Course syllabus

Machine Design

(Design of Machine Element)

Course Title: Machine Design (Design of Machine Element)

Semester Credit Hours: 2 credit hrs.

Course Delivery Method: Face to face and Electronic learning

Instructor: Dr. Rafil M. Laftah

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Lectures: Monday 8:30 – 10:30

Course Objectives:

1-The course is designed to teach the student how to analyze mechanical systems and select proper machine elements.

2-The course teaches the student how to design machine elements by specifying their type, geometry, material, and heat treatment and how to integrate these elements to build a mechanical system.

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Machine Design

(Design of Machine Element)

Required Textbooks: Shigly's Mechanical Engineering Design; by Budynas & Nisbett; 10th or 11th Edition; McGraw-Hill.

Course Description: The Machine Design course includes various materials needed to design mechanical elements. Initially students will be familiar with some concepts and definitions, and then they will be introduced to general considerations & procedure of machine design: general principles of machine design, reliability and statistical considerations, engineering materials & their mechanical properties, factor of safety, fits & tolerances, deflections and stress analysis for the different types of elements, buckling, static strength and failure theories, fatigue strength and failure theories. Finally, the students will be introduced to the basic design principles of some machine elements and their selection (power screws, fasteners and weldments). Also, the students will be introduced to the ethical and social impacts of mechanical design.

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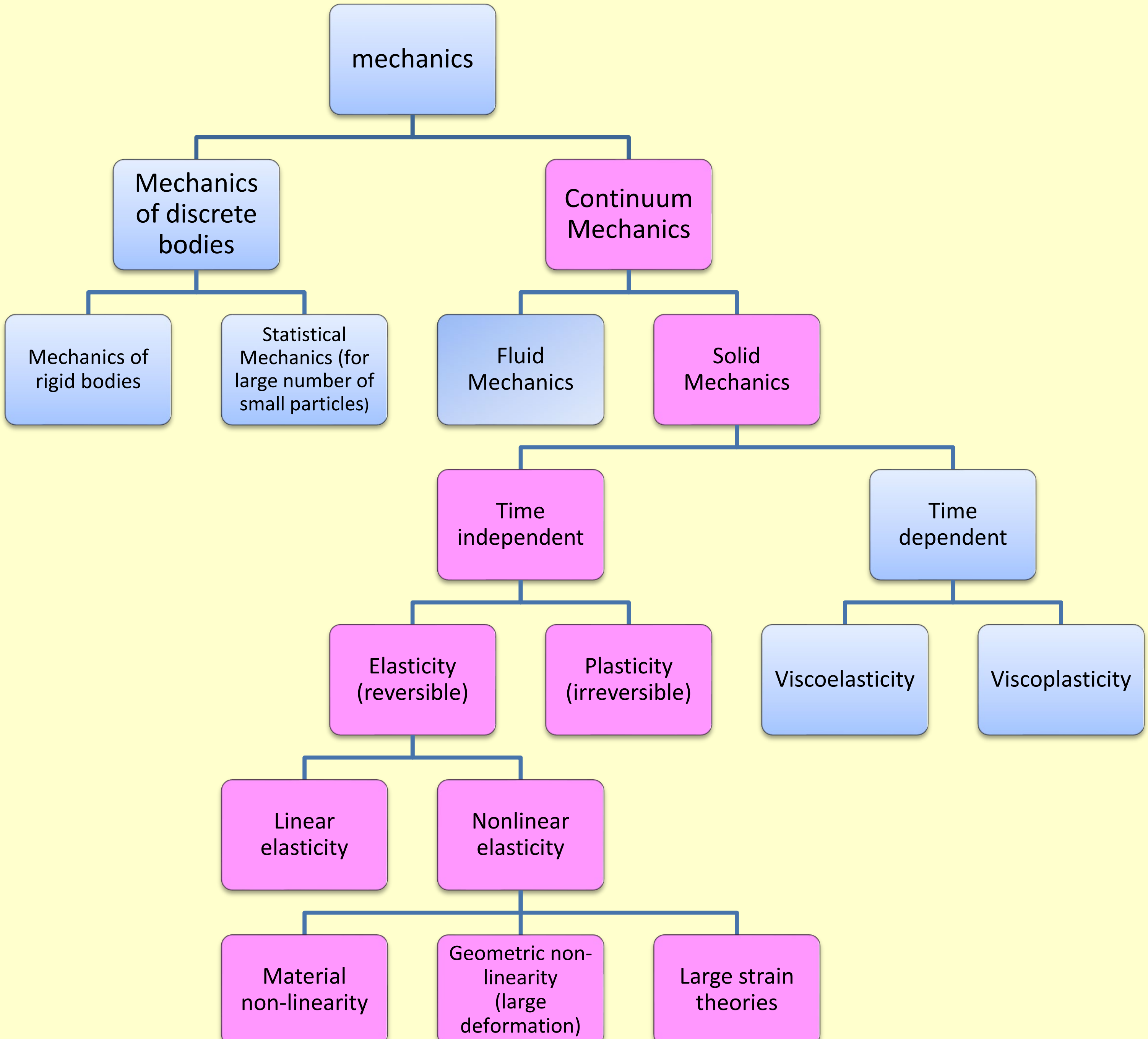
Course outline and Tentative Schedule:

Week	Date	Topics
1	23-Jan	Introduction (theories of stress and strain)
2	30-Jan	
3	6-Feb	Materials
4	13-Feb	Theories of Failure (Failure Criteria)
5	20-Feb	
6	27-Feb	
7	6-Mar	Introduction to Fracture Mechanics
8	13-Mar	Design for Fatigue Strength
9	20-Mar	
10	27-Mar	Springs Design
11	3-Apr	
12	10-Apr	Design of Shaft
13	17-Apr	Riveted and welded Joints
14	24-Apr	Threaded Fasteners
15	1-May	Power screws

Grading Policy:

Actions	Weighting of grades	Final grades
Homework	10 %	90-100 % Excellent
Midterm exam	20 %	80-89 V. good
Final exam	70%	70-79 Good
Total weight	100 %	60-69 Fair
		< 60 Fail


mechanics
Mechanics of discrete bodies
Continuum Mechanics
Mechanics of rigid bodies
Statistical Mechanics (for large number of small particles)
Fluid Mechanics
Solid Mechanics
Time independent
Time dependent
Elasticity (reversible)
Plasticity (irreversible)
Viscoelasticity
Viscoplasticity
Linear elasticity
Nonlinear elasticity
Material non-linearity
Geometric non-linearity (large deformation)
Large strain theories

Introduction

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

Introduction

Classifications of Machine Design

The machine design may be classified as follows:

1. Adaptive design: In most cases, the designer's work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The designer only makes minor alternation or modification in the existing designs of the product.

2.Development design: This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

3. New design: This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design. The designs, depending upon the methods used, may be classified as follows:

Introduction

(a) **Rational design:** This type of design depends upon mathematical formulae of principle of mechanics.

(b) **Empirical design**: This type of design depends upon empirical formulae based on the practice and past experience.

(c) **Industrial design**: This type of design depends upon the production aspects to manufacture any machine component in the industry.

(d) **Optimum design**: It is the best design for the given objective function under the specified constraints. It may be achieved by minimizing the undesirable effects.

(e) **System design:** It is the design of any complex mechanical system like a motor car.

(f) **Element design**. It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.

(g) **Computer aided design:** This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimization of a design.

Introduction

General Considerations in Machine Design

Following are the general considerations in designing a machine component:

1. **Type of load and stresses caused by the load**: The load, on a machine component, may act in several ways due to which the internal stresses are set up. The various types of load and stresses are discussed later.
2. **Motion of the parts or kinematics of the machine**: The successful operation of any machine depends largely upon the simplest arrangement of the parts which will give the motion required.

The motion of the parts may be:

(a) Rectilinear motion which includes unidirectional and reciprocating motions.

(b) Curvilinear motion which includes rotary, oscillatory and simple harmonic.

(c) Constant velocity.

(d) Constant or variable acceleration.

Introduction

General Considerations in Machine Design

Following are the general considerations in designing a machine component:

3. Selection of materials: It is essential that a designer should have a thorough knowledge of the properties of the materials and their behaviour under working conditions. Some of the important characteristics of materials are: strength, durability, flexibility, weight, resistance to heat and corrosion, ability to cast, welded or hardened, machinability, electrical conductivity, etc. The various types of engineering materials and their properties are discussed later.

4. Form and size of the parts: The form and size are based on judgment. The smallest practicable cross-section may be used, but it may be checked that the stresses induced in the designed cross-section are reasonably safe. In order to design any machine part for form and size, it is necessary to know the forces which the part must sustain. It is also important to anticipate any suddenly applied or impact load which may cause failure.

Introduction

5. Frictional resistance and lubrication: There is always a loss of power due to frictional resistance and it should be noted that the friction of starting is higher than that of running friction. It is, therefore, essential that a careful attention must be given to the matter of lubrication of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings.

6. Convenient and economical features: In designing, the operating features of the machine should be carefully studied. The starting, controlling and stopping levers should be located on the basis of convenient handling. The adjustment for wear must be provided employing the various take up devices and arranging them so that the alignment of parts is preserved. If parts are to be changed for different products or replaced on account of wear or breakage, easy access should be provided and the necessity of removing other parts to accomplish this should be avoided if possible. The economical operation of a machine which is to be used for production or for the processing of material should be studied, in order to learn whether it has the maximum capacity consistent with the production of good work.

Introduction

7. Use of standard parts: The use of standard parts is closely related to cost, because the cost of standard or stock parts is only a fraction of the cost of similar parts made to order. The standard or stock parts should be used whenever possible; parts for which patterns are already in existence such as gears, pulleys and bearings and parts which may be selected from regular shop stock such as screws, nuts and pins. Bolts and studs should be as few as possible to avoid the delay caused by changing drills, reamers and taps and also to decrease the number of wrenches required.

8. Safety of operation: Some machines are dangerous to operate, especially those which are speeded up to insure production at a maximum rate. Therefore, any moving part of a machine which is within the zone of a worker is considered an accident hazard and may be the cause of an injury. It is, therefore, necessary that a designer should always provide safety devices for the safety of the operator. The safety appliances should in no way interfere with operation of the machine.

Introduction

9. Workshop facilities: A design engineer should be familiar with the limitations of this employer's workshop, in order to avoid the necessity of having work done in some other workshop. It is sometimes necessary to plan and supervise the workshop operations and to draft methods for casting, handling and machining special parts.

10. Number of machines to be manufactured: The number of articles or machines to be manufactured affects the design in a number of ways. The engineering and shop costs which are called fixed charges or overhead expenses are distributed over the number of articles to be manufactured. If only a few articles are to be made, extra expenses are not justified unless the machine is large or of some special design. An order calling for small number of the product will not permit any undue expense in the workshop processes, so that the designer should restrict his specification to standard parts as much as possible.

Introduction

11. Cost of construction: The cost of construction of an article is the most important consideration involved in design. In some cases, it is quite possible that the high cost of an article may immediately bar it from further considerations. If an article has been invented and tests of handmade samples have shown that it has commercial value, it is then possible to justify the expenditure of a considerable sum of money in the design and development of automatic machines to produce the article, especially if it can be sold in large numbers. The aim of design engineer under all conditions should be to reduce the manufacturing cost to the minimum.

12. Assembling: Every machine or structure must be assembled as a unit before it can function. Large units must often be assembled in the shop, tested and then taken to be transported to their place of service. The final location of any machine is important and the design engineer must anticipate the exact location and the local facilities for erection.

Introduction

General Procedure in Machine Design

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows:

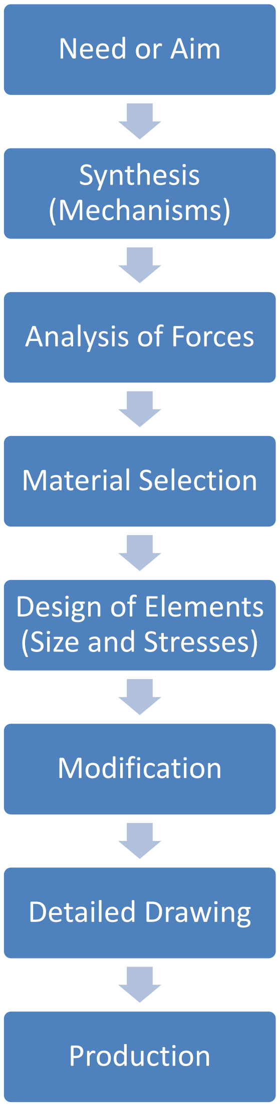


Fig. 1: General Machine Design Procedure

Introduction

1. Recognition of need: First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.

2. Synthesis (Mechanisms): Select the possible mechanism or group of mechanisms which will give the desired motion.

3. Analysis of forces: Find the forces acting on each member of the machine and the energy transmitted by each member.

4. Material selection: Select the material best suited for each member of the machine.

5. Design of elements (Size and Stresses): Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than the permissible limit.

6. Modification: Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

Introduction

7. Detailed drawing: Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

8. Production: The component, as per the drawing, is manufactured in the workshop. The flow chart for the general procedure in machine design is shown in Fig.1.

Note: When there are number of components in the market having the same qualities of efficiency, durability and cost, then the customer will naturally attract towards the most appealing product. The aesthetic and ergonomics are very important features which gives grace and lustre to product and dominates the market.

Introduction

Engineering materials and their properties

The knowledge of materials and their properties is of great significance for a design engineer. The machine elements should be made of such a material which has properties suitable for the conditions of operation. In addition to this, a design engineer must be familiar with the effects which the manufacturing processes and heat treatment have on the properties of the materials. Now, we shall discuss the commonly used engineering materials and their properties in Machine Design.

Classification of Engineering Materials

The engineering materials are mainly classified as:

1. Metals and their alloys, such as iron, steel, copper, aluminum, etc.
2. Non-metals, such as glass, rubber, plastic, etc.

The metals may be further classified as:

(a) Ferrous metals and (b) Non-ferrous metals.

The **ferrous metals** are those which have the iron as their main constituent, such as cast iron, wrought iron and steel.

The **non-ferrous metals** are those which have a metal other than iron as their mam constituent, such as copper, aluminum, brass, tin, zinc, etc.

Introduction

Selection of Materials for Engineering Purposes

The selection of a proper material, for engineering purposes, is one of the most difficult problems for the designer. The best material is one which serves the desired objective at the minimum cost. The following factors should be considered while selecting the material:

1. Availability of the materials,
2. Suitability of the materials for the working conditions in service, and
3. The cost of the materials.

The important properties, which determine the utility of the material, are physical, chemical and mechanical properties. We shall now discuss the physical and mechanical properties of the material in the following articles.

Physical Properties of Metals

The physical properties of the metals include luster, colour, size and shape, density, electric and thermal conductivity, and melting point. The following table shows the important physical properties of some pure metals.

Introduction

Mechanical Properties of Metals

The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load. These mechanical properties of the metal include strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and hardness. We shall now discuss these properties as follows:

1.**Strength**: It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called stress.

2.Stiffness: It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.

3.Elasticity: It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.

Introduction

4.Plasticity: It is property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

5. Ductility: It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminium, nickel, zinc, tin and lead.

6. Brittleness: It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads snap off without giving any sensible elongation. Cast iron is a brittle material.

Introduction

7. Malleability: It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminum.

8.Toughness: It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed upto the point of fracture. This property is desirable in parts subjected to shock and impact loads.

9. Machinability: It is the property of a material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of the material. It may be noted that brass can be easily machined than steel.

Introduction

10.Resilience: It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit. This property is essential for spring materials.

11. Creep: When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called creep. This property is considered in designing internal combustion engines, boilers and turbines.

12.Fatigue: When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as *fatigue. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.

Introduction

13. Hardness: It is a very important property of the metals and has a wide variety of meanings. It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal. The hardness is usually expressed in numbers which are dependent on the method of making the test. The hardness of a metal may be determined by the following tests:

(a) Brinell hardness test,

(b) Rockwell hardness test,

(c) Vickers hardness (also called Diamond Pyramid) test.

Standards and Codes

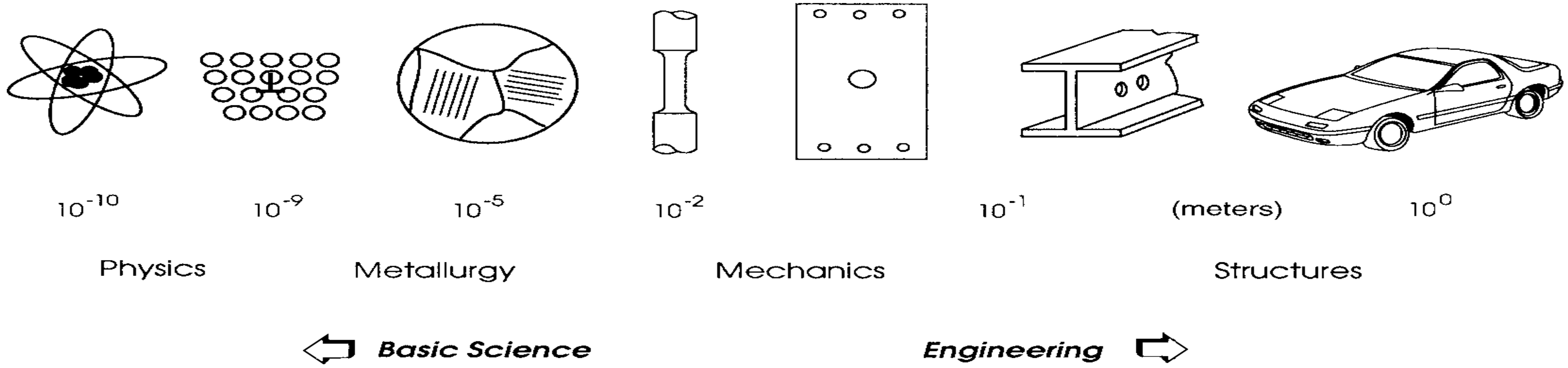
standards and Codes are made to organize and unify the engineering work. Imagine; what if there was no standard for bolts?

- A standard is a set of specifications for parts, materials, or processes intended to achieve uniformity, efficiency and specific quality.
- A code is a set of specifications for the analysis, design, manufacture, and construction of something.

Examples of organizations that establish standards and design codes: AISI, AGMA, SAE, ASTM, ISO.

Materials

Chapter two



Materials

Choosing the appropriate material is an important step in mechanical design.

Material Strength and Stiffness

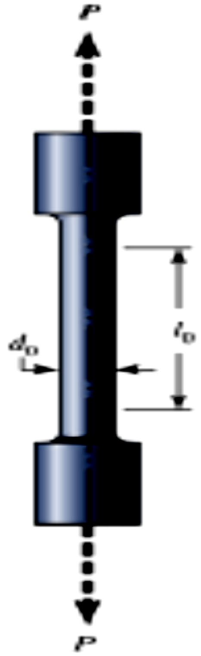
Standard tensile test is used to determine many of the mechanical properties of the materials. Load, P, and deflection, $\delta = (l - lo)$, are recorded and from that the engineering stress vs. strain curve is determined.

Engineering stress: $\sigma = \frac{P}{A}$

Strain: $\epsilon = \frac{l - l_o}{l_o}$

d_o: original diameter

l_o: gauge length (original length)

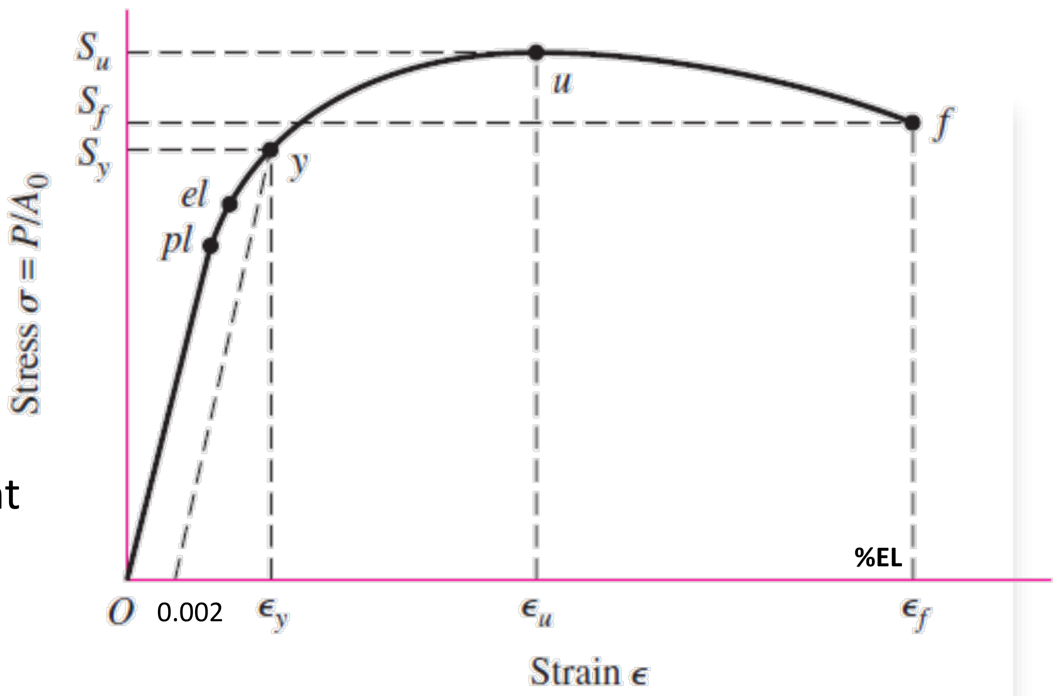


- Elastic region: no permanent deformation takes place (if the load is removed, the specimen goes back to its original length).

-Stress & strain are related linearly by Hook's law:

$$\sigma = E\epsilon$$

E: Young's modulus or modulus of elasticity
(slope of the linear part)



- **Proportional limit**: The curve starts to deviate from straight line.
- **Yield strength**, S_y: The end of elastic deformation, plastic (permanent) deformation starts.

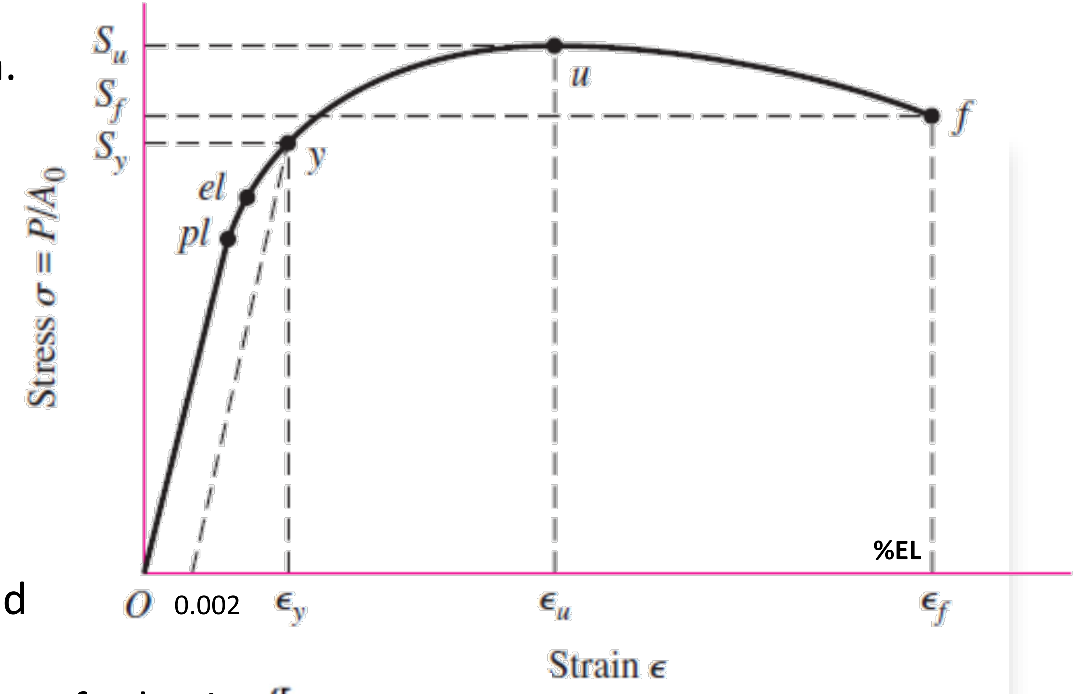
- Usually determined using the 0.002 yield criterion.

- **Ultimate strength**, S_u: Is the maximum stress reached on the "engineering" stress strain diagram.
- **Necking starts after** Su(for ductile materials) until fracture at point "f".
- **The elongation at fracture is referred to as (%EL)**.

- **Strength** of a material usually refers to the ultimate strength.
- **Stiffness** refers to the amount of deformation the material shows under applied load. The lesser the deformation, the stiffer the material.

- In the elastic region, "stiffness" refers to Young's modulus.

The higher the E, the higher the stiffness.

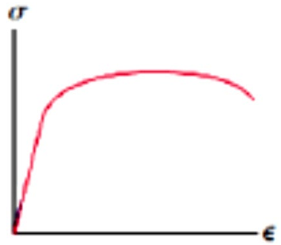


Ductile and Brittle Materials

Ductility measures the degree of plastic deformation sustained at fracture.

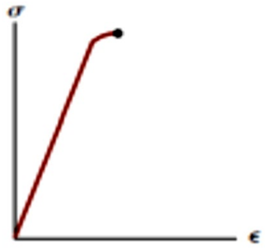
- **A Ductile material** is one that exhibits a large amount of plastic deformation before failure.

-It shows necking before fracture.



- **A Brittle material** exhibits little or no yielding before fracture.

- Usually defined as materials having %EL < 5% at fracture.
- It shows little or no necking before fracture.
-It is much stronger in compression than in tension.



Types of fracture

Failure in metallic materials can be divided into two main categories;

Ductile failure

Ductile fracture involves a large amount of plastic deformation and can be detected beforehand.

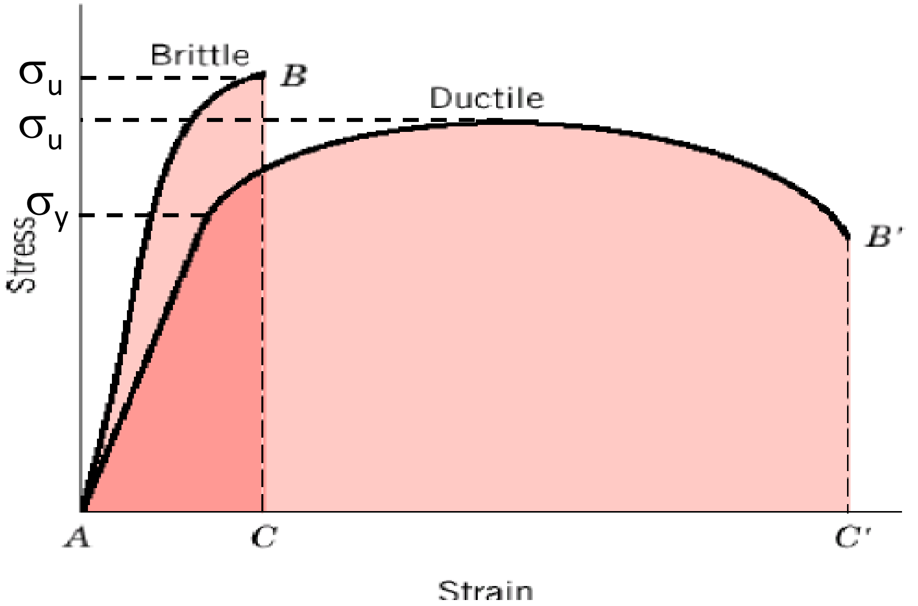
Brittle failure

Brittle fracture is more catastrophic and has been intensively studied.

Theories of brittle fracture

S_y: Yield stress from tensile test
S_u: Ultimate stress from tensile test

S_u: Ultimate stress from tensile test
(No yield)



Strength and Cold Work

Cold working is a process in which the material is **plastically deformed** at a temperature below the recrystallization temperature (no creep occurs).

- If a load was applied to the material such that the stress in the material exceeded the yield strength (but it is less than the ultimate strength) then the load was removed, the stress strain relationship during the unloading will follow a straight line parallel to the initial elastic line.

-The material will have some amount of elastic recovery and some permanent deformation. Knowing that the unloading line have a slope of E, we can find the amount of elastic strain ε_e as,

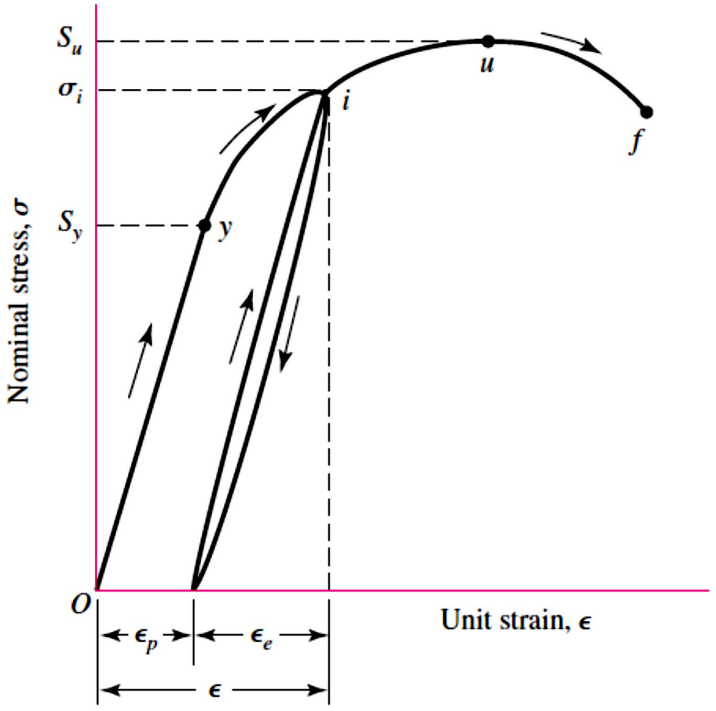
$$\varepsilon_e = \frac{\sigma_i}{E}$$

The total strain at point "i" is:

$$\varepsilon = \varepsilon_p + \varepsilon_e$$

ε_p: is the plastic (permanent) strain.

- If the material was reloaded again, the stress strain relation will follow the same unloading line and point "i" will be, approximately, the new yield point of the material.



Heat Treatment of Steel

Heat treatment refers to time & temperature-controlled process that relieves residual stresses and/or modifies material properties.

1. Annealing

When a material is cold (or hot) worked, residual stresses are introduced in the material. Also, the strength of the material is usually increased.

Annealing is a heat treatment process in which the residual stresses are removed, the material is softened (strength is reduced) and the material is made to be more ductile.

- The material to be annealed is heated to a temperature that is approximately 50˚C above the critical temperature and is held at this temperature for a sufficient time for the carbon to be dissolved and defused in the material. Then the object is allowed to cool-down slowly.

2. Quenching

Quenching is a process in which the material is hardened.

- The material is heated to a temperature above the critical temperature then the material is cooled at a fast rate (using water or oil) to a temperature below the critical temperature.

3. Tempering

When a material is quenched, residual stresses are introduced because of the uneven fast cooling.
Tempering is used to remove the residual stresses from the fully hardened material (which is very brittle) and it reduces hardness of the material.

- After the material has been quenched, tempering is done by reheating the material to some temperature below the critical temperature for a certain period of time, then allowing it to cool in still air.

4. Case Hardening

The purpose of case hardening is to produce a hard outer surface on a specimen of low carbon steel while maintaining the ductility in the core.

- Case hardening is usually done by increasing the carbon content at the surface by surrounding the surface by a carburizing material for a certain time at a certain temperature. Then the material is quenched down from this temperature and tempered.
- Another method of case hardening is to heat the surface using a flame (or induction coil) then quenching and tempering.

Classification of Solid Materials

Engineering materials fall into four major classes: Metals, Ceramics (including glasses), Polymers, and Composites.

1. Metals

Metals are combinations of metallic elements. They are strong and deformable, making them important materials in machine design.

2. Ceramics and Glasses

Ceramics are compounds of metallic and non-metallic elements, most frequently oxides, nitrides, and carbides. Glasses are similar to ceramics in composition but they have no clear crystal structure.

- Ceramics are stiff, hard, brittle, and much stronger in compression than in tension.

- Ceramics are made from ceramic slurry then it is fired to solidify.

3. Polymers

Polymers include plastic and rubber materials. Many polymers are organic compounds based on carbon & hydrogen. Polymers are of two basic types: thermoplastics & thermosets

- Thermoplastics are long chain molecules, sometimes with branches, where strength arises from interference between chains and branches. Thermoplastics are more ductile than thermosets and they soften significantly and melt at elevated temperatures.
- Thermosets have higher degree of cross-linking (like a sponge). They are more brittle, do not soften with temperature as much as thermoplastics and usually chemically degrade before melting.

- Mechanical properties of polymers depend largely on temperature and they cannot stand high temperatures (less than 250˚C).

4. Composites

Composite materials are formed from two or more materials that remain distinct from each other where each material contributes to the final properties. It combines the attractive properties of each component material.

- For example the graphite-reinforced epoxy acquires strength from the graphite fibers while the epoxy protects the graphite from oxidation and supports shear stress.
- The two main advantages of composites are tailor ability (strength is provided in the needed region and direction only) and light weight (for example the strength-to-weight ratio of graphite is about 40 times as that of steel).
- The three main types of composites are:

1. Particle reinforced: particles that have approximately the same dimensions in all directions are embedded into a matrix, such as concrete.
2. Short-fiber reinforced: short fibers oriented in arbitrary directions in a matrix, such as fiber glass castings.
3. Continuous-fiber reinforced: continuous fibers are constructed into a part by layers. It can be either unidirectional layers or woven-fabric layers.

- The most commonly used fibers are: Graphite (carbon), Glass, and Kevlar.
- The matrix is usually a thermoset (such as epoxy).
- Composites are manufactured by molding or layup, and then cured in an oven.
- The main disadvantages of composites are high cost, they cannot withstand high temperature (because of the polymer matrix), and they are relatively difficult to join.

Design against static load

Basics:

2. Ductile vs Brittle materials

Failure in structures leads to **lost of properties** and sometimes lost of human lives.

An oil tanker that fractured in a brittle manner by crack propagation around its girth. (Photography by Neal Boenzi. Reprinted with permission from The New York Times.)

Failed fuselage of the Aloha 737 aircraft in 1988.

Collapse of Point Pleasant suspension bridge, West Virginia, 1967. May-Aug 200?

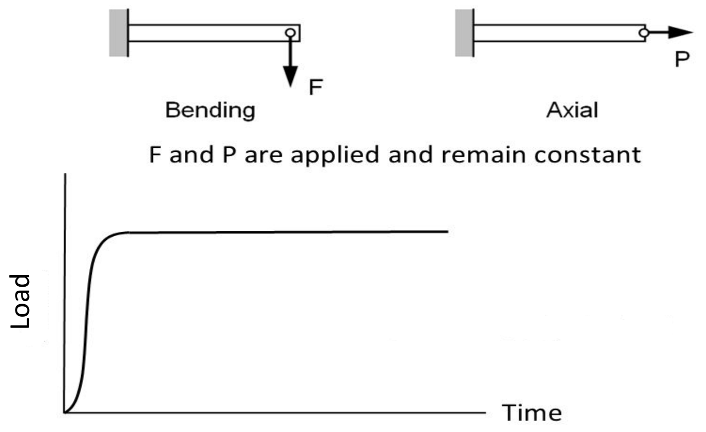
Design against static load

Basics:

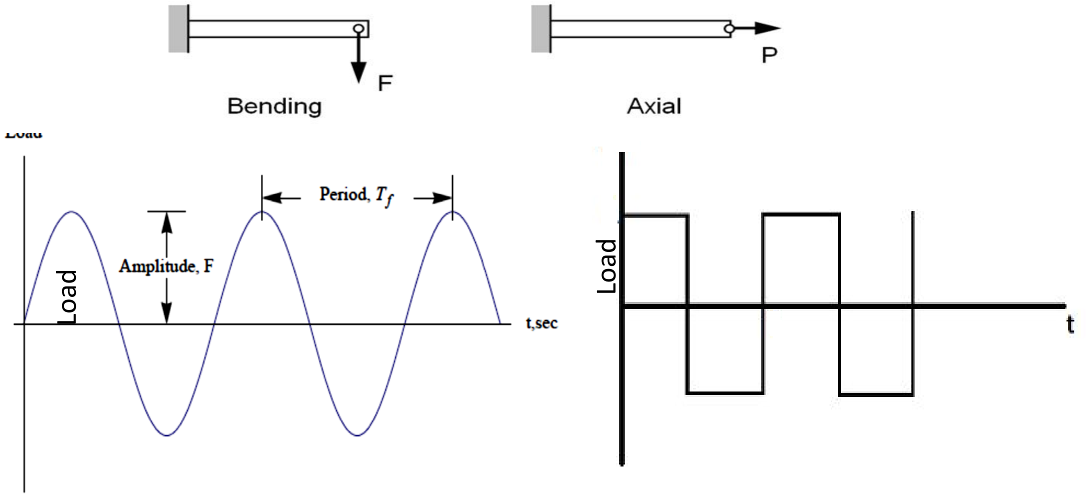
1. Static load vs dynamic load

- If load dose not vary with time called **Static** otherwise it is dynamic load.

Static Load



Dynamic Load



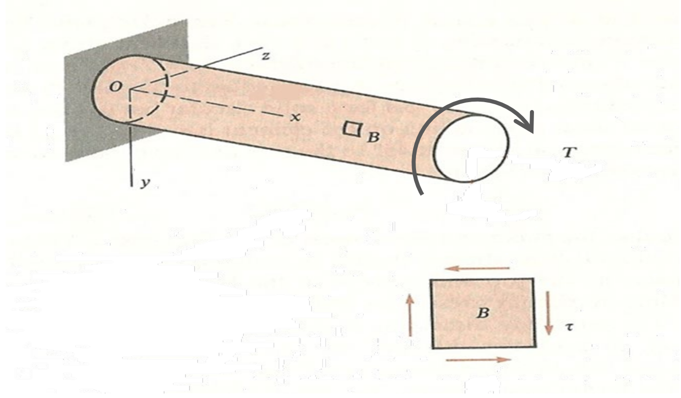
*For this chapter; load is always **static***

3. Principal Stresses

Case 1 shaft subjected to torque only

Take point B

Stress are $\tau = \frac{Tr}{J}$



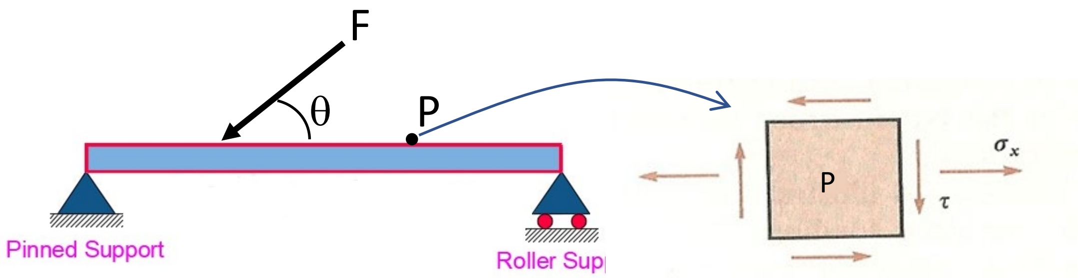
Case 2 beam under concentrated load

Stresses:

$$\sigma_x = \frac{F\cos\theta}{A} \pm \frac{My}{I}$$

$$\sigma_y = 0$$

$$\tau_{xy} = \frac{VA\bar{y}}{It}$$



Simply Supported Beam

In general (Two-Dimensional)

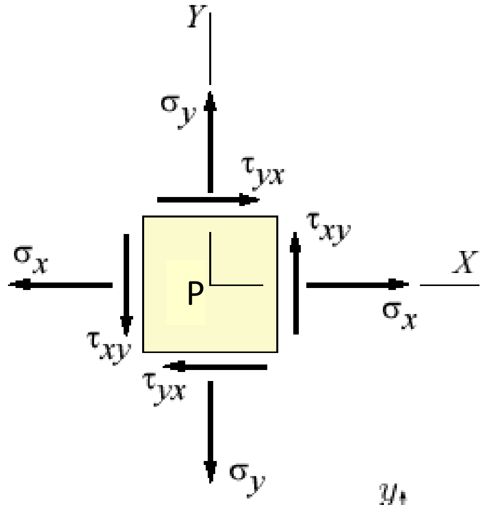
- Let a structure (member) subjected to external loads, stresses at a point (p) in general are:

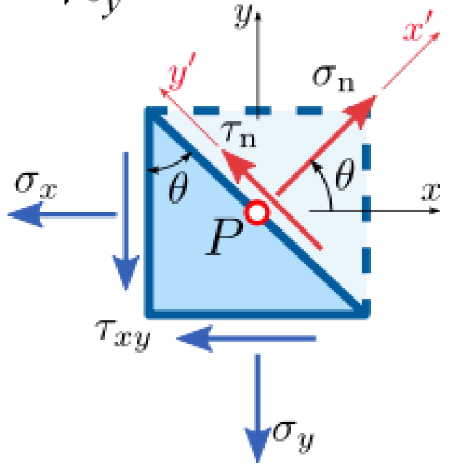
  (not enough)

  Need to check stresses (***normal and shear***) on oblique planes

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} cos2\theta + \tau_{xy} sin2\theta$$

$$\tau_\theta = -\left(\frac{\sigma_x - \sigma_y}{2}\right) sin2\theta + \tau_{xy} cos2\theta$$





To find Max. normal stress $\Longrightarrow \frac{d\sigma_\theta}{d\theta} = 0$

And to find Max. shear stress $\Longrightarrow \frac{d\tau_\theta}{d\theta} = 0$

This leads to principal stresses:

A- Principal Normal stresses:

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

B- Max. Shear Stress :

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Failure Theories

Chapter Three

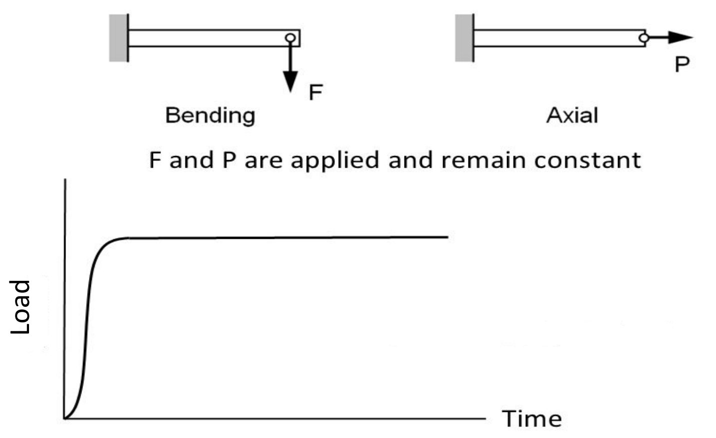
Design against static load

Basics:

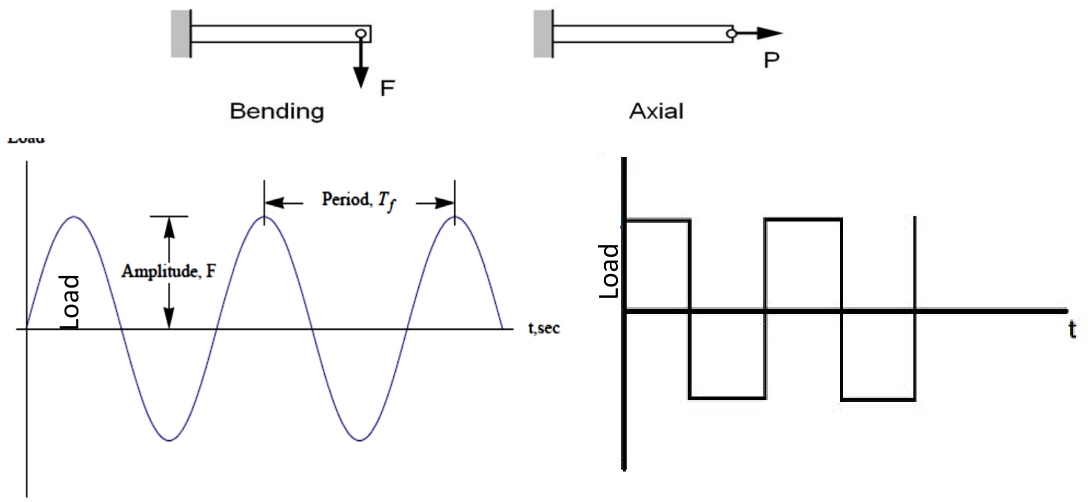
1. Static load vs dynamic load

- If load dose not vary with time called **Static** otherwise it is dynamic load.

