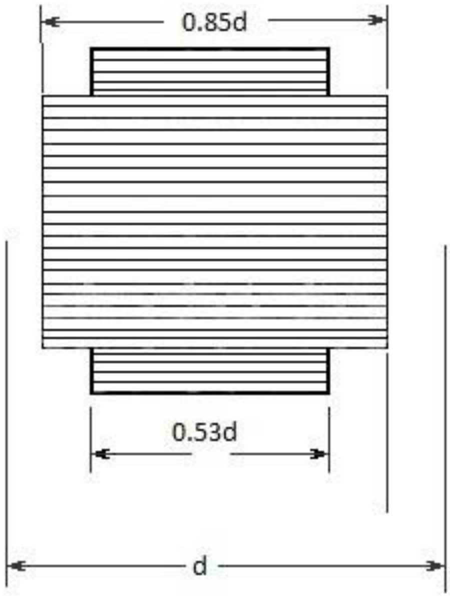
This method of construction involves the use of a double magnetic circuit. both the windings are place round placed round the central limb, the other two limbs acting simply as a low-reluctance flux path.

The choice of type (whether core or shell) will not greatly affect the efficiency *f* the transformer. The core type is generally more suitable for high voltage and small current while the shell-type is generally more low voltage and high current.

Core section



square
for small Tr.



cruciform .2 stepped core for medium Tr.

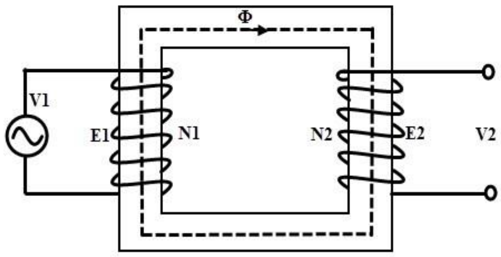
E.M.F Equation of a Transformer:

Consider that an alternating voltage V_1 of frequency f is applied to the primary as shown below .

The sinusoidal flux $\emptyset$ produced by the primary can be represented as:

$\emptyset = \emptyset_m sin\omega t$ $\qquad\qquad$ $\omega = 2\pi f$,$-cos\omega t = sin(\omega t - 90)$

The instantaneous $e.m.f$



$$e_1 = -N_1 \frac{d\emptyset}{dt} \quad = -N_1 \frac{d}{dt}(\emptyset_m sin\omega t)$$

$$= -\omega N_1 \emptyset_m cos\omega t = -2\pi f\, N_1 \emptyset_m cos\omega t$$

$$e_1 = 2\pi\, f\, N_1 \emptyset_m\, sin(\omega t - 90)$$

It is clear from the above equation that maximum value of induced $e.m.f$ in the primary is:

$$E_{m1} = 2\pi\, f\, N_1 \emptyset_m$$

The $r.m.s$ value E_1 of the primary $e.m.f$ is:

$$E_1 = \frac{E_{m1}}{\sqrt{2}} = \frac{2\pi\, f\, N_1 \emptyset_m}{\sqrt{2}}$$

$$E_1 = 4.44\, f\, N_1 \emptyset_m \quad \text{similarly} \quad E_2 = 4.44\, f\, N_2 \emptyset_m$$

(as the $e.m.f$ E_2 is produced by the same flux $\emptyset = \emptyset_m sin\omega t$ that cause E_1.

Thus the only difference of the two is the number of turns)

Note it is clear from the above that $e.m.\ f_s$ E_1 induced in the primary and E_2 induced in the secondary lag behind flux $\emptyset$ by 90°

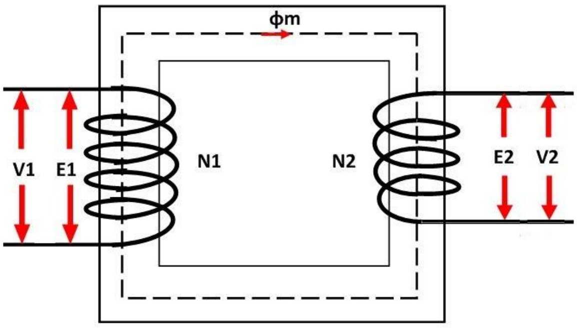
Ideal transformers:

For ideal transformers:

1- $E_1 = V_1$ and $E_2 = V_2$

As there is no voltage drop in the winding

$$\frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1}$$



2- There is no loss, therefore, volt-amperes input to the primary are equal to the output volt-amperes i.e.

$$I_1 V_1 = I_2 V_2$$

Or

$$\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{N_1}{N_2}$$

Hence currents are in the inverse ratio of voltage transformation ratio
This simply means that if we rise the voltage V ↑there is a corresponding decrease of current I ↓

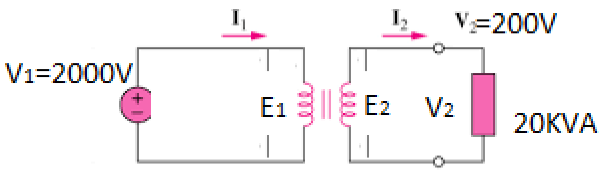
Example:

A 2000/200 V, 20 KVA transformer has 66 turns in the secondary.

Calculate

1- Primary turns.
2- Primary and secondary full load currents.

Neglect the losses



Solution

1- $\frac{V_1}{V_2} = \frac{2000}{200} = \frac{N_1}{N_2}$

$$N_1 = N_2 \times \frac{2000}{200} = 66 \times 10 = 660 \text{ turns}$$

2- $I_1 V_1 = I_2 V_2 = 20 \times 10^3$

$I_2 = \frac{20\times10^3}{200} = 100\ A \quad I_1 = \frac{20\times10^3}{2000} = 10\ A$

Practical Transformers

A Practical transformer differs from the idea transformer in many respects.

The Practical Transformers Has

1- iron loss
2- winding resistance
3- magnetic leakages, giving rise to leakage reactance.

Practical Transformers On No Load

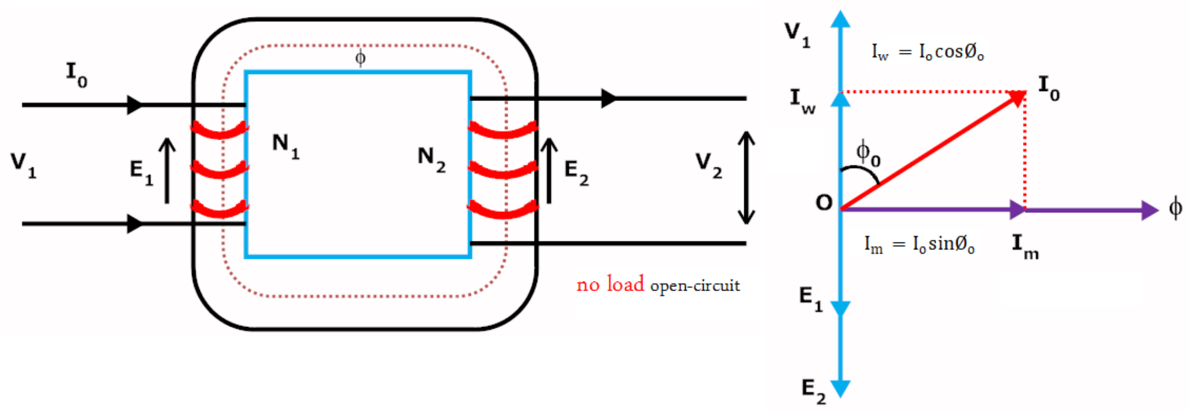
Consider a Practical transformer on no load I.e., secondary on open-circuit as shown in figure below.

The primary will draw a small current I_o to supply

1- The iron losses
2- a very small amount of copper loss in the primary.

Hence the primary no load current I_o is not 90° behind the applied voltage V_1 but lags it by an angle $\emptyset_o < 90^{\circ}$ as shown in the phasor diagram below.

