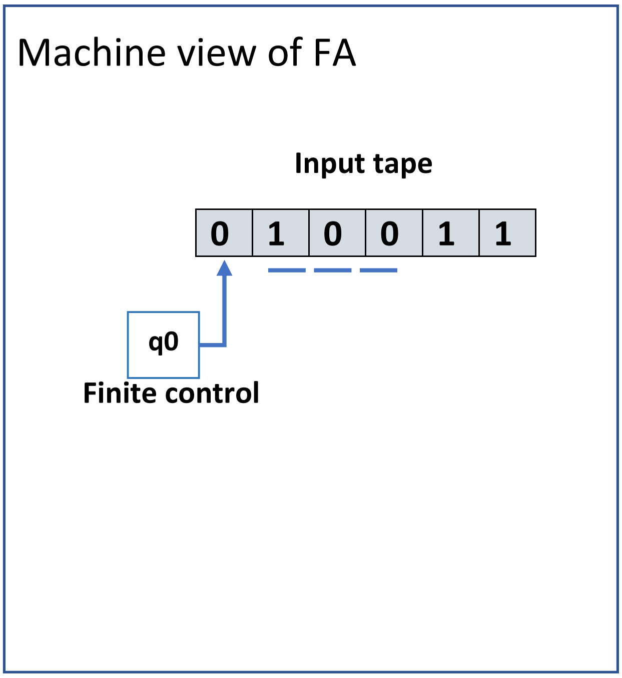
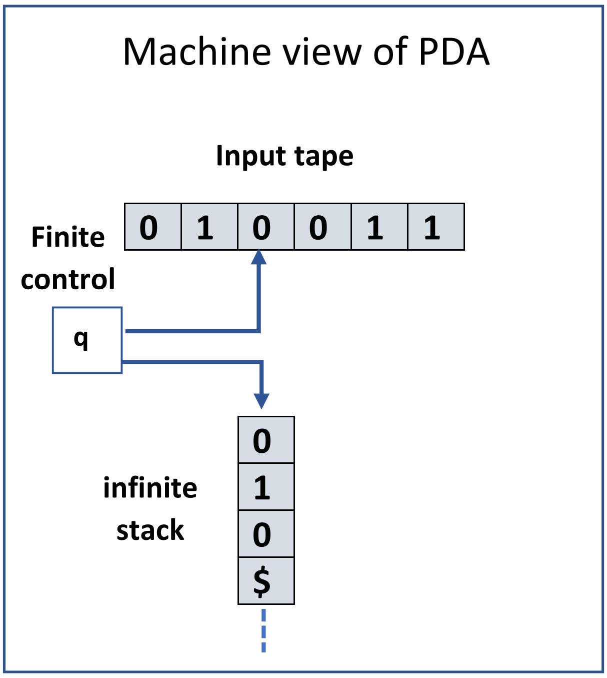
Push down automata (PDA)





A more powerful machine:

- Limitation of FA related to fact that they can only "remember" a bound amount of information.
- What is the simplest alteration that adds unbounded "memory" to our machine?
- Should be able to recognize, e.g, $\{0^n1^n : n>=0\}$

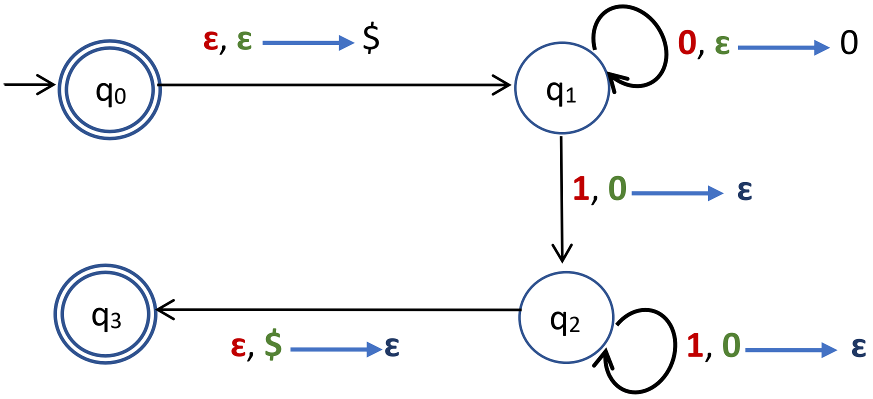
<u>PDF diagram</u>
Tape alphabet $\sum$
Stack alphabet $\Gamma = \sum$ and $