Static Load



Dynamic Load



*For this chapter; load is always **static***

Design against static load

Basics:

2. Ductile vs Brittle materials

Failure in structures leads to **lost of properties** and sometimes lost of human lives.

*An oil tanker that fractured in a brittle manner by crack propagation around its girth. (*Photography by Neal Boenzi. Reprinted with permission from The New York Times.)

Failed fuselage of the Aloha 737 aircraft in 1988.

Collapse of Point Pleasant suspension bridge, West Virginia, 1967.

Types of fracture

Failure in metallic materials can be divided into two main categories;

Ductile failure

Ductile fracture involves a large amount of plastic deformation and can be detected beforehand.

Brittle failure

Brittle fracture is more catastrophic and has been intensively studied.

Theories of brittle fracture

S_y: Yield stress from tensile test
S_u: Ultimate stress from tensile test

S_u: Ultimate stress from tensile test
(No yield)

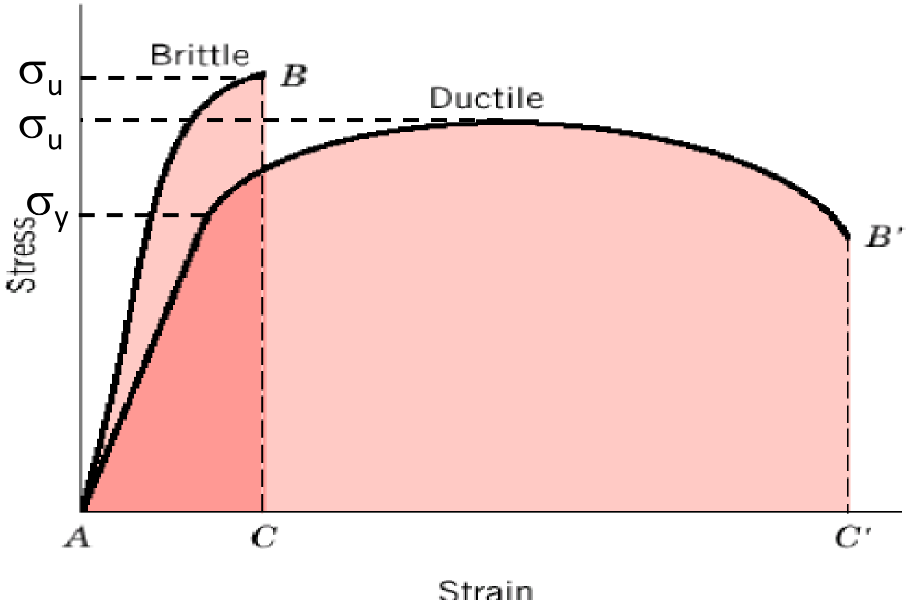
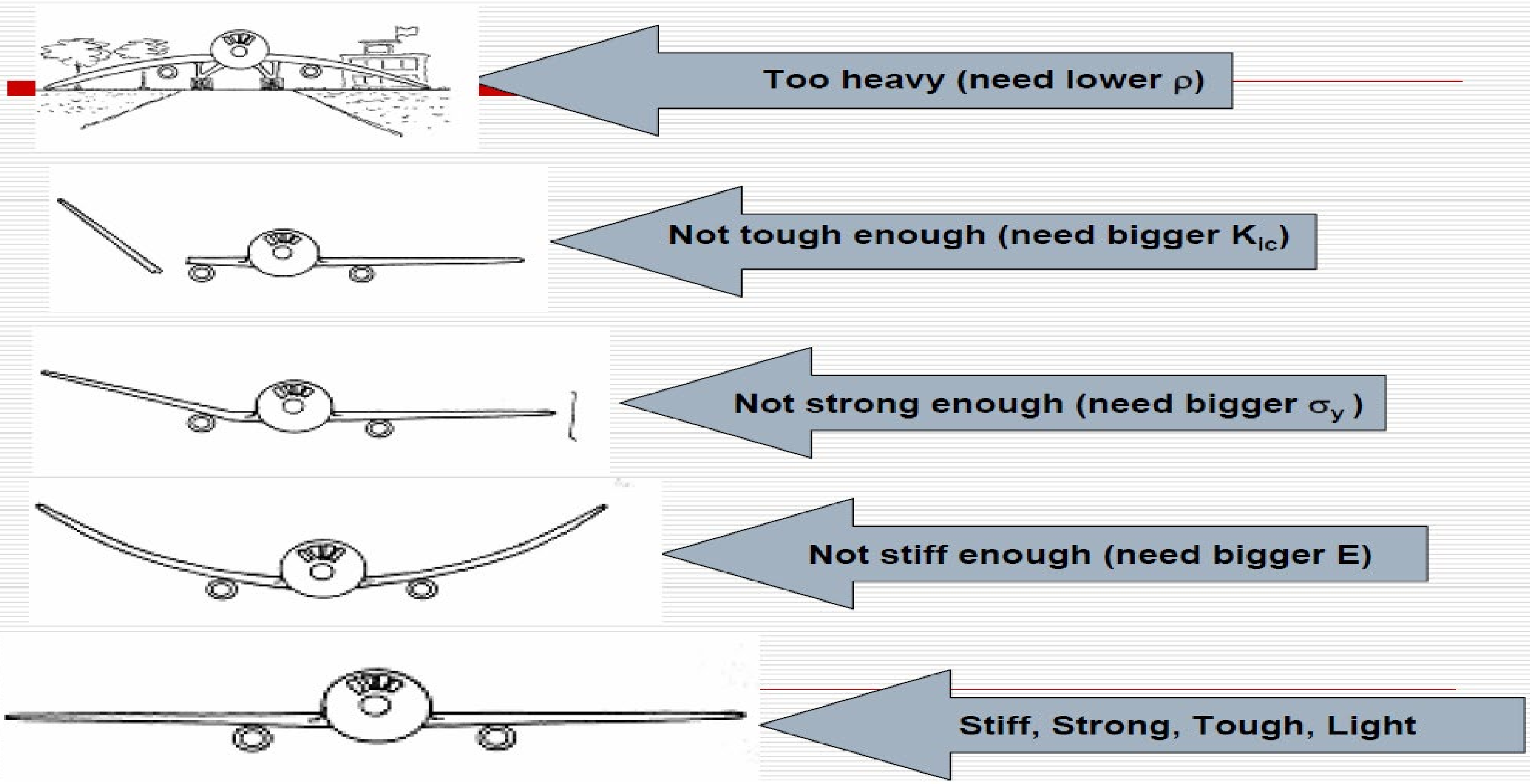


Illustration of Mechanical properties

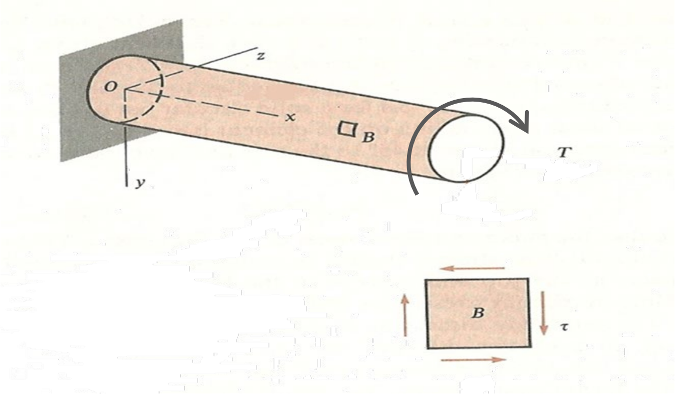


3. Principal Stresses

Case 1 shaft subjected to torque only

Take point B

Stress are $\tau = \frac{Tr}{J}$



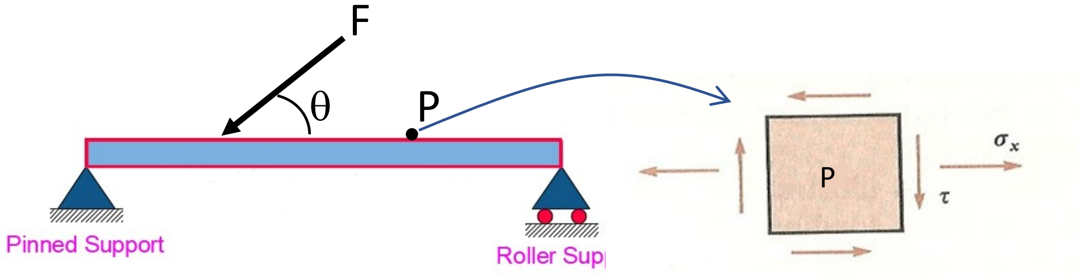
Case 2 beam under concentrated load

Stresses:

$$\sigma_x = \frac{F\cos\theta}{A} \pm \frac{My}{I}$$

$$\sigma_y = 0$$

$$\tau_{xy} = \frac{VA\bar{y}}{It}$$

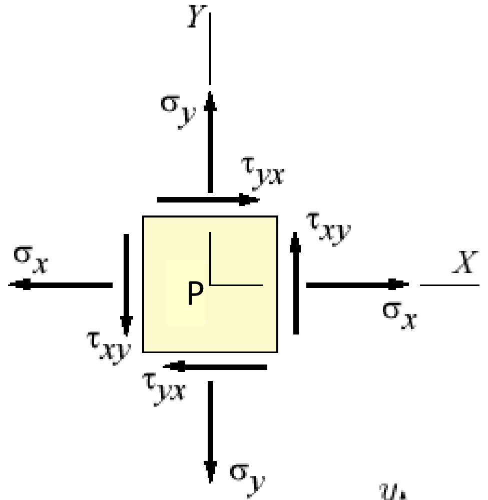


Simply Supported Beam

In general (Two-Dimensional)

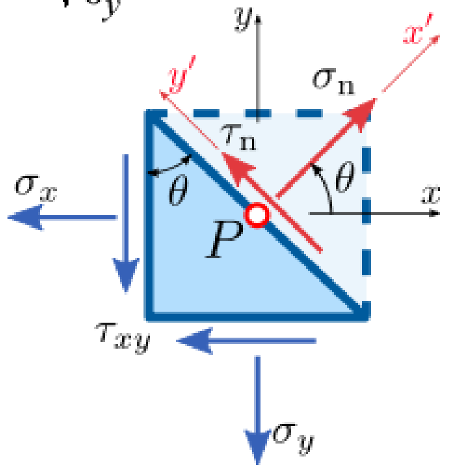
- Let a structure (member) subjected to external loads, stresses at a point (p) in general are:

(not enough)



Need to check stresses (***normal and shear***) on oblique planes

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} cos2\theta + \tau_{xy} sin2\theta$$



$$\tau_\theta = -\left(\frac{\sigma_x - \sigma_y}{2}\right) sin2\theta + \tau_{xy} cos2\theta$$

To find Max. normal stress $\Longrightarrow \frac{d\sigma_\theta}{d\theta} = 0$

And to find Max. shear stress $\Longrightarrow \frac{d\tau_\theta}{d\theta} = 0$

This leads to principal stresses:

A- Principal Normal stresses:

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

B- Max. Shear Stress :

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Failure Theories

- Too many theories, choose:

A- For ductile materials (yield criteria)

1- Rankine (max. principal stress)
2- Tresca (max. shear stress)
3- Von-mises (max. distortion energy)

B- For brittle materials (fracture criteria)

1- Rankine
2- Coulomb-Mohr theory
3-Modified Mohr theory

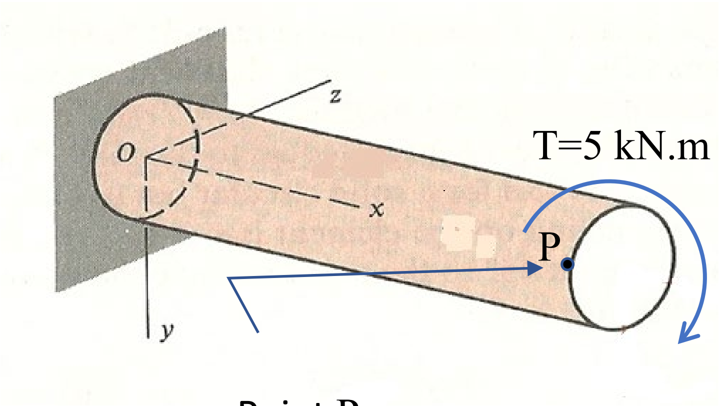
Failure Theories For Ductile Materials

- Explanation through following example:

Ex: A shaft is subjected to torque 5 kN.m. If material has a yield point of 350 Mpa. Find the diameter of shaft? Use safety factor 2.5.

Solution:

1- It is a design problem since we are looking for dimensions.



Point P
on surface

2- Stresses are only shear at point P

$$\tau = \frac{T\frac{d}{2}}{\frac{\pi}{32}d^4} = \frac{16\,T}{\pi d^3}$$

Failure Theories For Ductile Materials

1 Rankine theory: Failure occurs when maximum principal stress just exceeds yield point of tensile test.

Now, since $\begin{Bmatrix} \sigma_x = 0 \\ \sigma_y = 0 \\ \tau_{xy} = \frac{16T}{\pi d^3} = \frac{16\times5\times10^3}{\pi d^3} \end{Bmatrix}$ eq.(1)

And principal stress:

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \text{eq. (2)}$$

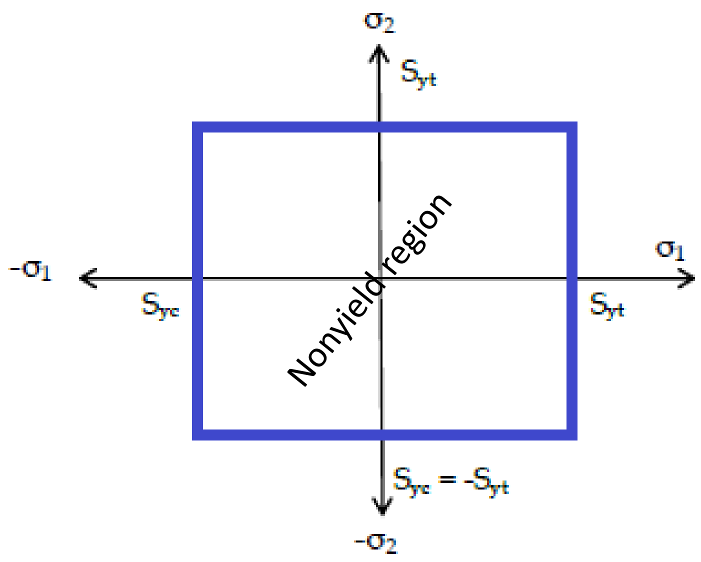
Substitute eq.(1) in eq.(2)

$$\sigma_1 = \frac{16\times5\times10^3}{\pi d^3}$$

Using failure theory above $\sigma_1 = \frac{S_y}{safety\ factor\ (SF)}$

$$\frac{16\times5\times10^3}{\pi d^3} = \frac{350}{2.5}$$

d= 56.6 mm ***Ans.***



Failure Theories For Ductile Materials

2 Tresca theory: Failure occurs when maximum shear stress just exceeds yield point of torsional test.

Note that experiments showed that:

$$S_{yield\ of\ torsional\ test} = \frac{1}{2} S_{yield\ of\ tensile\ test}$$

Or

$$S_{sy} = \frac{1}{2} S_y$$

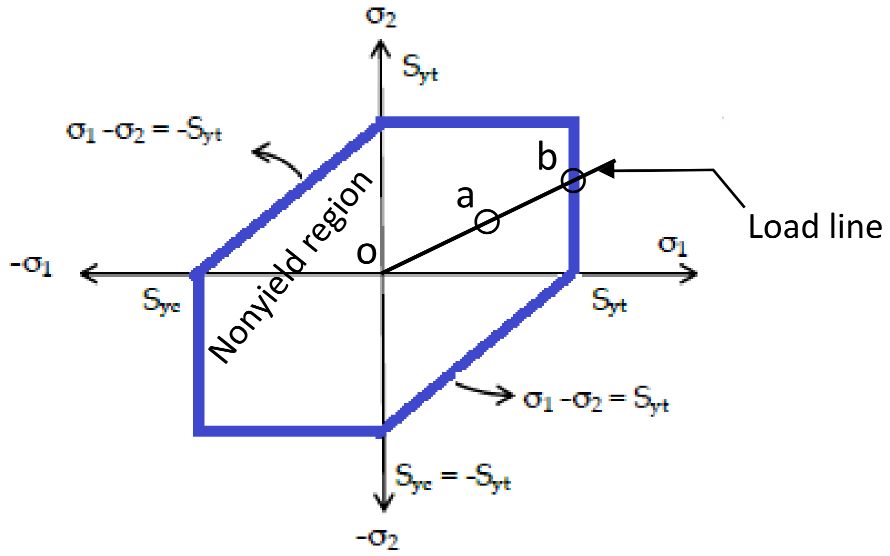
shear

yield

Back to example:

$$\sigma_x = \sigma_y = 0\ ;\ \tau_{xy} = \frac{16 \times 5 \times 10^3}{\pi d^3}$$

Max. shear stress $\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{16 \times 5 \times 10^3}{\pi d^3}$



Using above theory

$$\frac{16 \times 5 \times 10^3}{\pi d^3} = \frac{1}{2} \times \frac{350}{2.5}$$

d=71 mm ***Ans***.

Failure Theories For Ductile Materials

3. Von-Mises theory: Failure occurs when distortion energy/unit volume for stress state just exceeds distortion energy/unit volume of yield point of tensile test.

Now, for 3D complex stress stat, distortion energy is:

$$U_d = \frac{1+v}{3E}\left\{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right\} \qquad (1)$$

And distortion energy at yield point of tensile test is:

$$U_d = \frac{1+v}{3E} S_y^2 \qquad (2)$$

Where, $v: Poisson's\ ratio \quad and \quad E: Elasticity\ modulas$

Equate eq.(1) and eq.(2)

$$\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2} = S_y^2 \qquad (3)$$

For 2-dimensional

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = S_y^2 \qquad (4)$$

Failure Theories For Ductile Materials

Now back to example:

$$\begin{Bmatrix} \sigma_x = 0 \\ \sigma_y = 0 \\ \tau_{xy} = \frac{16T}{\pi d^3} = \frac{16\times5\times10^3}{\pi d^3} \end{Bmatrix}$$

Calculate:

$$\sigma_1 = +\frac{16 \times 5 \times 10^3}{\pi d^3}$$

$$\sigma_2 = -\frac{16 \times 5 \times 10^3}{\pi d^3}$$

Substitute in eq.(4)

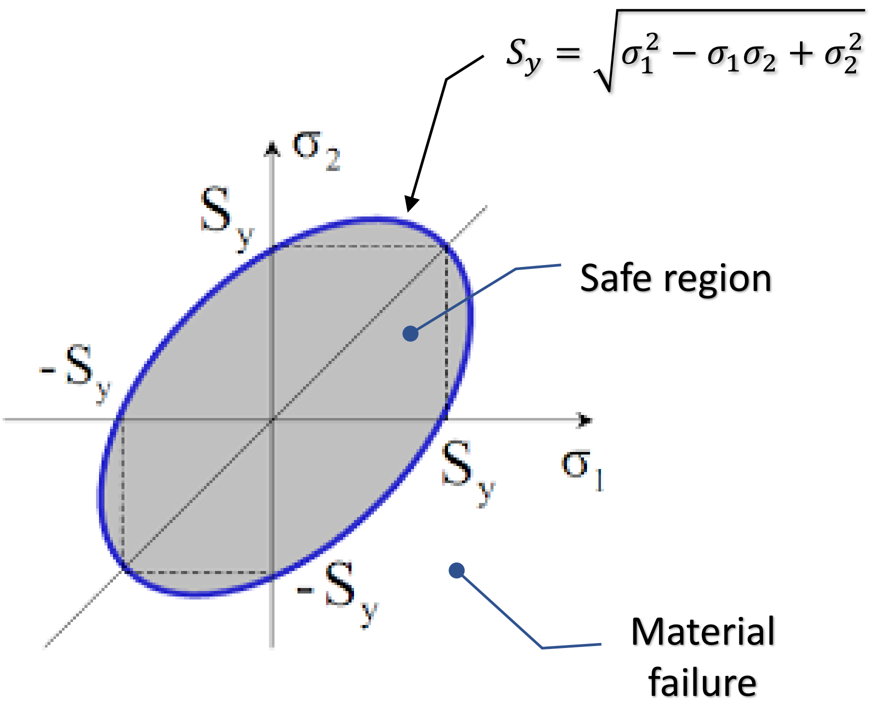
$$3\left(\frac{16\times5\times10^3}{\pi d^3}\right)^2 = S_y^2$$

Or

$$\frac{16 \times 5 \times 10^3}{\pi d^3} = \frac{S_y}{\sqrt{3}} = \frac{350}{\sqrt{3} \times 2.5}$$

Safety factor

d= 68 mm Ans.



Failure Theories For Ductile Materials

Q: What theory to choose??

Rankine → 56.6 mm

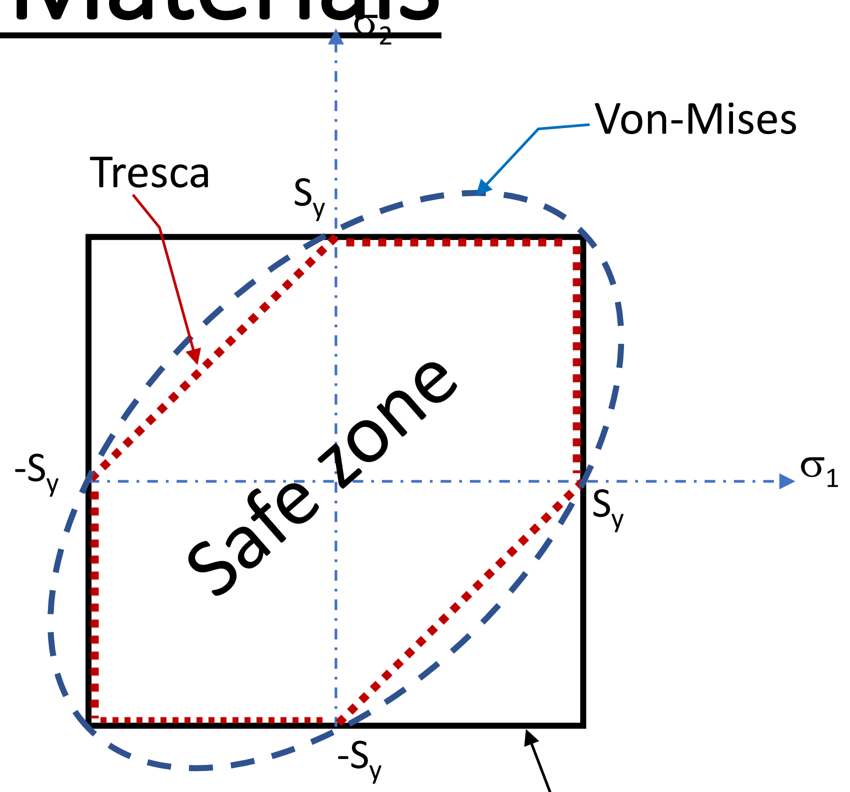
Trasca → 71 mm

Von-Mises → 68 mm

1- If your complex stress state line inside graph of theory → your design is safe. Other-wise unsafe zone.

2- Most conservative theory is Tresca.

3-Experimental data mostly coincides with Von-Mises



Graphs of theories

Failure Theories For Ductile Materials

The main items for design are:

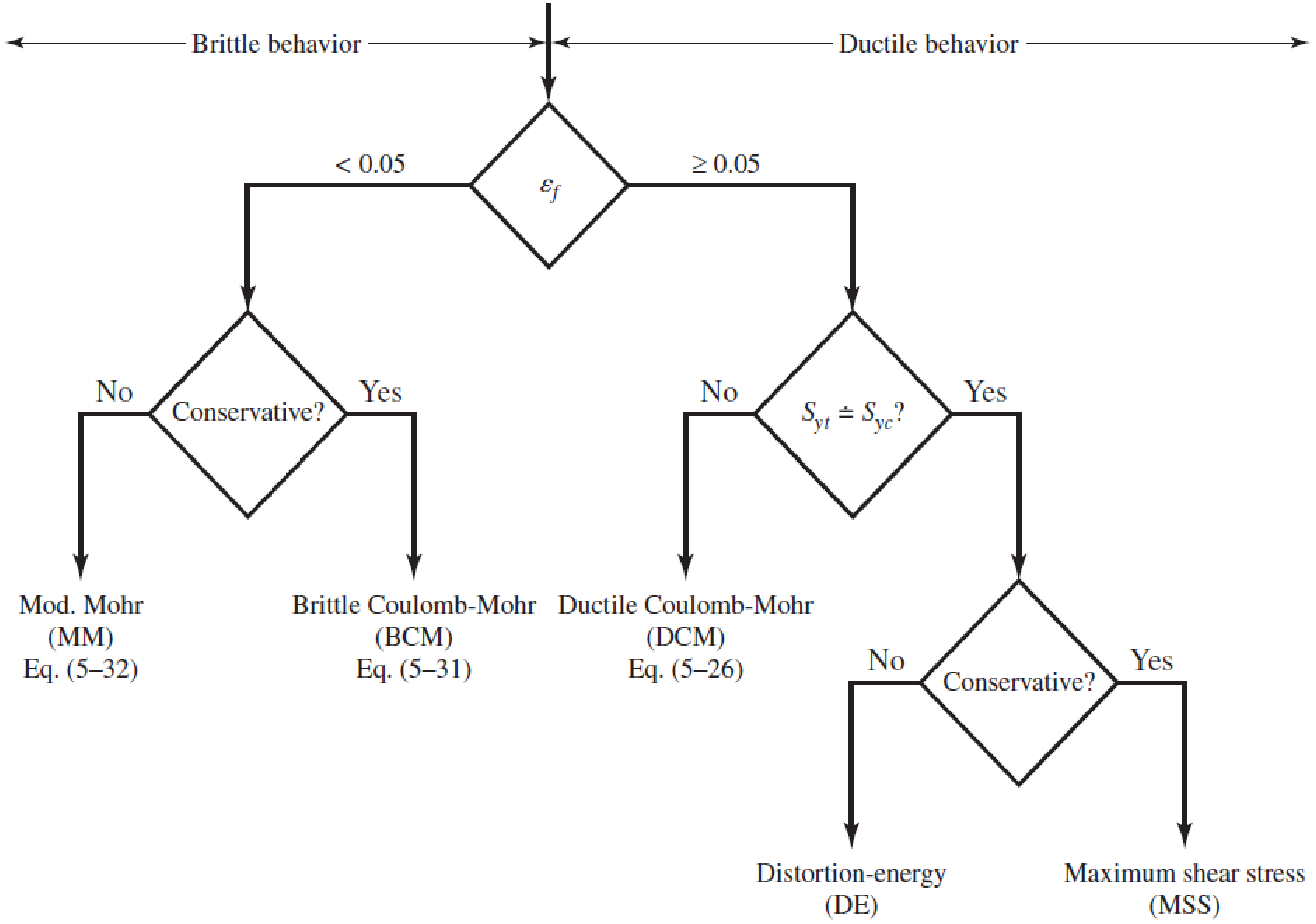
1-Loads

2-Cross section

3-Dimensions

4-Materials

5-Safety factor



Failure Theories For Ductile Materials

Ex#.2: State of stress at a point for a material is shown in fig.1. Find safety factor using (a) Tresca theory (b) Von-Mises theory. Use tensile yield strength of material 400 Mpa.

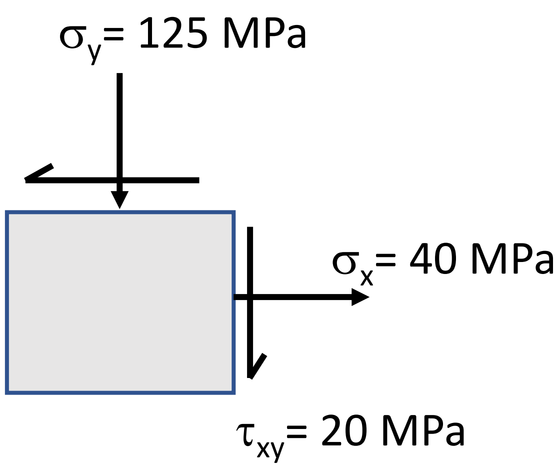


Fig. 1

Sol. : use equations/ Mohr's circle

$$\sigma_1 = 42.3\ MPa$$
$$\sigma_2 = -127.38\ MPa$$

(a) Tresca theory $\frac{\sigma_1 - \sigma_2}{2} = \frac{S_y}{2 \times n}$ $\Rightarrow safety$ factor, n=2.336

(b) Von-Mises theory

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \left(\frac{S_y}{n}\right)^2 \Rightarrow n = 2.63$$

Failure Theories For Ductile Materials

Ex#.3: A cantilever rod is loaded as shown in fig.1,

If Sy= 300 MPa, find diameter of rod using

(a) Rankine theory (b) Tresca theory

(c) Von-Mises theory

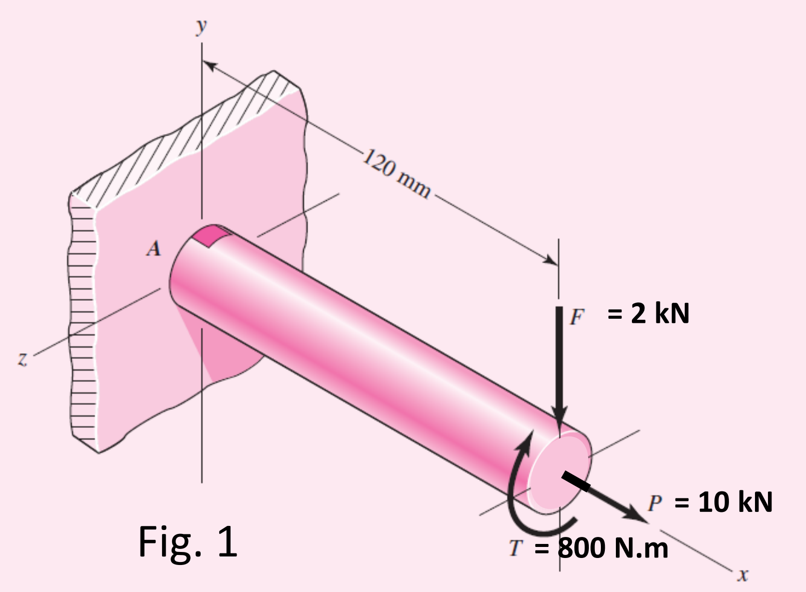


Fig. 1

Sol.: Most critical point is A

1) Direct axial stress $\sigma = \frac{P}{\frac{\pi}{4}d^2}$

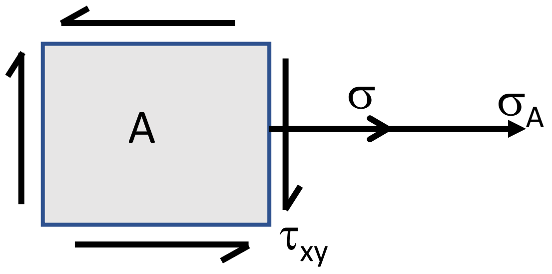
2) Bending stress $\sigma_A = \frac{32\, F\times l}{\pi d^3}$

3) Shear (due to torsion) $\tau = \frac{16\, T}{\pi d^3}$

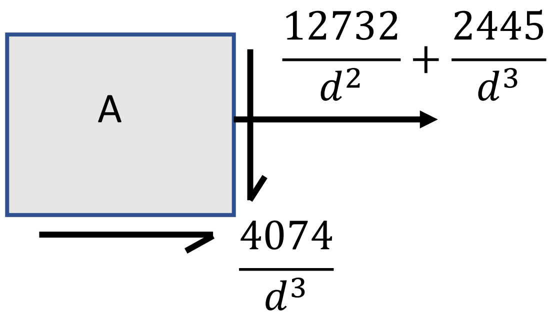
4) Transvers shear $\frac{QA\bar{y}}{It}$ is neglected (small at point A)

Failure Theories For Ductile Materials

$$\sigma_{1,2} = \frac{1}{2}\left(\frac{12732}{d^2} + \frac{2445}{d^3}\right) \pm \sqrt{\frac{1}{4}\left(\frac{12732}{d^2} - \frac{2445}{d^3}\right)^2 + \left(\frac{4074}{d^3}\right)^2}$$



a) Rankine theory set $\sigma_1 = S_y \Rightarrow d = 26.67\ mm \quad (by\ trail\ and\ error)$



b) Tresca theory set $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{S_y}{2} \Rightarrow d = 30.6\ mm$

c) Von-Mises theory set $\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = S_y^2 \Rightarrow d = 29.36\ mm$

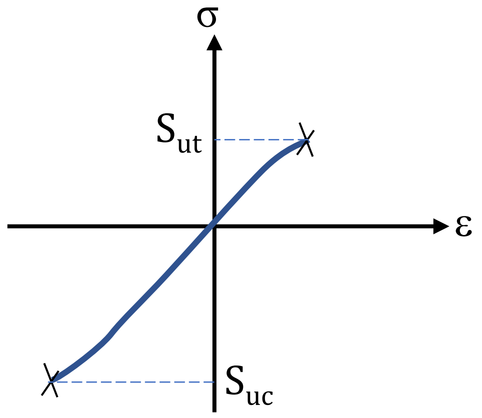
Failure Theories For Brittle Materials

A Characteristics of brittle material :

1. Do not have yield point (strength); (σ-ε) is smooth curve up to failure by fracture.
2. Compression strength S_{uc} many times grater tensile S_{ut} ⟹ $S_{uc} > S_{ut}$
3. Ultimate torsional strength S_{us} (rupture modulus) $= S_{ut}$

B Failure theories (Brittle materials):

1. Rankine theory
2. Mohr-Coulomb theory (internal friction theory)
3. Modified Mohr-Coulomb theory



Fracture failure criteria = Brittle failure theories

Factor of safety = function (ultimate strength, S_u)

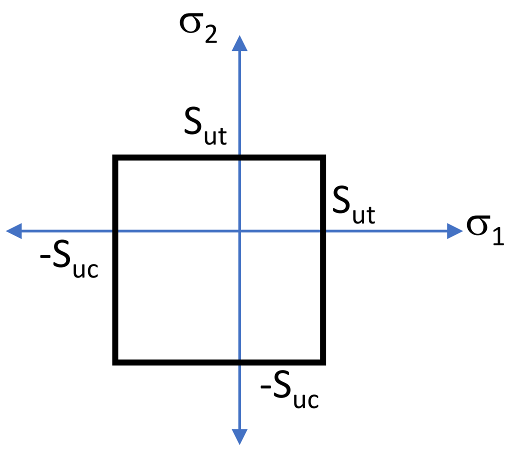
$S_{ut} \neq S_{uc}$ (usually)

Failure Theories For Brittle Materials

1. Rankine as previous; but only use S_{ut} and S_{uc} (not yield S_y)

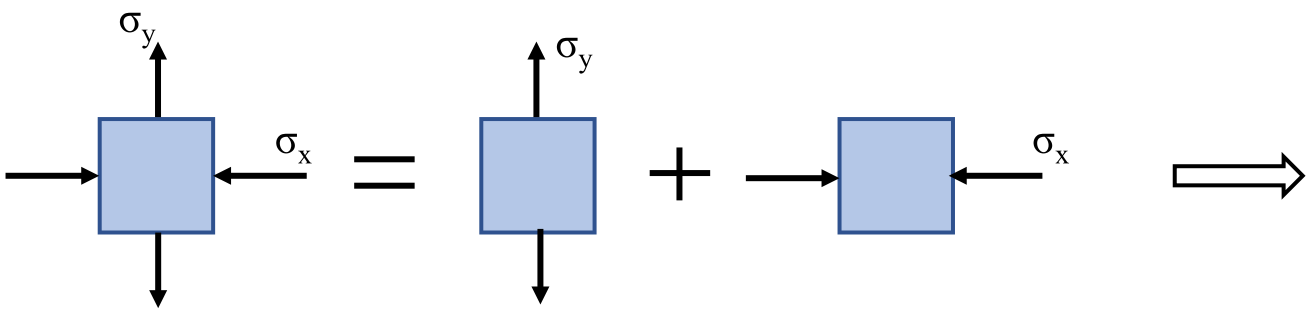
Safety factor (n)

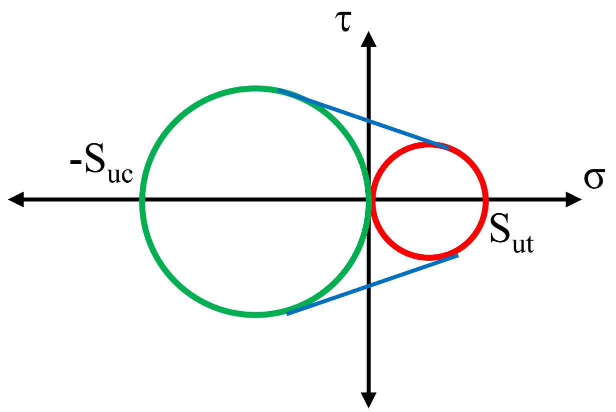
$$n = \frac{S_{ut}}{\sigma_1} \quad \text{or} \quad n = \frac{S_{uc}}{|\sigma_3|}$$



Stress envelope

2. Coulomb-Mohr theory: Failure (fracture) occurs when complex stress state produce a circle tangent to the envelope of two test circles (tensile test for S_{ut} and compressive test for S_{uc}).





Failure Theories For Brittle Materials

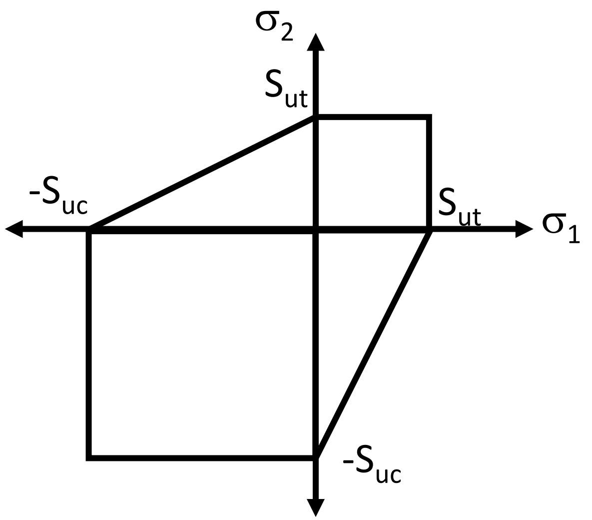
$$If \quad \sigma_1 > \sigma_2 > \sigma_3$$

and $\sigma_1 > 0 \;,\quad \sigma_3 < 0$

$$\frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} = 1$$

Safety factor is: $$n = \frac{1}{\frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}}}$$

Note: Use S_{uc} +ve value

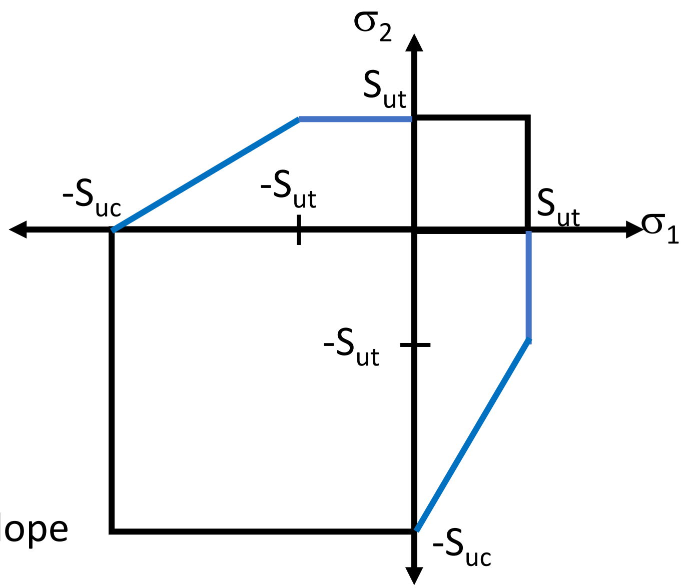


Stress envelope

3. Modified Mohr theory: Close to experimental results

$$\frac{(S_{uc} - S_{ut})}{S_{ut}}\frac{\sigma_1}{S_{uc}} - \frac{\sigma_3}{S_{uc}} = 1$$

Safety factor is: $$n = \left[\frac{(S_{uc} - S_{ut})\sigma_1}{S_{ut}S_{uc}} - \frac{\sigma_3}{S_{uc}}\right]^{-1}$$



Stress envelope

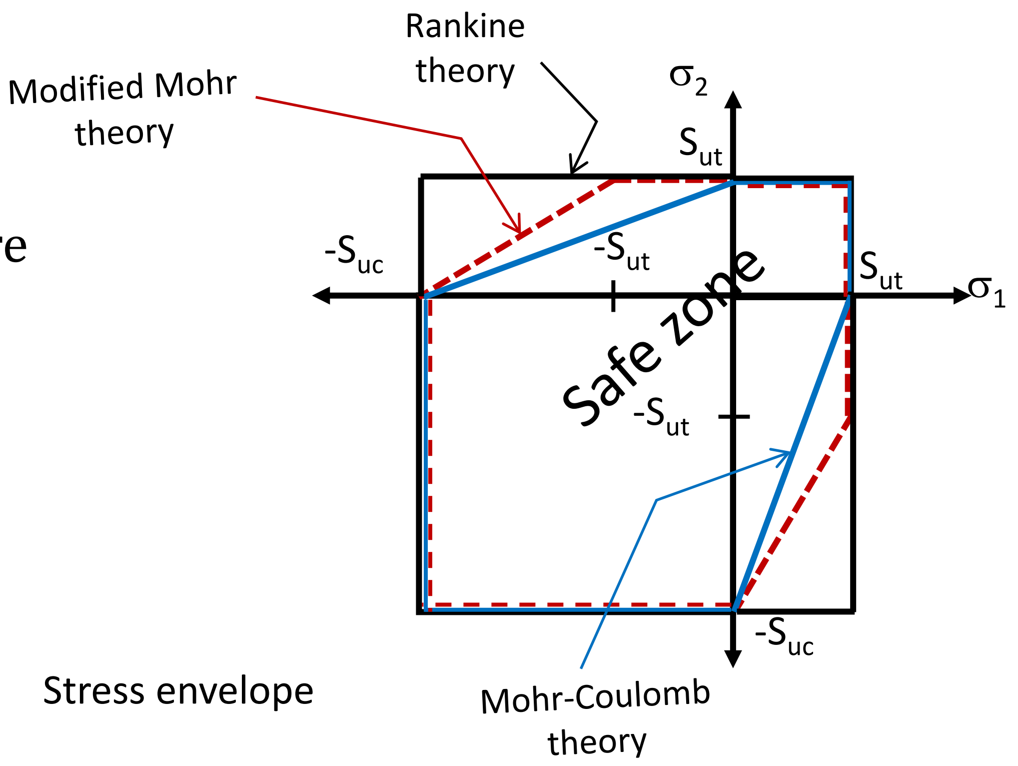
Failure Theories For Brittle Materials

Graphically

1- Inside zone ⟹ safe design ; outside ⟹ failure

2- at 1st and 3th quadrant: Rankine coincides with Mohr-Coulomb theory

3- Experimental results coincides with Modified Mohr theory especially at 4th quadrant.