No load input power, $w_o = V_1 I_o \cos\emptyset_o$



As seen from the phasor diagram, the no-load primary current I_o can be resolved into two rectangular components

1- The component I_w in phase with the applied voltage V_1. This is known as active or working or iron loss component and supplies the iron loss and a very small primary copper loss.

$$I_w = I_o \cos\emptyset_o$$

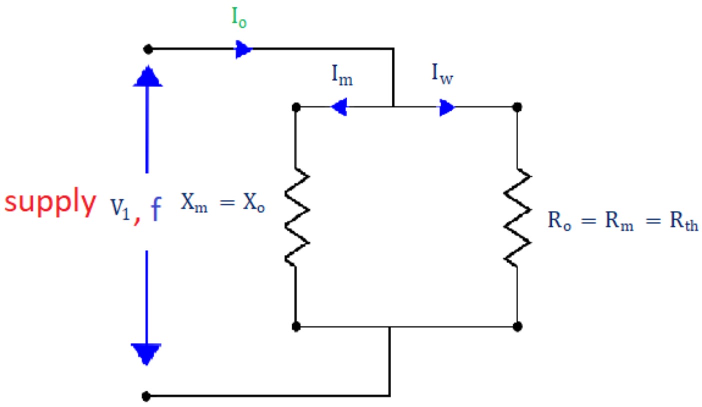
2- The component I_m lagging behind V_1 by 90° and is known as magnetizing component. It is this component which produces the mutual flux $\emptyset$ in the core.

$$I_m = I_o \sin\emptyset_o$$

Clearly, I_o is phasor sum of I_m and I_w.

$$I_o = \sqrt{I_m{}^2 + I_w{}^2}$$

No load power factor, $\cos\emptyset_o = \frac{I_w}{I_o}$



The no load primary copper loss (i.e. $I_o{}^2 R_1$) is very small and may be neglected. Therefore, the no load primary input power is practically equal to the iron loss in the transformer i.e.,

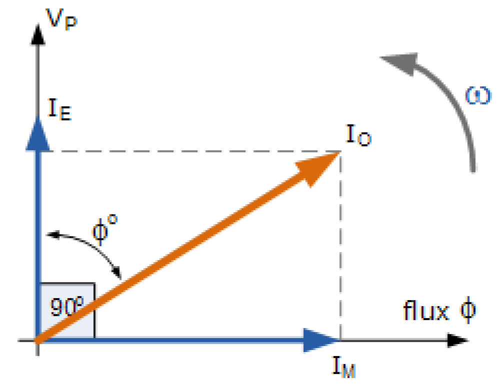
No load input power, W_o = Iron loss = core loss

$$V_1 I_o \cos\emptyset_o = I_w{}^2 R_0$$

Note.

At no load, there is no current in the secondary so that $V_2 = E_2$.

On the primary side, the drops due to I_o are also very small because of the smallness of I_o. Hence, we can say that at no load, $V_1 = E_1$.



$$I_E = I_O \cos\phi$$

$$I_M = I_O \sin\phi$$

$$I_O = \sqrt{I_M^2 + I_E^2}$$

Example: A 230/ 2300 V transformer takes no load current of 5 A at 0.25 power factor lagging. find

1- The core loss
2- Magnetizing current

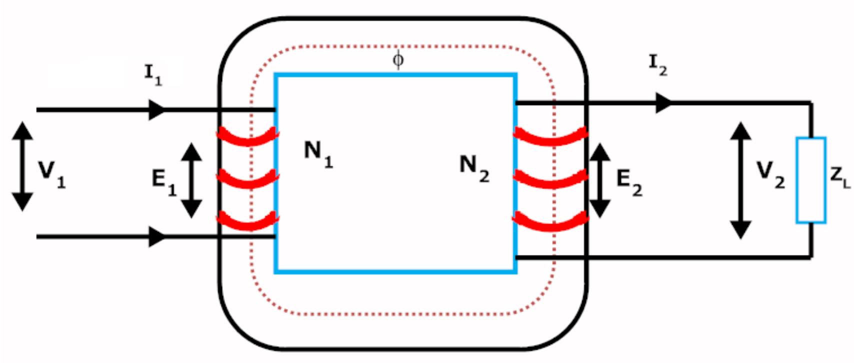
Solution

1- core loss $w_o = V_1 I_o \cos\emptyset_o \quad = 230 \times 5 \times 0.25 = 287.5$ w

2- Iron-loss current $I_w = I_o \cos\emptyset_o = 5 \times 0.25 = 1.25$ A

Magnetizing current $I_m = \sqrt{{I_o}^2 - {I_w}^2} = \sqrt{5^2 - 1.25^2} = 4.85$ A

Practical Transformer On Load



The secondary current I_2 set up an m.m.f $N_2 I_2$ which produces a flux in the opposite direction to the flux $\emptyset$ originally set up in the primary by the magnetizing current. This will reduce the flux in the core from the magnetizing original value and hence E_1. Since applied voltageV_1 is kept fixed, E_1 must remain unchanged. This is possible only if the flux remains fixed. Hence mutual flux $\emptyset$ remains fixed whether a load is connected or not. In order to fulfill this condition, the primary must develop an m.m.f which exactly counter balances the secondary m.m.f $N_2 I_2$ .Hence the primary current I_1 must follow such that:

A.T. at no load $= I_o N_1$

I_2' load component of primary current

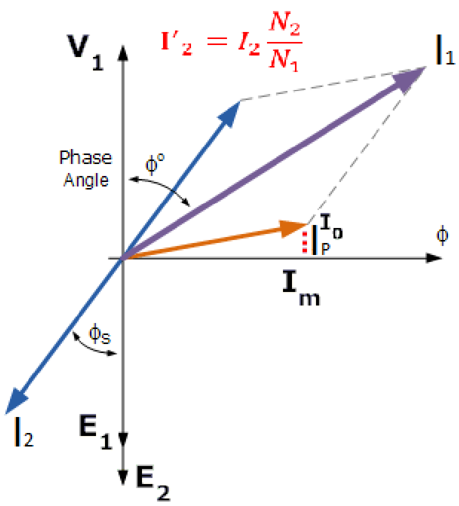
A.T. at to load on secondary $= I_2 N_2$

A.T. at to load on primary $= I_2 N_2$

$= I_2 N_1$

$$I_2' = I_2 \frac{N_2}{N_1}$$

primary current$=$ $I_1 = I_o + I_2'$

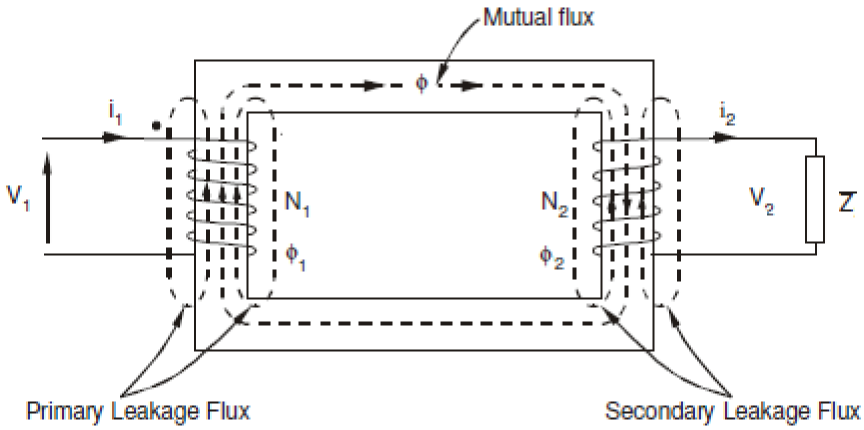


* The power input, therefore, automatically increases with the output.

* The flax in the core of the transformer is constant, therefore the iron losses in the core are constant also.

* I_2' is 180 out of phase with I_2

Magnetic Leakage in Transformers



Both primary and secondary currents produce flux. The flux $\emptyset$ which links both the windings is the useful flux and is called mutual flux. However, primary current would produce some flux $\emptyset_{L_1}$ which would not link the secondary winding. Similarly, secondary current would produce some flux $\emptyset_{L_2}$ that would not link the primary winding. The flux such as $\emptyset_{L_1}$ or $\emptyset_{L_2}$ which links only one winding is called leakage flux. The leakage flux paths are mainly through the air.

