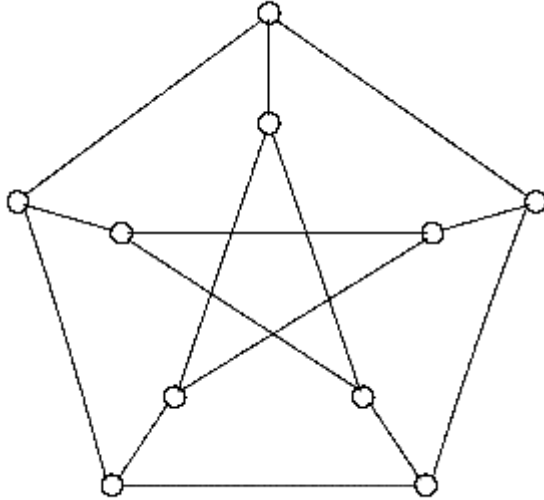


## Some special graphs:

### 1- Petersen graph

The graph shown in Figure



### 2- Complete graph

In this graph any vertex adjacent with each others , write as  $K_n$  , where n is the number of the vertices .

Example :  $K_2$

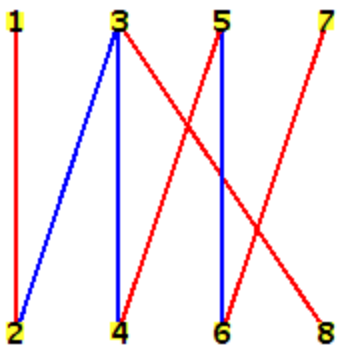
**Remark:**

$$\varepsilon(K_n) = \begin{cases} n(n-1) & \text{if } K_n \text{ is directed} \\ \frac{1}{2} n(n-1) & \text{if } K_n \text{ is undirected} \end{cases}$$

### 3 – Bipartite graph

A bipartite graph is a graph that the vertex – set can be partition into two sub sets X and Y such that the end – vertices of any edge one in X and the other one in Y .

**Example:**



#### 4- complete bipartite graph

A complete bipartite graph is a bipartite  $G(X \cup Y, E)$  in which each vertex of  $X$  is jointed by an edge to vertex of  $Y$ . If  $|X| = m$  and  $|Y| = n$  then the complete bipartite undirected graph denoted by  $K_{m,n}$ .

See Figure for an example to the complete bipartite graph for  $m = n=3, K_{3,3}$ .

**Example :-**  $G = (V(G), E(G), \psi_G)$  is an undirected graph, where

$$V(G) = \{z_1, z_2, z_3, z_4, z_5, z_6\}$$

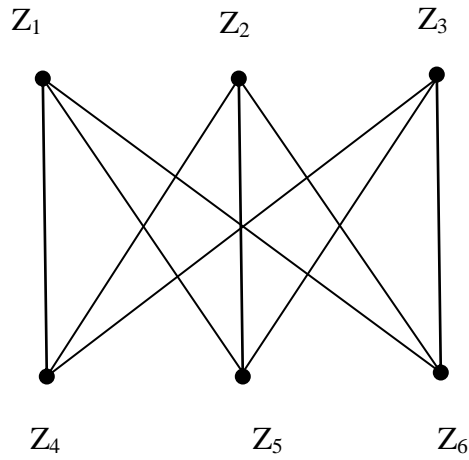
$$E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$$

and  $\psi_G$  is defined by

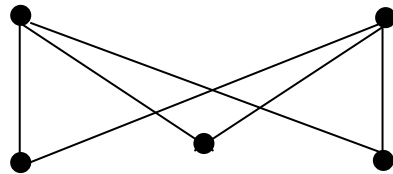
$$\psi_G(e_1) = z_1z_5, \quad \psi_G(e_2) = z_1z_4, \quad \psi_G(e_3) = z_1z_6,$$

$$\psi_G(e_4) = z_2z_4, \quad \psi_G(e_5) = z_3z_4, \quad \psi_G(e_6) = z_3z_6,$$

$$\psi_G(e_7) = z_2z_5, \quad \psi_G(e_8) = z_3z_5, \quad \psi_G(e_9) = z_2z_6,$$