Failure Theories For Brittle Materials

Ex#1: A small 6 mm diameter pin was designed of material near BS 300 Cast Iron(S_{ut}=293 MPa; S_{uc}=965 MPa). The pin take an axial compressive load of 3.5 kN combined with a torsional load of 9.8 kN.m; find safety factor for each of three failure theories?

Sol. Graphical solution

Step 1: Find the stress

- Axial stress $\sigma_x = \frac{F}{A} = \frac{4F}{\pi d^2} = \frac{4\times3.5\times10^3}{\pi 6^2} = -124\ MPa$

•Torsional shear stress $\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16\times9.8\times10^3}{\pi 6^3} = 231 MPa$

Step 2: Find principal stress (equation, Mohr circle)

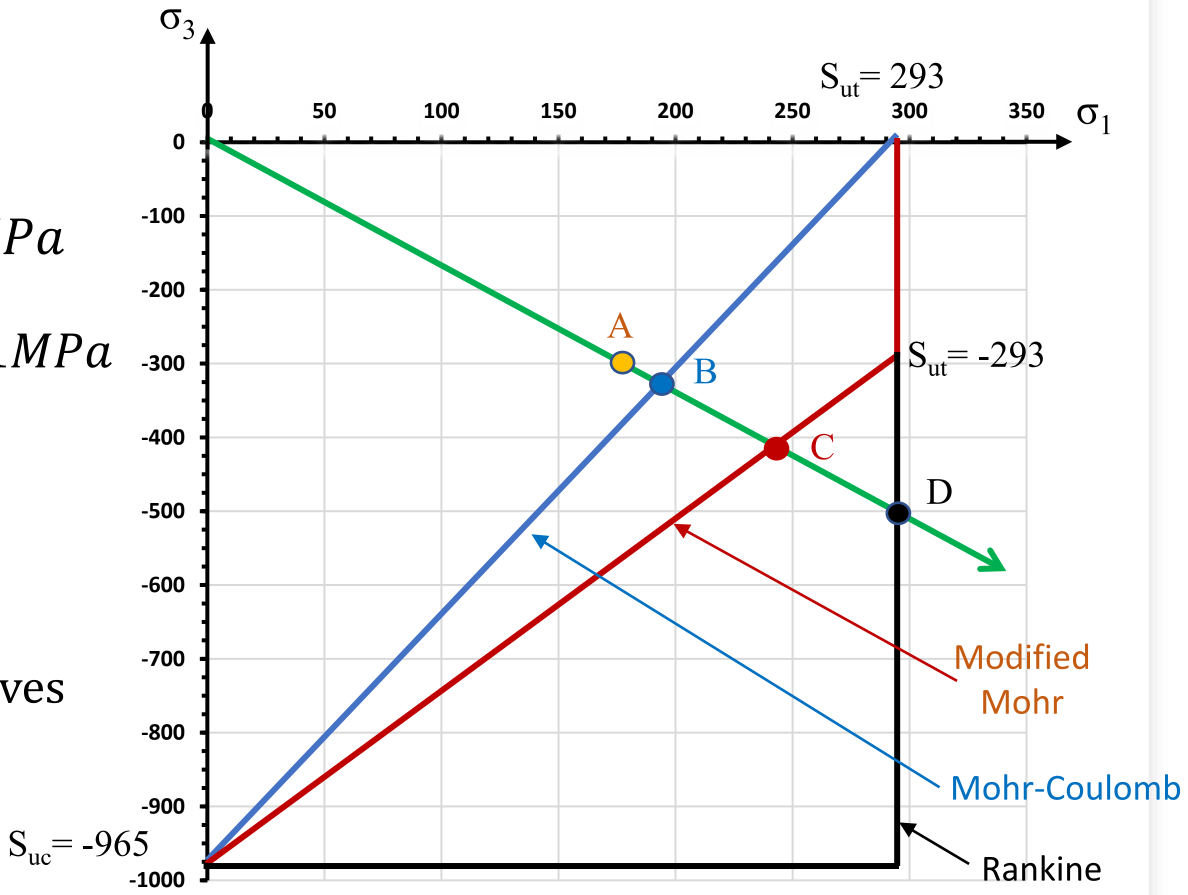
σ_1=177 MPa σ_2=0 σ_3= - 301 MPa

Step 3: Use graph paper (scale), plot 4th quadrant.

a- plot Rankine, Mohr-Coulomb and Modified Mohr curves

b- plot point A=(σ_1, σ_3)

A: (177, -301)



Failure Theories For Brittle Materials

c- Draw line OA through all failure corves $\overrightarrow{OA}$, locate points B,C and D

d- $n = \frac{OD}{OA} = 1.6 \quad (Rankine)$

$n = \frac{OC}{OA} = 1.35 \quad (Modified\ Mohr)$

$n = \frac{OB}{OA} = 1.05 \quad (Mohr - Coulomb)$

Or use horizontal projections

Ex#2: The stress state at a point is shown in fig.1

for a material (same as previous example) BS 300 Cast Iron

(S_{ut}=293 MPa; S_{uc}=965 MPa).Find safety factor using Rankine, Mohr-Coulomb and Modified Mohr theories.

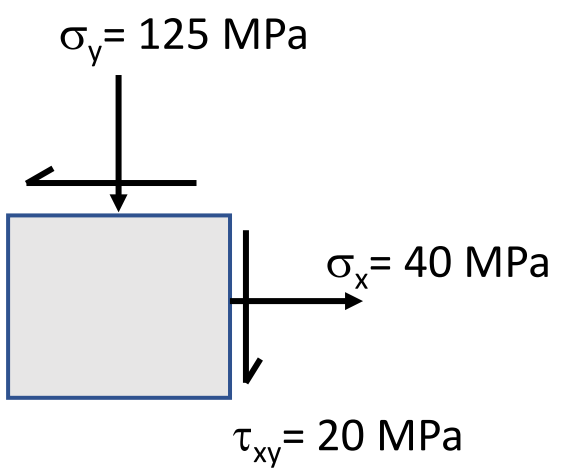


Fig. 1

Sol. Using equations

σ_x=40 MPa σ_y=- 125 MPa τ_{xy}= - 20 Mpa

σ_1=42 MPa σ_2=0 σ_3= - 127.3 MPa

Failure Theories For Brittle Materials

Sol. Using equations

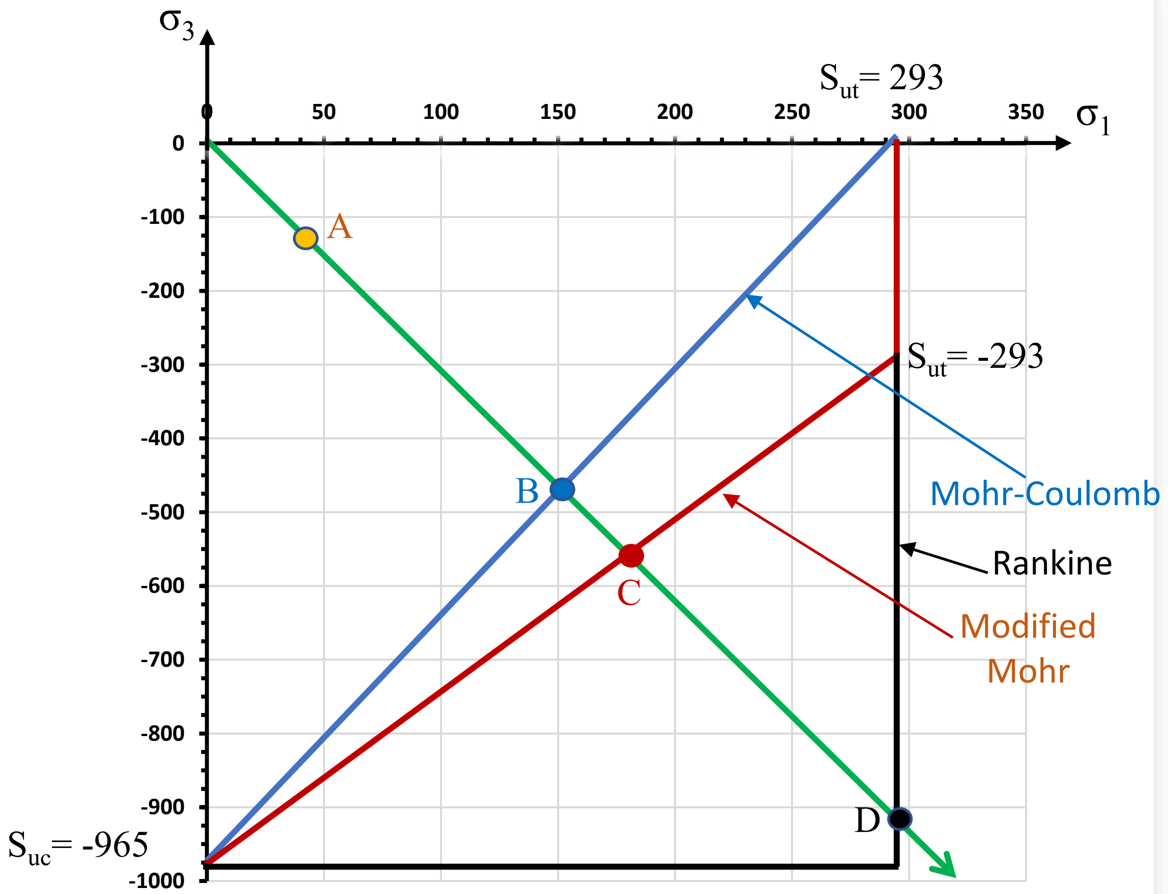
σ_x=40 MPa σ_y=- 125 MPa τ_{xy}= - 20 MPa

σ_1=42 MPa σ_2=0 σ_3= - 127.3 MPa

$$n = \frac{OD}{OA} = 6.976 \quad (Rankine)$$

$$n = \frac{OB}{OA} = 3.636 \quad (Mohr - Coulomb)$$

$$n = \frac{OC}{OA} = 4.315 \quad (Modified\ Mohr)$$



Introduction to Fracture Mechanics

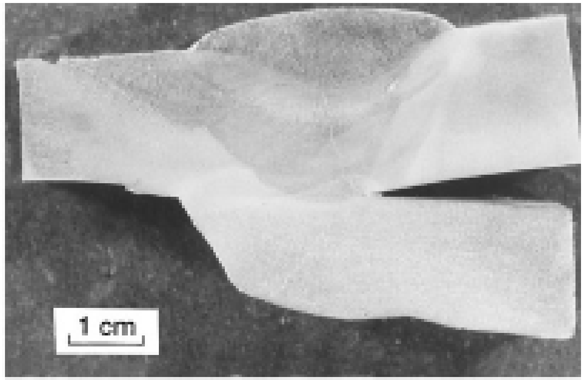
Chapter Four

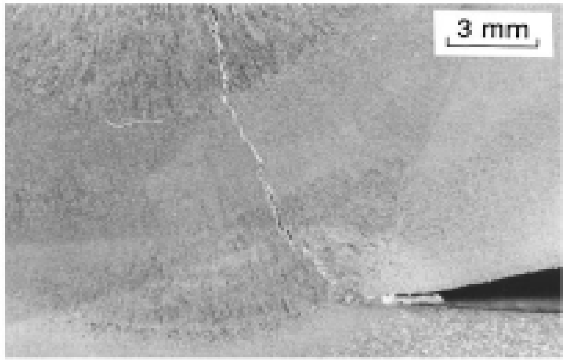
1.Introduction

Traditionally, the structural design approaches are based on the concept that the structures must have enough strength, stiffness, and stability to resist the loads.

For the strength criteria, the applied stress must be less than the yielding or ultimate strength of the material. However, when a crack is occurred in a component of the structure, it can cause the failure (in the form of fracture) at stresses well below the material's yielding strength. In this case, a special methodology called fracture mechanics can be used in design to minimize the possibility of failure.

Fracture mechanics is important in engineering design since cracks and crack-like flaw occur more frequently than we might expect. For example, the periodic inspections of large commercial aircraft frequently reveal cracks that must be repaired. Also, they are commonly occurred in ship structures, in bridge structures and in pressure vessel and piping. (as shown below)





1.Introduction

Generally, fracture process can be categorized into three stages.

1. Crack initiation – micromechanics and dislocation theory
2. Crack extension – slow crack growth
3. Fast crack propagation

Thus, in fracture mechanics, a preexistent crack is assumed. Fracture mechanics is used to study the growth behavior of crack and residual strength of cracked structures, and to evaluate the life.
The growth of a crack and its corresponding stress can be shown as in Fig. 1

The following questions are important in designing a structure using fracture mechanics.

1. What is the maximum permissible crack size?
2. What is the residual strength as a function of crack size?
3. How long does it take from the maximum detectable crack size to the maximum permissible crack size?
4. During the period available for crack detection, how often should the structure be inspected for crack?

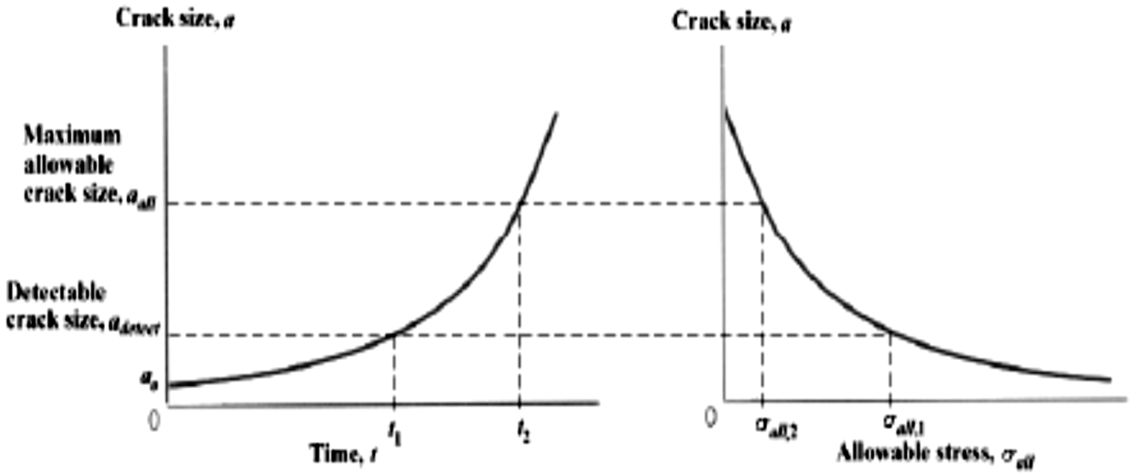


Fig. 1

2.Fracture Modes

Once a crack has been initiated, subsequent crack propagation may occur in several ways depending on the relative displacement of the particles in the two faces (surfaces) of the crack. There are three fundamental modes of fracture acting on the crack surface displacement as shown in Fig. 2.

1. Opening mode (Mode I) - The stress acts perpendicular to the crack growth direction and the crack growth plane. The crack surfaces move directly apart.

2. Shearing mode (Mode II) - The stress acts parallel to the crack growth direction and the crack growth plane. The crack surfaces move (slide) normal to the crack edge and remain in the plane of the crack.

3. Tearing mode (Mode III) - The stress acts perpendicular to the crack growth direction and parallel to the crack growth plane. The crack surfaces move parallel to the crack edge and remain in the plane of the crack.

2.Fracture Modes

The most general cases of crack surface displacements are obtained by superposition of these basic three modes.

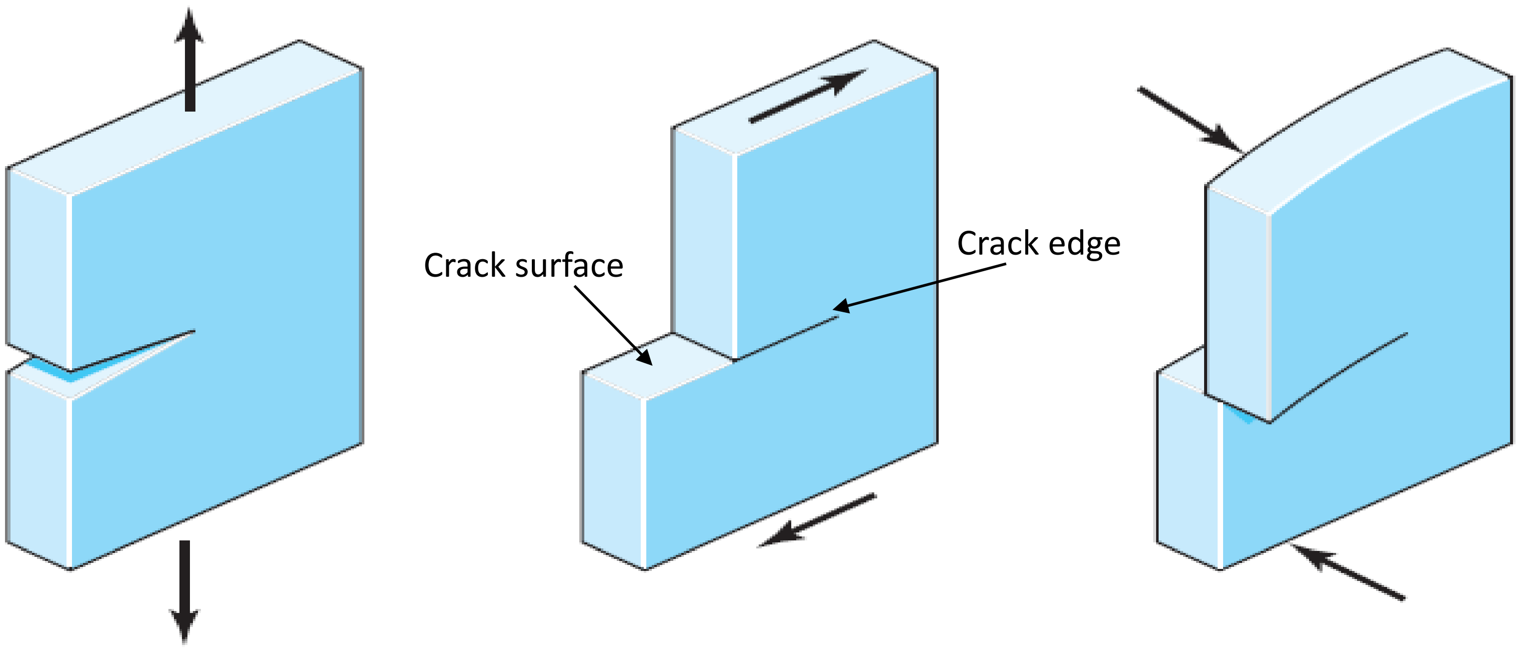


Fig. 2

(a) Mode I — **Opening**

(b) Mode II — **Sliding**

(c) Mode III — **Tearing**

3. Stress and Displacement Field at the Crack Tip

The foundation of fracture mechanics was first established by Griffith in 1921 using the stress field calculations for an elliptical flaw in a plate developed by Inglis in 1913. For the infinite plate loaded by an applied uniaxial stress s in Fig. 4, the maximum stress occurs at (±a, 0) and is given by:

$$\sigma_{y\,max} = \sigma\left(1 + 2\frac{a}{b}\right)$$

In 1950, Irwin showed that the local stresses near the crack tip, as the curvature at the crack tip goes to zero as shown in Fig. 4, are of the form:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta)$$

and we can see that $\sigma_{ij} \to \infty$ as $r \to 0$. Thus, the stress field is a singular stress field with a singularity of $\sqrt{r}$. The term K is called the ***stress intensity factor***, which defines the intensities or magnitudes of the singular stress around the crack tip. The expression of K depends on the fracture modes.

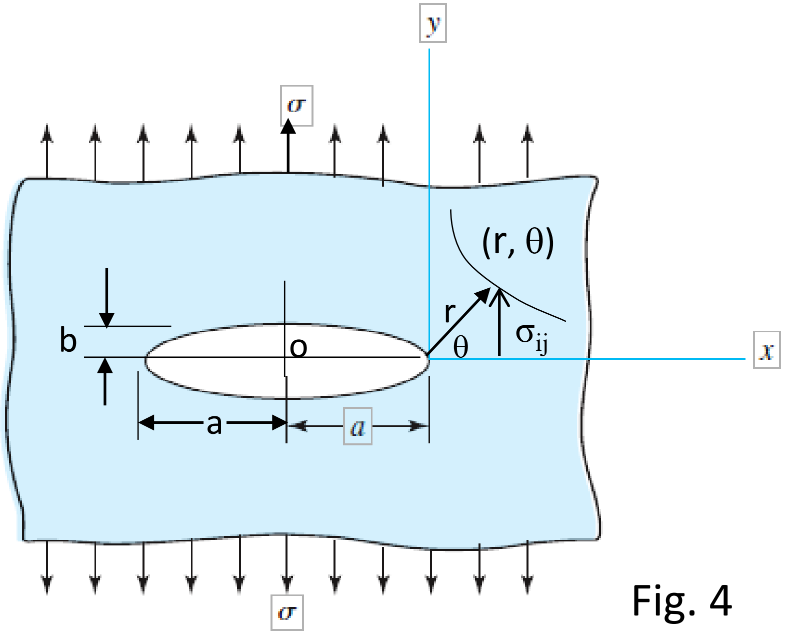


Fig. 4

3.1 Opening Mode (Mode I)

The stresses at a point having a distance r and angle θ from the crack tip and for Mode I as shown in Fig.5 are:

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$$

Plane stress $\sigma_z = \tau_{yz} = \tau_{xz} = 0$

Plane strain $\sigma_z = v(\sigma_x + \sigma_y)\,; \qquad \tau_{yz} = \tau_{xz} = 0$

Where $K_I = \lim_{r\to 0} \sigma_y\big|_{\theta=0}\left(\sqrt{2\pi r}\right) = \sigma_o\sqrt{\pi a}$

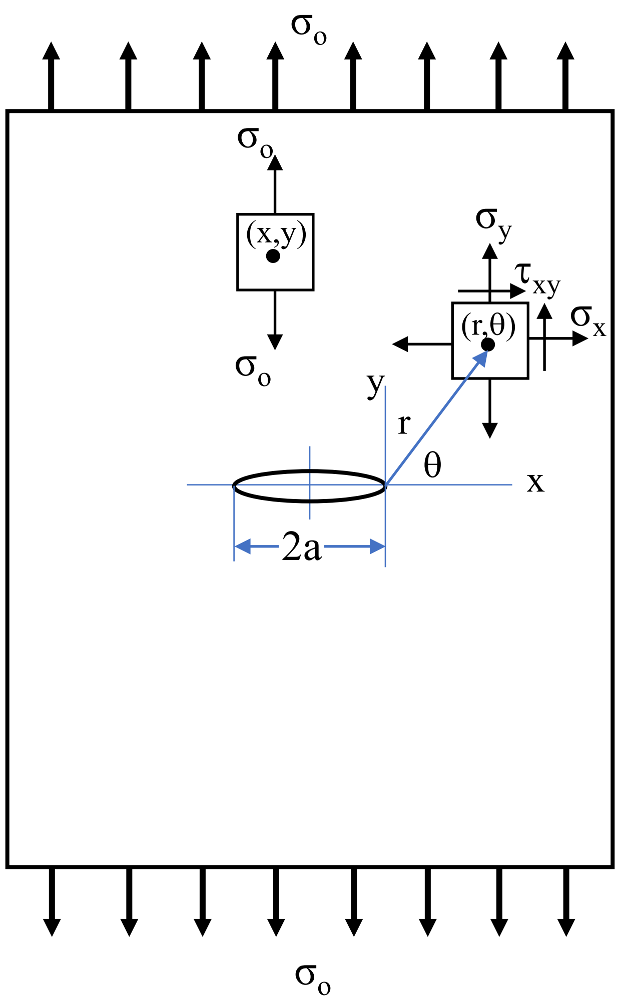


Fig.5

3.1 Opening Mode (Mode I)

The displacements at a point having a distance r from the crack tip and angle θ with the x axis are:

$$u_x = \frac{K_I}{8G}\sqrt{\frac{r}{2\pi}}\left[(2\mu-1)cos\frac{\theta}{2} - cos\frac{3\theta}{2}\right]$$

$$u_x(\theta) = u_x(-\theta)$$

$$u_y = \frac{K_I}{8G}\sqrt{\frac{r}{2\pi}}\left[(2\mu-1)sin\frac{\theta}{2} - sin\frac{3\theta}{2}\right]$$

$$u_y(\theta) = -u_y(-\theta)$$

Where $$\mu = \begin{cases} 3-4v & plane\ strain \\ \frac{3-v}{1+v} & plane\ stress \end{cases}$$

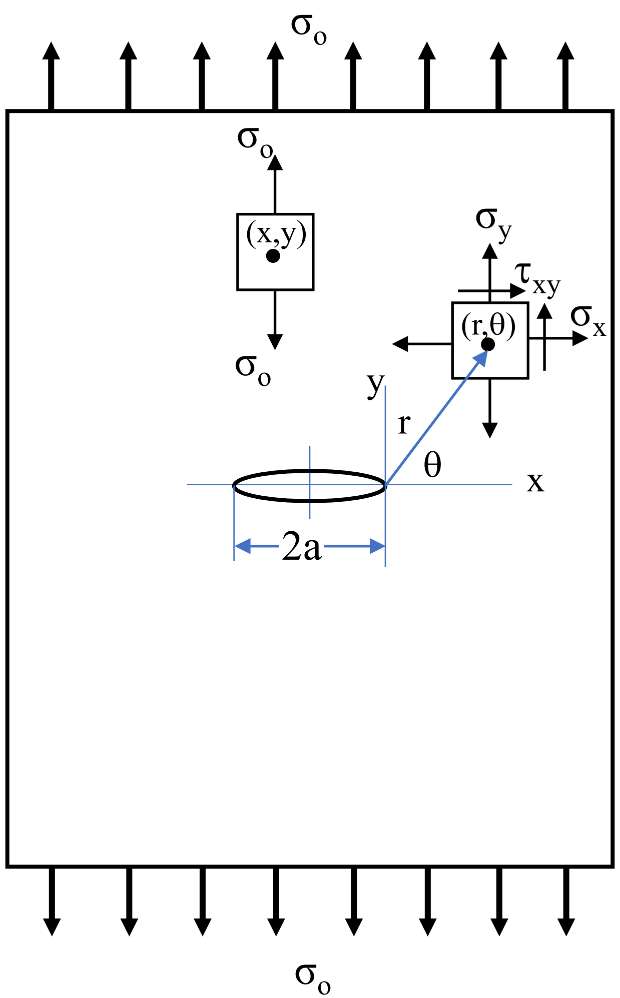


Fig.5

3.2 Sliding Mode (Mode I I)

The stresses at a point having a distance r from the crack tip and angle θ with the x axis for Mode II as shown in Fig.6 are:

$$\sigma_x = -\frac{K_{II}}{\sqrt{2\pi r}} sin\frac{\theta}{2}\left(2 + cos\frac{\theta}{2} cos\frac{3\theta}{2}\right)$$

$$\sigma_y = \frac{K_{II}}{\sqrt{2\pi r}} cos\frac{\theta}{2} sin\frac{\theta}{2} cos\frac{3\theta}{2}$$

$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} cos\frac{\theta}{2}\left(1 - sin\frac{\theta}{2} sin\frac{3\theta}{2}\right)$$

Plane stress $\sigma_z = \tau_{yz} = \tau_{xz} = 0$

Plane strain $\sigma_z = v(\sigma_x + \sigma_y)$; $\tau_{yz} = \tau_{xz} = 0$

Where $K_{II} = \tau_o\sqrt{\pi a}$

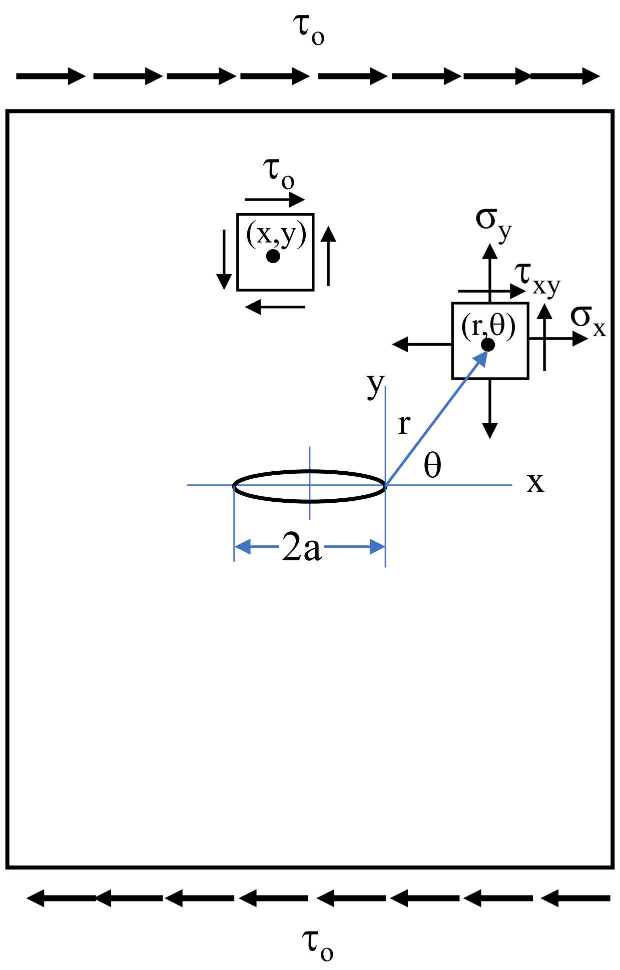


Fig.6

3.2 Sliding Mode (Mode I I)

The displacements at a point having a distance r from the crack tip and angle θ with the x axis for Mode II are:

$$u_x = \frac{K_{II}}{8G}\sqrt{\frac{2r}{\pi}}\left[(2\mu + 3)sin\frac{\theta}{2} + sin\frac{3\theta}{2}\right]$$

$$u_x(\theta) = -u_x(-\theta)$$

$$u_y = \frac{K_I}{8G}\sqrt{\frac{2r}{\pi}}\left[-(2\mu - 3)cos\frac{\theta}{2} - cos\frac{3\theta}{2}\right]$$

$$u_y(\theta) = u_y(-\theta)$$

Where $\mu = \begin{cases} 3 - 4v & plane\ strain \\ \frac{3-v}{1+v} & plane\ stress \end{cases}$

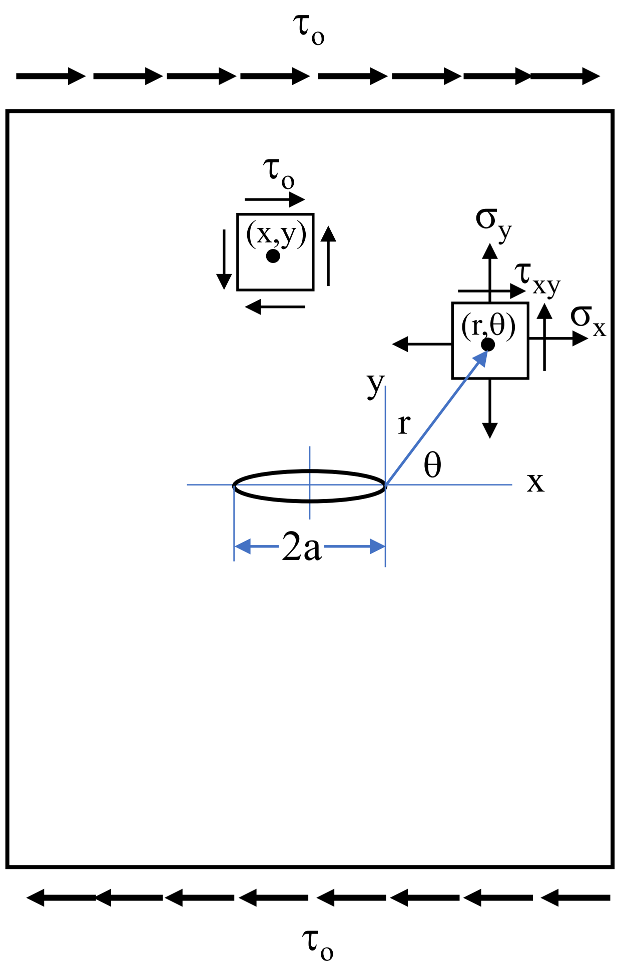


Fig.6

3.3 Tearing Mode (Mode I I I)

The stresses at a point having a distance r from the crack tip and angle θ with the x axis for Mode III as shown in Fig.7 are:

$$\tau_{xz} = -\frac{K_{III}}{\sqrt{2\pi r}} sin\frac{\theta}{2}$$

$$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} cos\frac{\theta}{2}$$

$$\sigma_x = \sigma_y = \sigma_z = \tau_{xy} = 0$$

Where $$K_{III} = \tau_o\sqrt{\pi a}$$

The displacements at a point having a distance r from the crack tip and angle θ with the x axis for Mode III are:

$$u_z = \frac{K_{III}}{G}\sqrt{\frac{2r}{\pi}}\sin\theta$$

$$u_x = u_y = 0$$

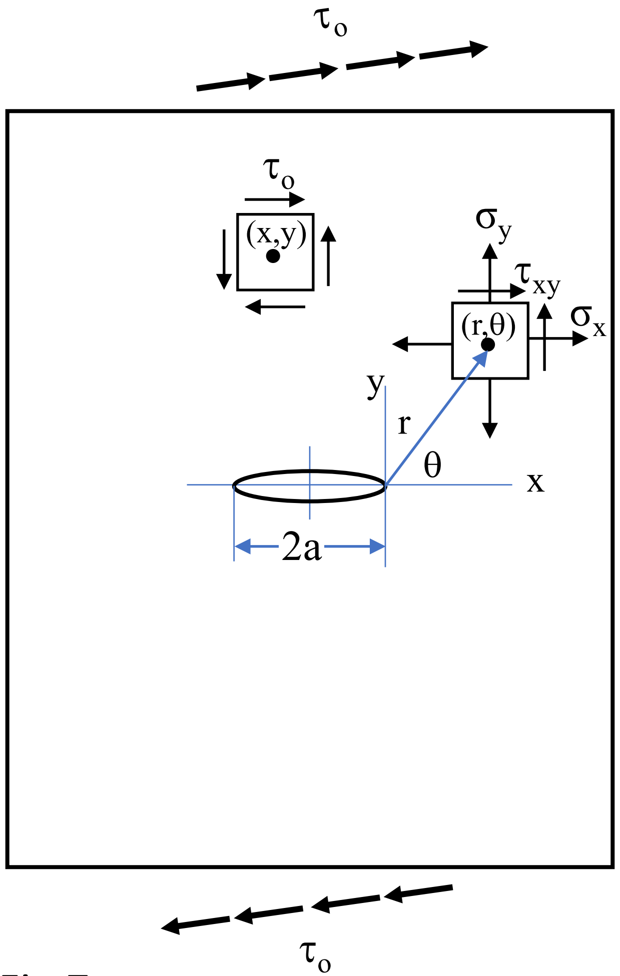


Fig.7

4. Stress Intensity Factor (SIF or K)

Stress intensity factors are needed to measure the intensity or magnitude of the singular stress field in the vicinity of an ideally sharp crack tip in a linear elastic and isotropic material. This approach is called linear-elastic fracture mechanics (LEFM). The factors do depend on loading condition, crack size, crack shape, and geometric boundaries. The general form of the stress intensity factors is given by:

$$K = f.\sigma\sqrt{\pi a}$$

Where, σ = applied stress

a = effective crack length

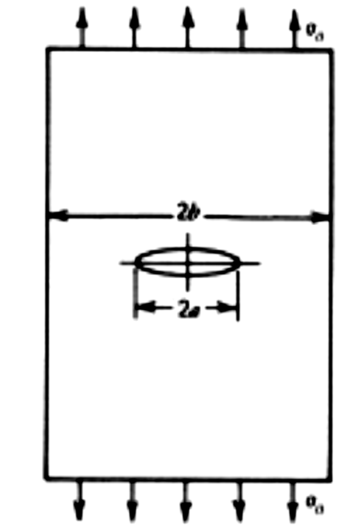
f = correction factor. For infinity plate, *f* =1.

Thus, stress intensity factor K has a unit in ksi in or MPa m . The solutions of the stress intensity factors have been obtained for wide variety of problems and published in a handbook form. The followings are the typical solution for SIF:

a. Center crack

$$K_I = \sigma_o\sqrt{\pi a}.\,f\left(\frac{a}{2b}\right)$$

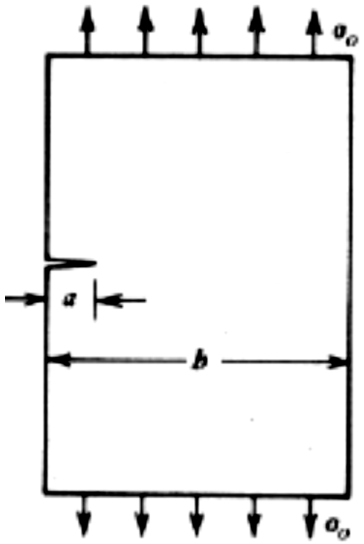
$$f\left(\frac{a}{2b}\right) = \sqrt{sec\left(\frac{\pi a}{2b}\right)}$$



b. Single edge crack

$$K_I = \sigma_o\sqrt{\pi a}.\,f\left(\frac{a}{b}\right)$$

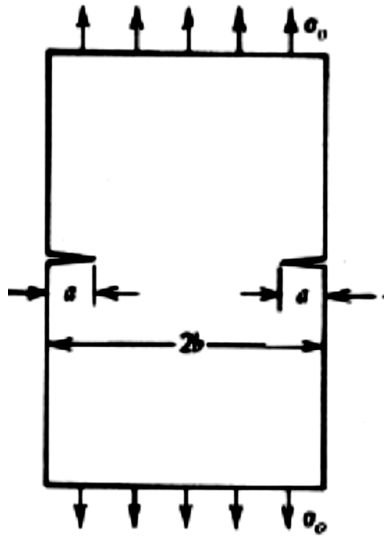
$$f\left(\frac{a}{b}\right) = 1.12 - 0.231\left(\frac{a}{b}\right) + 10.55\left(\frac{a}{b}\right)^2 - 21.72\left(\frac{a}{b}\right)^3 + 30.39\left(\frac{a}{b}\right)^4$$



c. Double edge crack

$$K_I = \sigma_o \sqrt{\pi a}.\, f\left(\frac{a}{b}\right)$$

$$f\left(\frac{a}{b}\right) = 1.12 + 0.203\left(\frac{a}{b}\right) - 1.197\left(\frac{a}{b}\right)^2 + 1.930\left(\frac{a}{b}\right)^3$$

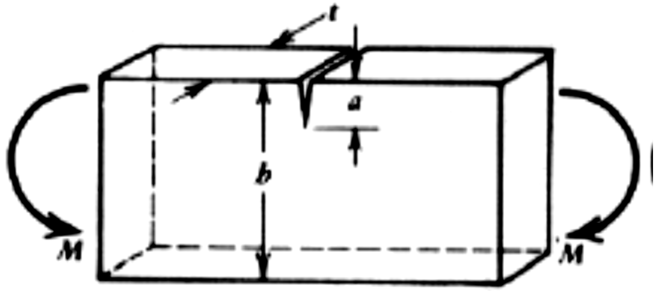


d. Crack under bending

$$K_I = \sigma_o \sqrt{\pi a}.\, f\left(\frac{a}{b}\right)$$

$$\sigma_o = \frac{6M}{b^2 t}$$

$$f\left(\frac{a}{b}\right) = 1.12 - 1.40\left(\frac{a}{b}\right) + 7.33\left(\frac{a}{b}\right)^2 - 13.08\left(\frac{a}{b}\right)^3 + 14\left(\frac{a}{b}\right)^4$$



5. Superposition of SIF

Stress intensity factor for combined loading can be obtained by the superposition method, that is, by adding the contribution to K from the individual load components. It is valid only for combination of the same mode of failure. Consider an eccentric load applied at a distance e from a centerline of a member with a single edge crack as shown in Fig. 6.8. This eccentric load is statically equivalent to the combination of a centrally applied tension load and a bending moment.