$\emptyset_{L_1} \propto I_1$

$$N_1\emptyset_{L_1} = L_1 I_1$$

Also

$\emptyset_{L_2} \propto I_2$

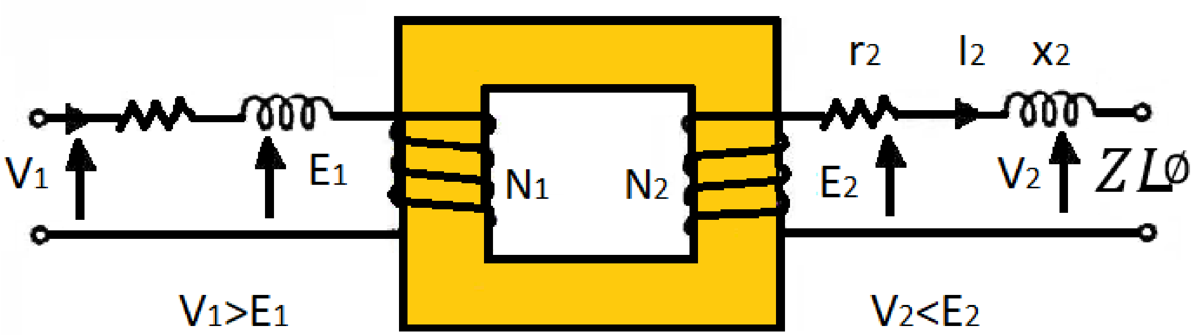
$$N_2\emptyset_{L_2} = L_2 I_2$$

Where

L_1 = leakage inductance on primary winding
L_2 = leakage inductance on secondary winding

In other words, the effect of primary leakage flux $\emptyset_1$ is to introduce an inductive reactance X_1 in series with the primary winding

Similarly, the secondary leakage flux $\emptyset_2$ introduces an inductive reactance X_2 in series with the secondary winding.



r_1 = primary winding resistance

r_2 = secondary winding resistance

$x_1 = \omega L_1 = 2\pi f L_1$ (leakage reactance) on primary

$x_2 = \omega L_2 = 2\pi f L_2$ (leakage reactance) on secondary

$V_1 = -E_1 + I_1(r_1 + jx_1)$ ....(1

$E_2 = V_2 + I_2(r_2 + jx_2)$ ....(2

$\frac{E_1}{E_2} = \frac{N_1}{N_2}$ .... (3

$I_1 = I_o + I_2'$................................... (4

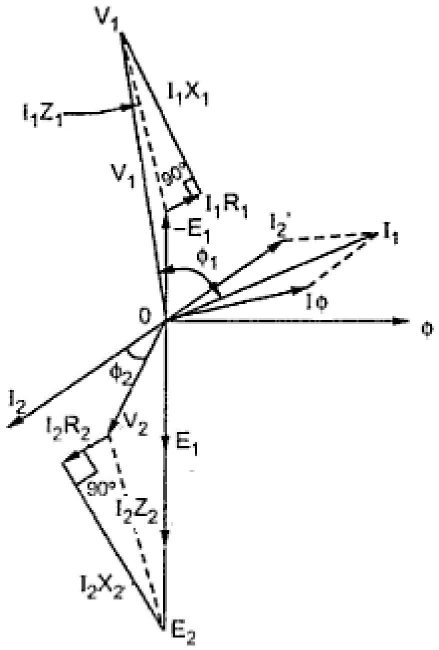
$I_2' = I_2 \frac{N_2}{N_1}$....................................... (5

practical transformer having winding resistance and leakage reactance. There is voltage drop in r_1 and x_1 so that primary e.m.f. E_1 is less than the applied voltage V_1. Similarly, there is voltage drop in r_2 and x_2 so that secondary terminal voltage V_2 is less than the secondary e.m.f. E_2.

The current I_1 must meet two requirements:

1- It must supply the no-load current I_o to meet the iron losses in the transformer and to provide flux in the core.
2- It must supply a current I_2' to counteract the demagnetizing effect of secondary currently I_2

Phasor diagram



Both E_1 and E_2 lag the mutual flux $\emptyset$ by 90°. The current I'2 represents the primary current to neutralize the demagnetizing effect of secondary current I_2.Now $I_2' = k\, I_2$ and is opposite to I_2. Also I_o is the no-load current of the transformer ($I_o = I_m + I_w$) . The Phasor sum of I_2' and I_o gives the total primary current I_1.

Note that counter e.m.f that opposes the applied voltage V_1 is $-E_1$. Therefore, if we add I_1R_1 (in phase with I_1) and $I_1\ X_1$ (90° ahead of I_1) to $-E_1$, we get the applied primary voltage V_1. The phasor E_2 represents the induced voltage in the secondary by the mutual flux $\emptyset$.

The secondary terminal voltage V_2 will be what is left over after subtracting I_2R_2 and I_2X_2 from E_2.

Load power factor = $cos\emptyset_2$

Primary power factor = $cos\emptyset_1$

Input power to transformer, $P_1 = V_1 I_1\ cos\emptyset_1$

Output power to transformer, $P_2 = V_2 I_2\ cos\emptyset_2$

Example: The primary of a 1000/250 V transformer has a resistance of $0.15\ \Omega$

And leakage reactance of $0.8\ \Omega$.

Find the primary induced $\mathrm{e.m.f}$ when the primary current is 60 A at 0.8 $\mathrm{p.f.}$ lagging

Solution

Primary impedance, $z_1 = 0.15 + j0.8 = 0.814 \angle 79.6^\circ\ \Omega$

Power factor angle, $\emptyset_1 = \cos^{-1} 0.8 = 36.9^\circ$

Taking applied voltage as the reference phasor, we have, $V_1 = 1000\angle 0^\circ$

$-E_1 = V_1 - I_1 Z_1$

$$= 1000\angle 0^\circ - 60 \angle -36.9^\circ \times 0.814 \angle 79.6^\circ$$

$$= 1000\angle 0^\circ - 48.4 \angle 42.7^\circ$$

$$= 1000 - (36 + j33) = 964 - j33 = 964.5 \angle -2^\circ\ V$$

-: primary $\mathrm{e.m.f}\ E_1 = -964 + j33 = 964.5\ \angle 178^\circ\ V$

Example: The voltage on the secondary of a single phase transformer is 200 V

When suppling a load of 8 kw at a $\mathrm{p.f.}$ of 0.8 lagging. The secondary resistance is $0.04\ \Omega$ and secondary leakage reactance is $0.8\ \Omega$. Calculate the induced $\mathrm{e.m.f}$ in the secondary .

