Graph theory (ر350)

Le1

Definition : A graph is an ordered triple $(V(G),\ E(G),\ \psi_G)$ consisting of a non-empty set $V(G)$ of vertices, a set $E(G)$ of edges, and a mapping $\psi_G : E(G) \to V(G) \times V(G)$, is function define as : $\psi(e) = (x, y)$.

The two vertices x and y are called end vertices of the edge.

Remarks:

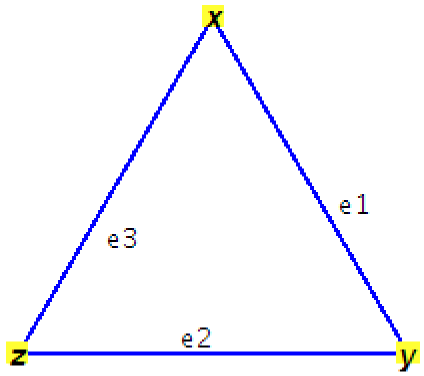
1- If $V \times V$ is set of ordered pairs then the graph is called directed graph (digraph) .
2- If $V \times V$ is set of unorder pairs then the graph is called undirected graph.
3- In digraph , the edge **e** denoted by $\psi(e) = (x, y)$.
4- In undirected graph , the edge **e** denoted by $\psi(e) = x\,y$ or $\psi(e) = \{x\ , y\ \}$.
5- The digraph denoted by **D or G** .
6- The undirected graph denoted by **G .**

Definition : The edge $\psi(e) = (x, y)\ or\ \ \psi(e) = x\,y$ is called loop if $x = y$.

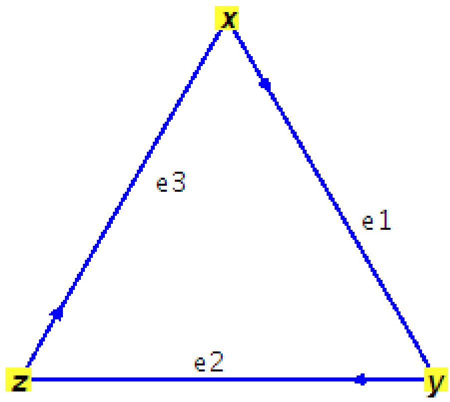
Definition : The two edges $\psi(e_1) = (x_1, y_1)$, $\psi(e_2) = (x_2, y_2)(or$ $\psi(e_1) = x_1y_1\ \psi(e_2) = x_2y_2)$ are called Parallel if $x_1 = y_1$ and $x_2 = y_2$.

Definition : The graph is called simple if contains neither **Parallel** nor **loop** .

Example :

1- 

G



D

$V(G)=\{x,y\ ,z\}=V(D)$ and edge set $E(G)=\{e_1,e_2,e_3\}\ =E(D)$.

Where G is undirected graph and D is digraph .

2- The edge is loop , the two edges are parallel.

3- Where not parallel .

4- Let G be a graph where

$V(G)=\{\ x_1\ x_2\ x_3\ :\ x_i\ \in\{0,1\},i=1,2,3\}$

$E(G)=\{\ (x_1\ x_2\ x_3\ ,y_1\ y_2\ y_3\ ):\ \sum_{i=1}^{3}|x_i-\ y_i|=1\ \}$

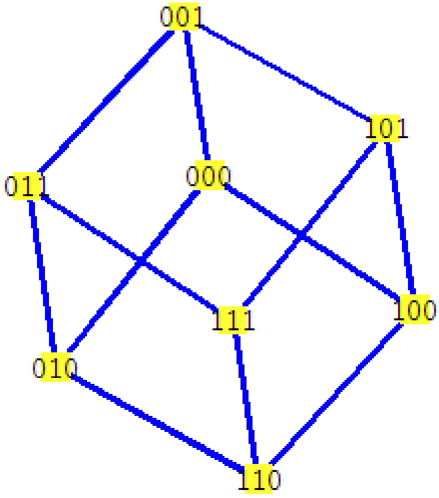
Solution : Then $V(G)=\{\ 000,001,010,100,011,101\ ,110,111\ \}$

$(000,001\ )\in E(G)$

$|0-0|+|0-1|+\ |0-1|=1$

$(001\,,010\,) \notin E(G)$

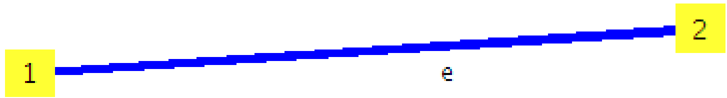
$|0-0| + |0-1| + |1-0| = 2 \neq 1$



$Q3$

Remarks :

1. The end vertices of the edge **e is** called incident with **e** .



2. If there is vertex $\boldsymbol{x}$ in common between two edges e_1 *and* e_2 then the edges e_1 *and* e_2 are called incident with vertex $\boldsymbol{x}$.

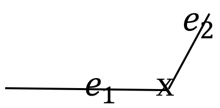
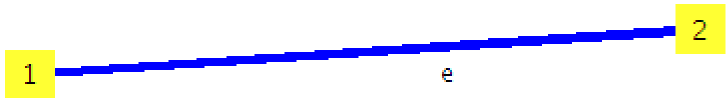


Fig. 1

3. The end vertices of the edge **e** **is** called adjacent .



4. If there is vertex $\boldsymbol{x}$ in common between two edges e_1 *and* e_2 (the edges adjacent) . (see Fig.1).

Definition : 1- The order of the graph G is the number of the vertices (write as : $\mathcal{V} = |V(G)|$) .

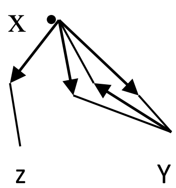
2- The size of the graph G is the number of the edges (write as : $\mathcal{E} = |E(G)|$) .

Remark :

1- $\mathrm{E}(x, y)$ is the set of edges from x to y .
2- $\mu\ (x, y) = |\mathrm{E}(x, y)|$ (*عدد الحافات بين*).

Example:

1-



$\mathrm{E}(x, y)$ = { 1 , 2 }

$\mathrm{E}(y, x)$ = {3}

$\mathrm{E}(x, z) = \{4\}$

$\mathrm{E}(z, x) = \varphi$

Remarks :

1- We can induced from any digraph undirected graph called underlying graph of digraph **D** by removing the orientation of the all directed edges.

2- We can obtained two digraphs from undirected graph.

1. Symmetric digraph of the graph G . In this digraph we write any undirected edge $\psi_G\,(e) = x\,y$ as two directed edges $\psi_D\,(e) = (x\,, y\,)$ and $\psi_D\,(e^{`}) = (\,y, x)$.
2. Oriented graph of a graph G in this digraph we write any undirected edge. $\psi_G\,(e) = x\,y$ as directed edge $\psi_D\,(e) = (x, y)$ or $\psi_D\,(e) = (y, x\,)$