The stress intensity factor for the centrally applied tension load is:

$$K_I^a = \frac{P}{bt}\sqrt{\pi a}.\, f^a\left(\frac{a}{b}\right)$$

The stress intensity factor for the bending moment is:

$$K_I^b = \frac{6M}{b^2 t}\sqrt{\pi a}.\, f^b\left(\frac{a}{b}\right)$$

Thus, the total stress intensity factor of this case is:

$$K_I = K_I^a + K_I^b = \frac{P}{bt}\left(f^a\left(\frac{a}{b}\right) + \frac{6e}{b} f^b\left(\frac{a}{b}\right)\right)\sqrt{\pi a}$$

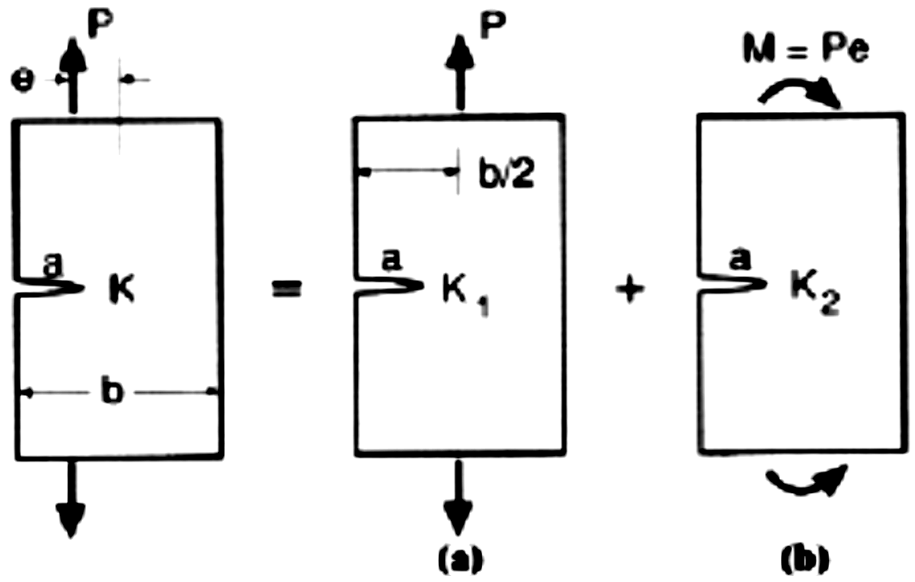


Fig.8

6. Fracture Toughness (critical SIF)

Fracture toughness K_{IC} is the critical value of the stress intensity factor K . If the tress intensity factor K occurred in a given material is less than the fracture toughness, the material will have ability to resist the crack without brittle fracture.

Fracture toughness is a material parameter, but it depends on both temperature and specimen thickness.

$$K_{IC} = \sigma_c \sqrt{\pi a} . f\left(\frac{a}{b}\right)$$

$$\sigma_c = \frac{K_{IC}}{\sqrt{\pi a} f\left(\frac{a}{b}\right)}$$

$$K_I = \sigma \sqrt{\pi a_c} . f\left(\frac{a}{b}\right)$$

Where a_c = critical crack size

It should be noted that in order to ensure that the state of stress is plane strain for each of the cases in Section 4, the magnitudes of the crack half-length a and the thickness t should satisfy

$$a, t \geq 2.5\left(\frac{K_{IC}}{\sigma_y}\right)^2$$

Example

Determine the stress intensity factor for the edge-cracked beam having the crack half-length a of 44.5 mm. When subjected to a moment of 136 kN.m. It the beam was made of an extremely tough steel that has $\sigma_y = 1345$ MPa and a K_{IC} of 165 Mpa√m. The width of the beam is 102 mm. and the depth of the beam is 305 mm in. If the moment applied to the beam was increased to 542 kN.m, would this beam fail?

Solution:

The flexural stress due to the moment 136 kN.m is: $\sigma = \frac{My}{I} = \frac{M(b/2)}{(tb^3/12)} = \frac{6M}{tb^2} = \frac{6\times136\times10^3}{102\times10^{-3}\times(305\times10^{-3})^2} = 86\ MPa$

Since the crack half-length a of the beam is 44.5 mm, $\frac{a}{b} = \frac{44.5}{305} = 0.1459$

$$f\left(\frac{a}{b}\right) = 1.12 - 1.40\left(\frac{a}{b}\right) + 7.33\left(\frac{a}{b}\right)^2 - 13.08\left(\frac{a}{b}\right)^3 + 14\left(\frac{a}{b}\right)^4 = 1.329$$

The stress intensity factor for the edge-cracked beam is:

$$K_I = f\left(\frac{a}{b}\right)\sigma\sqrt{\pi a} = 1.329 \times 86 \times \sqrt{\pi \times 44.5 \times 10^{-3}} = 42.73\ MPa\sqrt{m}$$

The flexural stress due to the moment 542 kN.m is: $\sigma = \frac{6M}{tb^2} = \frac{6\times542\times10^3}{102\times10^{-3}\times(305\times10^{-3})^2} = 342.7\ MPa$

$$K_I = 1.329 \times 342.7 \times \sqrt{\pi \times 44.5 \times 10^{-3}} = 170.29\ MPa\sqrt{m}$$

Since the stress intensity factor is larger than K_{IC} 165 MPa√m and the flexural stress is less than the yielding strength $\sigma_y = 1345$ MPa, the beam does fail by fracture.

7. Strain Energy Release Rate and Its Equivalent to SIF

Consider a cracked member under a Mode I as shown in Fig.9a. Assume that the behavior of the material is linear elastic under the action of load P . As a result of the elastic deformation, the strain energy stored in the member as shown in Fig.9a is:

$$U = \frac{1}{2} P v$$

where v is the displacement at the loading point.

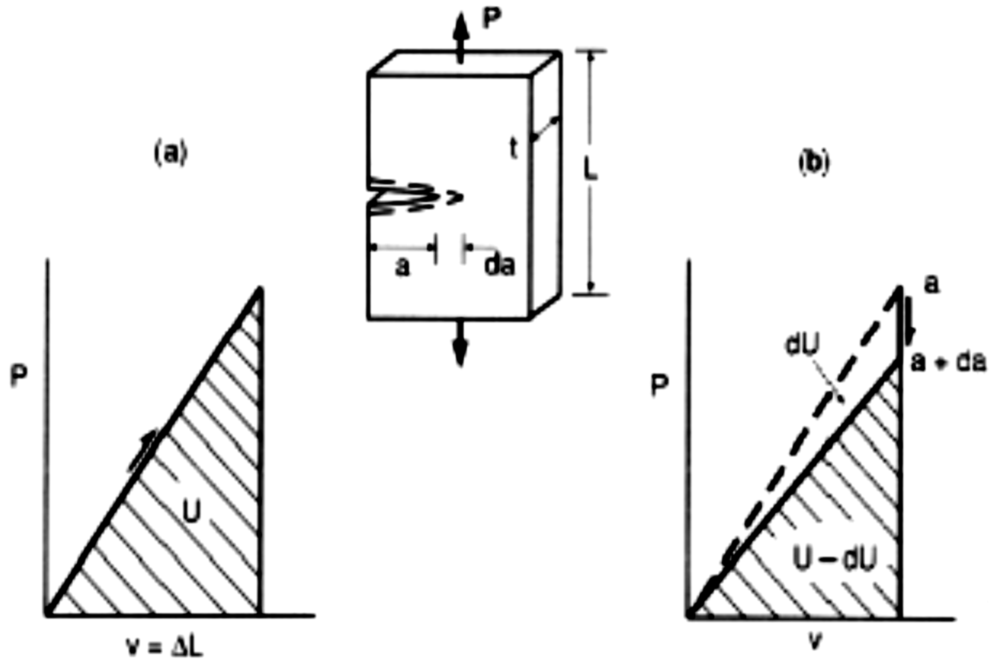


Fig. 9

If the crack moves ahead by a small distance da , while the displacement is held constant, the stiffness of the member decreases as shown in Fig. 6.9b. This results in the decreasing in the strain energy by the amount of dU , that is, U decreases due to a release of this amount of energy.

The strain energy release rate (G) is defined as the rate of change of strain energy with increase in crack area .

$$G = -\frac{\partial U}{\partial A} = -\frac{1}{t}\frac{\partial U}{\partial a}$$

where t = thickness of the plate. Since G has a unit in $\frac{N.m}{m^2} = \frac{N}{m}$,G is sometimes considered as a crack driving force.

7. Strain Energy Release Rate and Its Equivalent to SIF

$$\left.\begin{aligned} G_I &= \frac{K_I^2}{E} \\ G_{II} &= \frac{K_{II}^2}{E} \\ G_{III} &= \frac{1+v}{E} K_{III}^2 \end{aligned}\right\} \text{for plane stress} \qquad \left.\begin{aligned} G_I &= \frac{1-v^2}{E} K_I^2 \\ G_{II} &= \frac{1-v^2}{E} K_{II}^2 \\ G_{III} &= \frac{1+v}{E} K_{III}^2 \end{aligned}\right\} \text{for plane strain}$$

8. Plastic Zone Size

Irwin has shown that the local stress at the crack tip as shown in Fig.10 is in the form of

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta)$$

This means that at the crack tip or $r \to 0$, the local stress is infinite or $\sigma_{ij} \to \infty$. However, real materials can not support these theoretical infinite stresses. Thus, upon loading, the crack tip becomes blunted and a region of yielding or microcracking forms. This region of yielding is called **plastic zone**.

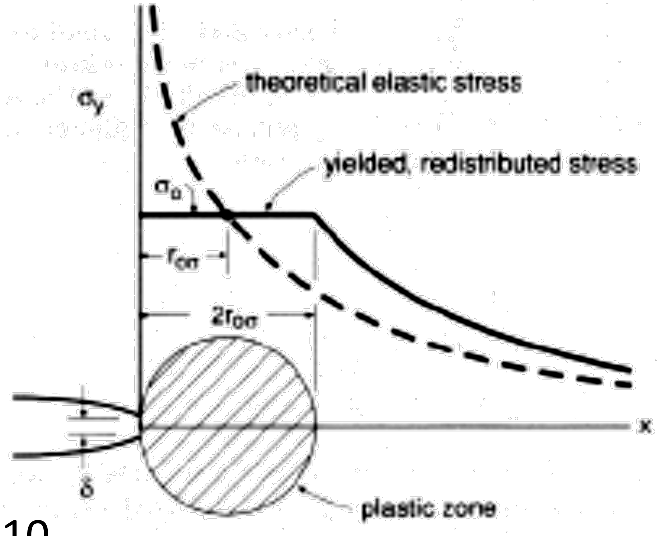


Fig.10

8. Plastic Zone Size

For any cases of Mode I loading, the stresses near the crack tip are:

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} cos\frac{\theta}{2}\left(1 - sin\frac{\theta}{2} sin\frac{3\theta}{2}\right)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} cos\frac{\theta}{2}\left(1 + sin\frac{\theta}{2} sin\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} cos\frac{\theta}{2} sin\frac{\theta}{2} cos\frac{3\theta}{2}$$

Plane stress $\sigma_z = \tau_{yz} = \tau_{xz} = 0$

Plane strain $\sigma_z = v(\sigma_x + \sigma_y)$; $\tau_{yz} = \tau_{xz} = 0$

Where $K_I = \sigma_o\sqrt{\pi a}$

For plane stress, the state of stress at the plane of the crack where the angle θ = 0^o is:

$$\sigma_x = \sigma_y = \frac{K_I}{\sqrt{2\pi r}}$$

$$\sigma_z = \tau_{yz} = \tau_{xz} = \tau_{xy} = 0$$

Since all shear stress along the plane of the crack are zero, σ_x, σ_y, and σ_z are the principal normal stresses.

8. Plastic Zone Size

The maximum shear stress and the maximum octahedral shearing stress criteria estimate the yielding at :

$$\sigma_x = \sigma_y = \sigma_{ys}$$

Where σ_{ys} is the yielding strength. Therefore, we obtain the radius of the plastic zone for the plane stress in the form of

$$r_y = \frac{1}{2\pi}\left[\frac{K_I}{\sigma_{ys}}\right]^2$$

For the plane strain, the radius of the plastic zone can be determined by using the equation

$$r_y = \frac{1}{6\pi}\left[\frac{K_I}{\sigma_{ys}}\right]^2$$

It should be noted that the radius of the plastic zone for the plane strain is smaller than one of the plane stress. This is due to the fact that the stress σ_z for the plane strain is nonzero, and this elevates the value of $\sigma_x = \sigma_y$ necessary to cause yielding, in turn decreasing the plastic zone size relative to that for plane stress.

Thus, for different materials, the one having a lower σ_{ys} will have a larger r_y. The plastic zone size for plane stress condition is larger than that of the plane strain condition.

8. Plastic Zone Size

For cyclic loading, the cyclic plastic zone size can be determined by:

For plane stress, $$r_y = \frac{1}{8\pi}\left[\frac{K_I}{\sigma_{ys}}\right]^2$$

For plane strain, $$r_y = \frac{1}{24\pi}\left[\frac{K_I}{\sigma_{ys}}\right]^2$$

Hence, we can see that $r_y|_{cyclic} < r_y|_{static}$.

Example

A large plate made of 4140 steel (σ_{ys} = 620 MPa) containing a 5 mm center crack is subjected to a tensile stress of σ_o = 207 MPa.

a) Determine the plastic zone size.

b) Are the LEFM's assumptions violated?

c) If the yielding strength of the material is reduced by a factor of 2.0, calculate the plastic zone size. Are the LEFM's assumptions violated

Example

Solution

a) This problem is a plane stress problem. The plastic zone size can be determined by using the equation

$$r_y = \frac{1}{2\pi}\left[\frac{K_I}{\sigma_{ys}}\right]^2$$

The stress intensity factor,

$$K_I = \sigma_o\sqrt{\pi a}.f\left(\frac{a}{2b}\right)$$

$$f\left(\frac{a}{2b}\right) = \sqrt{sec\left(\frac{\pi a}{2b}\right)} = 1$$

$$K_I = 1 \times 207 \times \sqrt{\pi \times \frac{5}{2} \times 10^{-3}} = 18.34\ MPa\sqrt{m}$$

Thus, the plastic zone size is

$$r_y = \frac{1}{2\pi}\left[\frac{K_I}{\sigma_{ys}}\right]^2 = \frac{1}{2\pi}\left[\frac{18.34}{620}\right]^2 = 0.1393\ mm$$

Example

Solution

b) The assumptions remain valid since the plastic zone size is small relative to the crack size and cracked body ($r_y < a$).

c) If the yielding strength of the material is reduced by a factor of 2.0, calculate the plastic zone size.

Are the LEFM's assumptions violated

$$\sigma_{ys} = \frac{620}{2} = 310\ MPa$$

$$r_y = \frac{1}{2\pi}\left[\frac{K_I}{\sigma_{ys}}\right]^2 = \frac{1}{2\pi}\left[\frac{18.34}{310}\right]^2 = 0.557\ mm$$

The assumptions are violated since the plastic zone size is quite large compared to the crack size

(about 22% of the half crack size).

Design for Fatigue Strength

Chapter Five

1.Introduction

What is fatigue?

Fatigue is a form of failure that occurs is it structures subjected to dynamic and fluctuating stresses (e.g., bridges, aircraft and machine components).

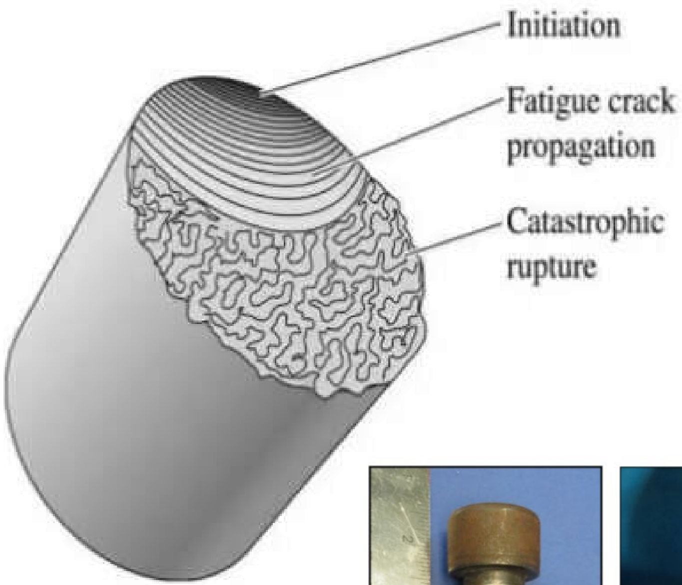
Under this circumstance, it is possible for failure to occur at a stress level considerably lower than tensile or yield strength for a static load.

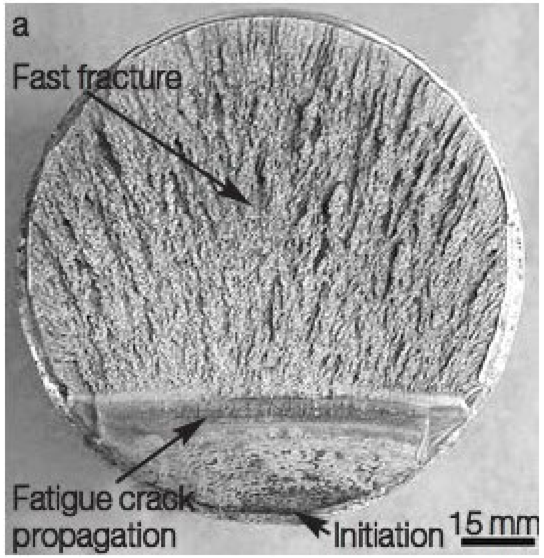
The term **fatigue** is used because this type of failure normally occurs after a lengthy period of repeated stress or strain cycling.

It is the single largest cause of failure in metal, estimated to comprise approximately 90% of all metallic failures; polymer and ceramic (except for glasses) are also susceptible to this type of failure.

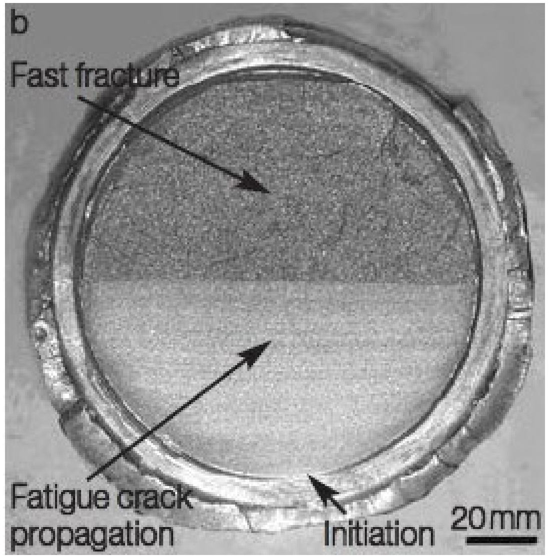
Fatigue failure is defined as the tendency of the material to fracture using progressive brittle cracking under repeated alternating or cyclic stresses of an intensity considerably below the normal strength.

Appearance of typical fatigue fracture surface

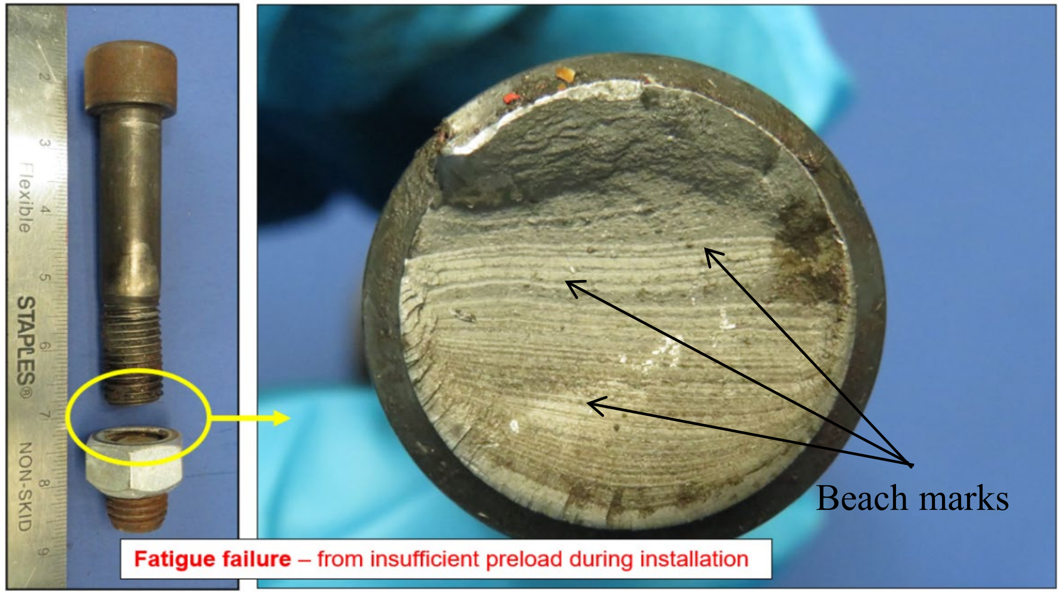




High applied load



Low applied load



1.Introduction

Where dose fatigue failure occur?

- Fatigue failure occurs when a material fails at a load lower than the determined yield strength and is usually caused by stress from cyclical loading over a period of time.

What is the reason for fatigue failure?

- Most fatigue failures are caused by cyclic loads significantly below the loads that would result in the yielding of the material.
- The failure occurs due to the cyclic nature of the load which causes microscopic material imperfections (flaws) to grow into a microscope crack (initiation phase).

What are the three stages of fatigue failure?

1- Initiation

2-Propagation

3-Final ruptures

1.Introduction

What type of stresses cause fatigue failure?

- Fatigue is a failure mechanism that involves the cracking of materials structural components due to cyclic (or fluctuating) stress.
- While applied stresses may be tensile, compressive or torsional, crack initiation and the propagation are due to the tensile component

Fatigue failure

- The process occurs by the initiation and the propagation of cracks and ordinarily the fracture surface is perpendicular to the direction of applied tensile stress.
- When cyclic stresses are applied to a material, even though the stresses do not cause plastic deformation, the material may fail due to fatigue.
- Furthermore, fatigue is catastrophic and insidious, occurring very suddenly and without warning.
- Fatigue failure is brittle like in nature even in normally ductile metals, in that there is very little, if any, plastic deformation associated with failure.

1.Introduction

- According to Linear-Elastic Fracture Mechanics (LEFM), fatigue failure develops in three stages:

Stage1: Development of one or more micro cracks(the size of two to five grains) due to the cyclic local plastic deformation.

Stage2: The cracks progress from micro cracks to larger cracks (macro cracks) and keep growing making a smooth plateau-like fracture surfaces with beach marks.

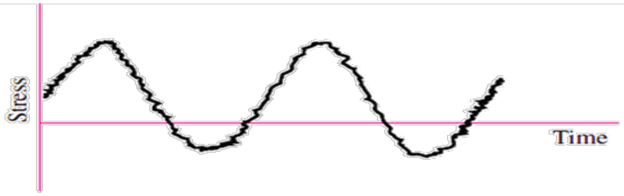
Stage3: Occurs during the final stress cycle where the remaining material cannot support the load, thus resulting in a sudden fracture (can be brittle or ductile fracture).

- Fatigue failure is due to crack formation and propagation. Fatigue cracks usually initiate at locations with high stresses such as discontinuities (hole, notch, scratch, sharp corner, crack, inclusions, etc.).
- Fatigue cracks can also initiate at surfaces having rough surface finish or due to the presence of tensile residual stresses. Thus, all parts subjected to fatigue loading are heat treated and polished in order to increase the fatigue life.

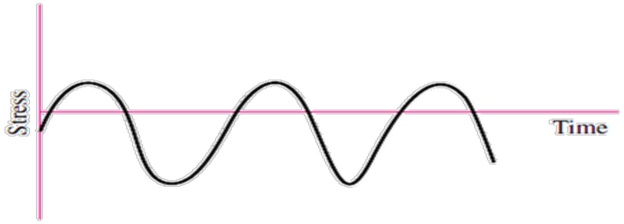
2.Stress Cycle

A typical stress cycle is shown in figure below where the maximum, minimum, mean and variable stresses are indicated. The mean and variable stresses are given by:

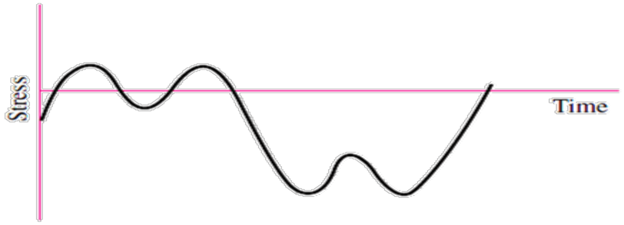
$$\sigma_{mean} = \frac{\sigma_{max}+\sigma_{min}}{2} \quad ; \quad \sigma_{alternating} = \frac{\sigma_{max}-\sigma_{min}}{2}$$



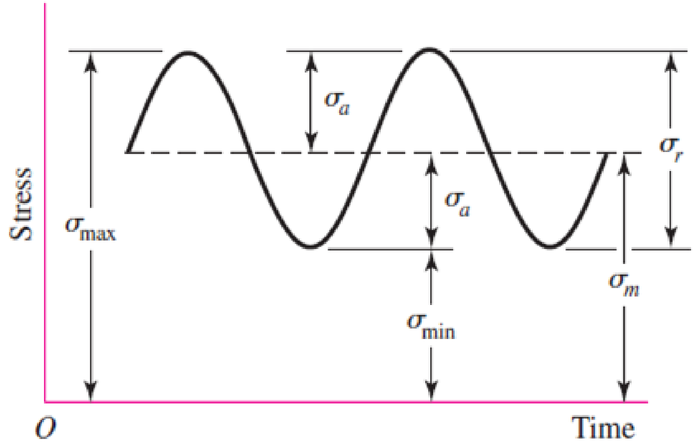
fluctuating stress with high-frequency ripple



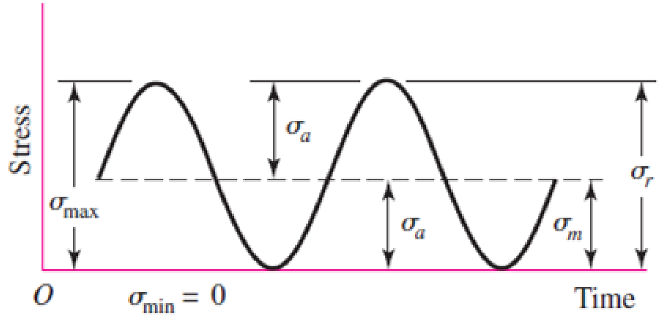
nonsinusoidal fluctuating stress



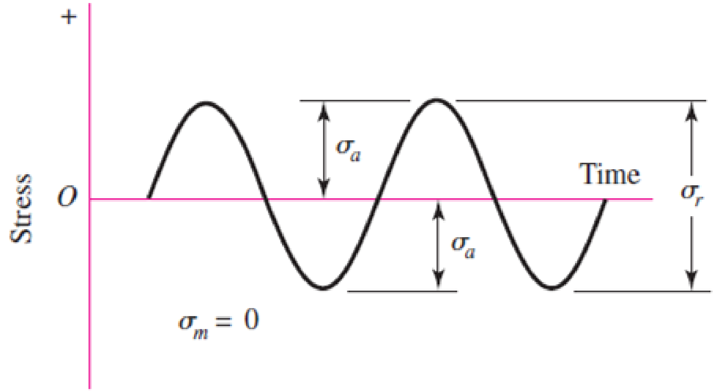
nonsinusoidal fluctuating stress



sinusoidal fluctuating stress



repeated stress



completely reversed sinusoidal stress

2.Stress Cycle

The following relations are derived from previous figures:

a) Mean or average stress

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

b) Reversed stress component or alternating or variable stress

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

For repeated loading, the stress varies from maximum to zero (i.e. $\sigma_{min} = 0$) in each cycle.

$$\sigma_m = \sigma_a = \frac{\sigma_{max}}{2}$$

c) Stress ratio, $R = \frac{\sigma_{min}}{\sigma_{max}}$. For completely reversed stresses, $R = -1$ and for repeated stresses, R = 0. It may be noted that R cannot be greater than unity.

3. Fatigue Life Methods

Fatigue failure is a much more complicated phenomenon than static failure where much complicating factors are involved. Also, testing materials for fatigue properties is more complicated and much more time consuming than static testing. Fatigue life methods are aimed to determine the life (number of loading cycles) of an element until failure.

There are three major fatigue life methods where each is more accurate for some types of loading or for some materials. The three methods are: **stress-life method**, **strain-life method**, and **linear-elastic fracture mechanics method.**

The fatigue life is usually classified according to the number of loading cycles into:

- Low cycle fatigue ($1 \leq N \leq 10^3$) and for this low number of cycles, designers sometimes ignore fatigue effects and just use static failure analysis.
- High cycle fatigue ($N > 10^3$)

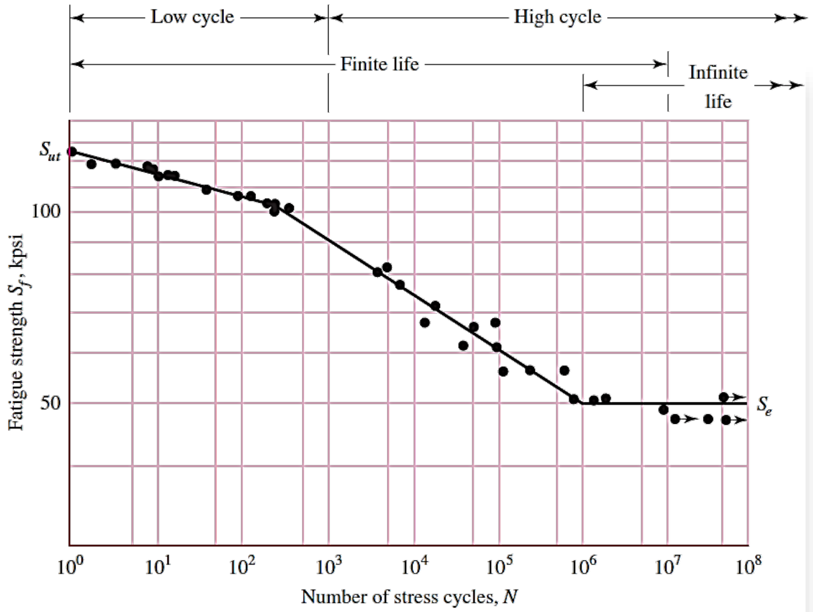
A) Stress Life Method

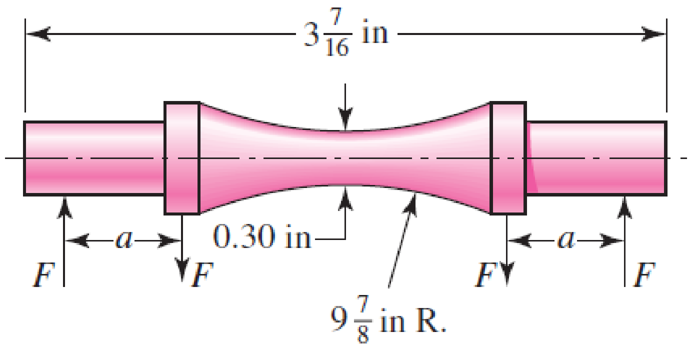
This method relates the fatigue life to the alternating stress level causing failure, but it does not give any explanation to why fatigue failure happens.

The stress-life relation is obtained experimentally using Moore high-speed rotating beam test.

- The test is conducted by subjecting the rotating beam to a pure bending moment (of a fixed known magnitude) until failure occurs.(Due to rotation, the specimen is subjected to an alternating bending stress)

- The data obtained from the tests is used to generate the fatigue strength vs. fatigue life diagram which is known as the S-N diagram.
- The first point on the S-N diagram is the ultimate strength which corresponds to failure in half a cycle.
- The alternating stress amplitude is set to be below the ultimate strength and the test is run until failure. The stress level and the number of cycles until failure give a data point on the chart.
- The testing continues and each time the stress amplitude is reduced (such that the specimen will live longer) and a new point is obtained.
- For steel alloys, the low-cycle fatigue and the high-cycle fatigue(finite and infinite) can be recognized as having different slopes. (the yare straight lines, but keep in mind it is a log-log curve)
- For steels, if we keep reducing the stress amplitude (for each test), we will reach to a stress level for which the specimen will never fail, and this value of stress is known as the Endurance Limit (Se).
- The number of stress cycles associated with the Endurance Limit defines the boundary between Finite-life and Infinite-life, and it is usually between 10^6 to 10^7 cycles.





3. Fatigue Life Methods

B) Strain Life Method

This method relates the fatigue life to the amount of plastic strain suffered by the part during the repeated loading cycles.

When the stress in the material exceeds the yield strength and the material is plastically deformed, the material will be strain hardened and the yield strength will increase (if the part is reloaded again). However, if the stress direction is reversed (from tension to compression), the yield strength in the reversed direction will be smaller than its initial value. That means that the material has been softened in the reverse loading direction (this is referred to as Bauschinger Effect). Each time the stress is reversed, the yield strength in the other direction is decreased, and the material gets softer and it undergoes more plastic deformation until fracture occurs.

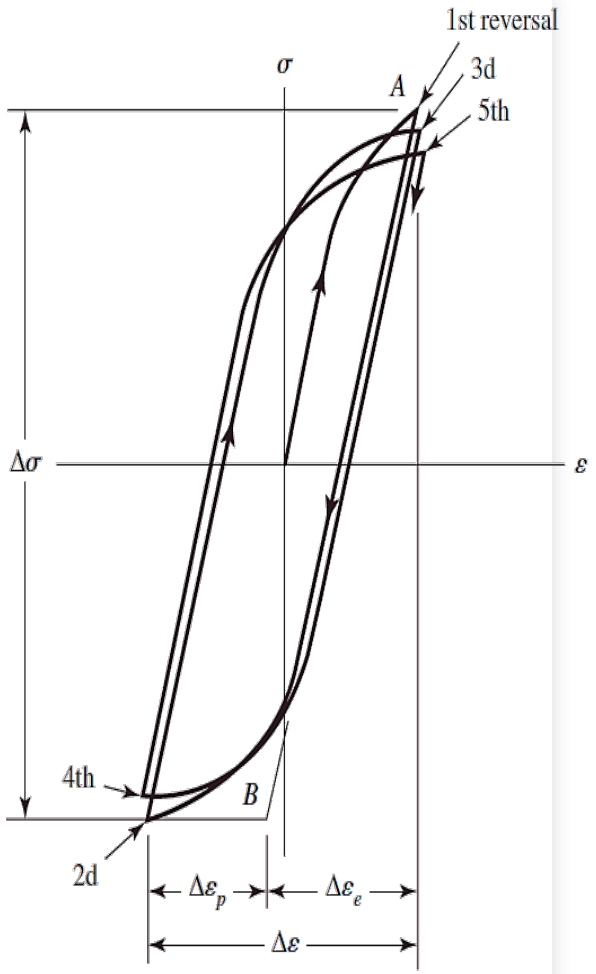


Fig. 1

- The strain-life method is applicable to Low-cycle fatigue.

From Fig.1, we see that the total strain is the sum of the elastic and plastic components. Therefore, the total strain amplitude is half the total strain range

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2}$$

The equation of the plastic-strain line in Fig. 2 is:

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon_F'(2N)^c$$

The equation of the elastic strain line is:

$$\frac{\Delta\varepsilon_e}{2} = \frac{\sigma_F'}{E}(2N)^b$$

Therefore, from first equation, we have for the total-strain amplitude

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_F'}{E}(2N)^b + \varepsilon_F'(2N)^c$$

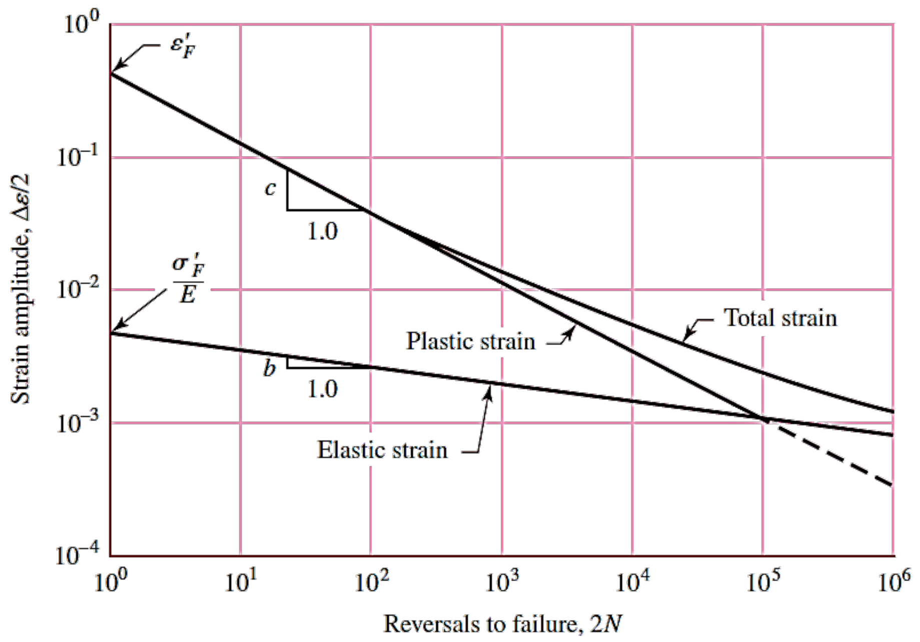


Fig. 2

3. Fatigue Life Methods

C) Linear Elastic Fracture Mechanics Method

This method assumes that a crack initiates in the material and it keeps growing until failure occurs.

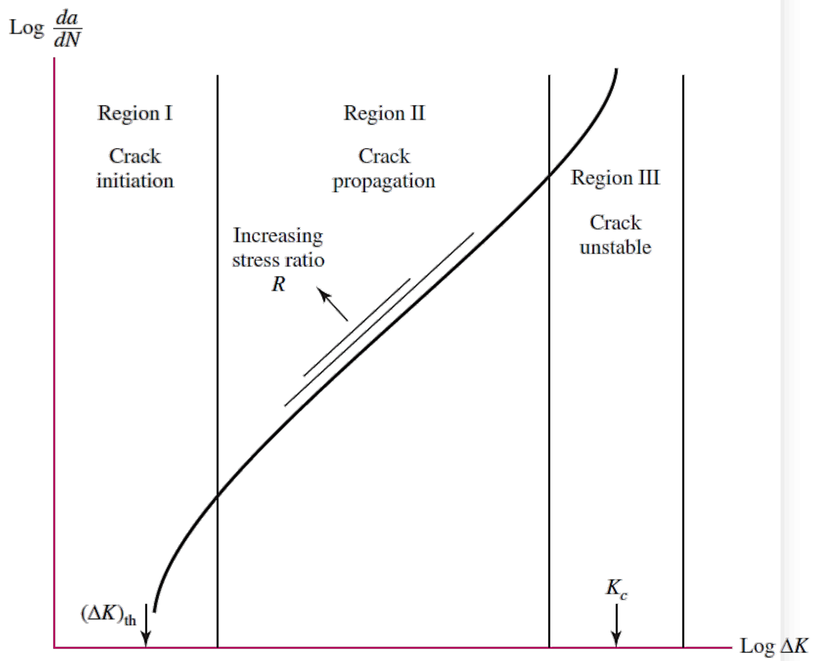
- The LEFM approach assumes that a small crack already exists in the material, and it calculates the number of loading cycles required for the crack to grow to be large enough to cause the remaining material to fracture completely.
- This method is more applicable to High-cycle fatigue.

The stress intensity factor is given by $K_I = f\sigma\sqrt{\pi a}$. Thus, for $\Delta\sigma$, the stress intensity range per cycle is:

$$K_I = f(\sigma_{max} - \sigma_{min})\sqrt{\pi a} = f\Delta\sigma\sqrt{\pi a}$$

When the rate of crack growth per cycle, da/dN is plotted as shown in Fig. 3, the data from all three stress range levels superpose to give a sigmoidal curve. The three stages of crack development are observable, and the stage II data are linear on log-log coordinates, within the domain of linear elastic fracture mechanics (LEFM) validity. A group of similar curves can be generated by changing the stress ratio R = $\sigma_{min}/\sigma_{max}$ of the experiment.