<u>Two ways to describe PDF</u>

1. Diagram
2. Formal definition

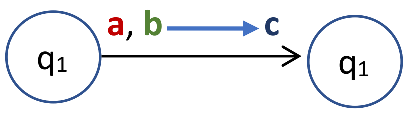
Example1: PDA

$\sum = \{0,1\}$
$\Gamma = \{0,1,\$\}$



Note: transition label:
(tape symbol read, stack symbol popped → stack symbol pushed)

<u>Taking a transition labelled:</u>



a ϵ (∑ U {ε})
b, c ϵ (Γ U {ε})

If the input symbol is a and the pop stack is b then
q1 to q2 , pop b, push c, advance read head

- read a from tape, or don't read from tape if a= ε
- pop b from stack, or don't pop from stack if b= ε
- push c onto stack, or don't push onto stack if c= ε

➢ In example 1 recognize $L= \{0^n1^n: n>=0\}$

:. Tape: 0011 is accept but 001 is not.

The formal definition of PDA

A PDA is a 6-tuple (Q, ∑, Γ, δ, S, F) where:

- Q is a finite set called the states.
- ∑ is a finite set called the tape alphabet
- Γ is a finite set called the stack alphabet
- δ:Q x (∑ U {ε}) x (Γ U {ε}) → δ(Q x (Γ U {ε})) is a function called the transition function
- S is an element of Q called the start state
- F is a subset of Q called the accepted states
- PDA M= (Q, ∑, Γ, δ, S, F) accepts string w ϵ ∑* if w can be written as $w_1\ w_2\ w_3 \ldots w_m$ ϵ (∑ U {ε})*, and there exist states $r_0\ r_1\ r_2 \ldots r_m$ and there exist strings $s_0\ s_1\ s_2 \ldots s_m$ in (Γ U {ε})*
  - $r_0 = q_0$ and $s_0 = \varepsilon$
  - $(r_{i+1}, b) \in \delta(r_i, w_{i+1}, a)$, where $s_i = a_t$, $s_{i+1} = b_t$ for some t ϵ Γ*

Example 1 in formal definition:

M= < Q, Σ, Γ ,S, F, δ >

Q= {q_0 , q_1 , q_2 , q_3 }

Σ= {0, 1}

Γ= {0,1,$}

S= q_0

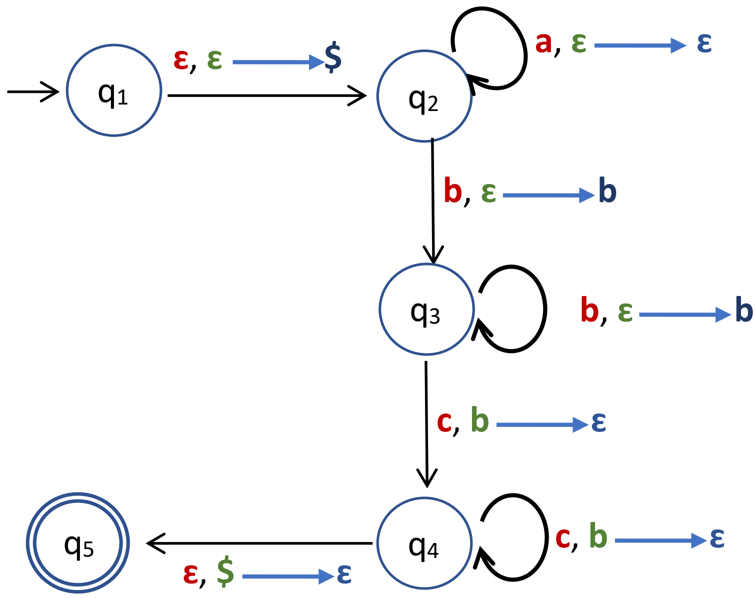
F={q_0, q_3}

- δ (q_0 , ε, ε) = {(q_1,$)}
- δ (q_1 , 0, ε) = {(q_1,0)}
- δ (q_1 , 1, 0) = {(q_2, ε)}
- δ (q_2 , 1, 0) = {(q_2, ε)}
- δ (q_3 , ε, $) = {($q_2$, ε)}

Example 2:

L= { $a^i b^j c^k$ | i, j, k>=0 and j=k}

Solution:



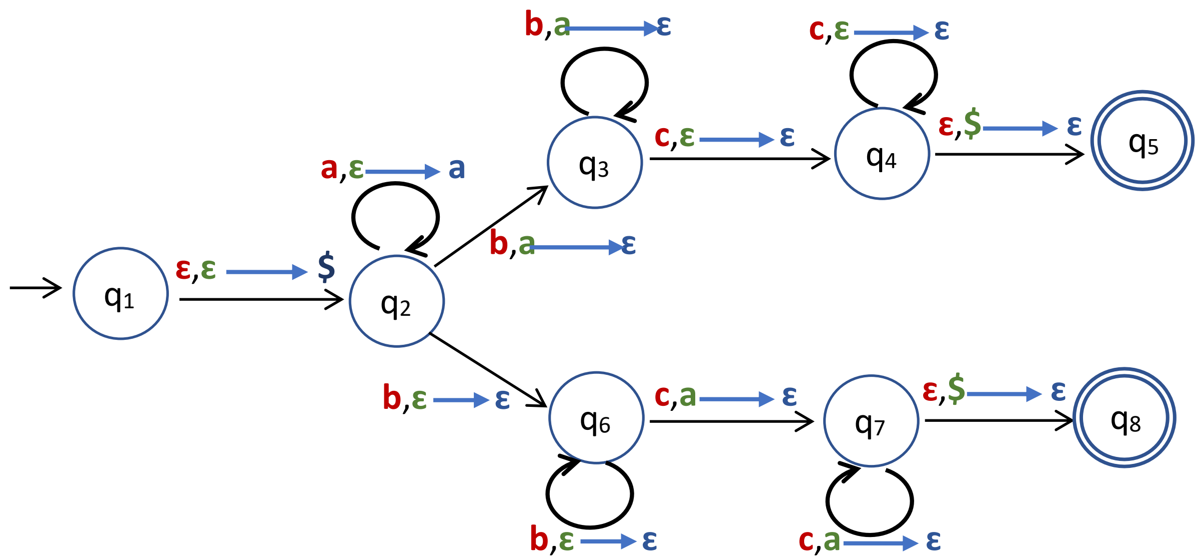
Example 3:

Construct a PDA for language

L= { $a^i b^j c^k$ | i, j, k>=0 and i=j or i=k }

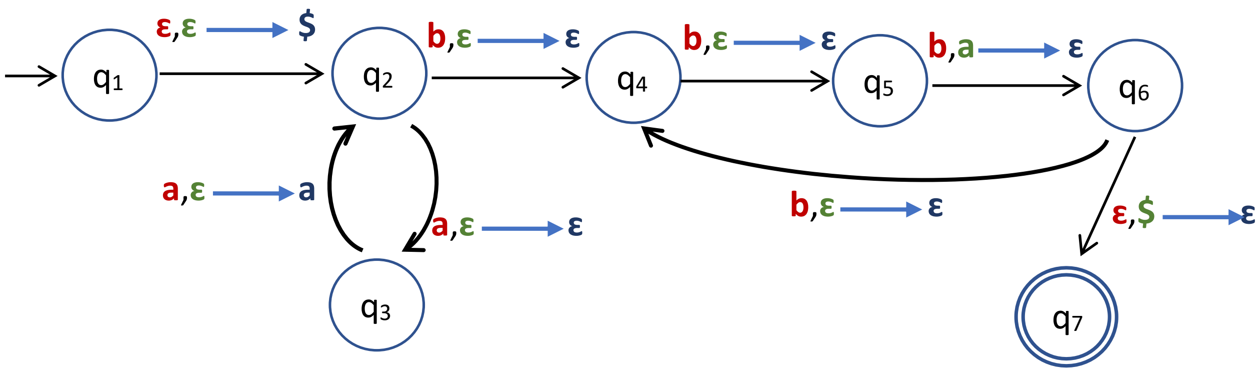
L1= { $a^i b^j c^k$ | i, j, k>=0 and i=j }   L2= { $a^i b^j c^k$ | i, j, k>=0 and i=k }

L= L1 **U** L2



Example 4:

Construct a PDA for language L={$a^{2n}b^{3n}$|n>=1}



n=1 aabbb

n=2 aaaabbbbbb

n=3 aaaaaabbbbbbbbb