Solution

Secondary current $I_2 = \frac{8\times 10^3}{200\times 0.8} = 50A$

Power factor angle, $\emptyset_2 = \cos^{-1} 0.8 = 36.9^\circ$

$$I_2 = 50 \angle -36.9^\circ\ A$$

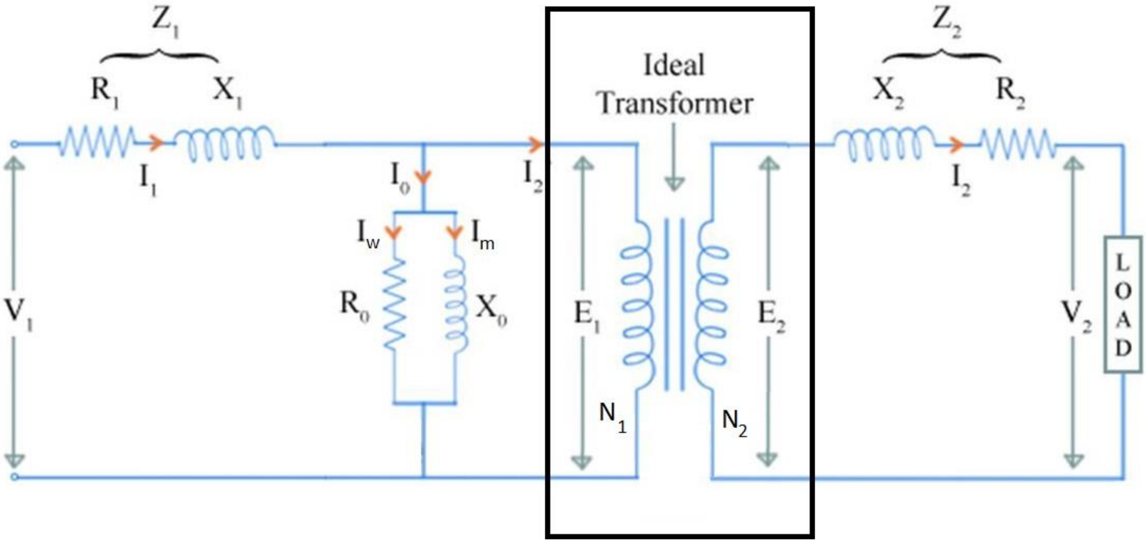
$$E_2 = V_2 + I_2 Z_2 = 200 \angle 0^\circ + 50 \angle -36.9^\circ \times 0.8 \angle 87.14^\circ$$

$$= 200\ \angle 0^\circ + 40 \angle 50.24^\circ = 227.67 \angle 7.8^\circ\ V$$

The secondary $\mathrm{e.m.f}\ E_2$ loads the secondary terminal voltage V_2 by 7.8°

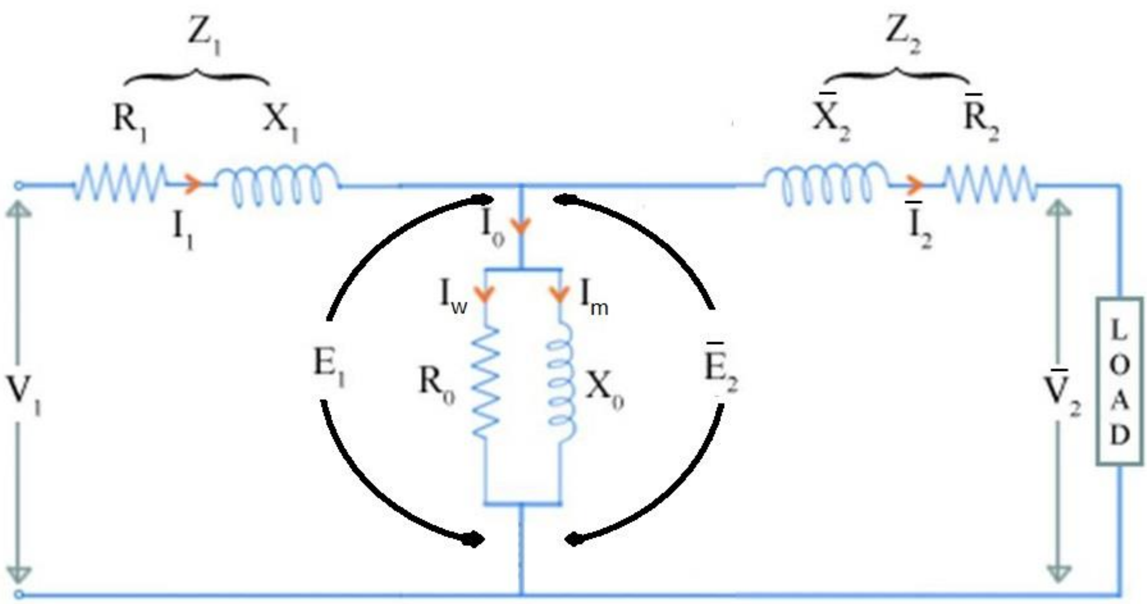
The equivalent circuit of a transformer

The figure below shows the equivalent circuit of a transformer on load.



1- The equivalent circuit Referred to primary

If all the secondary quantities are referred to the primary, we get the equivalent circuit of the transformer referred to the primary as shown in figure below



$\frac{E_1}{E_2} = \frac{N_1}{N_2}$, $E_2' = E_1 = E_2 \frac{N_1}{N_2}$ ....(1

then $V_2' = V_2 \frac{N_1}{N_2}$ ....(2

$I_1 = I_o + I_2'$, $I_2' = I_2 \frac{N_2}{N_1}$.................. (3

In The equivalent circuit the power and losses must be the same in the secondary winding

$$I_2^2(r_2 + jx_2) = {I_2'}^2(r_2' + jx_2')$$

$$I_2^2 r_2 = {I_2'}^2 r_2' \quad \Rightarrow r_2' = r_2 \frac{I_2^2}{{I_2'}^2} \quad \longrightarrow r_2' = r_2 (\frac{N_1}{N_2})^2$$

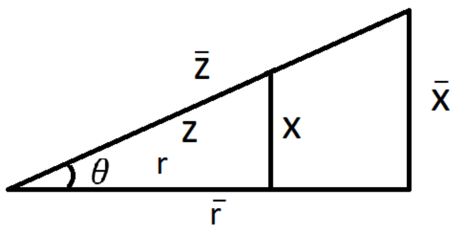
Similarly

$$X_2' = X_2 \left(\frac{N_1}{N_2}\right)^2 \quad , \quad Z_L' = Z_L (\frac{N_1}{N_2})^2$$

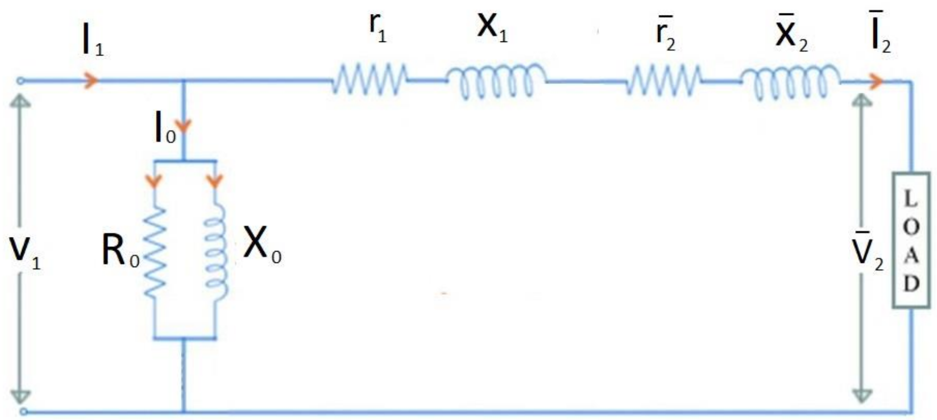
$\theta' = \theta$ prove that?

Because $\tan\theta' = \frac{x'}{r'} = \frac{x\left(\frac{N_1}{N_2}\right)^2}{r\left(\frac{N_1}{N_2}\right)^2} = \frac{x}{r} = \tan\theta$

$\theta' = \theta$



The no load current I_0 a transformer is small as compared to rated primary current. Therefore, voltage drops in R_1 and X_1 due to I_0 are negligible. the equivalent circuit shown in figure .above can be simplified by transferring the shunt circuit R_0 - X_0 to the input terminals as shown in figure below.