Here we present a simplified procedure for estimating the remaining life of a cyclically stressed part after discovery of a crack. This requires the assumption that plane strain conditions prevail. Assuming a crack is discovered early in stage II, the crack growth in region II of Fig. 3 can be approximated by the **Paris equation**, which is of the form



$$\frac{da}{dN} = C(\Delta K_I)^m$$

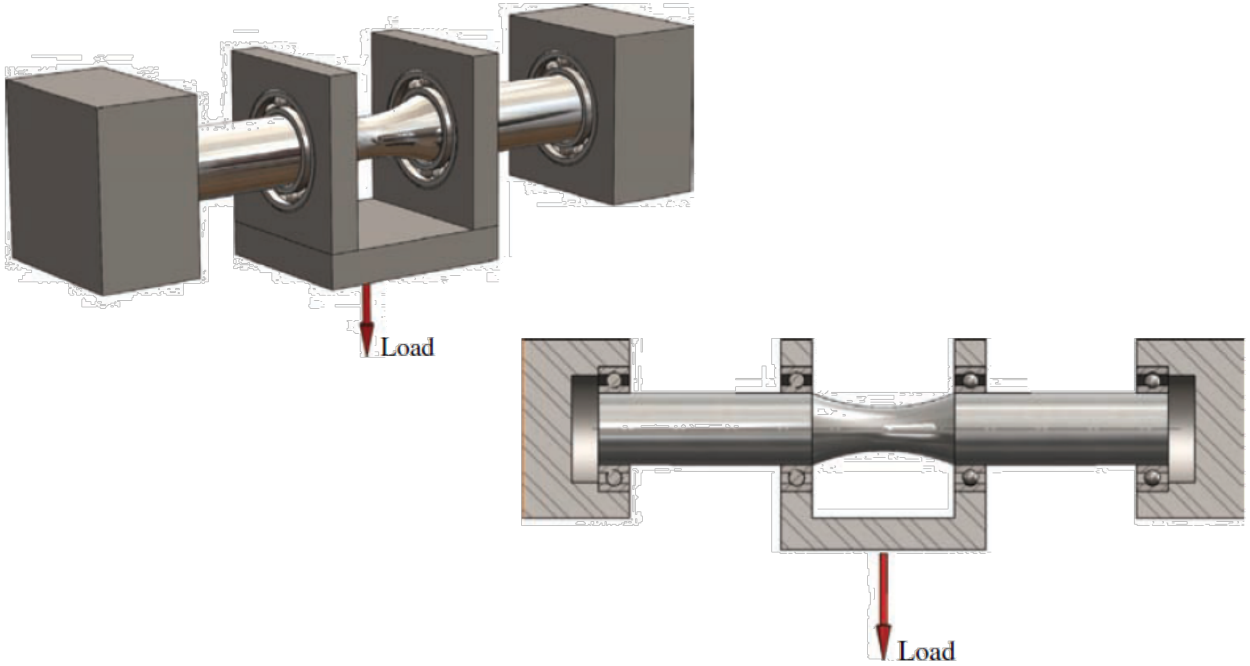
where C and m are empirical material constants (take from table 6.1)

$$\int_0^{N_f} dN = N_f = \frac{1}{C}\int_{a_i}^{a_f} \frac{da}{\left(f\Delta\sigma\sqrt{\pi a}\right)^m}$$

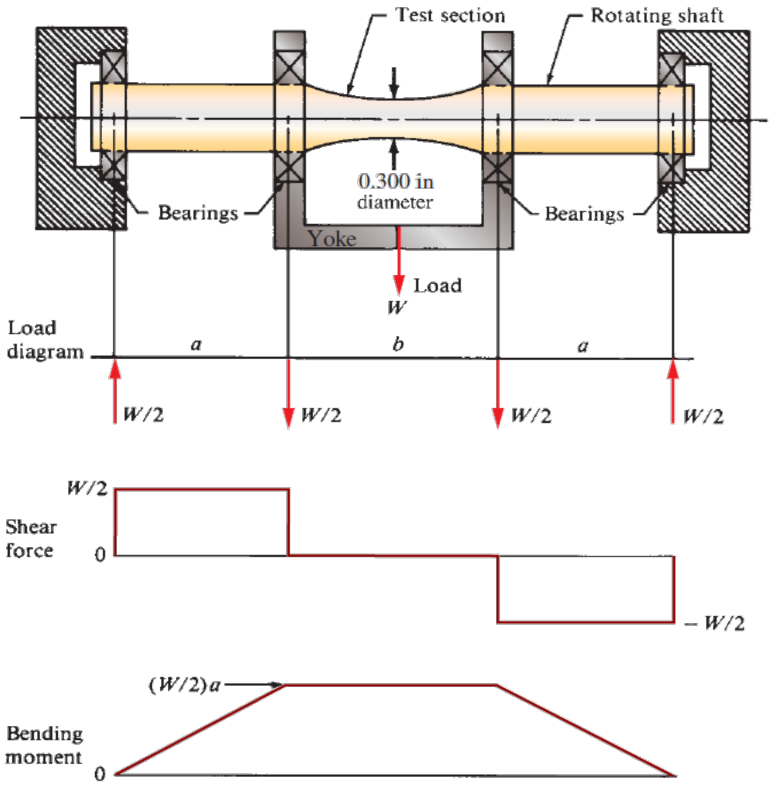
Here a_i is the initial crack length, a_f is the final crack length corresponding to failure, and N_f is the estimated number of cycles to produce a failure after the initial crack is formed.

4.Endurance Limit

It has been found experimentally that when a material is subjected to repeated stresses; it fails at stresses below the yield point stresses. Such type of failure of a material is known as fatigue. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The fatigue of material is affected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.



Moore fatigue test device; example of reversed bending



4.Endurance Limit

In order to study the effect of fatigue of a material, a rotating mirror beam method is used. In this method, a standard mirror polished specimen is rotated in a fatigue testing machine while the specimen is loaded in bending. As the specimen rotates, the bending stress at the upper fibers varies from maximum compressive to maximum tensile while the bending stress at the lower fibers varies from maximum tensile to maximum compressive. In other words, the specimen is subjected to a completely reversed stress cycle. A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in stress-cycle curve as shown in Figure (1). A little consideration will show that if the stress is kept below a certain value as shown by dotted line in Figure (1), the material will not fail whatever may be the number of cycles. This stress, as represented by dotted line, is known as endurance or fatigue limit (σ_e). It is defined as maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for infinite number of cycles (usually 10^7 cycles).

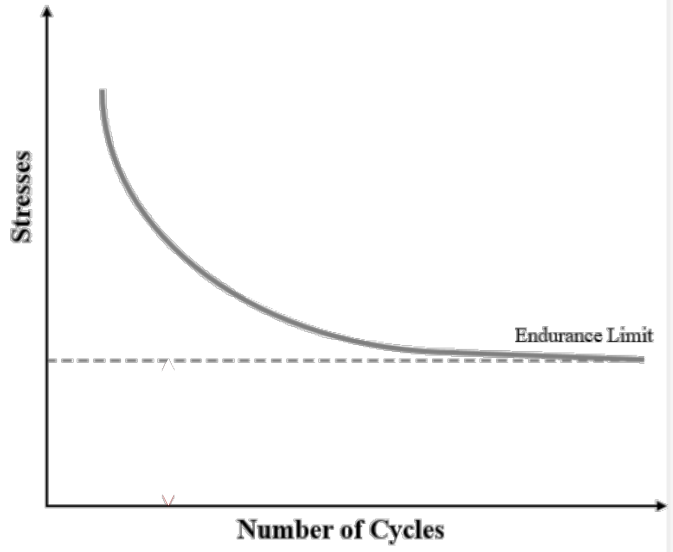


Fig. 1

4.Endurance Limit

It may be noted that the term endurance limit is used for reversed bending only while for other types of loading, the term **endurance strength** may be used when referring the fatigue strength of the material. It may be defined as the safe maximum stress which can be applied to the machine part working under actual conditions.

The loading is such that there is a constant bending moment over the specimen length and the bending stress is greatest at the center where the section is smallest. The arrangement gives pure bending and avoids transverse shear since bending moment is constant over the length.

Large number of tests with varying bending loads are carried out to find the number of cycles to fail. A typical plot of reversed stress (S) against number of cycles to fail (N) is shown in figure(2). The zone below 10^3 cycles is considered as low cycle fatigue, zone between 10^3 and 10^6 cycles is high cycle fatigue with finite life and beyond 10^6 cycles, the zone is considered to be high cycle fatigue with infinite life.

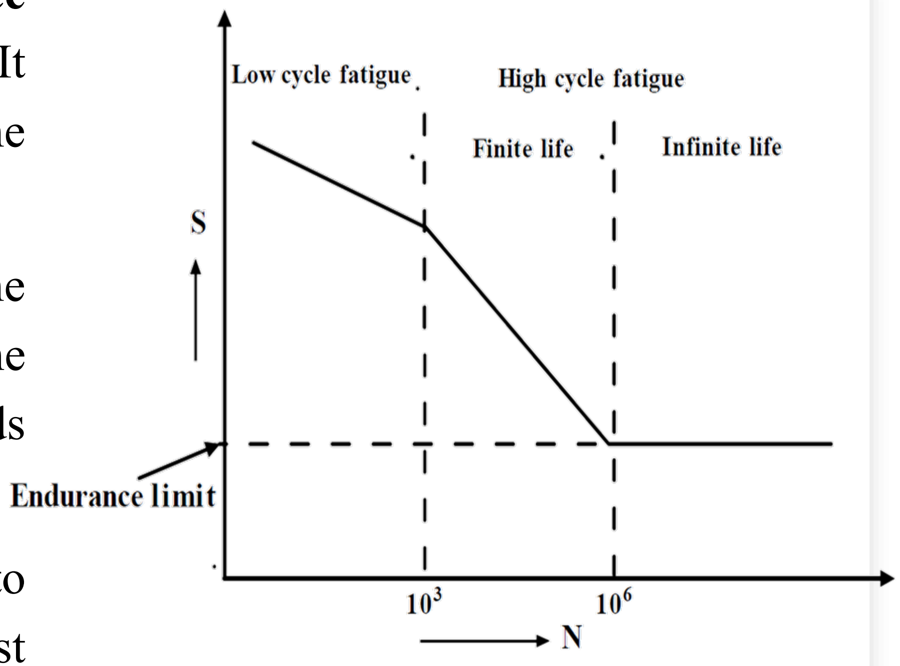


Fig.2

4.Endurance Limit

Important Remarks

1) $N<10^3$ low cycle fatigue

$\begin{Bmatrix} N > 10^3 \\ N < 10^6 \end{Bmatrix}$ high cycle fatigue

2) $N>10^6$ infinite life

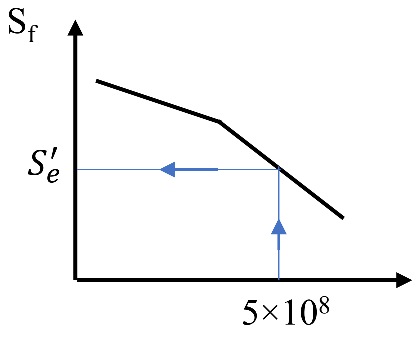
$N<10^7$ finite life

} overlap period $(10^6 - 10^7)$

3) For steels and titanium, if we keep reducing stress, we will reach to a stress level for which the specimen will never fail, this value of stress is call **Endurance limit** $\boldsymbol{S'_e}$.

4) Endurance limit define boundary between finite life and infinite life $\Rightarrow 10^6 < N < 10^7$

5) Ferrous steel and alloys have clear endurance limit but other materials ex. Aluminum do not have $\boldsymbol{S'_e}$, project back for 5×10^8.



6) Take specimen 1 test it

Take specimen 2 test it

} different results because of statistical nature.

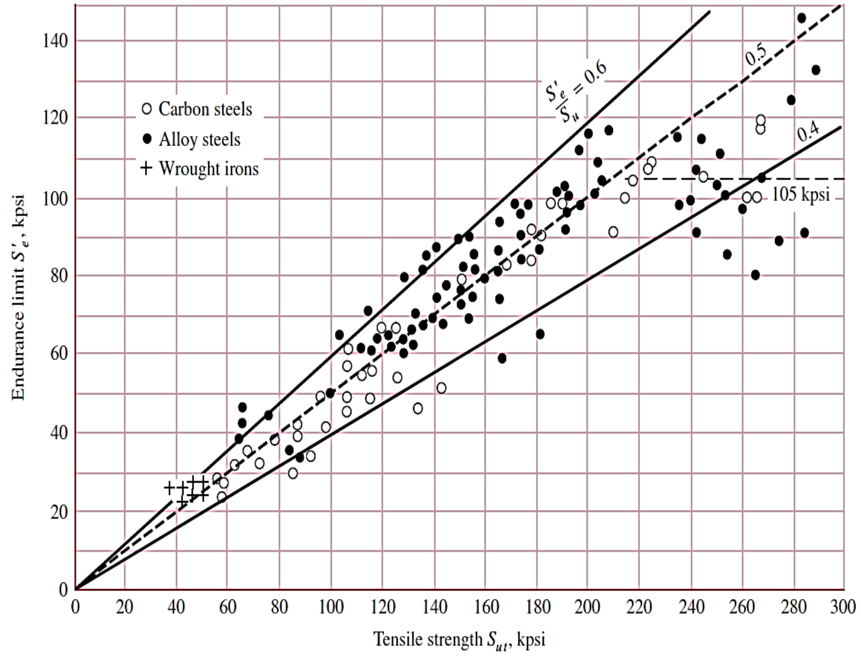
- An attempt is to relate S'_e to S_{ut} why?

  Fatigue test is time consuming and expensive.

4.Endurance Limit

The figure shows a plot of the **Endurance Limits** versus **Tensile Strengths** for a large number of steel and iron specimens.

- The graph shows a relation between the ultimate strength and endurance limit for ultimate strengths up to 1400 MPa (200 kpsi), then the endurance limit seems to have a constant value.
- The relationship between the endurance limit and ultimate strength for steels is given as:



$$S'_e = \begin{cases} 0.5\, S_{ut} & for \quad S_{ut} \le 1400\, MPa\ (200 kpsi) \\ 700\, MPa & for \quad S_{ut} > 1400\, MPa\ (200\ kpsi) \end{cases}$$

Notes:

1) For cast iron and cast steel

$$S'_e = \begin{cases} 0.45\, S_{ut} & for \quad S_{ut} \le 600\, MPa \\ 275\, MPa & for \quad S_{ut} > 600\, MPa \end{cases}$$

2) For Aluminum and Magnesium alloys S'_e: (30-40)% S_{ut}

For Plastic S'_e: (18-43)% S_{ut}

5. Endurance Limit Modifications Factors

The endurance limit is obtained from the rotating beam test. The test is conducted under closely controlled conditions (polished specimen of small size at a constant known temperature, etc.). It is not realistic to expect a machine element to have the exact same endurance limit value as that obtained from the rotating beam test because it has different conditions (size, surface finish, manufacturing process, environment, etc.).

Thus, some modification factors are used to correlate the endurance limit for a given mechanical element to the value obtained from tests:

$$S_e = k_a k_b k_c k_d k_e k_f S_e'$$

Where,

Se: The endurance limit at the critical location of a machine element with the geometry and conditions of use.

Se': The endurance limit obtained from the rotating beam test.

$\boldsymbol{k_a}$: Surface condition modification factor

$\boldsymbol{k_b}$: Size modification factor

$\boldsymbol{k_c}$: Load modification factor

$\boldsymbol{k_d}$: Temperature modification factor

$\boldsymbol{k_e}$: Reliability factor

$\boldsymbol{k_f}$: Miscellaneous-effects modification factor

5. Endurance Limit Modifications Factors

i) Surface Condition Factor (k_a)

The rotating-beam test specimens are highly polished. A rough surface finish will reduce the endurance limit because there will be a higher potential for crack initiation.

The surface condition modification factor depends on the surface finish of the part (ground, machined, as forged, etc.) and on the tensile strength of the material. It is given as:

$$k_a = a\, S_{ut}^b$$

Where the constants a & b depend on surface condition and are given in Table 6-2 (textbook).

ii) Size Factor (k_b)

The rotating-beam specimens have a specific (small) diameter (7.6 mm). Parts of larger size are more likely to contain flaws and to have more non-homogeneities.

The size factor is given as:

$$k_b = \begin{cases} 1.24d^{-0.107} & 2.79 \le d \le 51\, mm \\ 1.51d^{-0.157} & 51 < d \le 254\, mm \end{cases} \qquad \textbf{for bending and torsion}$$

Where, d: Is the dimeter mm

And $k_b = 1$ **for axial loading**

5. Endurance Limit Modifications Factors

Notes:

1) When a beam with circular cross-section is NOT rotating, an "effective diameter (d_e)" value is used instead of the actual diameter, where: $d_e = 0.37\, d$ **only for bending**
2) If structural element has rectangular cross section (h×b), $d_e = 0.808\sqrt{hb}$

iii) Loading Factor (k_c)

The rotating-beam specimen is loaded in bending. Other types of loading will have a different effect.

The load factor for the different types of loading is:

$$k_c = \begin{cases} 1 & bending \\ 0.85 & axial \\ 0.59 & torsion \end{cases}$$

iv) Temperature Factor (k_d)

When the operating temperature is below room temperature, the material becomes more brittle. When the temperature is high, the yield strength decreases, and the material becomes more ductile (and creep may occur).

5. Endurance Limit Modifications Factors

For steels, the tensile strength, and thus the endurance limit, slightly increases as temperature rises, then it starts to drop. Thus, the temperature factor is given as:

$$k_d = 0.9877 + 0.6507(10^{-3})T_c - 0.3414(10^{-5})T_c^{\,2} + 0.5621(10^{-8})T_c^{\,3} - 0.6246(10^{-11})T_c^{\,4} \qquad \text{For} \quad 40 \le T_c \le 540\ ^{\circ}C$$

The same values calculated by the equation are also given in Table 6-4 (textbook) where:

$$k_d = \frac{S_T}{S_{RT}}$$

v) Reliability Factor (k_e)

A reliability of (R=0.9) means that there is 90% chance that the structural member will perform its proper function without failure.

- Failure of 6 parts of every 1000 parts $\Rightarrow$ R= 1 − 6/1000 = 0.994 (99.4 % reliability)
- k_e is reliability factor = 1 − 0.08 z_a

5. Endurance Limit Modifications Factors

vi) Miscellaneous-Effects Factor (k_f)

It is used to account for the reduction of endurance limit due to all other effects (such as residual stress, corrosion, cyclic frequency, metal spraying, etc.).

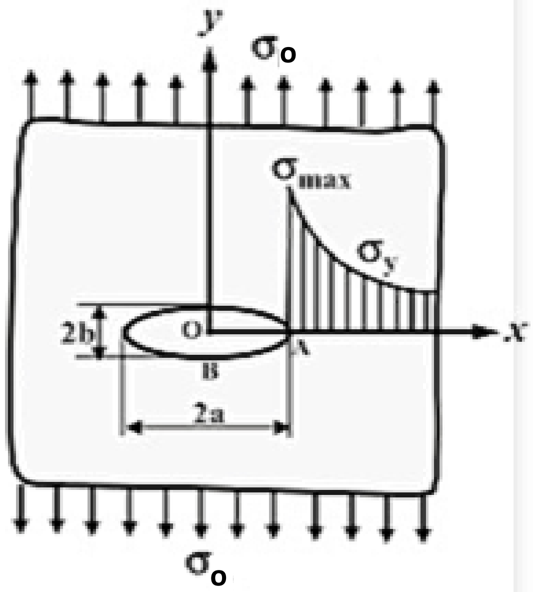
However, those effects are not fully characterized and usually not accounted for. Thus, we use ($k_f = 1$).

- Stress Concentration and Notch Sensitivity

a) Most machined parts have holes, grooves, notches, or other kind of irregularities or discontinuities which alter the stress distribution.

An infinite plate in tension with elliptical hole see stress distribution.

Define "Stress Concentration Factor" (theoretical one) to relate maximum stress at discontinuity to nominal stress σ_o.



$$K_t = \frac{\sigma_{max}}{\sigma_o} = \begin{cases} K_t = 1 + \frac{2a}{b} & for\ ellipse \\ K_t = 3 & for\ circle \end{cases}$$

5. Endurance Limit Modifications Factors

Notes:

1) K_t is called theoretical because it depends on geometry of discontinuity and not on material.
2) See data book (p.73-80) for practical cases such as (rectangular bar + hole), (bar + notch), (bar + fillets), (shaft + fillets) and (shaft + groove).
3) Stress concentration effect is very important in fatigue (dynamic loads) and in static load for brittle materials but very less effect in static load for ductile materials.

b) Some materials are sensitive to notch other are not, so define "Notch Sensitivity q"

such that: $q = 0 \Rightarrow$ materials has no sensitivity to notch or discontinuity

$q = 1 \Rightarrow$ full sensitivity (to gat the value of q see data book p.12)

c) Knowing K_t and q, find:

$$K_f = 1 + q(K_t - 1)$$ Fatigue Stress Concentration Factor

Then, miscellaneous-effect factor

$$k_f = \frac{1}{K_f}$$

Example

a) Corresponding to reliability 99% estimate endurance limit of a round rotating cold-drawn AISI 1018 steel shaft 30mm in diameter.

b) What is endurance limit of a non-rotating bar of same material and dimensions?

Solution

a) From data book table 17 p.83 $\Rightarrow$ $S_{ut} = 440$ MPa

Reliability 99% $\Rightarrow k_e = 0.814$;(p.11)

Surface factor; $k_a = aS_{ut}^{b}$

p.9 $\Rightarrow$ cold-drawn a=4.51 ; b= -0.265

$k_a = 4.51 \times (440)^{-0.265} = 0.899$

Load factor , bending $\Rightarrow k_c=1$

Size factor, $k_b=1.24\ d^{-0.107} = 1.24\ (30)^{-0.107} = 0.862$

Others factor $k_d=k_f = 1$

$$S_e' = \frac{1}{2}S_{ut} = \frac{1}{2} \times 440 = 220$$

$$S_e = 0.899 \times 0.862 \times 1 \times 1 \times 0.814 \times 1 \times 220$$
$$= 138.8\ MPa$$

b) If shat is not rotating,

$d_e=0.37\ d \Rightarrow d_e=0.37 \times 30 = 11.1$ mm

Size factor, $k_b=1.24\ d^{-0.107} = 1.24\ (11.1)^{-0.107} = 0.958$

$$S_e' = \frac{1}{2}S_{ut} = \frac{1}{2} \times 440 = 220$$

$$S_e = 0.899 \times 0.958 \times 1 \times 1 \times 0.814 \times 1 \times 220$$
$$= 154\ MPa$$

6.Design for High Cycle Fatigue (finite life : $10^3 \le N \le 10^6$)

In some design applications, the number of load cycles the element is subjected to is limited (less than 106), and therefore there is no need to design for infinite life using the endurance limit.

- In such cases, we need to find the Fatigue Strength associated with the desired life.
- For the High-cycle fatigue ($10^3 \rightarrow 10^6$), the line equation is $Sf = aN^b$ where the constants "a" (y intercept) and "b" (slope) are determined from the end points $(S_f)_{10^3}$ and $(Sf)_{10^6}$ as:

$$a = \frac{(S_f)^2}{S_e} \qquad \text{and} \qquad b = -\frac{\log\left(\sigma_f'/S_e\right)}{\log(2N_e)}$$

Where $\sigma_f{}'$ is the "True Stress at Fracture"; and for steels with HB ≤ 500, it is approximated as:

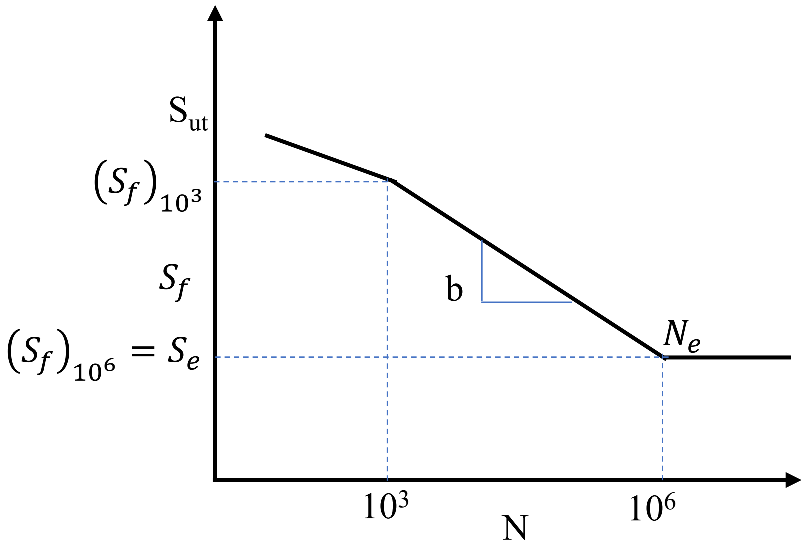
$$\sigma_f{}' = S_{ut} + 345\ Mpa$$

And $(S_f)_{10^3}$ can be related to S_{ut} as:

$$(S_f)_{10^3} = fS_{ut}$$

where f is found as:

$$f = \frac{\sigma_f'}{S_{ut}}\left(2 \times 10^3\right)^b$$



S_e is the modified Endurance Limit

6.Design for High Cycle Fatigue (finite life : $10^3 \leq N \leq 10^6$)

Using the above equations, the value of f is found as a function of Sut(using $N_e = 10^6$), and it is presented in graphical form in Figure 6-18 (textbook).

- If the value of (f) is known, the constant b can be directly found as:

$$b = -\frac{1}{3} log\left(\frac{f S_{ut}}{S_e}\right)$$

and a can be rewritten as:

$$a = \frac{(f S_{ut})^2}{s_e}$$

Thus, for $10^3 \leq N \leq 10^6$, the fatigue strength associated with a given life (N) is:

$$\left(S_f\right)_N = aN^b$$

and the fatigue life (N) at a given fatigue stress (σ) is found as:

$$N = \left(\frac{\sigma}{a}\right)^{\frac{1}{b}}$$

- Studies show that for ductile materials, the Fatigue Stress Concentration Factor (K_f) reduces for $N < 10^6$, however the conservative approach is to use K_f as is.

6.Design for High Cycle Fatigue (finite life : $10^3 \le N \le 10^6$)

Notes:

1) f : 'fraction" is found from chart p.7 data book. If $S_{ut} < 490$ MPa $\Rightarrow$ use $f = 0.9$ to be conservative.

2) $N = \left(\frac{\sigma}{a}\right)^{\frac{1}{b}}$: $N = f(\sigma)$

3) If chart f is not available $\Rightarrow$

$$\sigma_f' = S_{ut} + 345\ Mpa$$

$$b = -\frac{\log(\sigma_f'/S_e)}{\log(2 \times 10^6)}$$

$$f = \frac{\sigma_f'}{S_{ut}}\left(2 \times 10^3\right)^b$$

Example

A rotating shaft is supported by ball bearing at A and D and loaded by F=6.8 kN as shown in figure below. Estamite the life of the shaft. (Material : AISI 1050 CD)

Solution

Draw bending moment diagram

Most critical point is B why?

1) $M_B > M_C$, cross section at B < cross section at C, and stress concentration at B > at C.

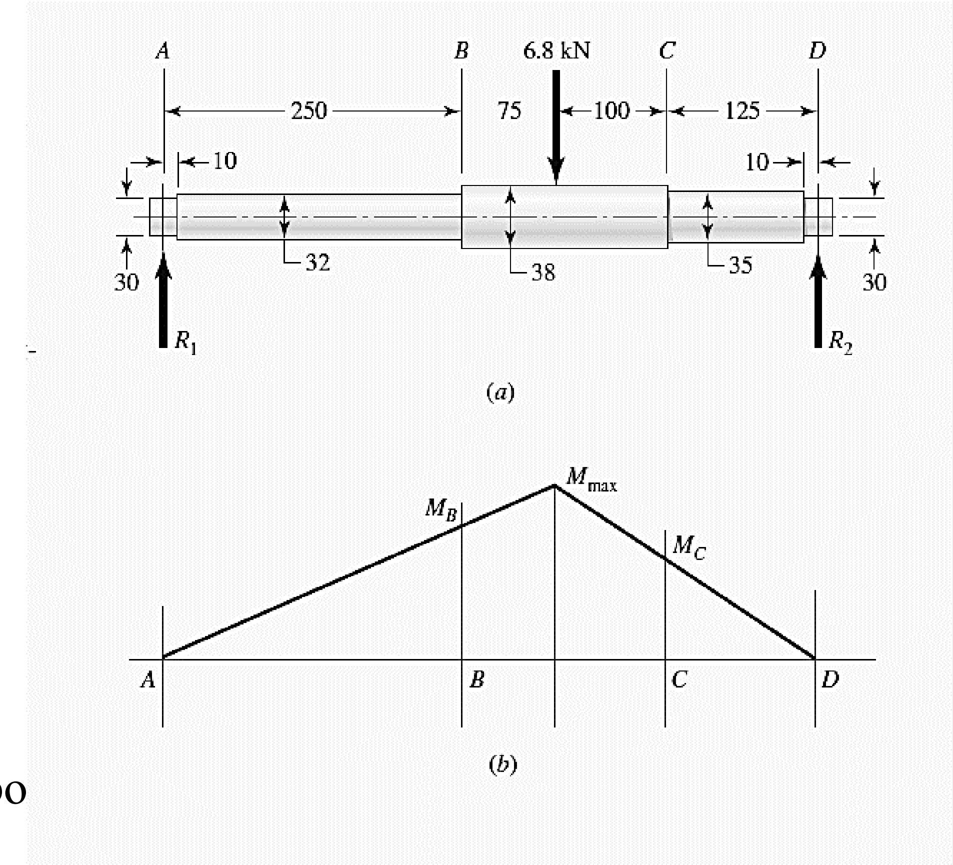
2) Under load $M_{max} > M_B$, but cross section is large and has low stress concentrations like B.

- Choose B location

From data book, S_{ut} = 690 MPa (AISI 1050 CD)

$$S'_e = \frac{690}{2} = 345\ MPa$$

Surface factor $k_a = aS_{ut}^b = 4.51(690)^{-0.265} = 0.798$ (p.9 data bo



Example

Size factor $k_b = 1.24d^{0.107} = 1.24(32)^{0.107} = 0.859$ (p.9 data book)

Load factor $k_c = 1$ (bending) (p.10 data book)

Reliability factor $k_e = 1$

Temperature factor $k_d = 1$

Stress concentration factor; data book p.77 : Fig A-9

$$K_t = 1.65 \quad at \begin{cases} \frac{D}{d} = \frac{38}{32} = 1.1875 \\ \frac{r}{d} = \frac{3}{32} 0.09375 \end{cases}$$

From data book p. 12; Fig.3.3 $\Rightarrow$ q = 0.83

$K_f = 1 + q(K_t - 1) = 1 + 0.83\,(1.65 - 1) = 1.54$

k_f (modifying factor) $= \frac{1}{K_f} = \frac{1}{1.54} = 0.649$

Correct modified endurance limit $S_e = k_a\, k_b\, k_c\, k_d\, k_e\, k_f\, S'_e$

$S_e = 0.798 \times 0.856 \times 0.649 \times 345 = 152.9$ MPa

If stress ≤ S_e (152.9 MPa) ⇒ infinite life

Otherwise, finite life.

bending moment at point B is

$M_B = R_1\, x = \frac{225 \times 6.8 \times 10^3}{550} \times 250 = 695.5\ N.m$

$$\sigma = \frac{My}{I} = \frac{32 M_B}{\pi d^3} = \frac{32 \times 695.5}{\pi (32 \times 10^{-3})^3} = 216\ MPa$$

Since $\sigma > S_e$ ⇒ finite life

Therefor, use : $S_f = aN^b \quad or\ \ N = \left(\frac{\sigma}{a}\right)^{\frac{1}{b}}$

From data book p.7 $f \cong 0.844$

$$a = \frac{(f S_{ut})^2}{S_e} = \frac{(0.844 \times 690)^2}{152.9} = 2218$$

$$b = -\frac{1}{3} log\left(\frac{f S_{ut}}{S_e}\right) = -\frac{1}{3} log\left(\frac{0.844 \times 690}{152.9}\right) = -0.1936$$

$$N = \left(\frac{216}{2218}\right)^{-0.1936} = 165.7 \times 10^3$$

7.Fatigue Failure Criteria for Fluctuating Stress

When a machine element is subjected to completely reversed stress (zero mean, $\sigma m = 0$), the endurance limit is obtained from the rotating-beam test (after applying the necessary modifying factors).
However, when the mean (or midrange) is non-zero, the situation is different and a fatigue failure criteria is needed.

- If we plot the alternating stress component (i.e., stress amplitude) (σ_a) vs. the mean stress component (σ_m), this will help in distinguishing the different fluctuating stress scenarios.

i) When $\sigma m = 0$ & $\sigma a \neq 0$, this will be a completely reversed fluctuating stress.

ii) When $\sigma a = 0$ & $\sigma m \neq 0$, this will be a static stress.

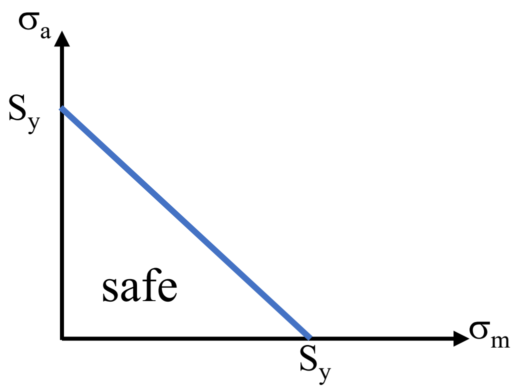
iii) Any combination of σm & σa will fall between the two extremes (completely reversed & static).

Different theories are proposed to predict failure in such cases:

1) Langer line (yield line)

It connects S_y on the σ_a axis with S_y on σ_m axis. But it is not realistic because S_y is usually larger than S_e.

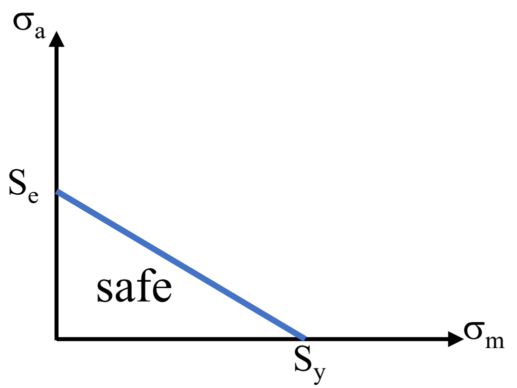
$(S_y > S_e)$



7.Fatigue Failure Criteria for Fluctuating Stress

2) Soderberg line: the most conservative, it connects S_e on σ_a axis with S_y on σ_m axis.

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}$$



Where,

S_e: modified endurance limit.

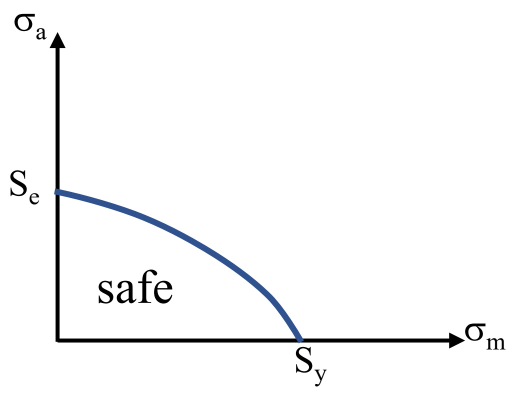
S_y: yield strength.

n: safety factor.

3) ASME elliptic line: Same as Soderberg, but it uses an ellipse instead of the straight line.

It fits experimental data better.

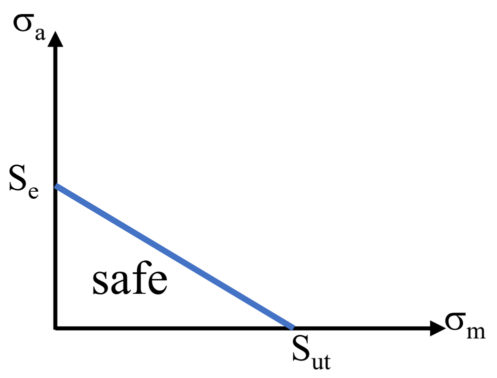
$$\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1$$



7.Fatigue Failure Criteria for Fluctuating Stress

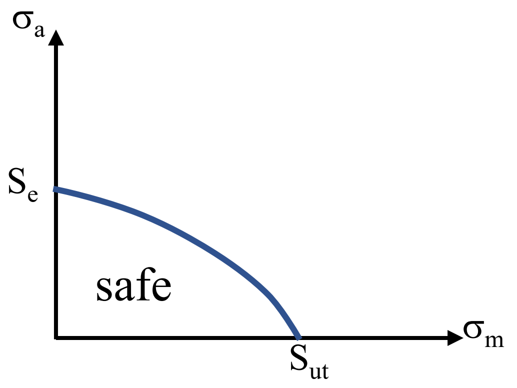
4) Goodman line: It considers failure due to static loading to be at S_{ut} rather than S_y, thus it connects S_e on σ_a axis with S_{ut} on σ_m axis using a straight line.

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$



5) Gerber line: Same as Goodman but it uses a parabola instead of the straight line.

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$



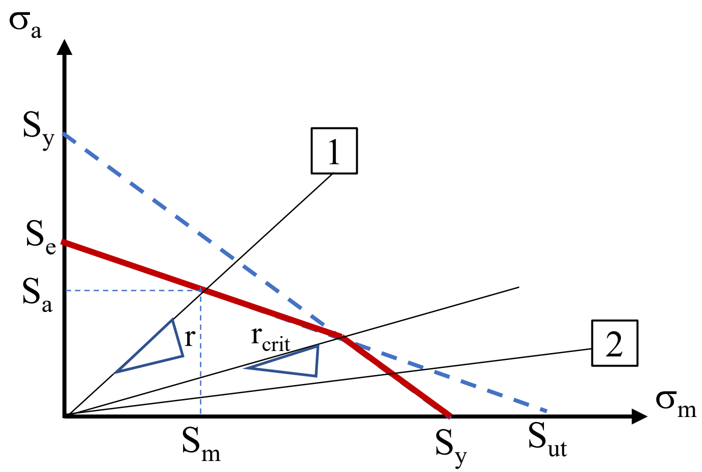
- The load line represents any combination of σ_a and σ_m. The intersection of the load line with any of the failure lines gives the limiting values S_a and S_m according to the line it intercepts.

7.Fatigue Failure Criteria for Fluctuating Stress

6) Modified Goodman (Goodman and Langer): It combines the Goodman and Langer lines.

- The slope of the loading line passing through the intersection point of the two lines is called the critical slope, and it is found as:

$$r_{crit} = \frac{S_a}{S_m}$$



Where, $S_m = \frac{(S_y - S_e)S_{ut}}{S_{ut} - S_e}$ and $S_a = S_y - S_m$

- According to the slope of the load line ($r = \frac{\sigma_a}{\sigma_m}$), it could intersect any of the two lines:

$r > r_{crit} \Rightarrow \boxed{1}$ $S_a = \frac{rS_eS_{ut}}{rS_{ut} + S_e}$ and $S_m = \frac{S_a}{r}$

$$n_f = \frac{S_a}{\sigma_a} = \frac{S_m}{\sigma_m} = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}}$$

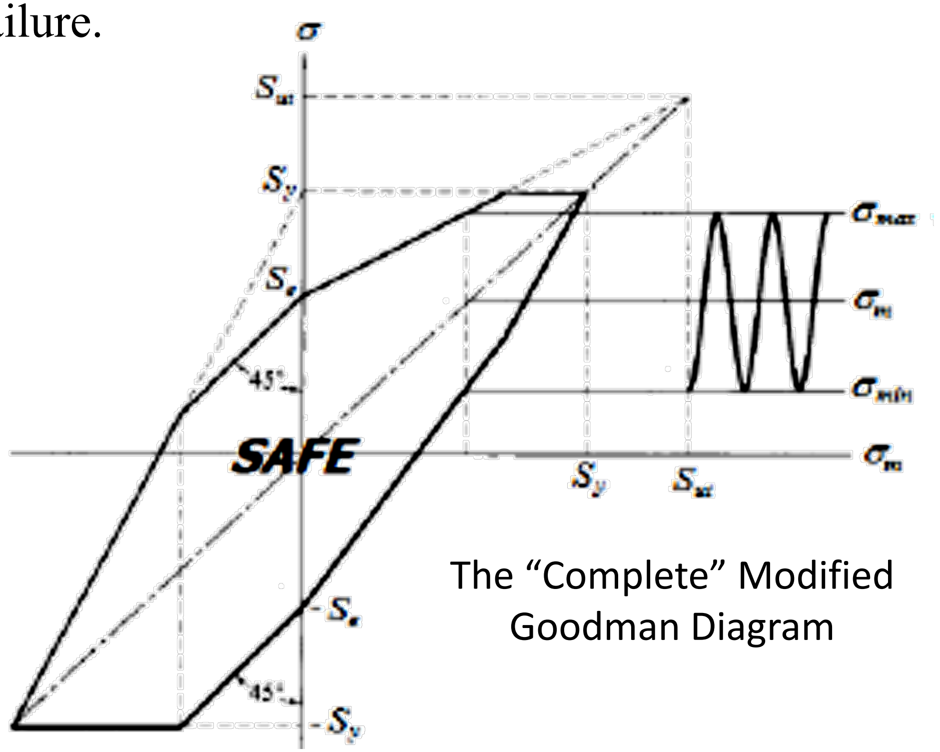
$r < r_{crit} \Rightarrow \boxed{2}$ $S_a = \frac{rS_y}{1+r}$ and $S_m = \frac{S_y}{1+r}$

$$n_s = \frac{S_a}{\sigma_a} = \frac{S_m}{\sigma_m} = \frac{S_y}{\sigma_a + \sigma_m}$$

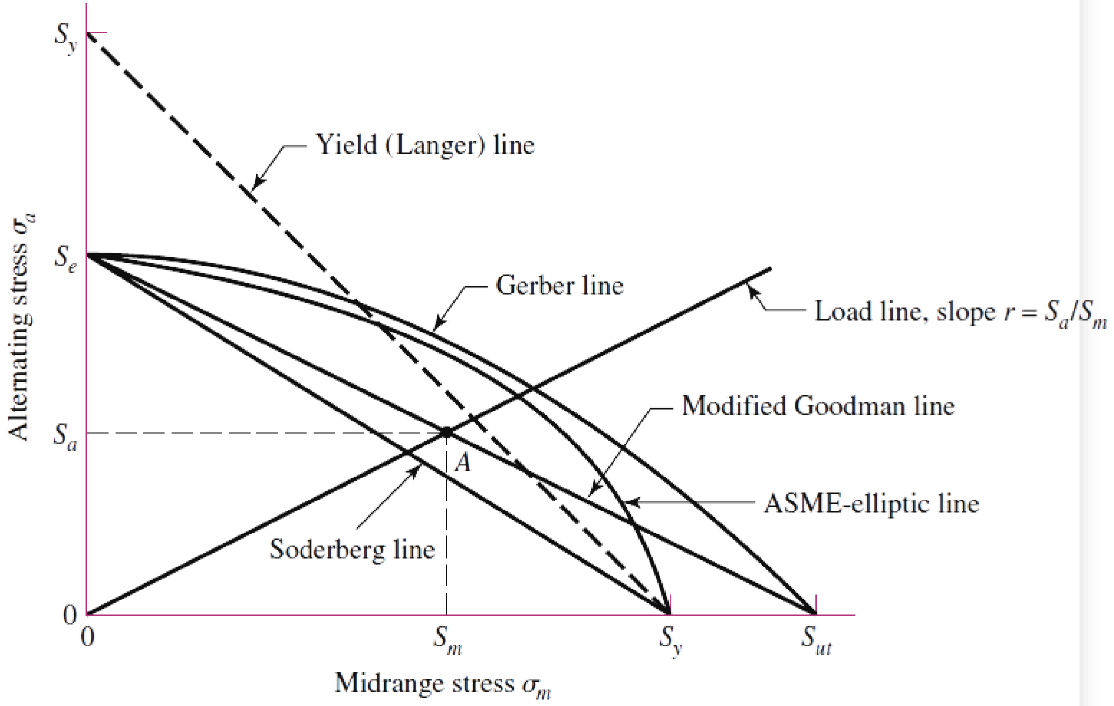
Where case $\boxed{2}$ is considered to be a static (yielding) failure.

7.Fatigue Failure Criteria for Fluctuating Stress

- If we plot the Modified Goodman on stress (σ) vs. mean stress (σ_m) axes, we obtain the complete Modified Goodman diagram where it defines a failure envelope such that any alternating stress that falls inside the diagram will not cause failure.



The “Complete” Modified Goodman Diagram



Example

A 40 mm diameter bar has been machined from AISI-1045 CD rod. The bar is subjected to a fluctuating tensile load varying from 0 to 100 kN. Because of the ends fillet radius, $K_f = 1.85$ is to be used. Find the critical mean and alternating stress values S_a & S_m and the fatigue factor of safety n_f according to the Modified Goodman fatigue criterion.