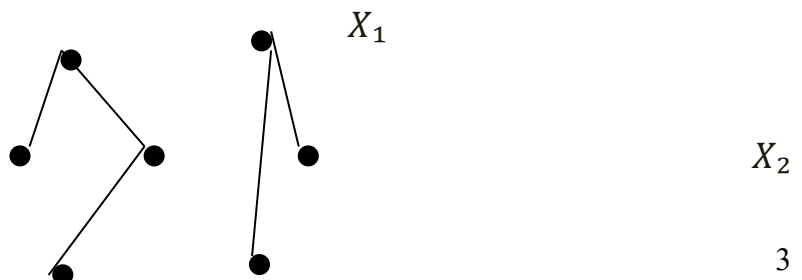
**Example:** to the complete bipartite graph for  $m = 2, n=3,$   
 $K_{2,3}$



### 5- K- Partite graph

In this graph we can partition the Vertex – set in to k- partite  $\{X_1, X_2, \dots, X_k\}$  such that in any edge in this graph , one end – Vertex in  $X_i$  and the other in  $X_j$  ( $i \neq j$ )

Example : 3- Partite



$X_3$ **6- Complete K- Partite graph**

The k- partite graph is called complete k- partite if any vertex in  $X_i$  adjacent with all vertices in  $X_1, X_2, \dots, X_{i-1}, X_{i+1}, \dots, X_k$ , denoted by  $K_n(k)$ .