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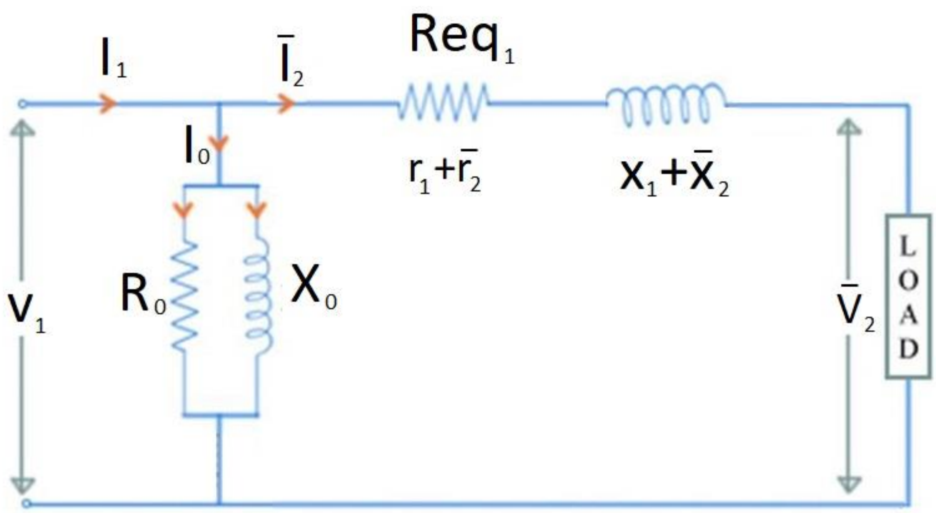
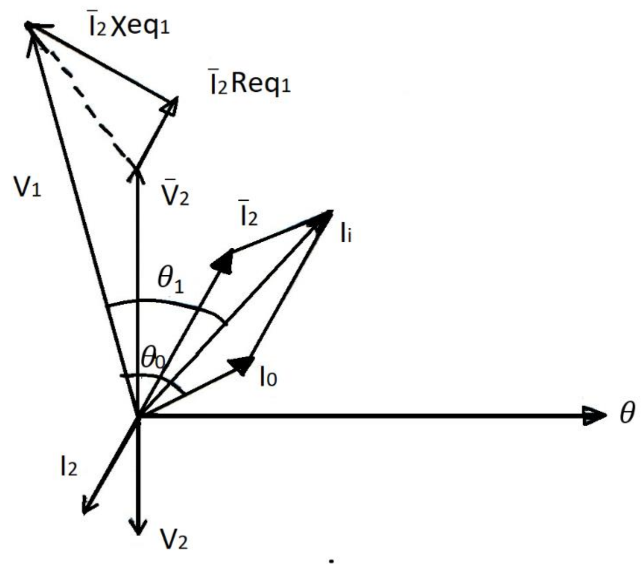


The simplified equivalent circuit referred to primary

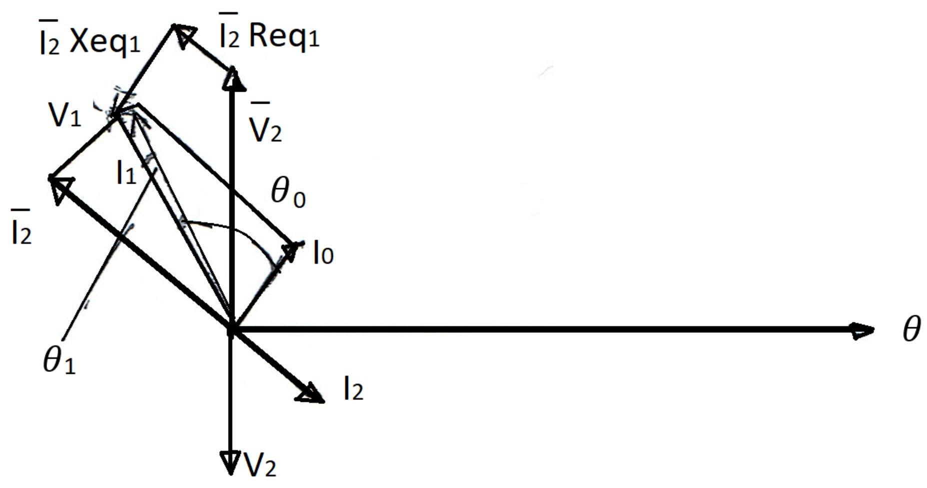
Phasor diagram

$$V_1 = V_2'+I_2'Z_{eq}$$

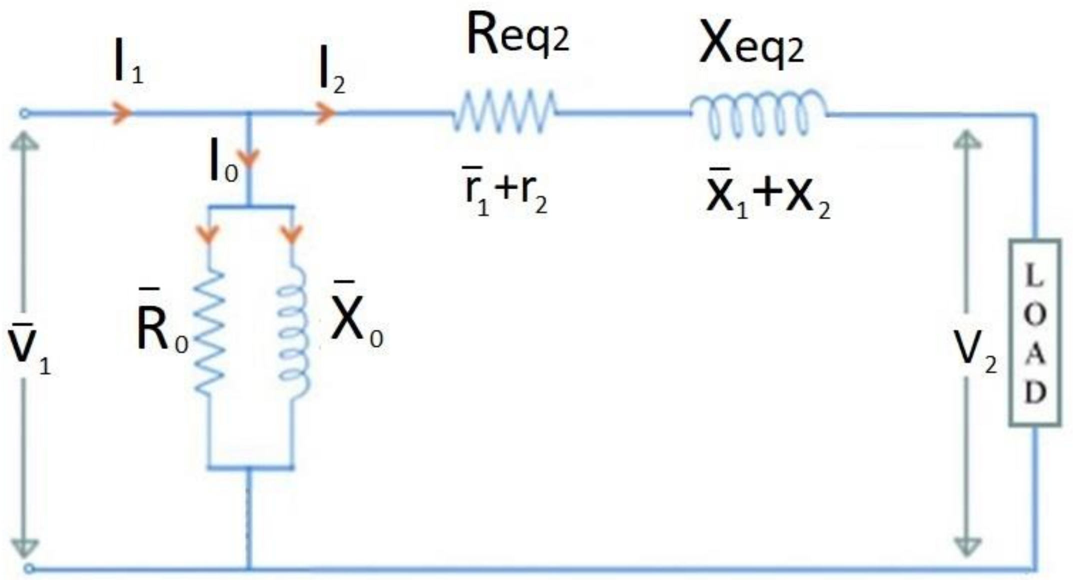
Lagging



Leading



2- The equivalent circuit Referred to secondary



$I_1' = I_1 \frac{N_1}{N_2}$, $I_0' = I_0 \frac{N_1}{N_2}$, $V_1' = V_1 \frac{N_2}{N_1}$

$r_1' = r_1 \left(\frac{N_2}{N_1}\right)^2$, $Req_2 = Req_1 \left(\frac{N_2}{N_1}\right)^2$

$X_1' = X_1 \left(\frac{N_2}{N_1}\right)^2$, $Xeq_2 = Xeq_1 \left(\frac{N_2}{N_1}\right)^2$

Voltage regulation

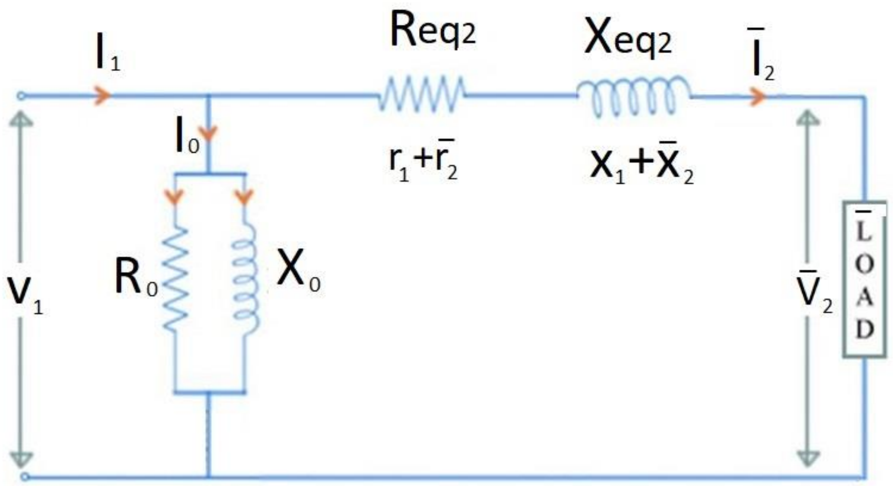
When a transformer is load with a constant primary voltage the secondary voltage decrease.