Solution

From data book table A-17 p.83 $S_{ut} = 630$ MPa and $S_y = 530$ MPa

$$S_e' = 0.5S_{ut} = 0.5 \times 630 = 315\ MPa$$

$k_a = 4.51(630)^{-0.265} = 0.817$

$k_b = 1$ since the loading is axial

$k_c = 0.85$ for axial loading

$$k_f = \frac{1}{K_f} = \frac{1}{1.85} = 0.541$$

$k_d = k_e = 1$

$$S_e = k_a k_b k_c k_d k_e k_f S_e' = 0.817 \times 1 \times 0.85 \times 1 \times 1 \times 0.541 \times 315 = 118.34\ MPa$$

$$\sigma = \frac{F}{A} \quad , \qquad A = \frac{\pi}{4} d^2$$

Example

$$\sigma_{max} = \frac{100 \times 10^3}{1275} = 79.6\ MPa \quad and \quad \sigma_{min} = 0$$

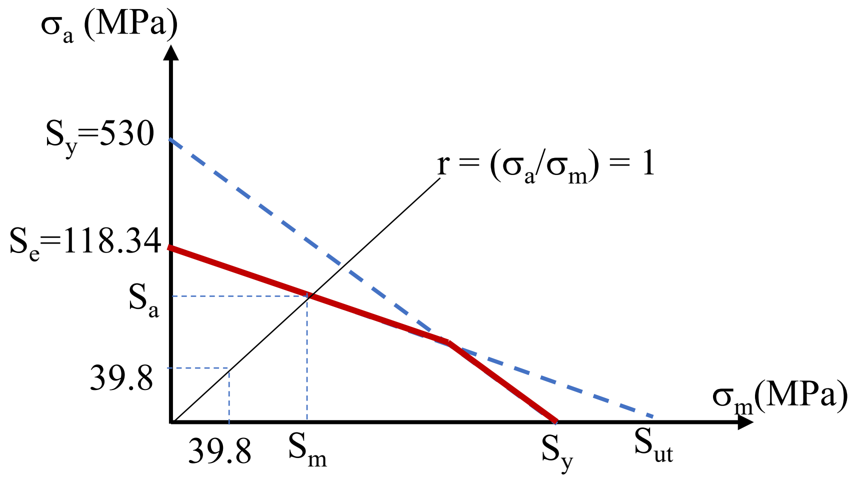
$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 39.8\ MPa \quad and \quad \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 39.8\ MPa$$

- The plot shows that the load line intersects the Goodman line:

$$S_a = \frac{rS_eS_{ut}}{rS_{ut} + S_e} = \frac{1 \times 118.34 \times 630}{1 \times 630 + 118.34} = 99.63\ MPa \quad and \quad S_m = \frac{S_a}{r} = \frac{99.63}{1} = 99.63\ MPa$$

$$n_f = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}} \quad or \quad n_f = \frac{S_a}{\sigma_a} = \frac{S_m}{\sigma_m}$$

$$n_f = \frac{1}{\frac{39.8}{118.34} + \frac{39.8}{630}} = 2.5 \quad or \quad n_f = \frac{99.63}{39.8} = 2.5$$



Example

For more conservative another solution: (K_f instead of k_f)

$$S_e = k_a k_b k_c k_d k_e S'_e = 0.817 \times 1 \times 0.85 \times 1 \times 1 \times 315 = 218.8\ MPa$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 39.8\ MPa \quad and \quad \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 39.8\ MPa$$

Appling K_f=1.85 to both components: $\sigma_a = K_f \sigma_a$ and $\sigma_m = K_f \sigma_m$

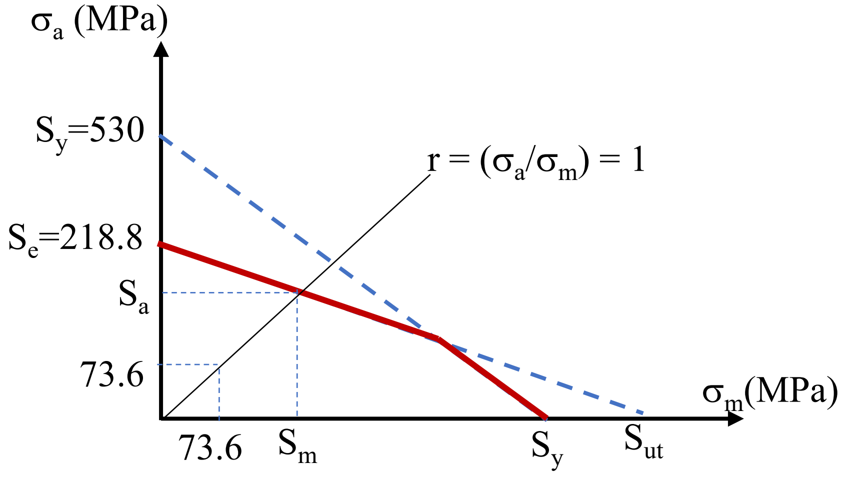
$$\sigma_a = \sigma_m = 1.85 \times 39.8 = 73.6\ MPa$$

- The plot shows that the load line intersects the Goodman line:

$$S_a = \frac{r S_e S_{ut}}{r S_{ut} + S_e} = \frac{1 \times 218.8 \times 630}{1 \times 630 + 218.8} = 162.4\ MPa \quad and \quad S_m = \frac{S_a}{r} = \frac{162.4}{1} = 162.4\ MPa$$

$$n_f = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}} \quad or \quad n_f = \frac{S_a}{\sigma_a} = \frac{S_m}{\sigma_m}$$

$$n_f = \frac{1}{\frac{73.6}{218.8} + \frac{73.6}{630}} = 2.21 \quad or \quad n_f = \frac{162.4}{73.6} = 2.21$$



8.Torsional Fatigue Loading

For shafts that are subjected to fluctuating shear stress with non-zero mean (due to pulsating torque), a fatigue criterion (ASME elliptic, Gerber, etc.) needs to be used.

- It should be noted that the endurance limit S_e already accounts for the torsional loading since $k_c = 0.59$ is used in such case.
- Similarly, the yield or ultimate strengths need to be corrected where the "shear yield strength" S_{ys} or the "shear ultimate strength" S_{us} need to be used and those are found as:

$$S_{ys} = 0.577\ S_y \qquad \text{and} \qquad S_{us} = 0.67\ S_{ut}$$

9. Combination of Loading Modes

The procedures presented earlier can be used for fatigue calculations for a component subjected to general fluctuating stress (or fully reversed stress, which is easier) under one of the three modes of loading; Axial, Bending or Torsion.

- For a component subjected to general fluctuating stress under combination of loading modes:

i) Construct an element for mean stress $(\sigma_x , \sigma_y \text{ and } \tau_{xy})_{mean} \Rightarrow \sigma_{xm}, \sigma_{ym}, \text{ and } \tau_{xym}$

ii) Find principal stresses $(\sigma_1 , \sigma_2)_{mean} \Rightarrow \sigma_{1m}, \sigma_{1m}$

9. Combination of Loading Modes

i) Construct an element for mean stress (σ_x , σ_y and τ_{xy})$_{mean}$ $\Rightarrow$ σ_{xm}, σ_{ym}, and τ_{xym}

ii) Find principal stresses (σ_1 , σ_2)$_{mean}$ $\Rightarrow$ σ_{1m}, σ_{1m}

iii) Use equivalent mean stress using Von-Mises $\Rightarrow$ $\sigma'_m = \sqrt{\sigma_{1m}^2 - \sigma_{1m}\sigma_{2m} + \sigma_{2m}^2}$

iv) Similarly, $\sigma'_a = \sqrt{\sigma_{1a}^2 - \sigma_{1a}\sigma_{2a} + \sigma_{2a}^2}$

v) Select fatigue failure criterion and apply σ'_m and σ'_a. (ex: Goodman $\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{1}{n}$)

Notes:

a) A simplification, if there exist τ_{xy} and one normal stress (σ_x or σ_y) use

$$\sigma'_m = \sqrt{\sigma_{xm}^2 + 3\tau_{xym}^2}$$

$$\sigma'_a = \sqrt{\sigma_{xa}^2 + 3\tau_{xya}^2}$$

No need for principal stresses to be calculated

9. Combination of Loading Modes

b) Do not use k_f to reduce endurance limit k_f "modifying factor" is not working since there is

$$\left.\begin{array}{ll} K_f & \text{for axial} \\ K_f & \text{for bending} \\ K_f & \text{for torsion} \end{array}\right\} \text{ which one to use?}$$

Instead; use K_f (fatigue stress concentration factor) to each mode loading, apply it to (σ_a and σ_m) of that mode. Ex:

$$\sigma_a' = \sqrt{\left[(K_f)_{bending}(\sigma_a)_{bending} + (K_f)_{axail}\frac{(\sigma_a)_{axial}}{0.85}\right]^2 + 3\left[(K_f)_{torsion}(\tau_a)_{torsion}\right]^2}$$

Similarly,

$$\sigma_m' = \sqrt{\left[(K_f)_{bending}(\sigma_m)_{bending} + (K_f)_{axail}(\sigma_m)_{axial}\right]^2 + 3\left[(K_f)_{torsion}(\tau_m)_{torsion}\right]^2}$$

- Note that the alternating component of the axial load is divided by 0.85 (i.e., kc for axial loading).

- The torsional alternating stress is not divided by its corresponding kc value (i.e., 0.59) since that effect is already accounted for in the von Mises stress.

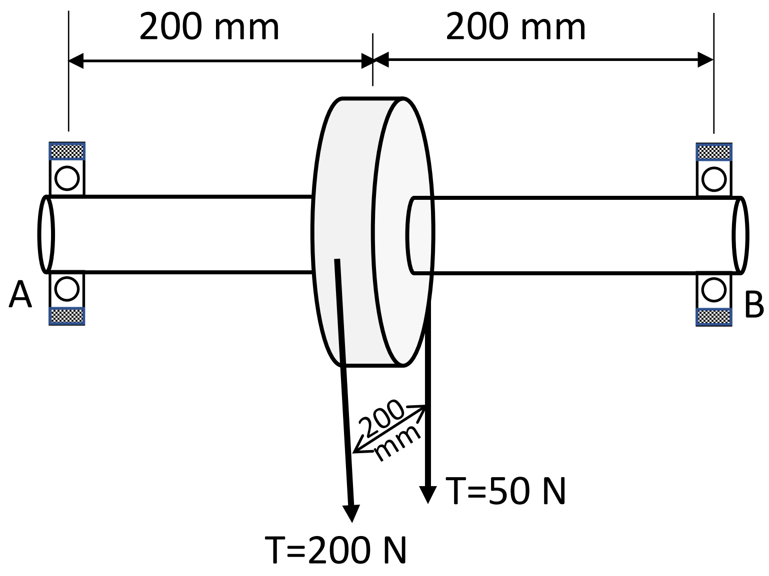
9. Combination of Loading Modes

c) The endurance limit is calculated assuming that the loading is bending (i.e., $k_c = 1$) (drop k_c).

d) Finally, a fatigue failure criterion (Gerber, Goodman, ASME-elliptic, etc.) is selected and applied as usual.

Example

For rotating shaft shown in figure below. Estimate the minimum shaft dimeter for a factor of safety of 3. the material is AISI 1050 CD.



Solution

A) Stresses

1- Bending stress: reactions at support A and B ⇒ R= 125 N,

moment at midshaft M=125×200= 25000N.mm=25 N.m

Since this type of loading is completely reversed.

$$\sigma_a = \frac{32M}{\pi d^3} = \frac{32 \times 25}{\pi d^3} = \frac{254.6}{d^3}$$

$$\sigma_m = 0$$

Example

2- Torsion stress: actually, torsion is not varying with time since torque is constant.

T= (200 – 50)×100/1000= 15 N.m

$\tau_a = 0$

$$\tau_m = \frac{16T}{\pi d^3} = \frac{16 \times 15}{\pi d^3} = \frac{76.4}{d^3}$$

$$\sigma'_a = \sqrt{\sigma_a^2 + 3\tau_a^2} = \frac{254.6}{d^3}$$

$$\sigma'_m = \sqrt{\sigma_m^2 + 3\tau_m^2} = \frac{132}{d^3}$$

B) Material data

AISI 1050 CD $\Rightarrow$ S_{ut} = 690 MPa , $S'_e = 345\ MPa$

Modified factors

$k_a = 4.51(690)^{-0.265} = 0.796$

k_b=1 (assume for now as no dimeter is available)

k_c=1 (just for bending, torque constant)

$k_d = k_e = k_f$=1

$$S_e = 0.796 \times 345 = 274.6\ MPa$$

Use Goodman criteria

$$\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{1}{n}$$

$$\frac{\frac{254.6}{d^3}}{274.6 \times 10^6} + \frac{\frac{132}{d^3}}{690 \times 10^6} = \frac{1}{3}$$

$\Rightarrow$ d =14.97 mm

Example

Now, according to new diameter, calculate:

$k_b = 1.24 \times d^{-0.107} = 1.24 \times 15^{-0.107} = 0.928$

$S_e = 0.796 \times 0.928 \times 345 = 254.9$ MPa

Once again

$$\frac{\frac{254.6}{d^3}}{254.9 \times 10^6} + \frac{\frac{132}{d^3}}{690 \times 10^6} = \frac{1}{3}$$

$$d = 15.28\, mm$$

Iterate until convergence

$k_b = 1.24 \times (15.28)^{-0.107} = 0.926$ stop

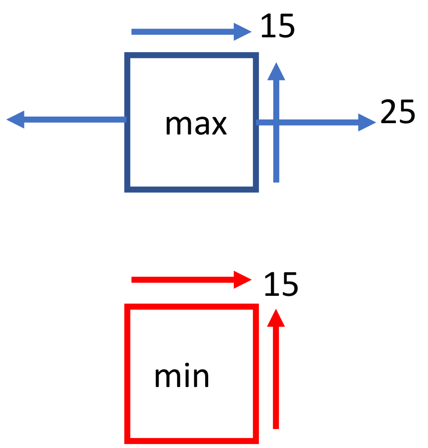
Example

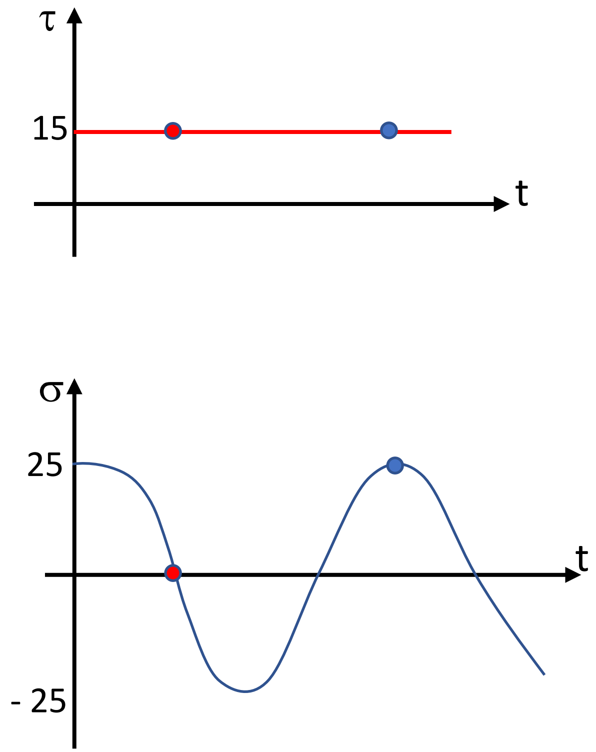
Steel bar: $S_e = 40$ ksi $S_{ut} = 80$ ksi

Steady torsional stress: 15 ksi

Alternating bending stress: 25 ksi

Factor of safety? (Goodman)





Solution

$$n = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1}$$

Torsion $\tau_a = 0$ ksi , $\tau_m = 15$ ksi

Bending $\sigma_a = 25$ ksi , $\sigma_m = 0$ ksi

$$\sigma'_{max} = \sqrt{\sigma^2 + 3\tau^3} = \sqrt{25^2 + 3 \times 15^2} = 36\ ksi$$

$$\sigma'_{min} = \sqrt{\sigma^2 + 3\tau^3} = \sqrt{0^2 + 3 \times 15^2} = 26\ ksi$$

$$\sigma'_m = \frac{36 + 26}{2} = 31\ ksi$$

$$\sigma'_a = \frac{36 - 26}{2} = 5\ ksi$$

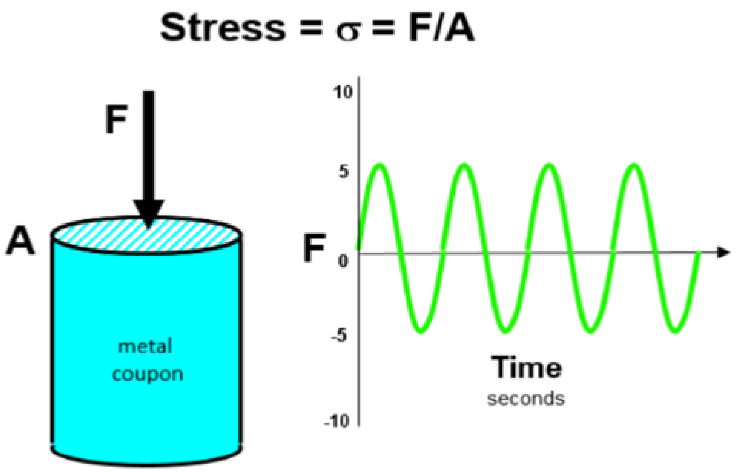
$$\Rightarrow \quad n = \left(\frac{5}{40} + \frac{31}{80}\right)^{-1} = 1.95$$

10.Cumulative Fatigue Damage (Miner's Rule)

The definition of failure for a physical part varies. It could mean that a crack has initiated on the surface of the part. It could also mean that a crack has gone completely thru the part, separating it. In this article, a conservative approach to failure will be used: a crack starts to appear on the surface of the part.

Applying a constant amplitude, cyclical stress to a metal coupon causes it to fail (a crack appears) after a specific number of cycles. When the crack appears, the accumulated damage is considered to be equal to 1. A brand-new part that has never had any stress loads applied is considered to have no accumulated damage (damage equal to 0).

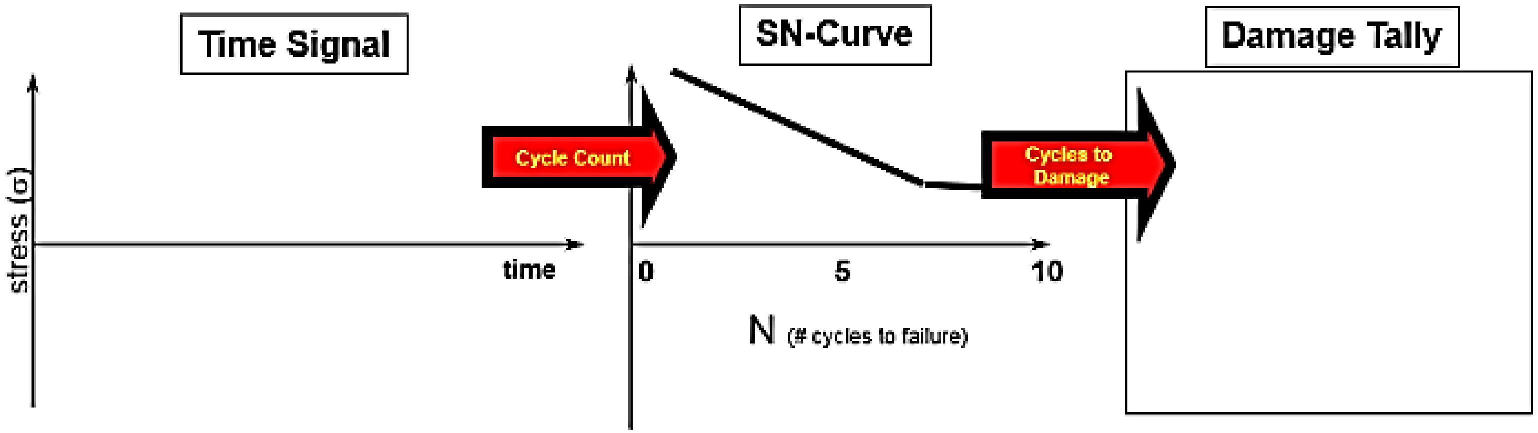
A stress is a force (F) applied over a surface area (A). Note that a complete cycle starts at a specific value (zero in this illustration), cycles above and below the initial value (or below and above), and then returns to the initial value.



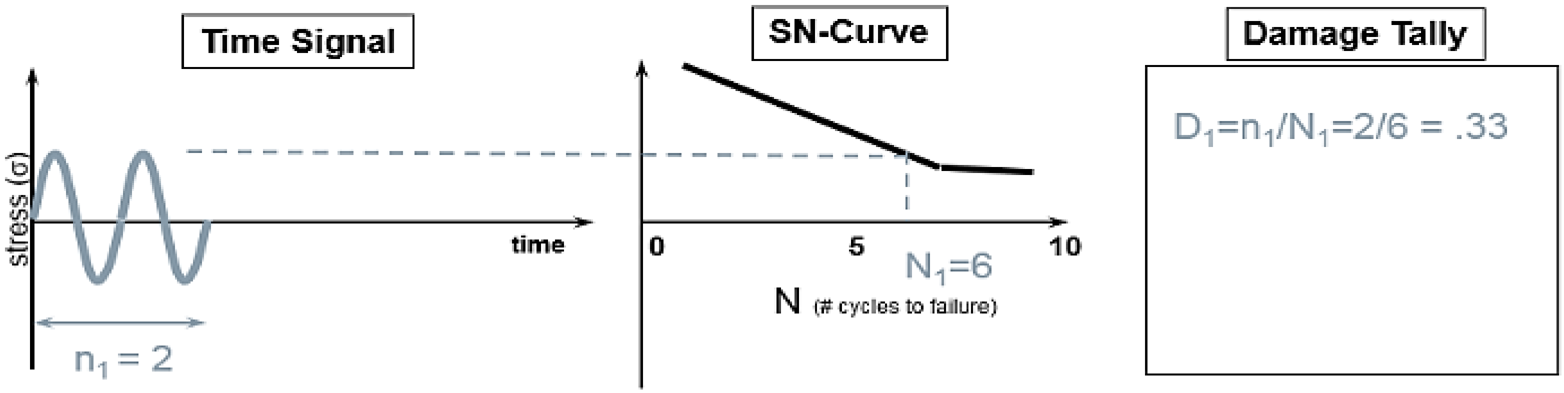
- Linear damage accumulation model.
- Failure when damage total = 1

11.Damage Accumulation Simple Example

When a physical part undergoes stress cycles, Miner's Rule works like this:

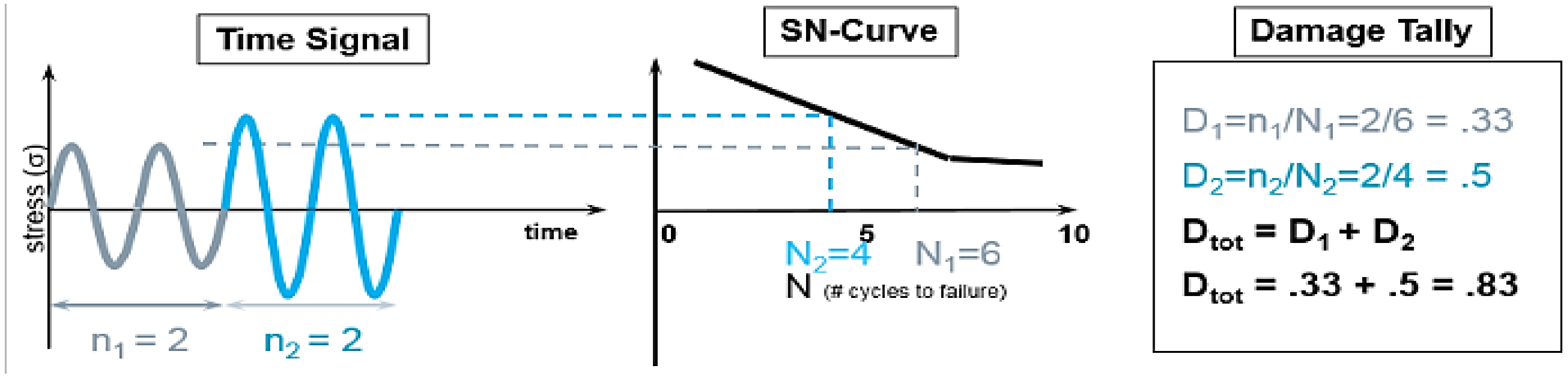


On the left graph, there is a loading time history. The SN-Curve is the middle graph, and a damage tally is kept on the right side.



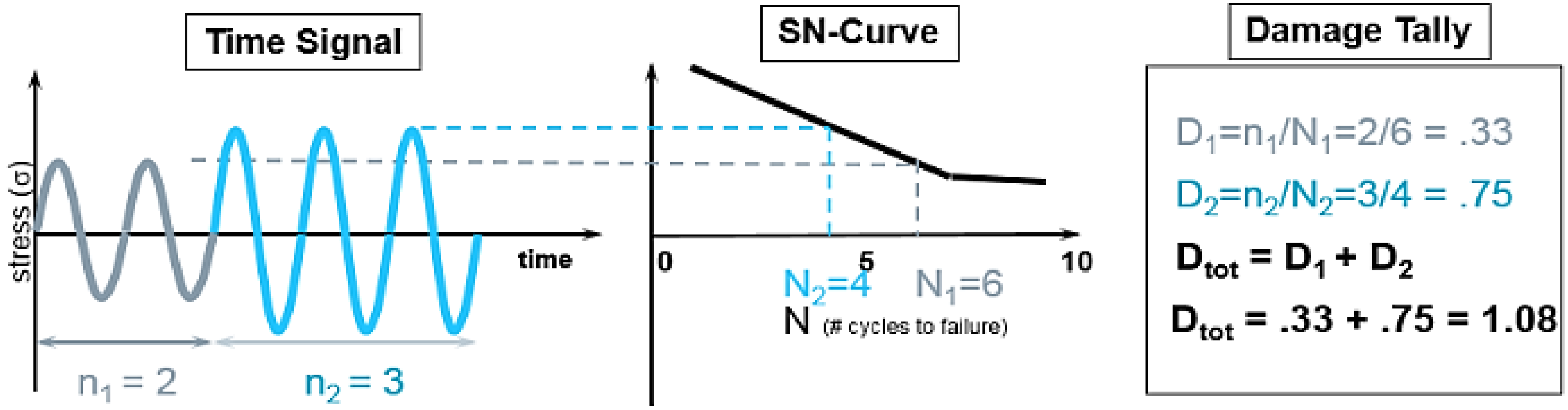
11.Damage Accumulation Simple Example

In this case, two cycles at a specific amplitude are applied to the part. At this amplitude the part could take 6 cycles before it would fail. Dividing two cycles by six cycles, the accumulated damage is 0.33. A third of the life of the part has been used.



Two more cycles of a higher amplitude are applied. At this higher amplitude, four cycles would be required for failure to occur. Dividing two cycles by four cycles, an additional 0.5 of damage has occurred. The total accumulated damage is now 0.83. According to Miner's Rule, no failure has occurred.

11.Damage Accumulation Simple Example



One more cycle of the higher amplitude is now applied. The accumulated damage is now 1.08. Failure has occurred!

The equation for Miner's Rule is:

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i}$$

Where,

D: is total damage. When damage is equal to one, failure occurs.

n: is the number of cycles of a given amplitude that the part or object is subjected to in the field or during its lifetime.

N: is the total number of cycles of a give amplitude that a material can survive, as determined by laboratory testing.

k: is the number of different amplitude levels of the cycles from the field or lifetime data.

Mechanical Springs

Chapter Four

DESIGN OF SPRINGS

Definition of spring

A spring is an elastic object used to store mechanical energy. Springs are elastic bodies (generally metal) that can be twisted, pulled, or stretched by some force. They can return to their original shape when the force is released. In other words, it is also termed as a resilient member.

A spring is a flexible element used to exert a force or a torque and, at the same time, to store energy. The force can be a linear push or pull, or it can be radial, acting similarly to a rubber band around a roll of drawings.

The torque can be used to cause a rotation, for example, to close a door on a cabinet or to provide a counterbalance force for a machine element pivoting on a hinge.

Objectives of spring

To provide Cushioning, to absorb, or to control the energy due to shock and vibration.

Car springs or railway buffers to control energy, springs-supports and vibration dampers

To Control motion

Maintaining contact between two elements (cam and its follower) Creation of the necessary pressure in a friction device (a brake or a clutch)

To Measure forces

Spring balances, gages

Commonly used spring materials

One of the important considerations in spring design is the choice of the spring material. Springs are commonly made with carbon content from 0.5% C to 1% C. Some of the common spring materials are given below.

Hard-drawn wire (0.6 – 0.7 C)

This is cold drawn, cheapest spring steel. Normally used for low stress and static load. The material is not suitable at subzero temperatures or at temperatures above 120^{o}C.

Oil-tempered wire (0.6 – 0.7 C)

It is a cold drawn, quenched, tempered, and general purpose spring steel. It is not suitable for fatigue or sudden loads, at subzero temperatures and at temperatures above 180^{o}C.

Chrome Silicon

This material can be used for highly stressed springs. It offers excellent service for long life, shock loading and for temperature up to 250^{o}C.

Chrome-Vanadium

This alloy spring steel is used for high stress conditions and at high temperature up to 220 0 C. It is good for fatigue resistance and long endurance for shock and impact loads.

Music wire (0.8 – 0.95 C)

This spring material is most widely used for small springs. It is the toughest and has highest tensile strength and can withstand repeated loading at high stresses. It cannot be used at subzero temperatures or at temperatures above 120^{o}C.

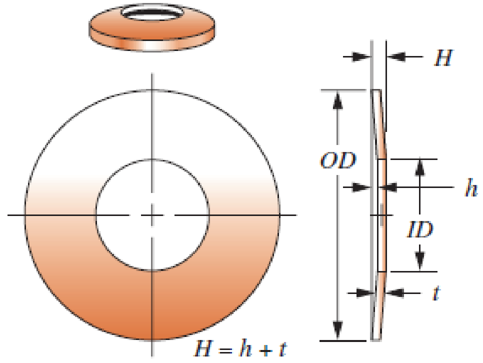
Stainless steel
Widely used alloy spring materials.

Phosphor Bronze / Spring Brass
It has good corrosion resistance and electrical conductivity. It is commonly used for contacts in electrical switches. Spring brass can be used at subzero temperatures.

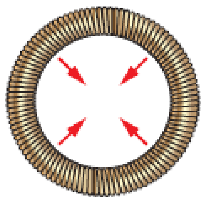
Springs can be classified according to the direction and the nature of the force exerted by the spring when it is deflected.

Uses	Types of Springs
Push	Helical compression spring, Belleville spring, Torsion spring, force acting at the end of torque arm. flat spring, such as a cantilever spring or leaf spring
Pull	Helical extension spring, Torsion spring, force acting at the end of torque arm. Flat spring, such as a cantilever spring or leaf spring, Draw bar spring (special case of the compression spring) constant – force spring.
Radial Torque	Garter spring, elastomeric band, spring clamp, Torsion spring, Power spring

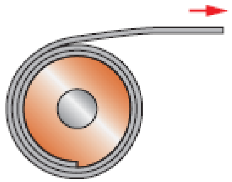
Several types of Springs



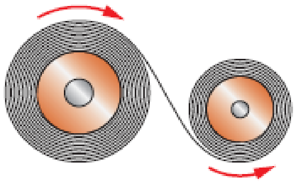
(*e*) Belleville spring



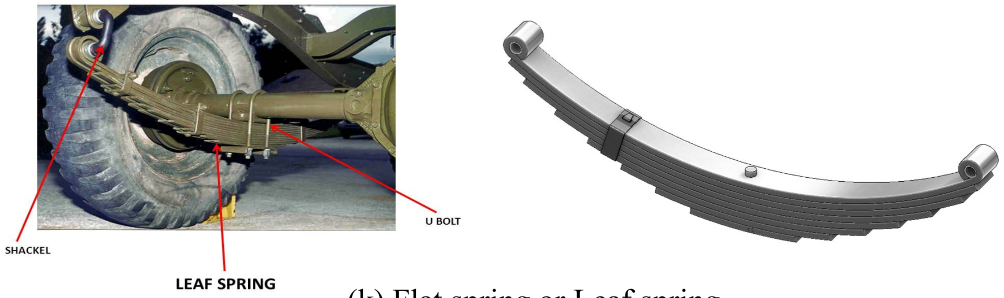
(*f*) Garter spring



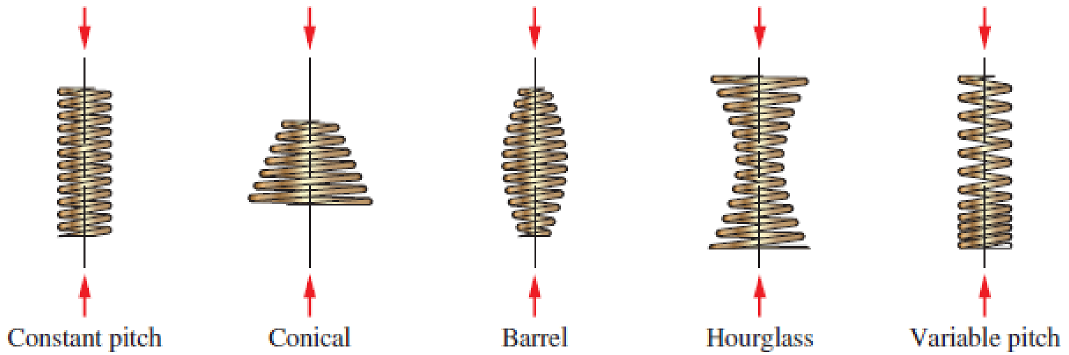
(*g*) Constant-force spring



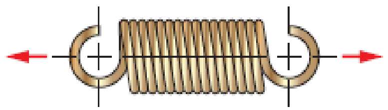
(*h*) Constant-force spring motor



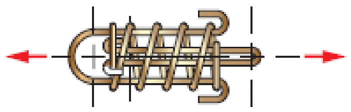
(k) Flat spring or Leaf spring



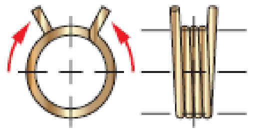
(*a*) Variations of helical compression springs



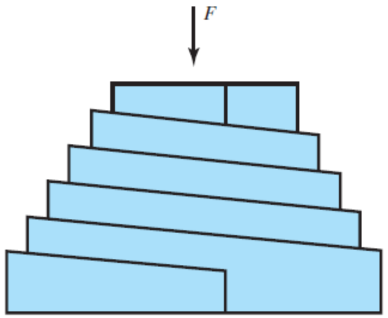
(*b*) Helical extension spring



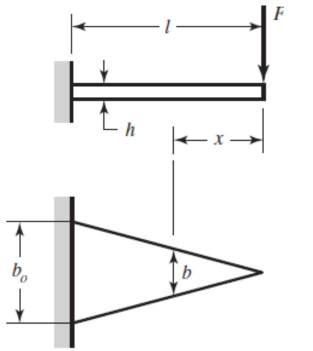
(*c*) Drawbar spring



(*d*) Helical torsion spring



(i) Volute spring



(j) Flat triangular spring

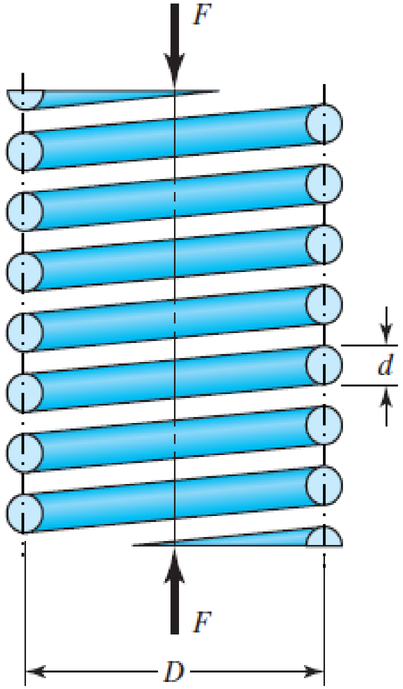
Several types of Springs

Spring Index

It is defined as the ratio of mean coil diameter to the wire diameter. It is denoted by "C"

Mathematically,

$$C = \frac{D}{d}$$



It is a real and positive number and doesn't have a unit.

Factors to be considered while designing springs

1. Space in which the spring will operate.
2. Values of forces and deflections.
3. Accuracy and reliability of springs.
4. Tolerances in specifications.
5. Environmental conditions.
6. Cost of spring.

Terminologies used in springs

1) Spring index:

$$C = \frac{D}{d}$$

Where,

D= Mean coil diameter (mm) , d= Wire diameter (mm)

2) Deflection: It is the distance moved by the spring under the action of load.

$$\delta = \frac{F}{k} \qquad \text{(mm)}$$

3) Stiffness of spring (spring rate or spring constant): It is defined as the ratio of load to deflection.

$$k = \frac{F}{\delta} \qquad \text{(N/mm or kN/mm)}$$

4) Shear stress factor:

$$K_s = 1 + \frac{1}{2C}$$

5) Wahl's correction factor: It is considered in designing because it takes case of both direct shear stress and curvature in spring. (dynamic load)

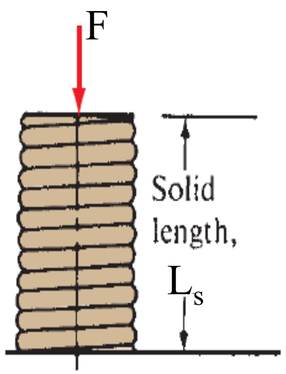
$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

Terminologies used in springs

6) Solid Length: It is the length of spring in fully closed condition.

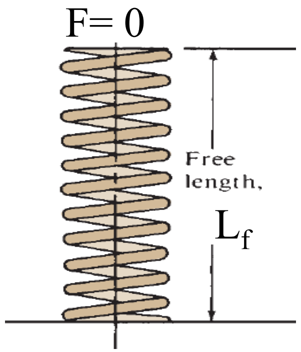
$$\therefore L_s = N_t\, d$$

Where, N_t= Total No. of coils, d= diameter of wire (mm)



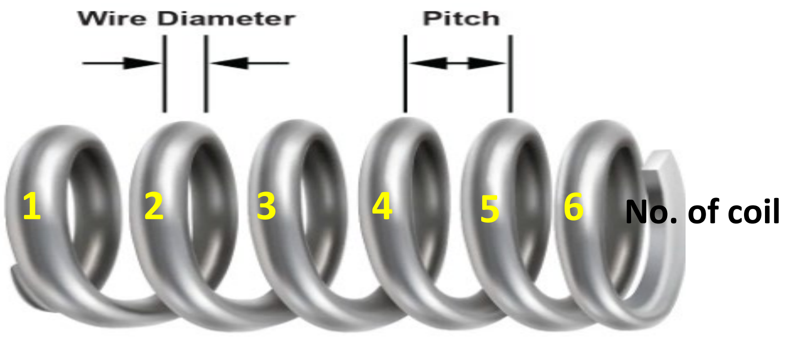
7) Free Length: It is the length of spring in unloaded condition.

$$L_f = L_s + \delta_{max} + 0.15\, \delta_{max}$$



8) Pitch of spring:

$$p = \frac{L_f}{N_t - 1}$$



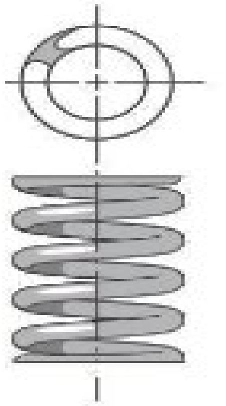
Terminologies used in springs

9) For squared and ground ends:

Total No. of coils = No. of active coils (N_a) + 2

Active coils are those which take part in deflection.

$$\therefore\ N_t = N_a + 2$$


Squared and ground ends

10) Deflection

$$\delta = \frac{8FD^3N_a}{Gd^4}$$

Where,

F= Load on the spring (N or kN)

D= Mean coil diameter (mm)

N_a = No. of active coils

G = Modulus of rigidity (N/mm^2)

d= Wire diameter (mm)

(1) Stresses in Helical springs

Consider a helical compression spring made of circular wire and subjected to an axial load F, as shown in Fig. 1, then stresses are:

Let, D = Mean diameter of the spring coil, d = Diameter of the spring wire

1) Direct shear stress: $\tau = \frac{F}{A}$

2) Torsional shear stress: $\tau = \pm\frac{Tr}{J}$

Use superposition ⇒ $\tau_{total} = \frac{F}{A} \pm \frac{Tr}{J}$

$$\tau_{max} = \frac{F}{A} + \frac{Tr}{J}$$ at inner surface of wire

But, $T = \frac{FD}{2}$, $r = \frac{d}{2}$, and $J = \frac{\pi}{32}d^4$

$$\Rightarrow \tau_{max} = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

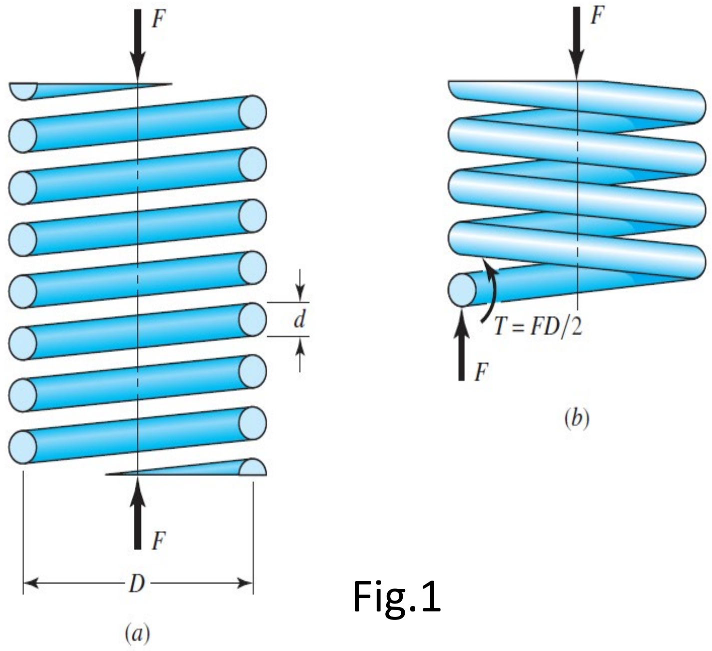


Fig.1

(1) Stresses in Helical springs

Define spring index $C = \frac{D}{d}$ (most springs C: 6 to 12)

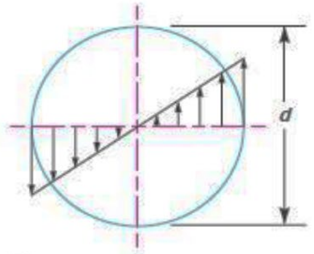
$$\tau_{max} = \left(\frac{2C+1}{2C}\right) \times \frac{8FD}{\pi d^3}$$

$$\tau_{max} = K_s \frac{8FD}{\pi d^3}$$

where K_s: shear stress factor

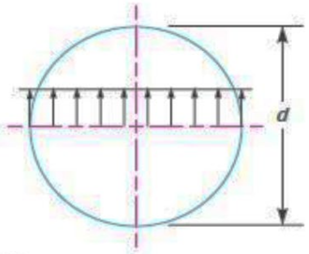
But shear stress is affected by wire curvature (spring is not straight). This curvature increases shear stress, and this is accounted for by another factor, Wahl's correction factor, K_w.

$$\tau_{max} = K_w \frac{8FD}{\pi d^3} \qquad \text{where, } K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$



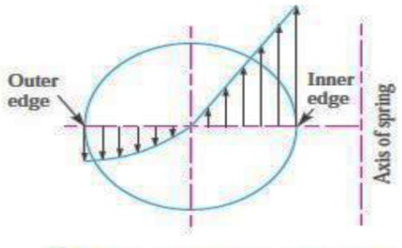
(a) Torsional shear stress diagram.

