Mechanics of Materials

SYLLABUS

Chapter (1): Simple Stress and Strain

Chapter (2): Thermal Stresses

Chapter (3): Composite Bars in Tension and Compression

Chapter (4): Statically indeterminate Members

Chapter (5): Thin-Walled Cylinder

Chapter (6): Torsion of Circular Shaft

Chapter (7): Plan Stress Analysis

REFERENCES

- Strength of Materials by Andrew Pytel and Ferdinand L. Singer
- Mechanics of materials by E.J Hearn
- Mechanics of materials E.P Popove

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Chapter (1): Simple stresses and strains

Simple Stresses:

Normal (Direct Stress)

One of the most fundamental types of stress that exists is normal stress, in which the stress acts perpendicular, or normal, to the cross-section of the load-carrying member.

The intensity of normal force per unit area is termed the normal stress and is expressed in units of force per unit area, N/m2. If the forces applied to the ends of the bar are such that the bar is in tension, then tensile stresses are set up in the bar; if the bar is in compression we have compressive stresses. The line of action of the applied end forces passes through the centroid of each cross section of the bar.

$$stress\ (\sigma) = \frac{load}{area} = \frac{P}{A}$$

The area of the cross-section of the load-carrying member is taken perpendicular to the line of action of the force.

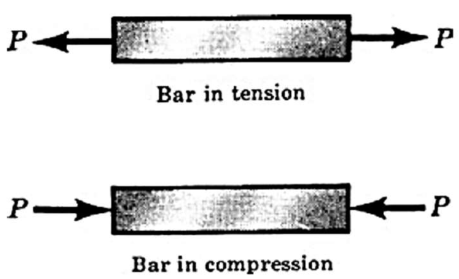

Fig. 1-1 Axially loaded bars.

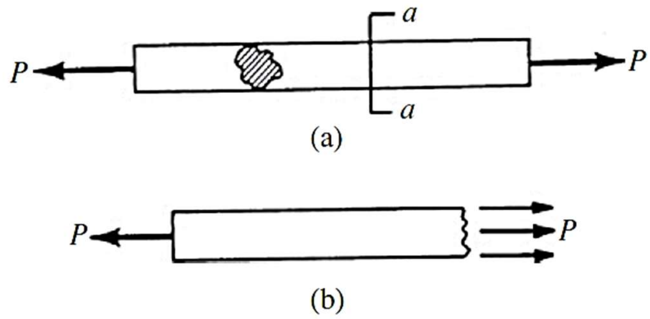

Fig. 1-2 Internal force.

Normal Strain

Let us suppose that the bar of Fig. 1-1 has tensile forces gradually applied to the ends. The elongation per unit length, which is termed normal strain and denoted by ε may be found by dividing the total elongation Δ by the length L, i.e.,

$$\epsilon = \frac{\Delta}{L}$$

The strain is usually expressed in units of meters per meter and consequently is dimensionless.

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The stress-strain curve shown in Fig. 1-3(a) may be used to characterize several strength characteristics of the material. They are:

Proportional Limit

The ordinate of the point P is known as the proportional limit, i.e., the maximum stress that may be developed during a simple tension test such that the stress is a linear function of strain. For a material having the stress-strain curve shown in Fig. 1-3(d), there is no proportional limit.

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Elastic Limit

The ordinate of a point almost coincident with P is known as the elastic limit, i.e., the maximum stress that may be developed during a simple tension test such that there is no permanent or residual deformation when the load is entirely removed. For many materials the numerical values of the elastic limit and the proportional limit are almost identical and the terms are sometimes used synonymously. In those cases where the distinction between the two values is evident, the elastic limit is almost always greater than the proportional limit.

Elastic and Plastic Ranges

The region of the stress-strain curve extending from the origin to the proportional limit is called the elastic range. The region of the stress-strain curve extending from the proportional limit to the point of rupture is called the plastic range.