The voltage regulation of a transformer is the arithmetic difference (not phasor difference) between the no-load secondary voltage oV_2 and the secondary voltage V_2 no load express as percentage of no-load voltage i.e.

$$\%\text{ age voltage regulation up} = \frac{oV_2 - V_2}{V_2} \times 100$$

$$\%\text{ age voltage regulation down} = \frac{oV_2 - V_2}{oV_2} \times 100$$

The way in which the secondary terminal voltage varies with load depends on the load current, the internal impedance and the load power factor



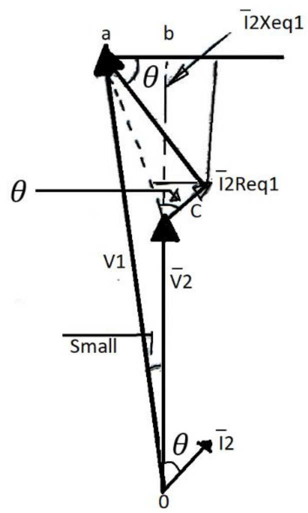
The regulation may also be explained in the term of primary values

$$\text{Regulation up } \% = \frac{V_1 - V_2'}{V_2'} \times 100$$

$$\text{Regulation down } \% = \frac{V_1 - V_2'}{V_1} \times 100$$

$$= \frac{oa - oc}{oa} \times 100 \quad = \frac{ob - oc}{oa} \times 100$$

$$= \frac{ob}{oa} \times 100$$



Regulation down %

$$= \frac{I_2' Req_1 \cos\theta + I_2' Xeq_1 \sin\theta}{V_1} \times 100$$

on leading power factor θ is negative

Regulation %

Leading

↓

$$= \frac{I_2' Req_1 \cos\theta \pm I_2' Xeq_1 \sin\theta}{V_1} \times 100$$

↑

Lagging

Percentage resistance and percentage reactance:

Regulation up %

$$= \frac{I_2' Req_1 \cos\theta \pm I_2' Xeq_1 \sin\theta}{V_2'} \times 100$$

$$= \frac{I_2' Req_1}{V_2'} \times 100 \cos\theta \pm \frac{I_2' Xeq_1}{V_2'} \times 100 \sin\theta$$

Where:

$$\frac{I_2' Req_1}{V_2'} = Req\% = \text{Percentage resistance}$$

$$\frac{I_2' Xeq_1}{V_2'} = Xeq\% = \text{percentage reactance}$$

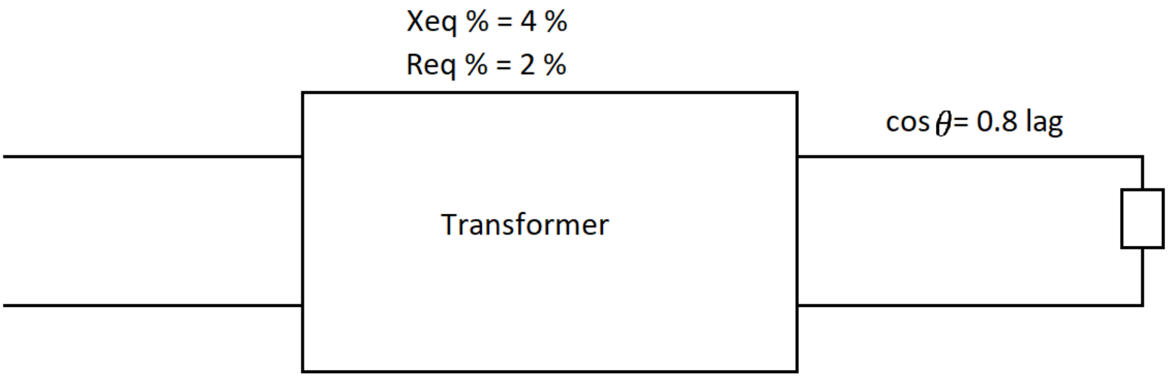
Usually Req% and Xeq% taken at full-load

$$\text{Req P.U.} = \frac{\text{Req\%}}{100}$$

$$\text{Xeq P.U.} = \frac{\text{Xeq\%}}{100}$$

$$\text{Regulation \%} = \text{Req\% } \cos\theta \pm \text{Xeq\% } \sin\theta$$

Example:



$$\text{Regulation} = 2 \times 0.8 + 4 \times 0.6 = 4\%$$

On leading power factor:

$$\text{Regulation \%} = 2 \times 0.8 - 4 \times 0.6 = 0.8\%$$

The negative sign means that the voltage will increase when the load is contacted

Regulation down $= 0 = \frac{I_2' Req_1 \cos\theta + I_2' Xeq_1 \sin\theta}{V_1} \times 100$

$\therefore \tan\theta = -\frac{Req}{Xeq}$

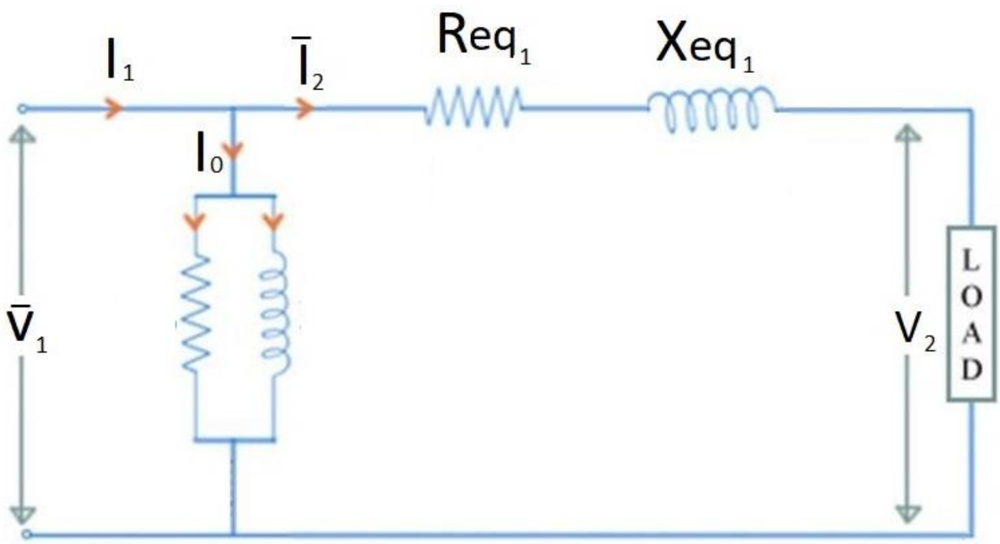
From the above the voltage regulation depends on:

1- Load current I_2
2- Internal impedance Req_1, Xeq_1
3- Load power factor $\cos\theta$

Kapp regulation diagram:

Kapp regulation diagram is used to find the regulation at constant load current and variable power factor.

Take I_2' as a reference:



Transformer efficiency:

A detailed analysis of transformer losses would take in account the dielectric loss and stray load loss, but for present purposes they will be assumed to be included in the iron loss and copper loss respectively. The iron losses are independent of load current if voltage and frequency are constant so they constitute the fixed loss. The copper losses I^2Req referred to either primary or secondary winding represent the variable losses. The output is $V_2 I_2 \cos\theta_2$

$$\text{Efficiency} = \eta = \frac{\text{Output}}{\text{Output+Losses}} = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + I_2^2 Req_2 + Wi}$$

$$= \frac{V_2 \cos\theta_2}{V_2 \cos\theta_2 + I_2 Req_2 + \frac{Wi}{I_2}}$$

From the above equation the efficiency is max when $I_2 Req_2 + \frac{Wi}{I_2}$ is minimum

$$f(I_2) = I_2\, Req_2 + \frac{Wi}{I_2}$$

$$\frac{d\, f(I_2)}{d\, I_2} = Req_2 - \frac{Wi}{I_2^2} = 0 \Rightarrow Wi = I_2^2 Req_2$$

.: the condition for maximum efficiency

Iron loss = Copper loss

(Constant) (Constant)