$\frac{Tr}{J}$



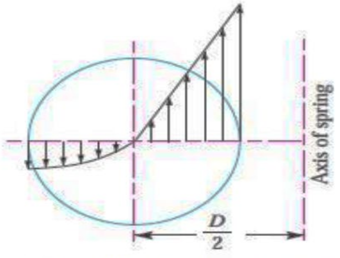
(b) Direct shear stress diagram.

$\frac{F}{A}$



(c) Resultant torsional shear and direct shear stress diagram.

$K_s \frac{8FD}{\pi d^3}$



(d) Resultant torsional shear, direct shear and curvature shear stress diagram.

$K_w \frac{8FD}{\pi d^3}$

(1) Stresses in Helical springs

Notes: Experimental shows

i) For static load, curvature effect is neglected; use K_s.

ii) For dynamic load, curvature is important, use K_w.

iii) Square/rectangle x-section wire is not recommended unless necessary (above eqs. not valid).

iv) New editions, use $K_B = \frac{4C+2}{4C-3}$ for (K_s $K_{curvature}$)

(Note: error K_w and K_B is small)

(2) Deflection in Helical springs

Using principles of strain energy

Total strain energy in the spring wire has two components shear and torsion:

$$U = \frac{F^2 L}{2AG} + \frac{T^2 L}{2JG}$$

Substitute: $T = \frac{FD}{2}$; $L = \pi D N$ (spring length); $A = \frac{\pi}{4} d^2$; and $J = \frac{\pi}{32} d^4$

(Note N=N_a is number of active coils)

$$\Rightarrow U = \frac{2F^2 D N}{G d^2} + \frac{4F^2 D^3}{G d^4}$$

Use the Castigliano's theorem to set the deflection 'y'

$$y = \frac{\partial U}{\partial F} = \frac{4FDN_a}{Gd^2} + \frac{8FD^3 N_a}{Gd^4}$$

Use $C = \frac{D}{d}$

$$y = \frac{8FD^3 N_a}{Gd^4}\left(1 + \frac{1}{2C^2}\right)$$

Very small

$$y \cong \frac{8FD^3 N_a}{Gd^4}$$

(2) Deflection in Helical springs

Spring stiffness (constant or rate)

$$k = \frac{F}{y} \quad \Rightarrow \quad k = \frac{Gd^4}{8D^3 N_a}$$

Notes:

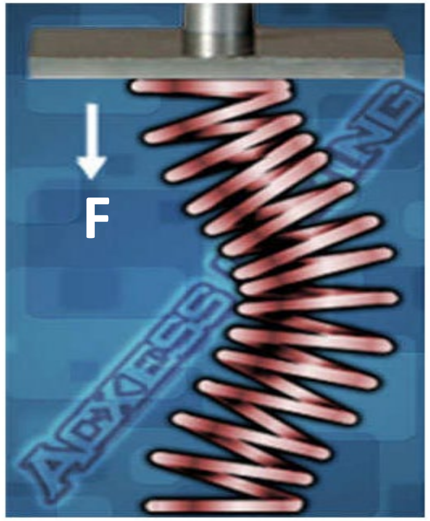
1) Four types of spring ends are used for compression springs (see data P31)
2) Active coils : $N_a = N_t - N_e$; where N_t: Total No. of coil, N_e: No. of end coil (inactive coil)
3) Full range of motion = $L_f - L_s$

(3) Stability of compression springs

Similar to columns, If the length of a compression spring is large relative to its diameter, the spring may **buckle** while it's being compressed as shown in figure.

(buckling is a sudden lateral deflection that occurs in members subjected to axial compressive loading). In general :

i) The use squared ends is better in stability of spring than using plain ends.
ii) To prevent buckling, the spring can be inserted inside a hole, or a rod is inserted Inside the spring.



(3) Stability of compression springs

Notes:

1) Clearance between hole and spring outside diameter:

Hole diameter = spring O.D + 1.5 mm

2) Clearance between rod and spring inside diameter:

Rod diameter = spring I.D – 1 mm

(4) Spring Strength

Tensile strength of wires depends on wire diameter.

$$S_{ut} = \frac{A}{d^m}$$ (A and m are material constants, see data book P.31)

- However, springs are subjected to **shear** not tension and we need to consider **yield strength** not ultimate.
- An approximate relation: $S_y = 0.75\ S_{ut}$
- Using Von Mises: shear yield strength $S_{ys} = 0.577\ S_y$
- Table 6.2 / data book P.32 give elastic constants E and G for different spring materials.

(4) Springs Strength

Important notes for static design:

1) Material: HD steel is 1st choice since it has lowest cost (cheapest).

2) Type ends: squared ends is 1st choice since gives good stability and has low cost.

3) Manufacturing: as-wound is 1st choice since it has low cost.

4) Safety: use design safety factor at solid length $n \geq 1.2$

5) Working range: A void closing spring to its solid length under maximum load.

   (75% of its possible compression distance, when $F: 0 \rightarrow F_s$)

6) Spring Index $4 \leq C \leq 12$

7) Number of active turns $3 \leq N_a \leq 15$

Example #1

A helical compression Spring made of music wire the wire (d=1.397 mm). The wire is to be inserted inside a hole with (20 mm) diameter. The free length of the spring is to be (100 mm) and it should have a squared end ground ends. The maximum load applied to spring (30 N). Find total number of coils and full range of motion. (Note: under load 30 N, spring compress 60 mm)

Solution

Spring outer diameter OD= hole diameter – clearance = 20 – 1.5 = 18.5 mm

Mean diameter D= OD – d = 18.5 – 1.397 = 17.103 mm

$$k = \frac{F}{y} = \frac{30}{60} = \frac{1}{2} \text{ N/mm}$$

$$k = \frac{d^4 G}{8 D^3 N_a}$$, let G= 80 GPa (check table 6.2)

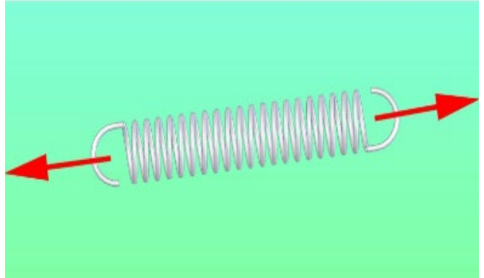
$$0.5 = \frac{1.397^4 \times 80 \times 10^3}{8 \times 17.103^3 \times N_a} \Longrightarrow N_a = 15.23\ turns, \quad N_a = 15$$

For squared and ground ends N_e=2 ⇒ N_t = 15 + 2 = 17 turns

Solid length (L_s) = d ´ N_t = 1.397 ´ 17 = 24.07 mm

Full range of motion = L_f – L_s = 100 – 24.07 = 75.93 mm

(5) Extension Springs



- Extension springs, also known as tension springs, are springs that can be stretched to increase their length. When extended, these springs are under tension. If load is removed $\Rightarrow$ Shorten back to the original length
- Hooks, threaded plugs…..etc. types of spring ends to transfer load to body of spring.
- Stresses in the body of spring is the same as compression springs.

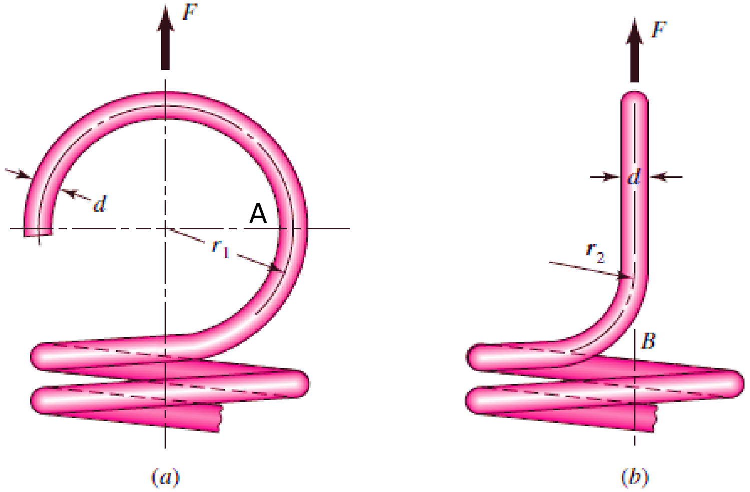
❖ Stresses at spring ends (Hooks):

i) Maximum tensile stress at A (inner fiber of ring)

$$\sigma_A = K_A \frac{16FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

$$K_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} \quad , \quad C_1 = \frac{2r_1}{d} \quad , \quad r_1 = \frac{D_{mean}}{2}$$

K_A = Bending stress correction factor for curvature



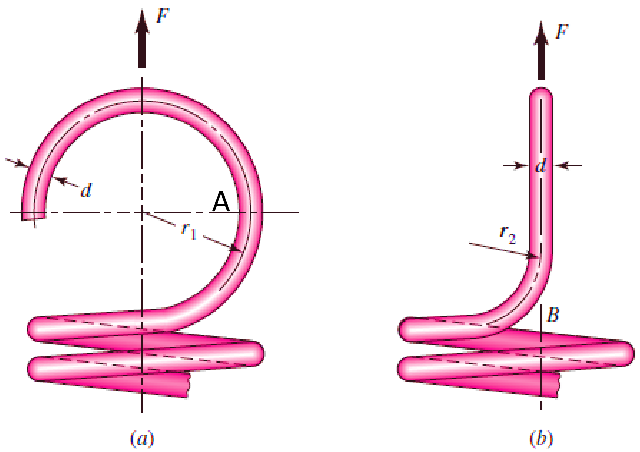
(5) Extension Springs

ii) Maximum shear stress at B (outer fiber of ring)

$$\tau_B = K_B \frac{8FD}{\pi d^3}$$

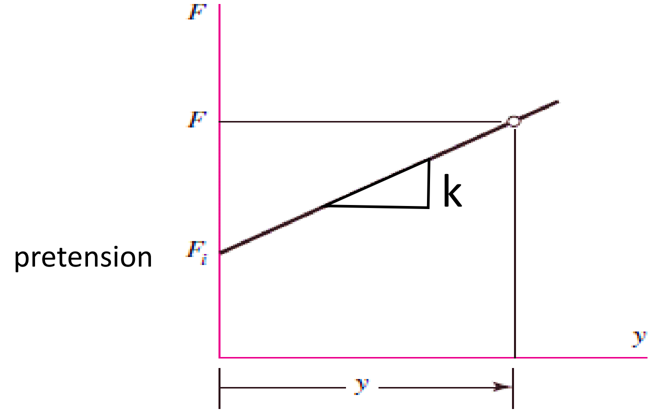
$$K_B = \frac{4C_2 - 1}{4C_2 - 4} \quad , \quad C_2 = \frac{2r_2}{d} \quad , \quad r_2 : curvature\ radius$$

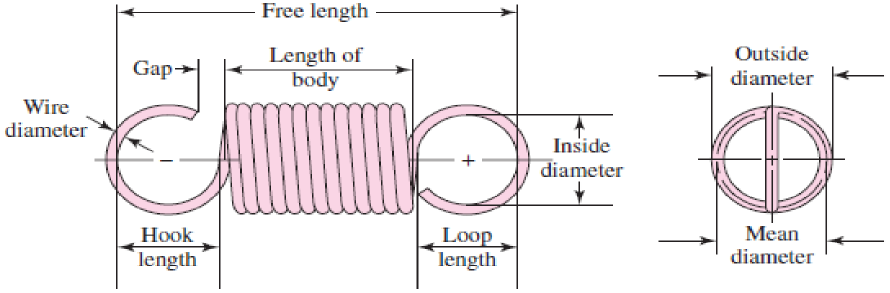
K_B = Shear correction factor for curvature



- Extension springs usually have "initial tension" (pretension)

$$F = F_i + ky$$



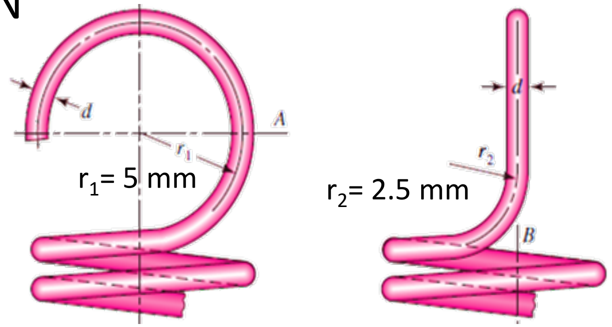


Example

The following specifications were taken from tension spring removed from junked washing machine:

D= 10 mm , d= 1.8 mm, N_a= 122 , distance between hook=244 mm, preload= 25 N

And material is hard drawn (HD) wire G= 79.3 GPa , S_{ut}= 1560 MPa. Determine:



a) Compute initial stress in the spring?

b) What is the force to cause spring body stressed to yield?

c) What is the spring stiffness?

d) What is the force to cause hook's ends torsionally stressed to yield strength?

e) What is the force to cause normal stress at hook's end to reach tensile yield strength?

Solution

a) $\tau = K_s \frac{8FD}{\pi d^3}$, $K_s = \frac{2C+1}{2C}$, $C = \frac{D}{d}$

$$\Rightarrow C = \frac{10}{1.8} = 5.56 \Rightarrow K_s = 1.09 \Rightarrow \tau = 1.09 \frac{8\times 25\times 10\times 10^{-3}}{\pi\times(1.8\times 10^{-3})^3} = 119\ MPa$$

b) $S_y = 0.75\, S_{ut} \Rightarrow S_{y\ tensile} = 0.75\times 1560 = 1170\ MPa \Rightarrow S_{y\ shear} = 0.58\, S_{y\ tensile} = 679\ MPa$

$$S_{y\ shear} = K_s \frac{8FD}{\pi d^3} \Rightarrow 679 = 1.09 \frac{8\times F\times 10}{\pi\times 1.8^3} \Rightarrow F = 142\ N$$

Example

Solution

c) $k = \frac{Gd^4}{8D^3N_a} = \frac{79.3\times10^9\times1.8^4}{8\times10^3\times122} = 853\ N/m$

d) $r_2 = 2.5 \Rightarrow C_2 = \frac{2r_2}{d} = 2.78 \Rightarrow K_B = \frac{4C_2-1}{4C_2-4} = 1.42$

$$S_{y\ shear} = K_B\frac{8FD}{\pi d^3} \Rightarrow 679 = 1.42\times\frac{8\times F\times10}{\pi\times1.8^3} \Rightarrow F = 109.5\ N$$

e) $r_1 = 5 \Rightarrow C_1 = \frac{2r_1}{d} = 5.55 \Rightarrow K_A = \frac{4C_1^2-C_1-1}{4C_1(C_1-1)} = 1.16$, $S_{y\ tensile} = K_A\frac{16FD}{\pi d^3} + \frac{4F}{\pi d^2}$

$$1170 = 1.16\frac{16\times F\times10}{\pi 1.8^3} + \frac{4\times F}{\pi 1.8^2} \Rightarrow F = 111\ N$$

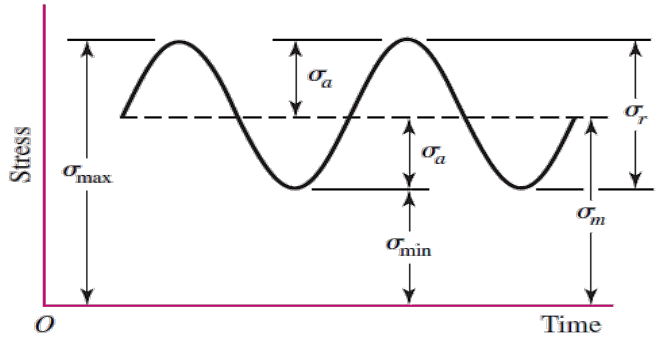
Notes:

1) N_a =122 too much.
2) Failure at hook because of shear (weakest) then hook (bending) then body of spring

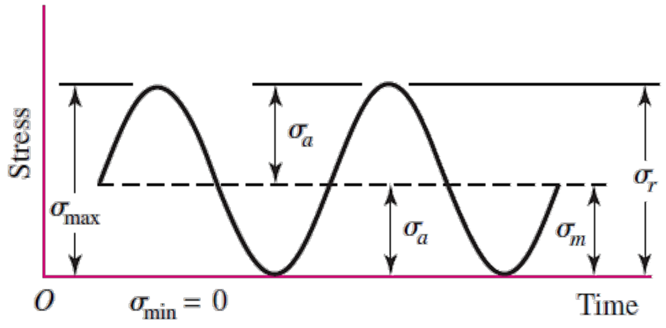
109 < 111 < 142 N

(6) Fatigue Loading

- Springs are almost always subjected to fatigue loading.
- Some springs are designed to operate without failure (infinite life).
- Preload is always exerted on helical springs such that usual condition is as shown below:



Usual Case (preload)



Worst Case (no preload)

- To design against fatigue:

1) Find $F_a = \frac{F_{max}-F_{min}}{2}$, $\tau_a = K_w \frac{8F_a D}{\pi d^3}$

$F_m = \frac{F_{max}+F_{min}}{2}$, $\tau_m = K_s \frac{8F_m D}{\pi d^3}$

(6) Fatigue Loading

2) Use Goodman Diagram $\left[\frac{\tau_a}{S_{es}} + \frac{\tau_m}{S_{us}} = \frac{1}{n}\right]$, $(S_{us} = 0.67 S_{ut})$

use $\tau_a = \frac{S_{es}}{n}$ (this equation is valid since load is shear only)

3) Zimmerli found that endurance limit for different types of springs material and sizes (diameter< 10 mm)

$$S'_{es} = \begin{cases} 310\ MPa & for\ Unpeened\ Springs \\ 465\ MPa & for\ Peened\ Springs \end{cases}$$

4) Above endurance limit should corrected for reliability and temperature only.

Example:

A 2.24 mm Helical compression Spring (music wire) has an outside coil diameter of 14.3 mm, free length of 105mm and 21 active coils. The spring to be assembled with a preload of 45 N and will operate to a maximum load of 225 N during use. Determine the safety factor guarding against fatigue failure based on a life of 50x 10^3 cycles and reliability of 99% .

(6) Fatigue Loading

Solution:

Mean coil diameter : D=OD – d = 14.3 – 2.24 = 12.06 mm

Spring index: $C = \frac{D}{d} = \frac{12.06}{2.24} = 5.38$

$$K_s = 1 + \frac{1}{2C} = 1.092 \;\; ; \quad K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C} = 1.285$$

$$F_a = \frac{F_{max}-F_{min}}{2} = \frac{225-45}{2} = 90\ N$$

$$F_m = \frac{F_{max}+F_{min}}{2} = \frac{225+45}{2} = 135\ N$$

$$\tau_a = K_w \frac{8F_aD}{\pi d^3} = 1.285 \times \frac{8\times90\times12.06}{\pi(2.24)^3} = 316\ MPa$$

$$\tau_m = K_s \frac{8F_mD}{\pi d^3} = 1.09 \times \frac{8\times135\times12.06}{\pi(2.24)^3} = 402.8\ MPa$$

(6) Fatigue Loading

Material data:

1) Unpeened spring $\Rightarrow S'_e = 310\ MPa$

for 99% reliability $\Rightarrow$ k_e = 0.814

Corrected endurance limit S_e= 0.814 x 310 = 252.3 Mpa

2) Music wire spring $\Rightarrow$ m= 0.145 ; A= 2211

$$S_{ut} = \frac{A}{d^m} = \frac{2211}{(2.24)^{0.145}} = 1967\ MPa \quad ; \quad S_{us} = 0.67 S_{ut} = 1318\ MPa$$

Now, $S_f = aN^b$, $a = \frac{(f \times S_{us})^2}{S_e} = 4135.4$; $b = -\frac{1}{3} log\left(\frac{f S_{us}}{S_e}\right) = -0.202$

$$S_f = 4135.4\left(50 \times 10^3\right)^{-0.202} = 464.86\ MPa$$

$$n = \frac{S_f}{\tau_a} = \frac{464.86}{316} = 1.47$$

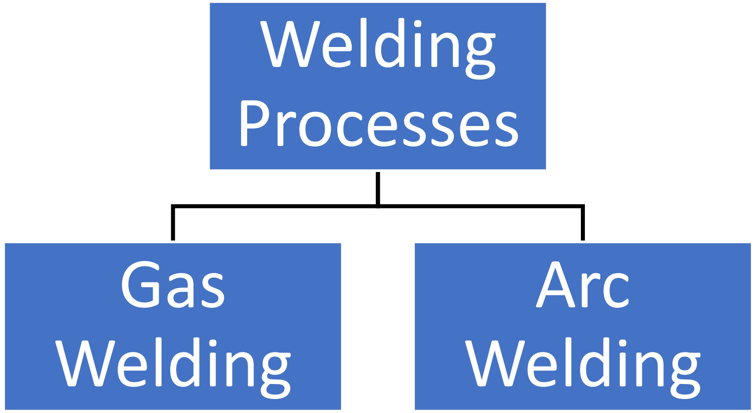
Design
of
Welded Joints
(Permanent Joints)

Chapter Two

1. Introduction to Welded Joints

A welded joint is a permanent joint which is obtained by the fusion of the edges of the two parts to be joined together, with or without the application of pressure and a filler material. The heat required for the fusion of the material may be obtained by burning of gas (in case of gas welding) or by an electric arc (in case of electric arc welding). The latter method is extensively used because of greater speed of welding. Welding is extensively used in fabrication as an alternative method for casting or forging and as a replacement for bolted and riveted joints. It is also used as a repair medium e.g. to reunite metal at a crack, to build up a small part that has broken off such as gear tooth or to repair a worn surface such as a bearing surface. Temperature control is the most important parameter associated with obtaining good welds.

- It is a permanent joint.
- Fusion of edges of two parts.
- Extensively used for fabrication.
- Replacement of bolted and riveted joints.



Arc welding is the most common method used with structural steel

Design of Welded Joints

Advantages

1. Lighter than riveted structures.
2. Provides maximum efficiency.
3. Alternation and addition are easy.
4. Great strength.
5. Circular shapes or any of this kind are easily welded.
6. Less time consuming.

Disadvantages

1. High skill labour and supervision is required.
2. Additional stresses get develop in the material.
3. Dissimilar metals can not be welded.

Selection of type of welded joint

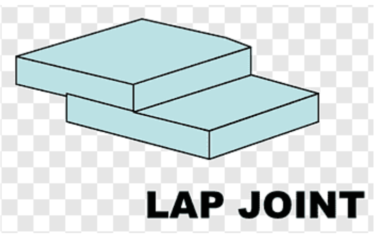
- Shape of the object to be welded.
- Thickness of the object.
- Direction of the forces

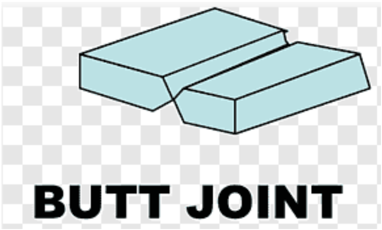
2. Types of Welded Joints

Following two types of welded joints are important from the subject point of view:

a. Lap joint or fillet joint.

b. Butt joint.





a. Lap Joint

The lap joint or the fillet joint is obtained by overlapping the plates and then then welding the edges of the plates. The cross-section section of the fillet is approximately triangular. The fillet joints may be:

1. Single transverse fillet
2. Double transverse Double transverse
3. Parallel fillet joints

The fillet joints are shown in Fig.1. A single transverse fillet joint has the disadvantage that the edge of the plate which is not welded can buckle or warp out of shape.

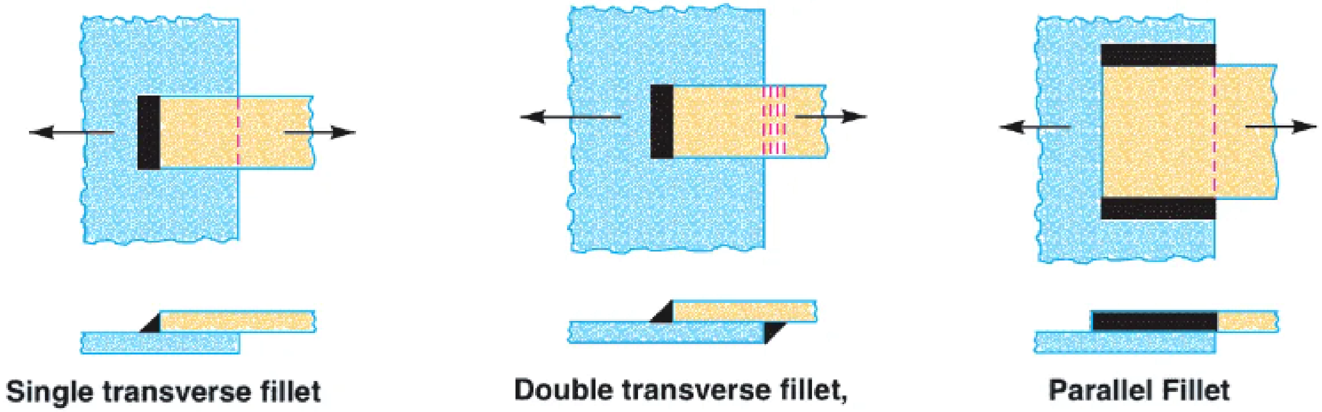


Fig.1

b. Butt Joint

The butt joint is obtained by placing the plates edge to edge as shown in Fig.2. In butt welds, the plate edges do not require beveling if the thickness of plate is less than 5 mm. On the other hand, if the plate thickness is 5 mm to 12.5 mm, the edges should be beveled to V or U groove on both sides.

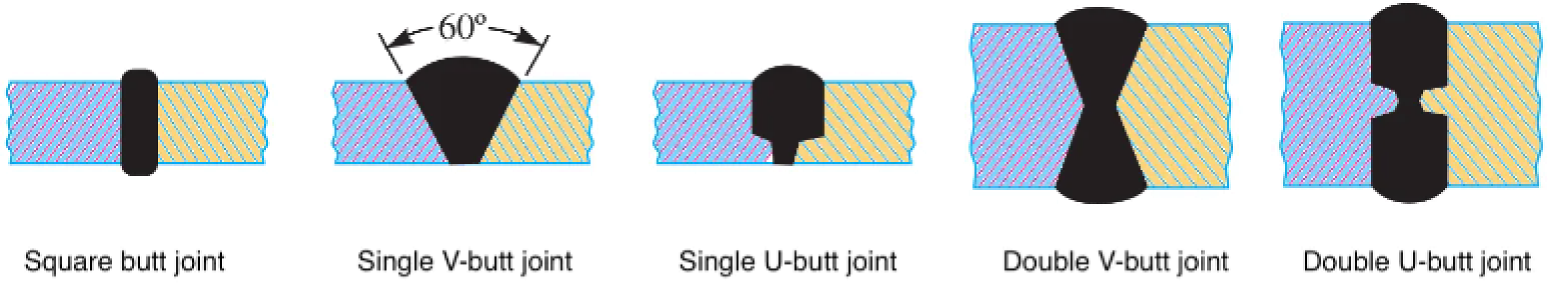


Fig.2

The butt joints may be:

1. Square butt joint 2. Single V-butt joint butt join 3. Single U-butt joint 4. Double V-butt joint 5. Double U-butt joint

The other type of welded joints are corner joint, edge joint and T-joint as shown in Fig.3.

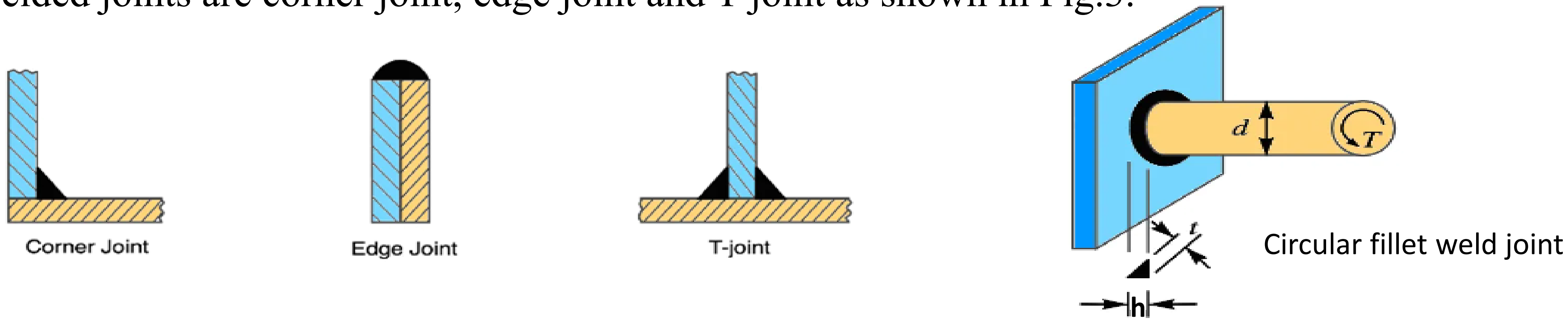
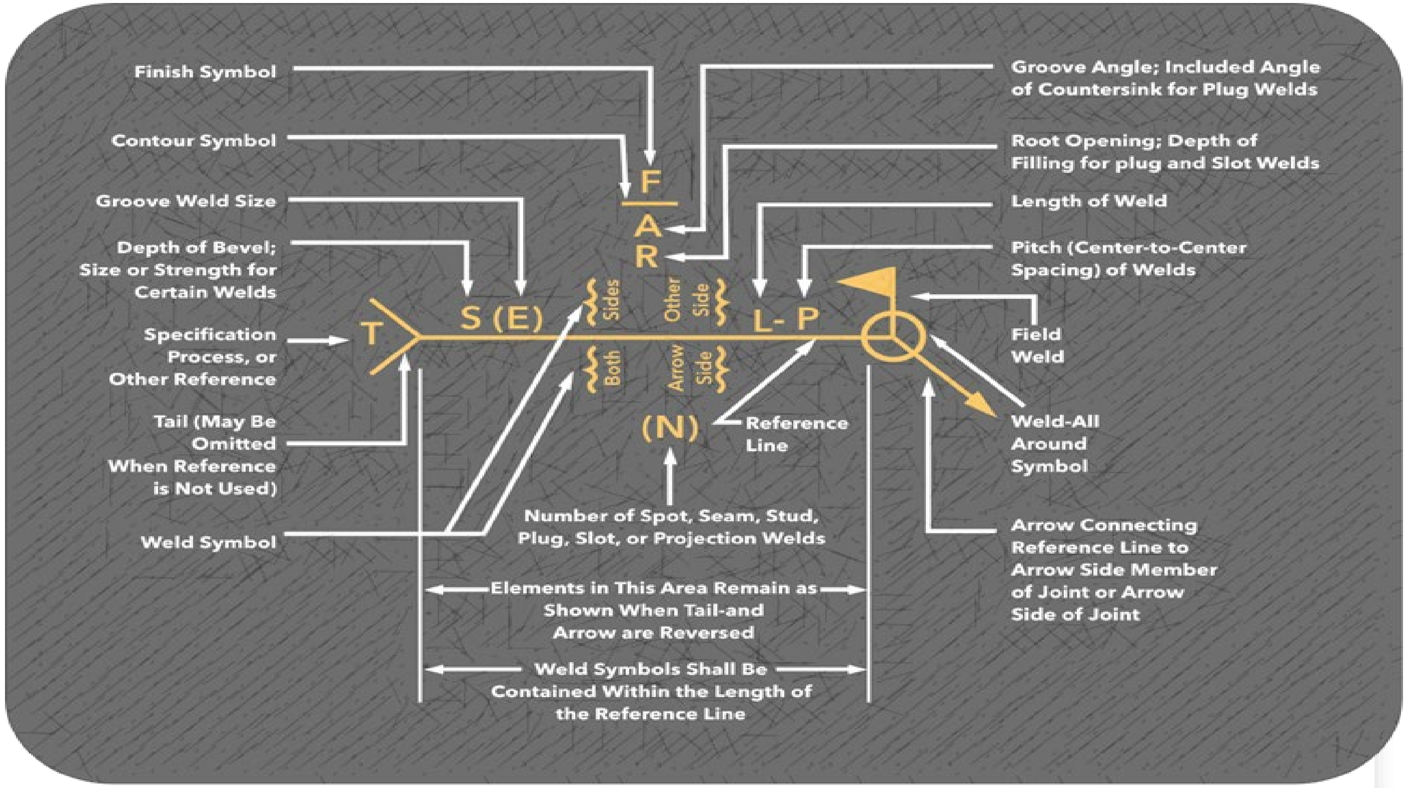


Fig.3

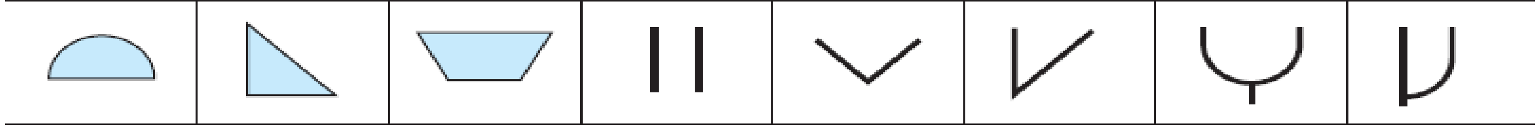
3. Basic Weld Symbols

Welding symbols are used on the drawings to indicate the type of specification of the weldments as standardized by the American Welding Society (AWS).

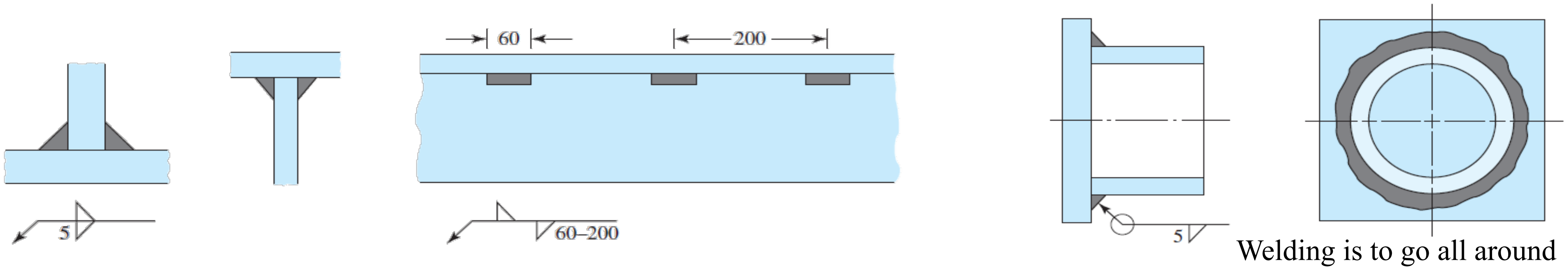
S. No.	*Particulars*	*Drawing representation*	*Symbol*
1.	Weld all round		
2.	Field weld		
3.	Flush contour		
4.	Convex contour		
5.	Concave contour		
6.	Grinding finish	G	G
7.	Machining finish	M	M
8.	Chipping finish	C	C



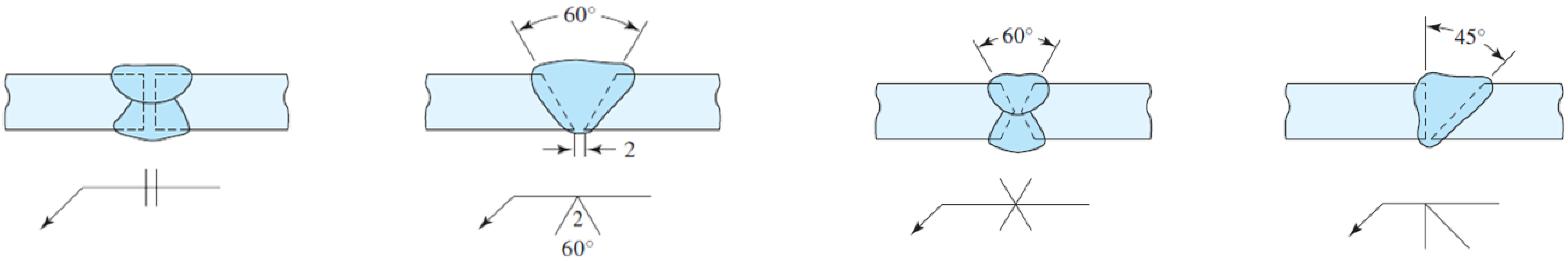
Type of weld							
Bead	Fillet	Plug or slot	Groove				
			Square	V	Bevel	U	J



3. Basic Weld Symbols



Example of Fillet welds



Example of Butt or groove welds

In general, there are two types of welds:

1. Fillet welds are used for machine elements (by designers)
2. Butt welds are usually used for pressure vessels.

3. Basic Weld Symbols

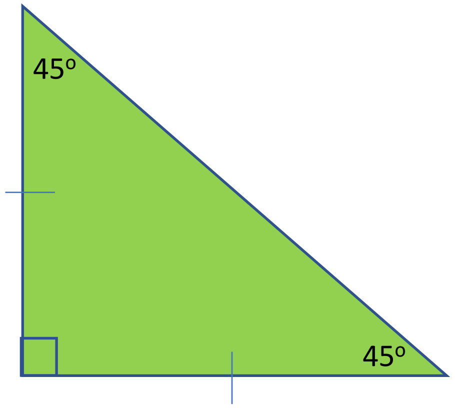
Since welding is associated with a significant increase in temperature, there will be some metallurgical changes in the parent materials in the vicinity of weldments.

Check: In design against failure, for parent metal use hot rolled materials properties even if materials is cold drawn (in the vicinity of welding).

- Residual stresses exist during welding.

Assumption in design of Lap or Fillet joint

- The section of the weld is right angle isosceles triangle



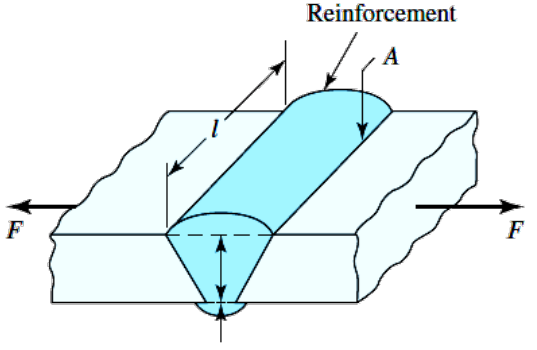
4. Stresses in a Weld

A. Butt weld: For a butt weld subjected to tensile force and shear force, the stresses are:

1. Normal stress:

$$\sigma = \frac{F}{hl}$$

l: length of weld
h: Throat of weld



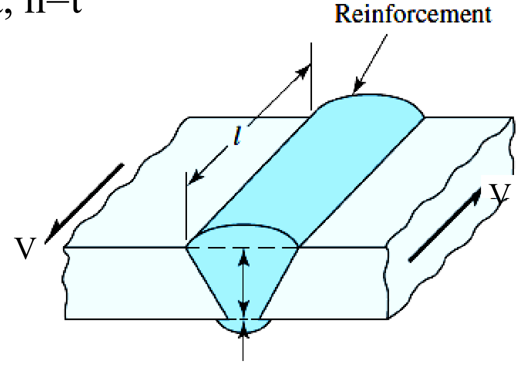
(a) Tensile loading

throat, h=t

2. Average shear stress:

$$\tau = \frac{V}{hl}$$

l: length of weld
h: Throat of weld



(b) Shear loading

4. Stresses in a Weld

B. Transverse Fillet weld: For a fillet weld loaded:

1. **Filled in tension**: for each weldments, two forces exist:

F_n: Normal force $= F\ cos\theta$
F_s: Shear force $= Fsin\theta$

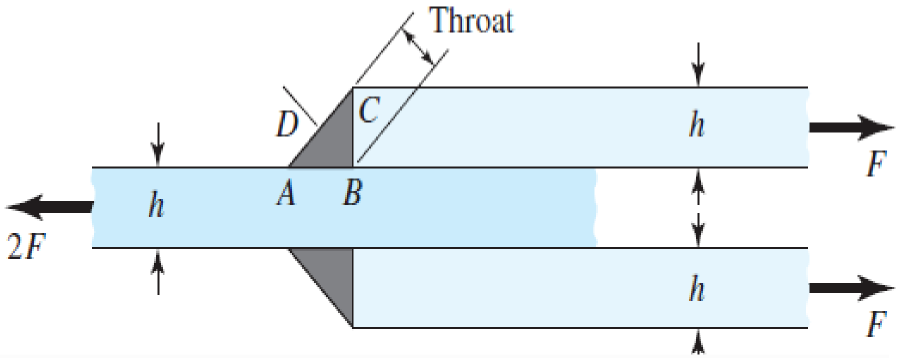
Note : F_n, F_s are $F(\theta)$

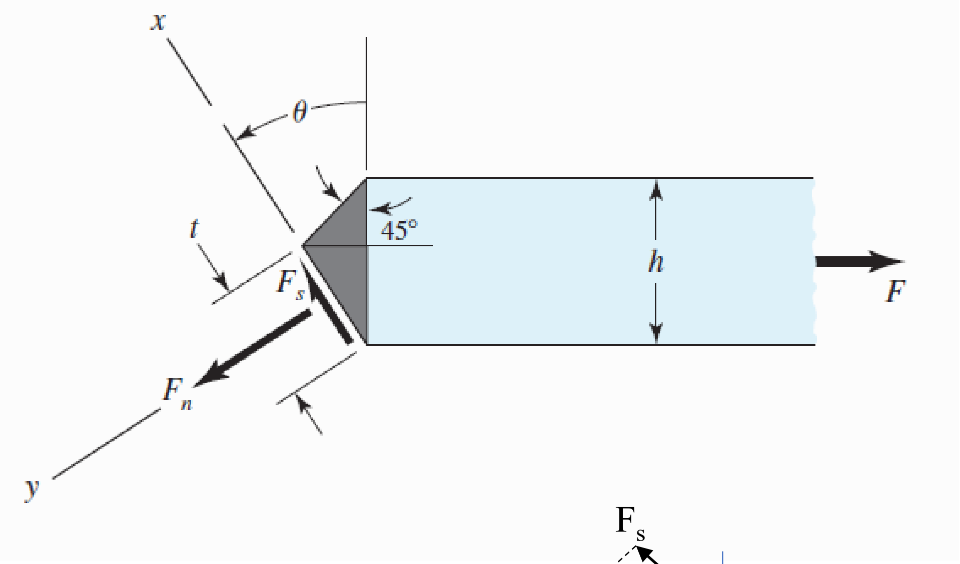
- For trigonometry

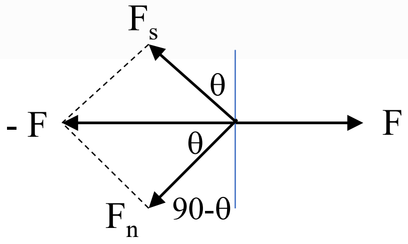
$$\text{Throat } t = \frac{h}{cos\theta + sin\theta}$$

$$\Rightarrow \quad \sigma = \frac{F_n}{A} = \frac{F_n}{lt} = \frac{Fcos\theta}{l\left(\frac{h}{cos\theta + sin\theta}\right)}$$

$$\tau = \frac{F_s}{A} = \frac{F_s}{lt} = \frac{Fsin\theta}{l\left(\frac{h}{cos\theta + sin\theta}\right)}$$







4. Stresses in a Weld

The Von Mises stress $\acute{\sigma}$ at angle θ is:

$$\acute{\sigma} = \sqrt{\sigma^2 + 3\tau^2} = \frac{F}{hl}\left[\left(cos^2\theta + cos\theta sin\theta\right)^2 + 3\left(sin^2\theta + cos\theta sin\theta\right)^2\right]^{1/2}$$

The largest von Mises stress occurs at θ= 62.5° with a value of $\acute{\sigma}$= 2.16 F/(*hl*). The corresponding values of τ and σ are τ= 1.196 F/(*hl*) and σ= 0.623 F/(*hl*).