Yield Point

The ordinate of the point Y in Fig. 1-3(a), denoted by syp, at which there is an increase in strain with no increase in stress, is known as the yield point of the material. After loading has progressed to the point Y, yielding is said to take place. Some materials exhibit two points on the stress-strain curve at which there is an increase of strain without an increase of stress. These are called upper and lower yield points.

Ultimate Strength or Tensile Strength

The ordinate of the point U in Fig. 1-3(a), the maximum ordinate to the curve, is known either as the ultimate strength or the tensile strength of the material.

Breaking Strength

The ordinate of the point B in Fig. 1-3(a) is called the breaking strength of the material.

Working Stress

The above-mentioned strength characteristics may be used to select a working stress. Frequently such a stress is determined merely by dividing either the stress at yield or the ultimate stress by a number termed the safety factor. Selection of the safety factor is based upon the designer’s judgment and experience. Specific safety factors are sometimes specified in design codes.

Yield Strength

The ordinate to the stress-strain curve such that the material has a predetermined permanent deformation or ''set'' when the load is removed is called the yield strength of the material. The permanent set is often taken to be either 0.002 or 0.0035 mm per mm. These values are of course arbitrary. In Fig. 1-3(d) a set ε_1 is denoted on the strain axis and the line O'Y is drawn parallel to the initial tangent to the curve. The ordinate of Y represents the yield strength of the material, sometimes called the proof stress.

Poisson's Ratio

When a bar is subjected to a simple tensile loading there is an increase in length of the bar in the direction of the load, but a decrease in the lateral dimensions perpendicular to the load. The ratio of the strain in the lateral direction to that in the axial direction is defined as Poisson's ratio. It is denoted by the Greek letter ν. For most metals it lies in the range 0.25 to 0.35. For cork, n is very nearly zero.

Shear Modulus

The material constant G is called the shear modulus of elasticity (or simply shear modulus), or the modulus of rigidity. The shear modulus has the same units as the modulus of elasticity (Pa). G is related to the modulus of elasticity E and Poisson's ratio ν by

$$G = \frac{E}{2(1+\nu)}$$

General Form of Hooke's Law

The simple form of Hooke's law has been given for axial tension when the loading is entirely along one straight line, i.e., uniaxial. Only the deformation in the direction of the load was considered

$$\epsilon = \frac{\sigma}{E}$$

In the more general case, an element of material is subject to three mutually perpendicular normal stresses σ_x, σ_y, σ_z, which are accompanied by the strains ε_x, ε_y, ε_z, respectively. By superposing the strain components arising from lateral contraction due to Poisson's effect upon the direct strains we obtain the general statement of Hook's law:

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] \qquad \epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] \qquad \epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

Stress-Strain Curve

As the axial load in Fig. 1-1 is gradually increased, the total elongation over the bar length is measured at each increment of load and this is continued until fracture of the specimen takes place. Knowing the original cross-sectional area of the test specimen, the normal stress,

$$\sigma = \frac{P}{A}$$

denoted by σ, may be obtained for any value of the axial load by the use of the relation

where P denotes the axial load in newtons and A the original cross-sectional area. Having obtained numerous pairs of values of normal stress σ and normal strain ε, experimental data may be plotted with these quantities considered as ordinate and abscissa, respectively. This is **the stress-strain curve** or diagram of the material for this type of loading. Stress-strain diagrams assume widely differing forms for various materials.

Figure 1-3(a) is the stress-strain diagram for a medium-carbon structural steel, Fig. 1-3(b) is for an alloy steel, and Fig. 1-3(c) is for hard steels and certain nonferrous alloys. For nonferrous alloys and cast iron the diagram has the form indicated in Fig. 1-3(d).