Let $X = \frac{I_2}{I_{2fl}}$

∴ copper losses = $I_2^2 Req_2 = X^2 I_{2fl}^2 Req_2 = X^2 W_{cufl}$

$$\eta = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + I_2^2 Req_2 + Wi}$$

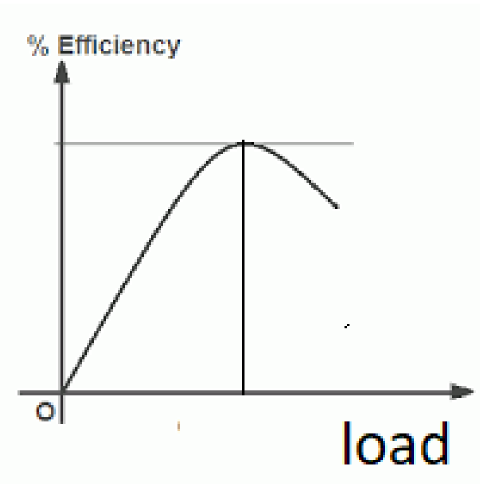
$$\eta = \frac{X V_2 I_{2FL} \cos\theta_2}{X V_2 I_2 \cos\theta_2 + X^2 W_{cufl} + Wi}$$

$$\eta = \frac{XVA \cos\theta_2}{XVA \cos\theta_2 + X^2 W_{cufl} + Wi}$$

At maximum efficiency

$$Wi = Wcu = I_2^2 Req_2 = X^2 I_{2fl}^2 Req_2 = X^2 W_{cufl}$$

$$X = \sqrt{\frac{Wi}{Wcufl}}$$



All – day efficiency

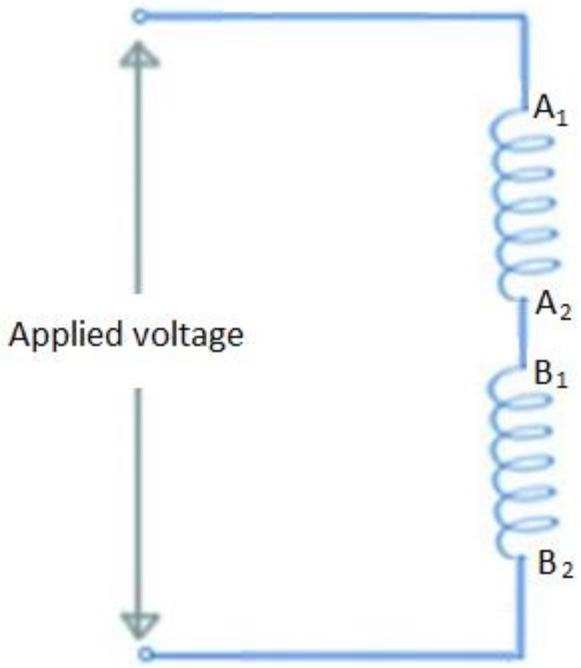
A more suitable method of assessing the efficiency of a transformer having a duty cycle is on an energy basis. The output and losses are calculated in KW hours over a 24- hours day. The all-day efficiency is then detained as:

All-day = $\frac{energy\ output\ in\ 24\ hours}{energy\ input\ in\ 24\ hours} = \frac{KWh\ output\ in\ 24\ hours}{KWh\ input\ in\ 24\ hours}$

Testing:

a- Polarity test:

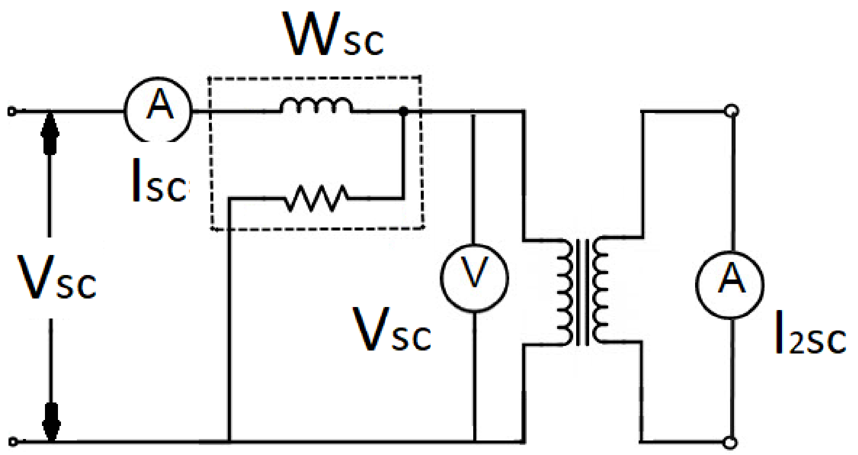
The recommended procedure is to connect primary and secondary winding in series in such a way that the e. m. fs are in phase.



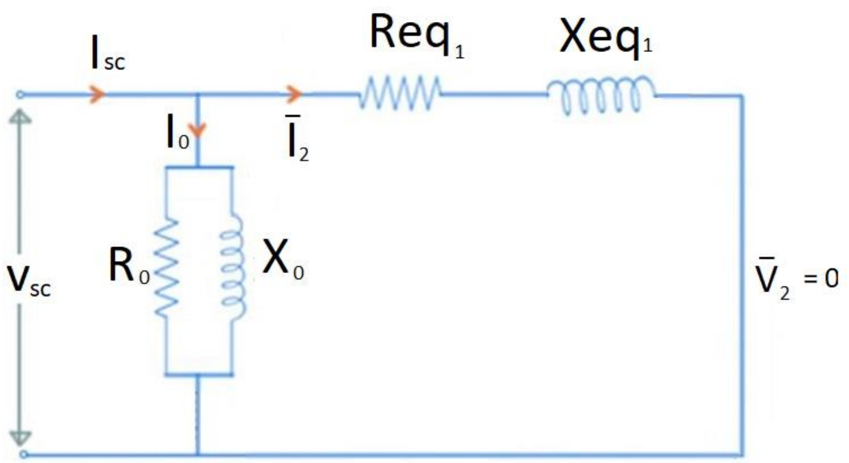
$$I = \frac{V - (E_1 + E_2)}{Z}$$

In this cave the current I will be low .whereas, if the polarity is not correct the voltage E_2 opposes E_2 and the current I is high

b- Short - Circuit Test:

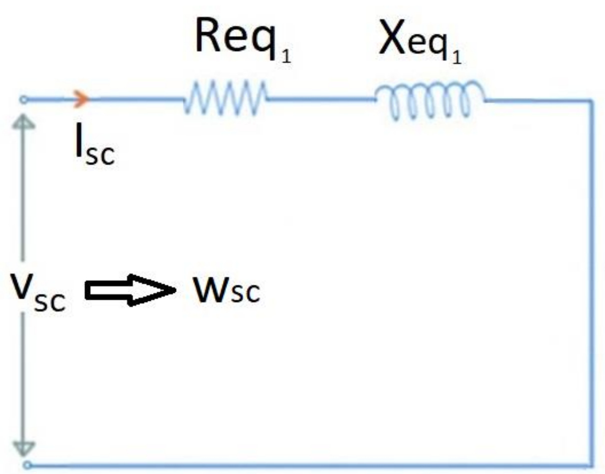


high current – Law voltage test



Req_1 and Xeq_1 are very small when compared with R_0 // X_0 and V_{sc} is a small voltage when compared with rated voltage. For these reasons the current through the magnetising branch I_0 is very small and can be neglected. R_0 // X_m can be taken to be open current