Instead of above complicated analysis. A simplify conservative approach is:

(1) Ignore σ

(2) Assume all external forces are carried by shear stress only on the smallest throat area

(the smallest value of t at θ=45°)

$$\tau = \frac{F}{lt} = \frac{F}{l(h\,cos45^o)} = 1.414\frac{F}{lh}$$

Note:

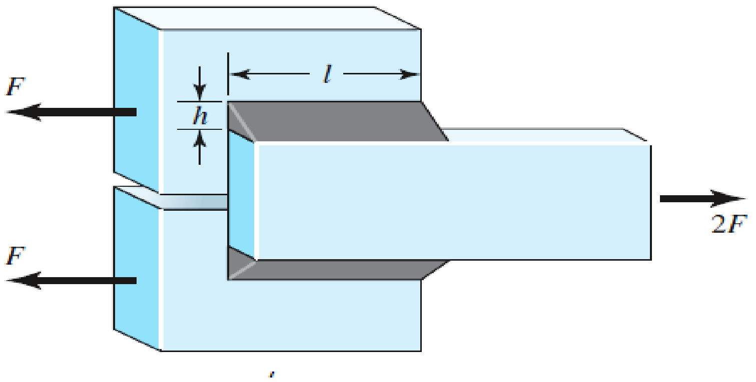
- Conservative because all external force is assumed to caused only shear stress, knowing that shear strength is almost two-third half of the normal strength.

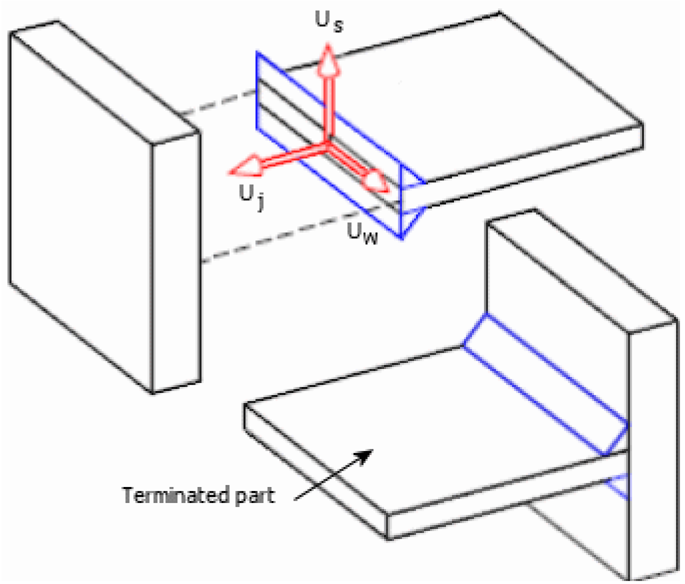
Ex. S_{ut}=610 MPa ⇒ S_{us}=2/3 *610=406 (S_{us}=0.67 S_{ut})

- $\tau = 1.414\frac{F}{lh} > 1.2\frac{F}{lh}$ safe design code.

2. Filled in Shear:

$$\tau = \frac{F}{lt} = \frac{F}{l(h\, cos45^o)} = \frac{F}{l(0.707h\,)} = 1.414\frac{F}{lh}$$





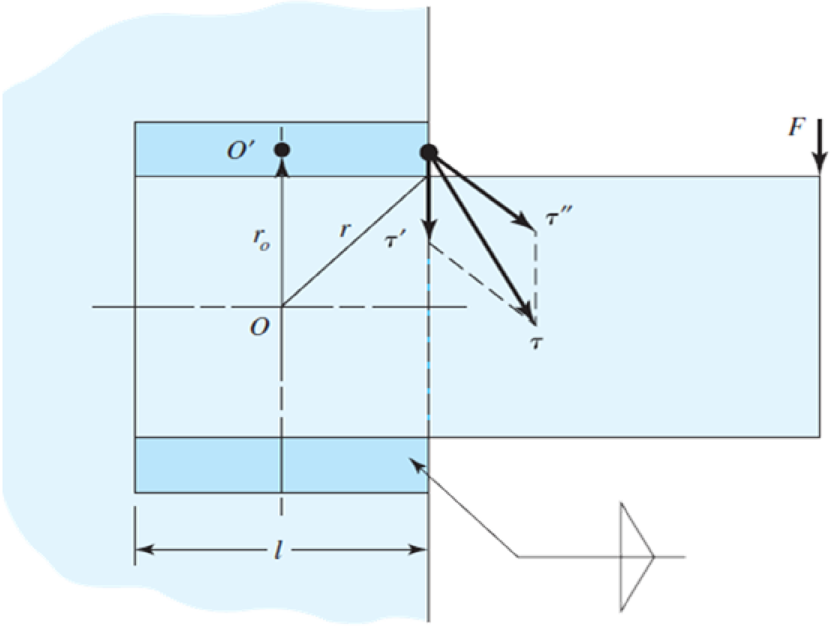
Note: If force is eccentric, two shear stress component exist:

a) τ': Primary shear stress

b) τ'' : Secondary shear stress

5. Torsion in Welded Joints

The figure below illustrates a cantilever welded to a column by two fillet welds each of length *l*. The reaction at the support of a cantilever always consists of a shear force V and a moment M.



a) Stress in the beam: as studied earlier

$$\sigma = \frac{My}{I}$$

$$\tau = \frac{VA\bar{y}}{It}$$

b) Stress in the weldments:

1. Primary shear stress (due to shear force)

$$\tau' = \frac{V}{A}$$

Where, V= Shear force

A= Total throat area of all welds

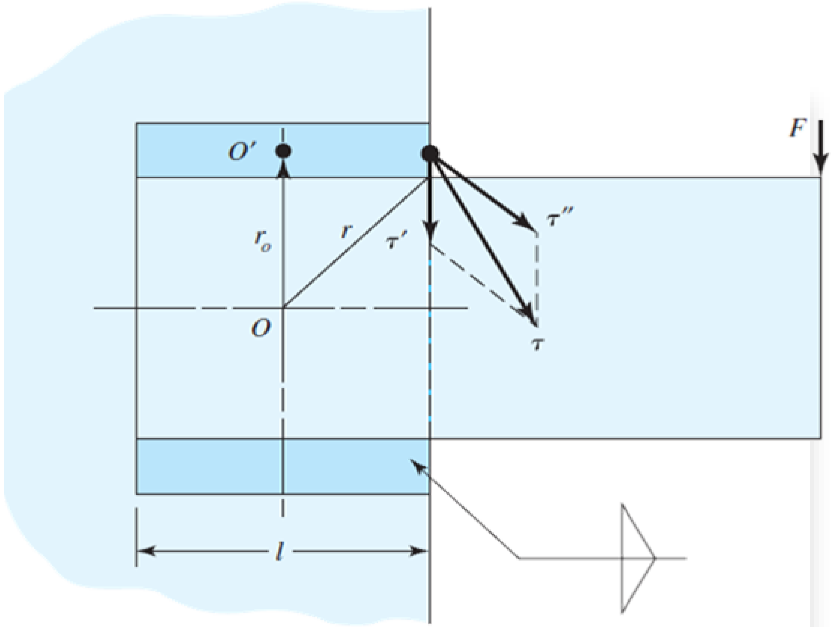
5. Torsion in Welded Joints

b) Stress in the weldments:

2. Secondary shear stress (due to twisting moment)

$$\tau'' = \frac{Mr}{J}$$

Where, r= Distance from centroid of weld group to point of interest.
J= Polar moment of inertia of weld group.
M= Moment.

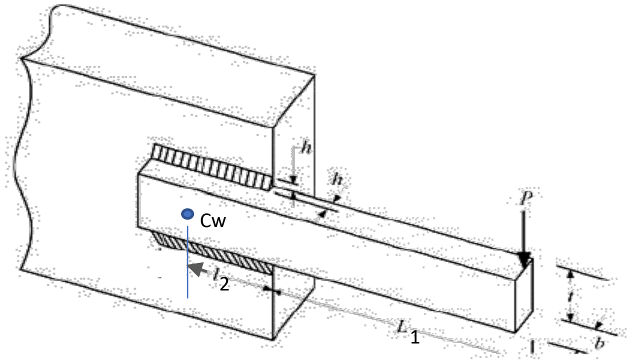


3. Add τ' and τ'' vectorially for maximum shear stress.

Notes: To make calculation easier:

a. Unite polar moment of inertia "J_u" for common weld shapes is given in table 5.4 (data book).
b. Location of centroid and throat area of weld group are given also in table 5.4 (data book).

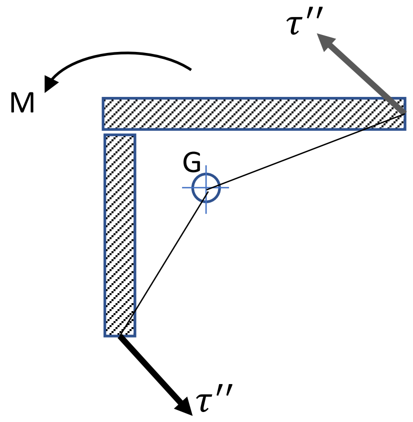
c. Moment is calculated with respect to centroid of weld group.
Exp: M= P(l_1+l_2)



5. Torsion in Welded Joints

d. $J = J_u \times 0.707\ h$ (since minimum throat width = 0.707 h)

e. Secondary shear stress direction is perpendicular to line from weld group center to point of required stress calculation and in the direction of moment. (as shown in figure below)



f. If in a problem, given

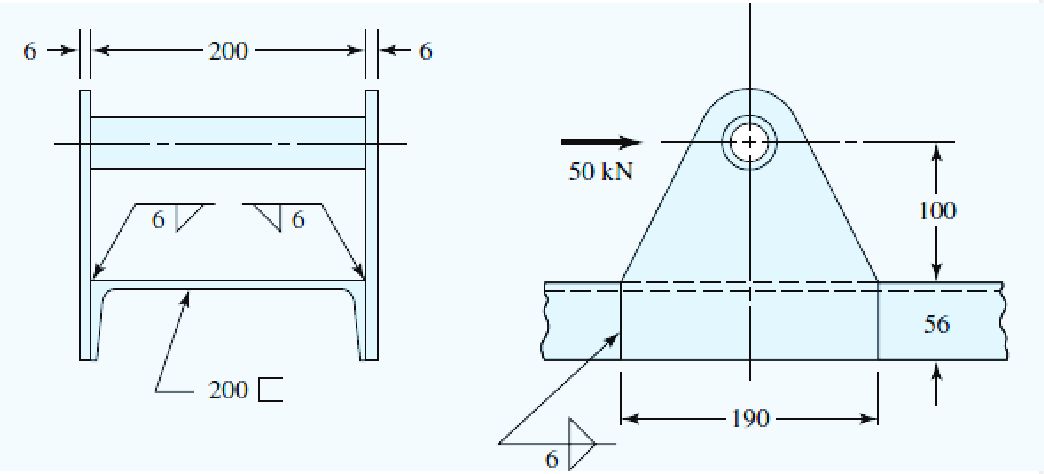
Allowable shear stress and weld size is unknown, do what?

Answer is to estimate weld size, then calculate τ' and τ'' and add for max. shear. compare allowable stress.

Repeat procedure until both equal.

Example:

A 50-kN load is transferred from a welded fitting into a 200-mm steel channel as illustrated in Figure below. Estimate the maximum stress in the weld.



Solution

See weld group

$F = \frac{50}{2} = 25kN$ (two weld groups)

-from data book ⇒ A= 0.707 h (2b+d)

= 0.707×6 (2×56+190)=1280 mm²

$$\tau' = \frac{V}{A} = \frac{25 \times 10^3}{1280} = 19.5\ MPa$$

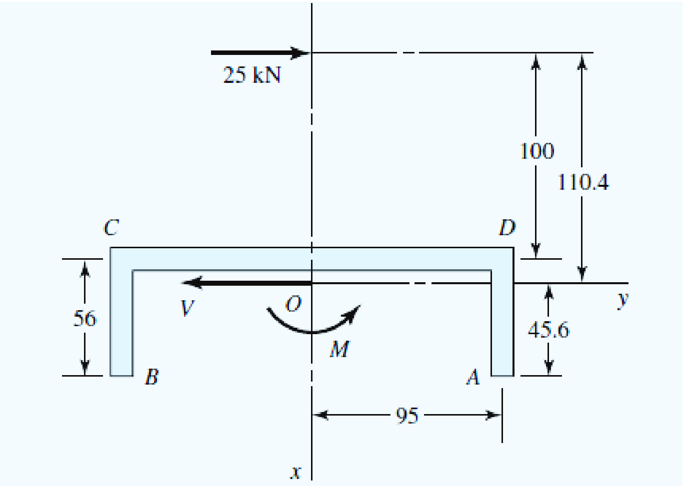
-Centroid (see weld group) $\bar{x} = \frac{190}{2} = 95\ mm$

$$\bar{y} = \frac{b^2}{2b+d} = \frac{56^2}{2 \times 56 + 190} = 10.4\ mm$$

- $J_u = \frac{8b^3+6bd^2+d^3}{12} - \frac{b^4}{2bd}$

$$J = 0.707 \times 6 \left[\frac{8\times56^3+6\times56\times190^2+190^3}{12} - \frac{56^4}{2\times56+190}\right]$$

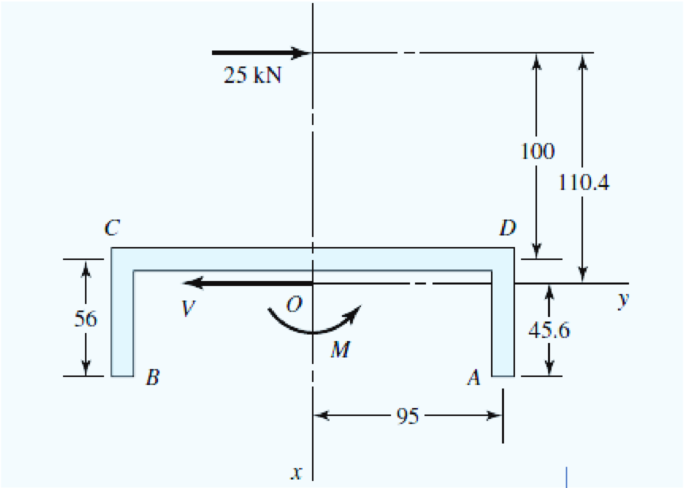
$$= 7.07 \times 10^6\ mm^4$$



Example:

A 50-kN load is transferred from a welded fitting into a 200-mm steel channel as illustrated in Figure below. Estimate the maximum stress in the weld.

Solution



$$M = F \times distance$$
$$= 25 \times (100 + 10.4) = 2760\ N.m$$

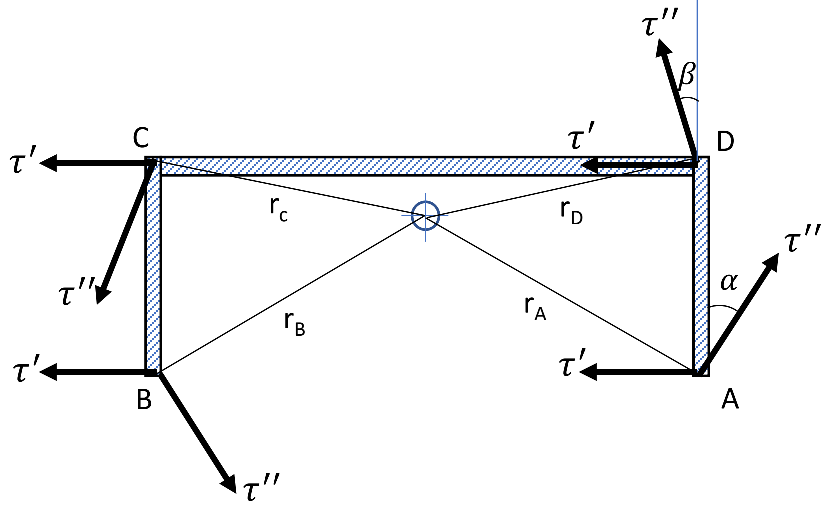
$$r_A = r_B = \sqrt{95^2 + (56 - 10.4)^2} = 105\ mm$$

$$r_C = r_D = \sqrt{95^2 + (10.4)^2} = 95.6\ mm$$

$$\tau_A'' = \tau_B'' = \frac{Mr}{J} = \frac{2760 \times 10^3 \times 105}{7.07 \times 10^6} = 41\ MPa$$

Note: τ_A'' & τ_B'' direction are shown below

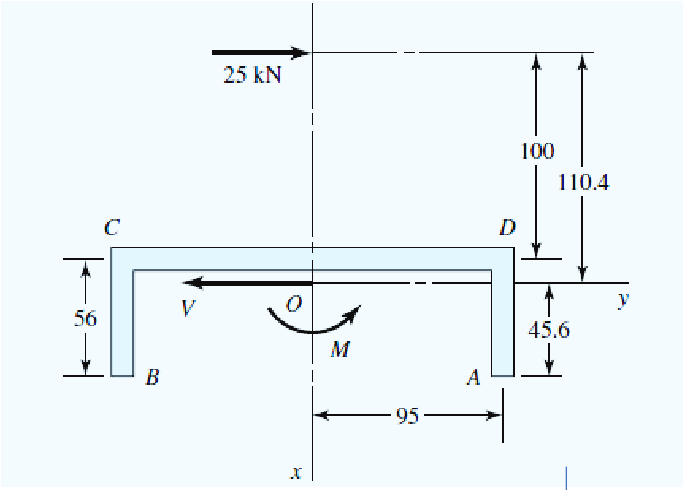
$$\tau_C'' = \tau_D'' = \frac{Mr}{J} = \frac{2760 \times 10^3 \times 95.6}{7.07 \times 10^6} = 37.3\ MPa$$



Example:

A 50-kN load is transferred from a welded fitting into a 200-mm steel channel as illustrated in Figure below. Estimate the maximum stress in the weld.

Solution



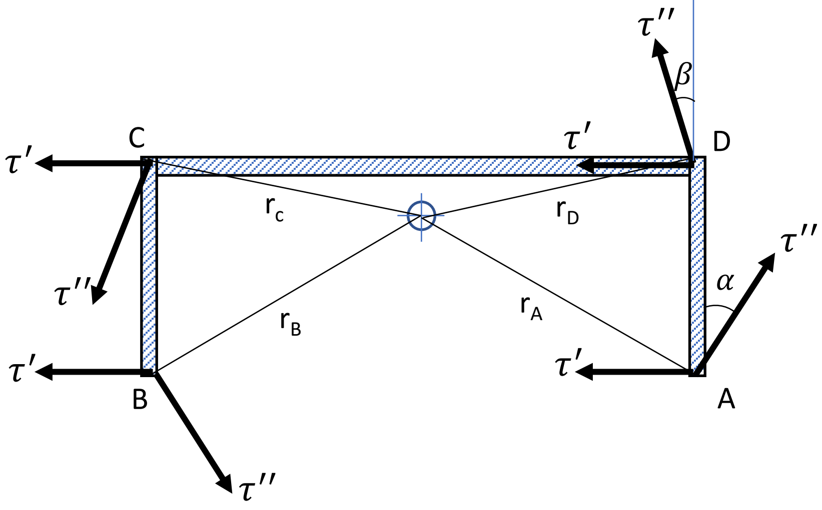
Draw on graphical paper and add vectorially

$$\tau_A = \tau_B = 37\ MPa$$

$$\tau_C = \tau_D = 44\ MPa$$

Or

Angles
$$\begin{cases} \alpha = tan^{-1}\left(\frac{45.6}{95}\right) = 25.6^o \\ \beta = tan^{-1}\left(\frac{10.4}{95}\right) = 6.25^o \end{cases}$$



$$\tau_A = \tau_B = \sqrt{(19.5 - 41sin25.6)^2 + (41cos25.6)^2} = 37\ MPa$$

$$\tau_C = \tau_D = \sqrt{(19.5 + 37sin6.25)^2 + (37cos6.25)^2} = 43.9\ MPa$$

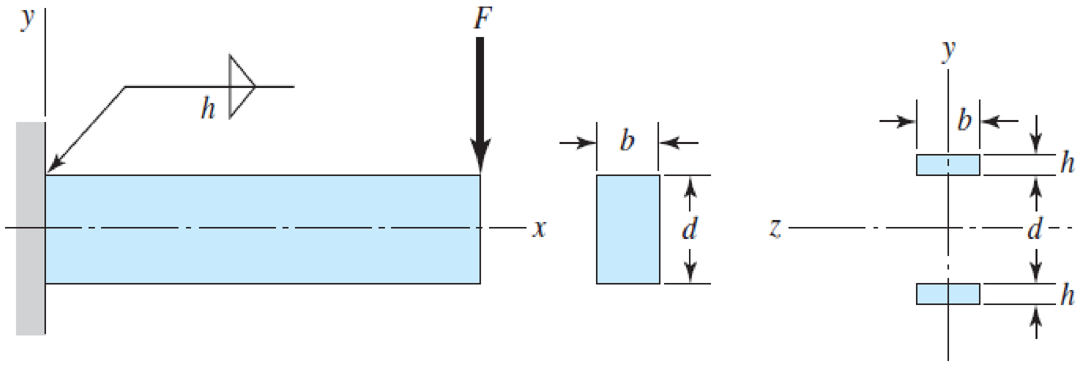
6. Bending in Welded Joints

For a cantilever beam welded to a support at top and bottom by fillet welds as shown in Figure. The force ‘F’ create both:

a) Primary shear stress (vertical)

$$\tau' = \frac{V}{A}$$

V: shear force
A: throat area of weld group



(*b*) Weld pattern

b) Secondary shear stress (horizontal)

The moment actually cause bending stress $\sigma = \frac{My}{I}$. However, according to the conservative approach we use, all force acting on fillet weld is assumed to be carried as shear force on the throat area of weld group. Thus,

$$\tau'' = \frac{My}{I}$$

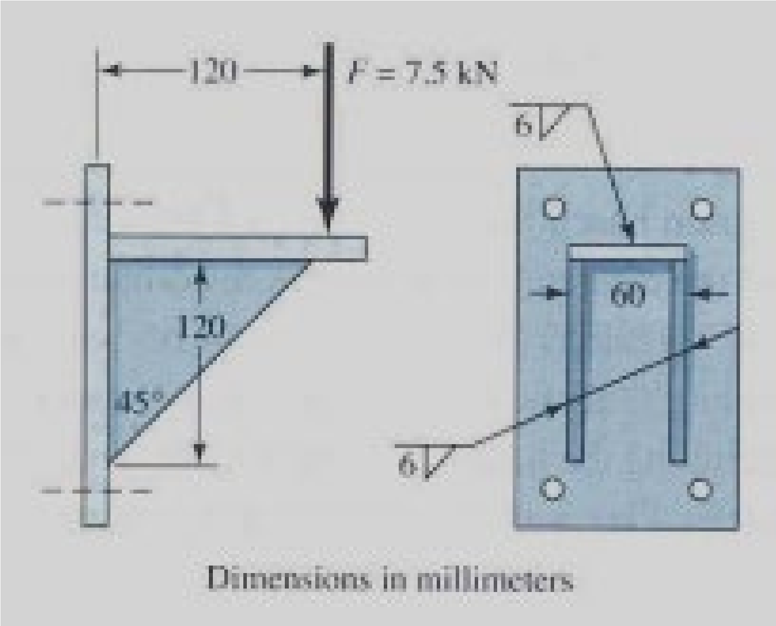
y: distance from N.A of weld group to point of interest.
I: 2nd moment of inertia of weld group about N.A.

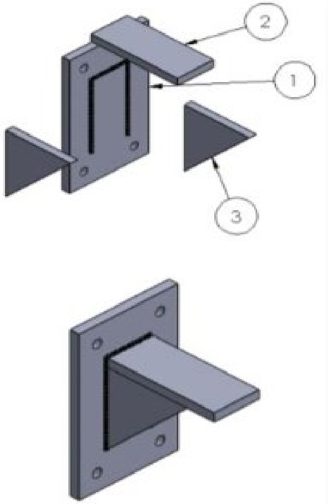
-See table 5.5 data book for: throat area, location of centroid and unit second moment of inertia of common fillet weld groups.

$$I = 0.707h \times I_u$$

Example

A force F=7.5 kN acts on the bracket shown in figure. Find the maximum stress in the weld metal





Solution

1) Primary shear: $\tau' = \frac{V}{A} = \frac{7.5\times10^3}{0.707\times6(60+2\times120)} = 5.84\ MPa$

Where, A=0.707h(b+2d) $\Rightarrow$ see data book p.28

Centroid: $\bar{Y} = \frac{d^2}{b+2d} = \frac{120^2}{60+2\times120} = 48\ mm$

Unit moment of inertia: $I_u = \frac{3}{2}d^3 - 2d^2\bar{Y} + (b+2d)\bar{Y}^2$

$= \frac{3}{2}120^3 - 2\times120^2\times48 + (60+2\times120)48^2$

$= 460684.8\ mm^3$

$I_{N.A} = 0.707\times6\times460684.8 = 1.954\times10^{-6}mm^4$

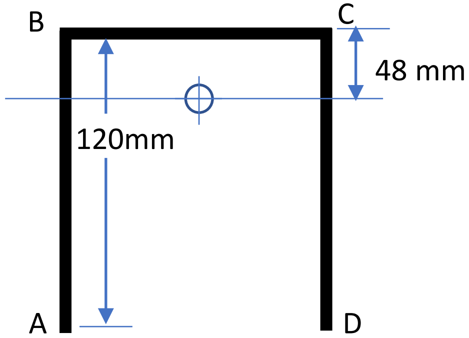
Moment : $M = F\times d = 7.5\times10^3\times120\times10^{-3} = 900\ N.m$

2) Secondary shear [@ points A and D]

$\tau'' = \frac{My}{I} = \frac{900\times(120-48)\times10^{-3}}{1.954\times10^{-6}} = 33.16\ MPa$

$\tau_A = \tau_D = \sqrt{\tau'^2 + \tau''^2}$

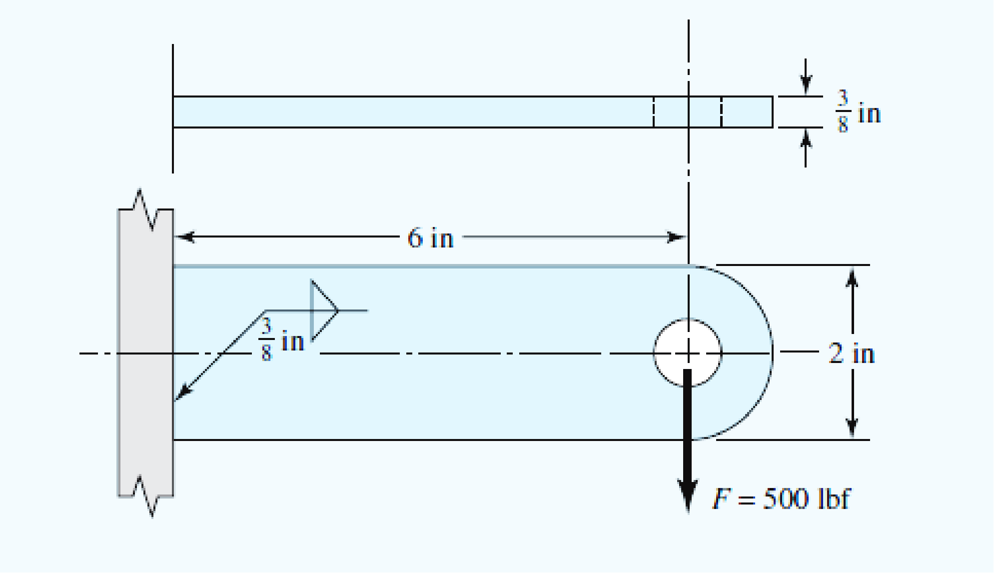
$= \sqrt{5.84^2 + 33.16^2} = 33.67\ MPa$



Example

A welded cantilever carrying 500 lbf is shown in figure below. The contriver is made of AISI steel and welded with a 3/8 in fillet weld. An E6010 electrode was used. Find:

a) Safety factor of weld metal.
b) Safety factor of cantilever metal (parent metal).



Solution

a) From table (5.1, p29) (σ_u = 62 kpsi , σ_y = 50 kpsi)

From (table 5.5, p.28)

A= 1.414bd = 1.414× 3/8× 2=1.06 in^2

I_u= $d^3/6 = 2^3/6$=1.33 in^3

I=0.707× h× I_u=0.707× 3/8× 1.33=0.353 in^4

$$\tau' = \frac{F}{A} = \frac{500}{1.06} = 472\; psi\; (0.472\; kpsi)$$

$$\tau'' = \frac{My}{I} = \frac{500 \times 6 \times 1}{0.353} = 8500\; psi\; (8.5\; kpsi)$$

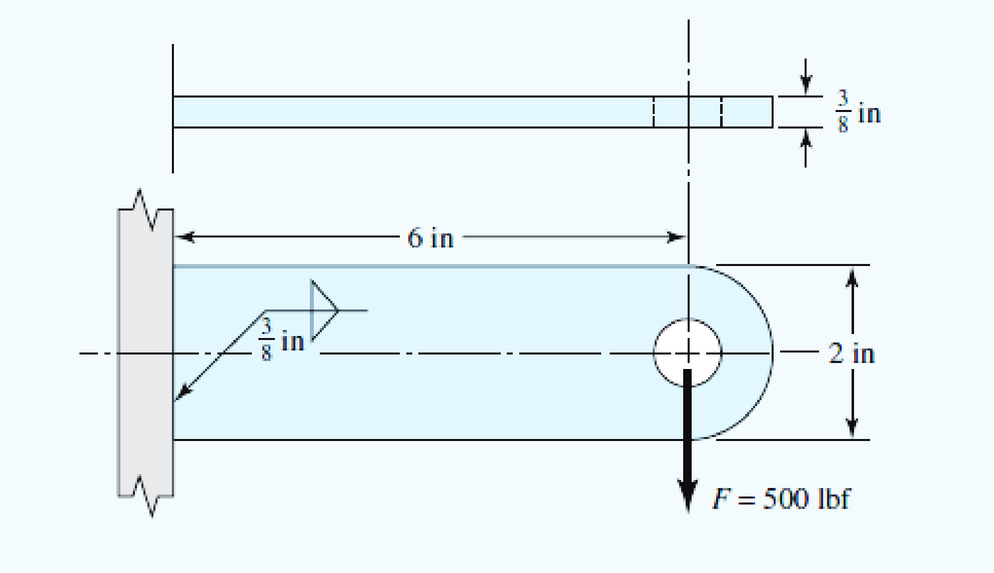
Resultant shear $\tau = \sqrt{\tau'^2 + \tau''^2} = \sqrt{0.472^2 + 8.5^2} = 8.513\; kpsi$

Safety factor $= \frac{S_{sy}}{\tau} = \frac{0.577 \times 50}{8.513} = 3.39$ (distortion energy)

Example

A welded cantilever carrying 500 lbf is shown in figure below. The contriver is made of AISI steel and welded with a 3/8 in fillet weld. An E6010 electrode was used. Find:

a) Safety factor of weld metal.
b) Safety factor of cantilever metal (parent metal).



Solution

b) From table (A-17, p83) for AISI 1018 HR

(S_u = 58 kpsi , S_y = 32 kpsi)

$$\sigma = \frac{My}{I} = \frac{500 \times 6 \times 1}{\frac{1}{12} \times \frac{3}{8} \times 2^3} = 12000\, psi\ (12\, kpsi)$$

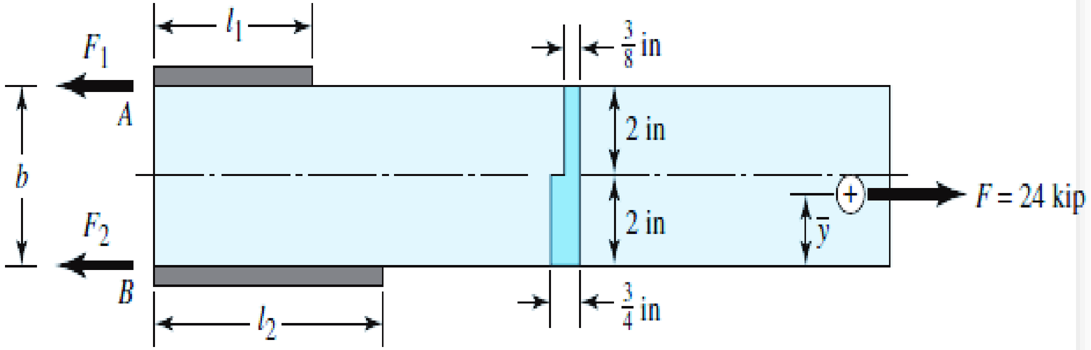
$$saftey\ factor\ n = \frac{S_y}{\sigma} = \frac{32}{12} = 2.67$$

(normal stress theory)

Note: Weld material is safer than parent material.

Example

A structural steel has (σ_{ut}=58 kpsi and σ_y=36 kpsi) are cross section as shown in figure below. A tensile force of 24 kip is acting through centroid of structure. Unsymmetrical weld track are used to compensate for eccentricity such that no moment to be resisted by weld. Find track lengths l_1 and l_2 for 3/8 fillet weld using E70xx electrode.



Solution

- Centroid of structure

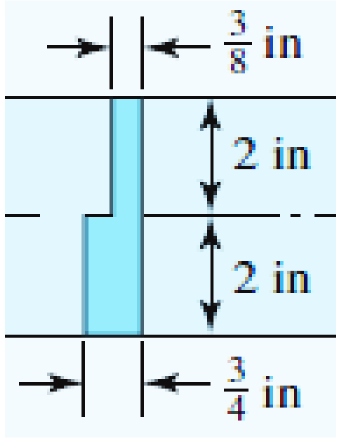
$$\bar{Y} = \frac{\sum yA_i}{\sum A_i} = \frac{1 \times 2 \times \frac{3}{4} + 3 \times \frac{3}{8} \times 2}{2 \times \frac{3}{4} + 2 \times \frac{3}{8}} = 1.67\ in$$

- Reactions F_1 and F_2

$$\sum M_B = 0 \implies F_1 \times 4 = 24 \times 1.67$$

$$F_1 = 10\ kip$$

$$F_2 = 14\ kip$$



- For weld materials,

The weld throat area have to be in the ratio $\frac{F_2}{F_1} = \frac{14}{10} = 1.4$

$\Rightarrow l_2 = 1.4\ l_1$

Example

From table 5-1, p29 E70xx (S_y= 57 kpsi, S_u=70 kpsi)

From table 5-2 p29

Permissible stress =0.3 × S_u= 0.3× 70 = 21 kpsi

Now, stress $\tau = \frac{F}{A} = \frac{24}{0.707\times\frac{3}{8}\times(l_1+1.4l_1)} = 21$

$$\Longrightarrow l_1 = 2.16'' \quad , \qquad l_2 = 1.4\, l_1 = 3.02''$$

For structure steel, (base metal adjacent to weld)

$$\tau = \frac{F}{h(l_1 + l_2)} = \frac{24}{\frac{3}{8}(l_1 + 1.4l_1)}$$

From table 5-2 p29

$$\tau_{all} = 0.4\, S_y = 0.4 \times 36 = 14.4\; kpsi$$

$$14.4 = \frac{24}{\frac{3}{8} \times (2.4l_1)} \Rightarrow l_1 = 2.22'' \, , \qquad l_2 = 3.11''$$

If AISC code is used, (table 5-2, p.29)

Tensile $\tau_{allowable} = 0.6S_y = 0.6 \times 36 = 21.6\; kpsi$

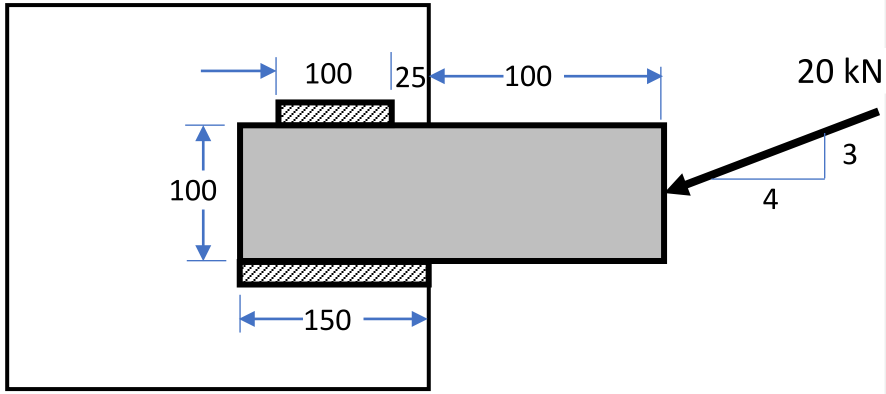
$$\sigma = \frac{F}{A} = \frac{24}{\frac{3}{4} \times 2 + \frac{3}{8} \times 2} = 10.7\; kpsi$$

Since $\sigma < \sigma_{allowable}$ $\Rightarrow$ OK

Example

A plate is to be welded to a steel frame by fillet welds as shown in figure below. If the plate is subjected to static inclined load of 20 kN. Fine the size of the weld using maximum shear stress theory.(S_y =350 MPa , n=3)

Solution



- Force

$$F_x = \frac{4}{5} \times 20 = 16 \text{ kN}$$

$$F_y = \frac{3}{5} \times 20 = 12 \text{ kN}$$

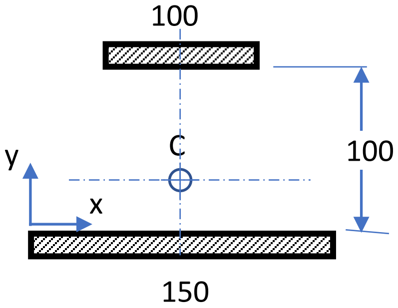
- Centroid

$$\bar{x} = 75\ mm \ \ \text{(symmetry)}$$

$$\sum yl = \bar{Y} \sum l$$

$$150 \times 0 + 100 \times 100 = \bar{Y} \times 250$$

$$\bar{Y} = 40\ mm$$



- Stress in weld due to horizontal force (16 kN)

$$(1)\ \ \tau'_1 = \tau_{av} = \frac{F_x}{A} = \frac{16 \times 10^3}{0.707h(250)} = \frac{90.5}{h}\ MPa$$

- Stress in weld due to vertical force (12 kN)

$$(1)\ \text{Primary shear} : \tau'_2 = \frac{F_y}{A} = \frac{12 \times 10^3}{0.707h(250)} \cong \frac{68}{h}\ MPa$$

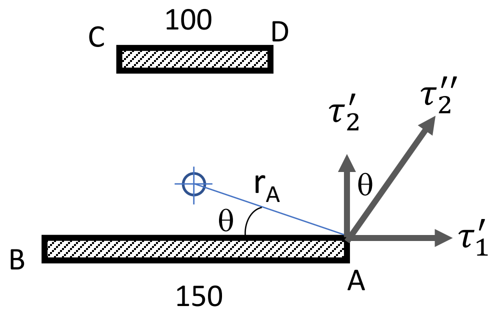
Example

A plate is to be welded to a steel frame by fillet welds as shown in figure below. If the plate is subjected to static inclined load of 20 kN. Fine the size of the weld using maximum shear stress theory.(S_y =350 MPa , n=3)

Solution

(2) secondary shear : $\tau_2'' = \frac{My}{J}$

$M = 12 \times (50 + 25 + 100) = 2100\ N.m$



$$J_u = \left(\frac{150^3}{12} + 150 \times 40^2\right) + \left(\frac{100^3}{12} + 100 \times 60^2\right) = 964583\ mm^3$$

$$J = 0.707 \times h \times 964583 = 681960\ h\ \ mm^4$$

Max. stress most likely at point A: $r_A = \sqrt{75^2 + 40^2} = 85\ mm$

$$\tau_2'' = \frac{My}{J} = \frac{2100 \times 85 \times 10^{-3}}{681960\ h \times 10^{-6}} = \frac{261.7}{h}\ MPa$$

Add vectorially τ_1' , τ_2' and τ_2''

$$tan\theta = \frac{40}{75} \Rightarrow \theta = 28^o$$

$$\tau_R|_A = \frac{1}{h}\sqrt{(90.5 + 261.7 \sin 28)^2 + (68 + 261.7 \cos 28)^2} = \frac{1}{h}\sqrt{213.3^2 + 299^2} = \frac{367}{h}$$

Using failure theory

$$\tau_R|_A = \frac{S_y}{2n} \quad \Rightarrow \frac{350}{2 \times 3} = \frac{367}{h} \Rightarrow h = 6.29\ mm$$