**Remark :**

$$1) \varepsilon(K_{m,n}) = m n.$$

$$2) \varepsilon(K_n(k)) = \frac{1}{2} k(k-1)n^2.$$

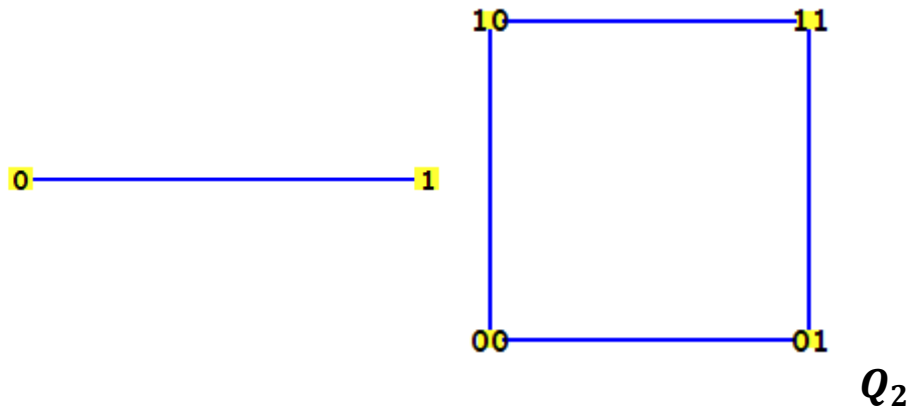
**7- n – Cube ( $Q_n$ )**

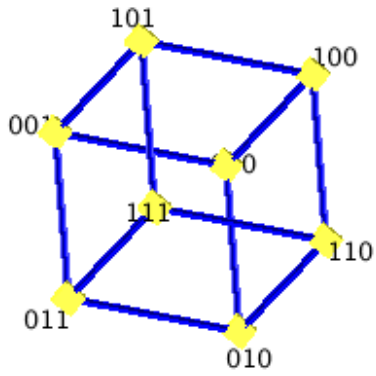
In this graph

$$V(Q_n) = \{x_1 x_2 \dots x_n : x_i \in \{0,1\}, i = 1,2, \dots, n\}$$

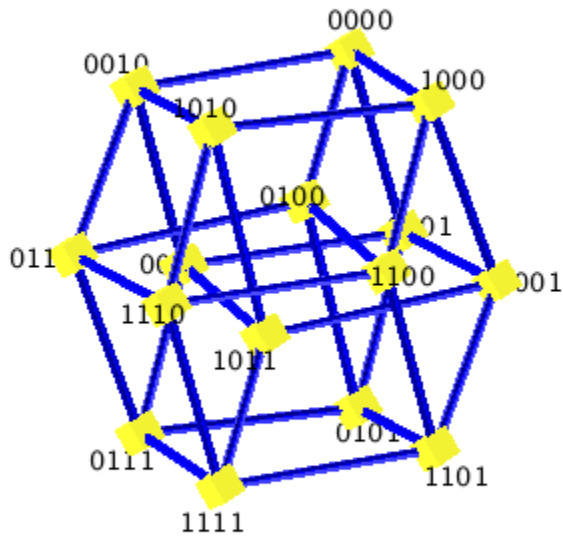
$$E(Q_n) = \{(x_1 x_2 \dots x_n, y_1 y_2 \dots y_n) : \sum_{i=1}^n |x_i - y_i| = 1\}, \text{ denoted by } Q_n.$$

$$\text{Remark : } v(Q_n) = 2^n, \varepsilon(Q_n) = n 2^{n-1}$$

**Example :  $Q_1$** 



$Q_3$



**Q4**

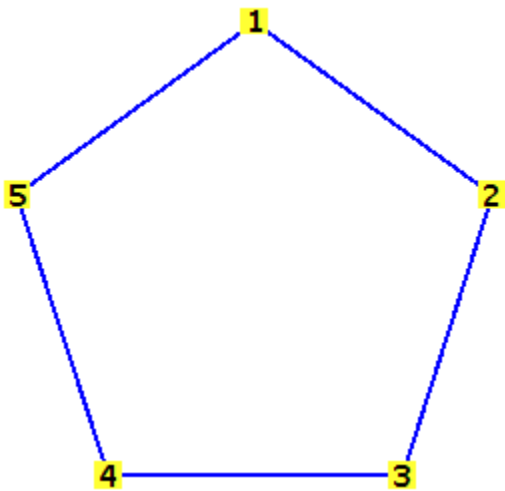
**Theorem :** The graph n- Cube is bipartite graph .

Proof:

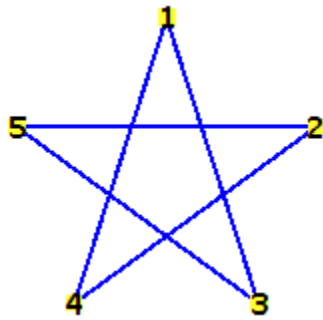
### 8- Complement graph

A graph  $G^c$  is called complement graph of the graph  $G$  if  $V(G^c) = V(G)$  and  $e \in E(G^c) \leftrightarrow e \notin E(G)$  , ( we write  $G^c$  as  $G'$  or  $\bar{G}$  )

Example :



$G$



$G^c$

**Definition :** If  $G \cong G^c$  then the graph  $G$  is called self-complementary .

**Remark:** Let  $D$  be a digraph such that

$$V(D) = \{x_1, x_2, \dots, x_n\}$$

$$E(D) = \{e_1, e_2, \dots, e_e\} \quad \text{but}$$

$$V(G) = \{x'_1, x''_1, x'_2, x''_2, \dots, x'_n, x''_n\},$$

$$E(G) = \{ = \{x'_i x''_j : (x_i, x_j) \in E(D)\} \quad \text{the graph } G \text{ is called}$$

**associated bipartite undirected graph with D .**

$$\text{i.e. } v(G) = 2v(D) \quad \text{and } \varepsilon(G) = \varepsilon(D) .$$

$$E(G) = \{e_1, e_2, \dots, e_e\}$$