$I_1 = I_2' = I_{sc}$



$w_{sc} = I_{sc}^2 Req_1$

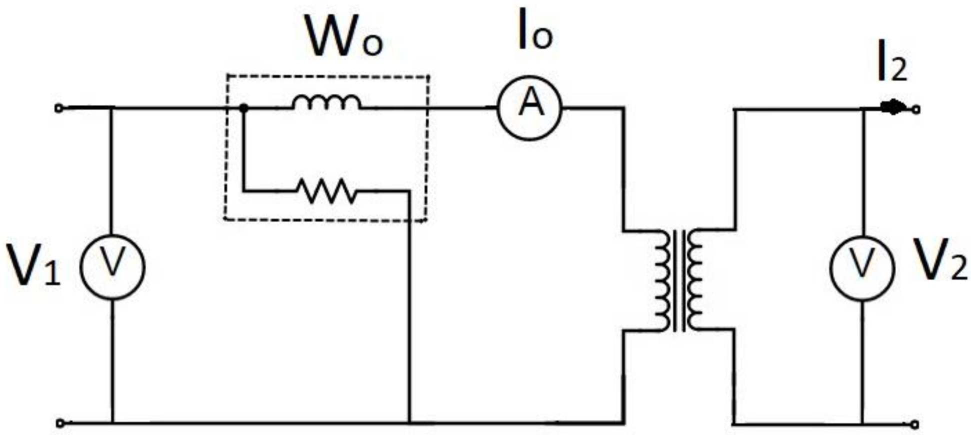
$Req_1 = \frac{w_{sc}}{I_{sc}^2}$

$Zeq_1 = \frac{V_{sc}}{I_{sc}} \Rightarrow Xeq_1 = \sqrt{Zeq_1^2 - Req_1^2}$

$\frac{I_2}{I_1} = \frac{I_2}{I_2'} = \frac{N_1}{N_2} = \frac{I_{2sc}}{I_{sc}}$

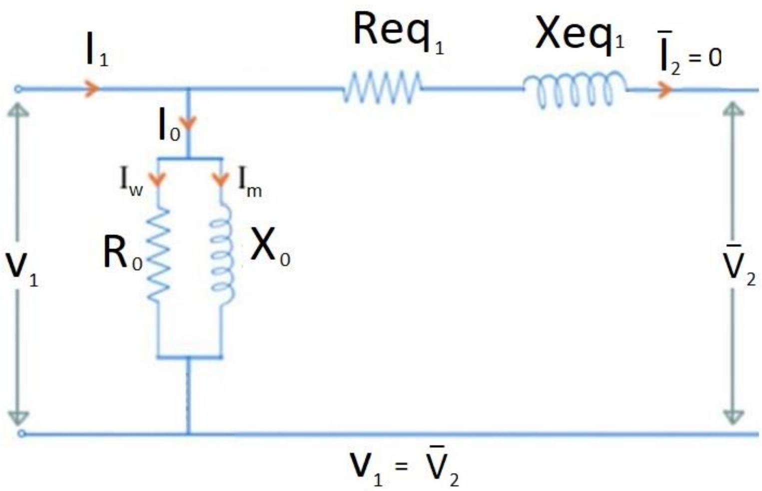
c- Open - Circuit Test or No- Load Test:

In this test a rated voltage is applied to the high voltage side keeping the secondary winding open –circuited



Law current – high voltage test

$I_o \cong 2.5\ \%\ of\ I_{fl}$



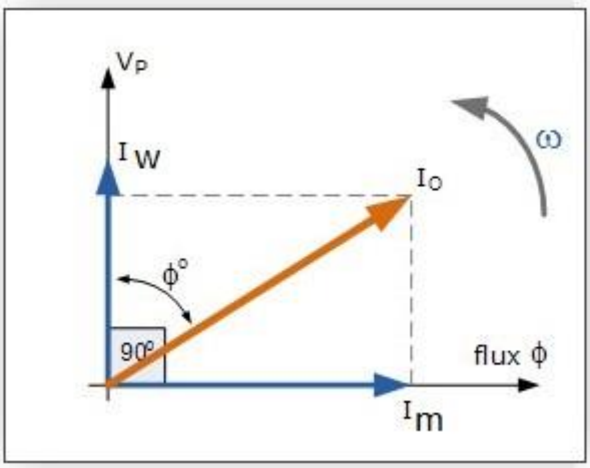
$$(\frac{V_2}{V_1})_{no\ load} \cong \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$W_0 = W_i =$ iron losses

$$\cos\theta_0 = \frac{W_0}{I_0 V_1}$$

$I_w = I_o \cos\emptyset_o$, $I_m = I_o \sin\emptyset_o$

$R_0 = \frac{V_1}{I_w}$, $X_m = \frac{V_1}{I_m}$



Calculation of efficiency from the Open-Circuit Test and Short-Circuit Test:

w_i = input power in watts on the open circuit test

= iron losses

w_{CUFL} = input power in watts on the Short - Circuit Test with full - load current

$= I_2'^2 Req_1 = I_2^2\, Req_2$

Then total loss on full – load = $w_i + w_{CUFL}$

and efficiency on full – load $= \dfrac{VA.PF}{VA.PF + w_i + w_{CUFL}}$

For any load equal to x full-load corresponding total loss = $w_i + X^2 w_{CUFL}$

and corresponding efficiency $= \dfrac{X.VA.PF}{X.VA.PF + w_i + X^2 w_{CUFL}}$

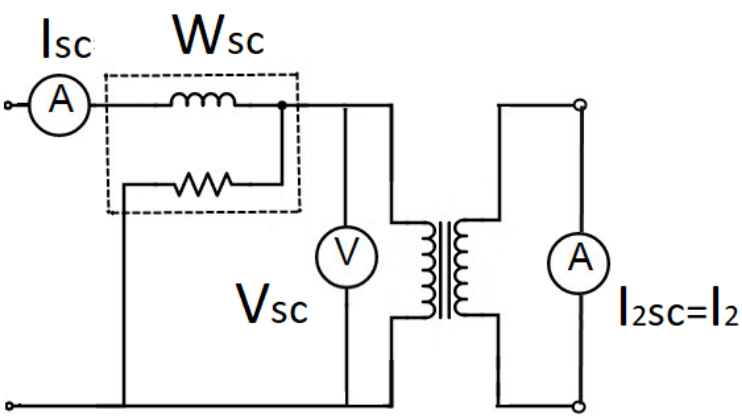
Calculation of the voltage regulation from the short circuit test

If the regulation required of I_2 with load angle θ_2 then the current I_2 is passed through the short current test in the secondary and I_{sc} , V_{sc} , W_{sc} are measured

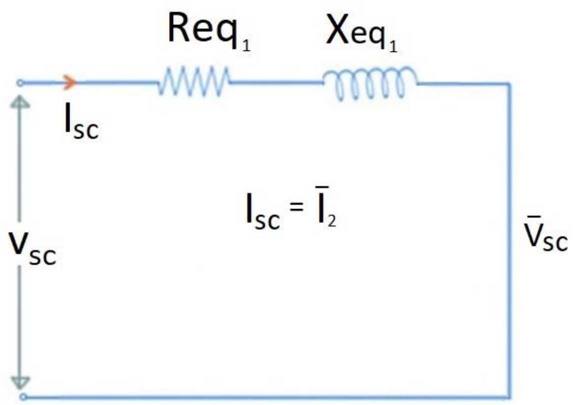
$Z_{sc} = \dfrac{V_{sc}}{I_{sc}}$

$Req_1 = \dfrac{V_{sc}}{I_{sc}} \cos\theta_{sc}$

$Xeq_1 = \dfrac{V_{sc}}{I_{sc}} \sin\theta_{sc}$



$$I_{sc} = I_2'$$



Regulation %

$$= \frac{I_2' \, Req_1 \cos\theta_2 \;\pm\; I_2' \, Xeq_1 \sin\theta_2}{V_1} \times 100$$

$$= \frac{I_{sc} \frac{V_{sc}}{I_{sc}} \cos\theta_{sc} \cos\theta_2 \;\pm\; I_{sc} \frac{V_{sc}}{I_{sc}} \sin\theta_{sc} \sin\theta_2}{V_1} \times 100$$

$$= \frac{V_{sc}}{V_1} (\cos\theta_{sc} \cos\theta_2 \;\pm \sin\theta_{sc} \sin\theta_2) \times 100$$

Regulation down %

Leading

$\downarrow$

$$= \frac{V_{sc}}{V_1} (\cos(\theta_{sc} \pm \theta_2) \;\times 100$$

$\uparrow$

Lagging