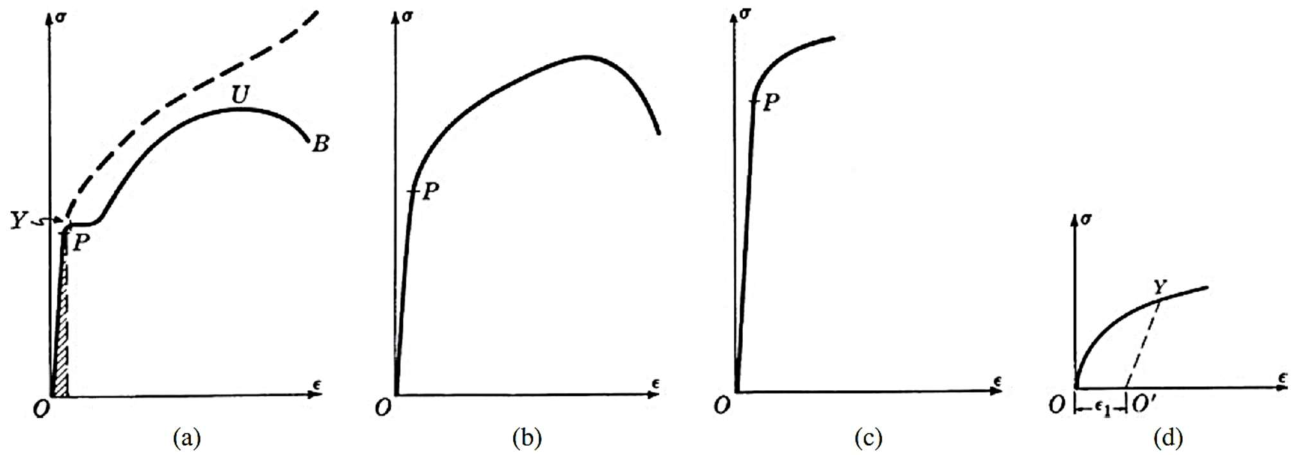

Fig. 1-3 Stress-strain diagrams.

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Ductile and Brittle Materials

Metallic engineering materials are commonly classified as either ductile or brittle materials. A ductile material is one having a relatively large tensile strain up to the point of rupture (for example, structural steel or aluminum) whereas a brittle material has a relatively small strain up to this same point. An arbitrary strain of 0.05 mm/mm is frequently taken as the dividing line between these two classes of materials. Cast iron and concrete are examples of brittle materials.

Hooke's Law

For any material having a stress-strain curve of the form shown in Fig. 1-3(a), (b), or (c), it is evident that the relation between stress and strain is linear for comparatively small values of the strain. This linear relation between elongation and the axial force causing it is called Hooke's law. To describe this initial linear range of action of the material we may consequently write

$$\sigma = E\epsilon$$

where E denotes the slope of the straight-line portion OP of each of the curves in Figs.1-3(a), (b), and (c).

The quantity E, i.e., the ratio of the unit stress to the unit strain, is the modulus of elasticity of the material in tension, or, as it is often called,

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Young's modulus. Values of E for various engineering materials are tabulated in handbooks. Since the unit strain ε is a pure number (being a ratio of two lengths) it is evident that E has the same units as does the stress, N/m2.

For many common engineering materials the modulus of elasticity in compression is very nearly equal to that found in tension. It is to be carefully noted that the behavior of materials under load as discussed through the course is restricted (unless otherwise stated) to the linear region of the stress-strain curve.

AXIAL DEFORMATION

In the linear portion of the stress-strain diagram, the tress is proportional to strain and is given by

$$\sigma = E\varepsilon$$

since $\sigma = P / A$ and $\varepsilon e = \delta / L$, then $P / A = E \delta / L$. Solving for δ,

$$\delta = \frac{PL}{AE} = \frac{\sigma L}{E}$$

Stresses on inclined planes

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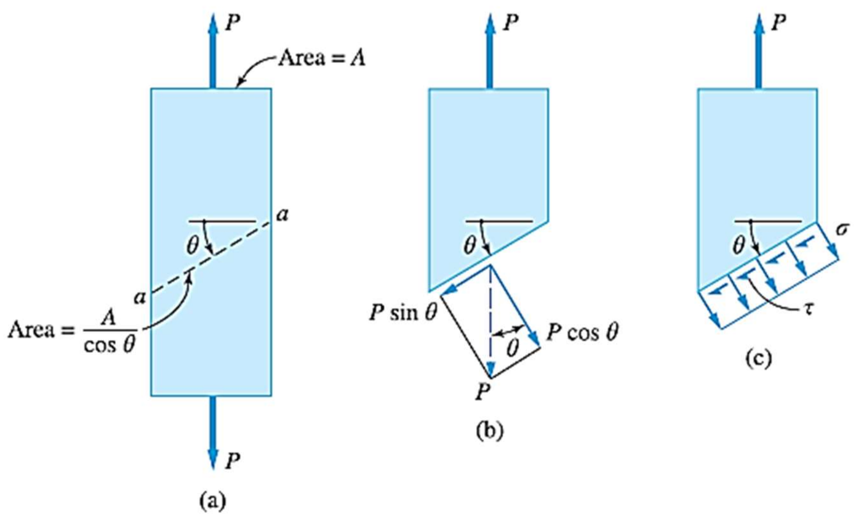
When a bar of cross-sectional area A is subjected to an axial load P, the normal stress P=A acts on the cross section of the bar. Let us now consider the stresses that act on plane a-a that is inclined at the angle θ to the cross section, as shown below. Note that the area of the inclined plane is A=cosθ: To investigate the forces that act on this plane, we consider the free-body diagram of the segment of the bar shown. Because the segment is a two-force body, the resultant internal force acting on the inclined plane must be the axial force P, which can be resolved into the normal component P cosθ and the shear component P sin θ. Therefore, the corresponding stresses, shown are:

$$\boxed{\sigma = \frac{P\cos\theta}{A/\cos\theta} = \frac{P}{A}\cos^2\theta}$$

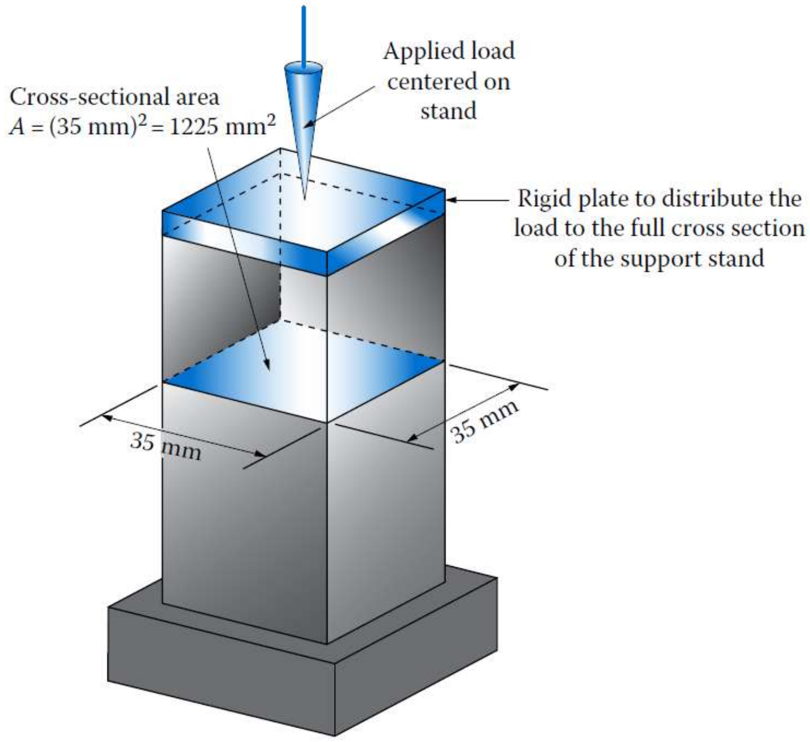
$$\boxed{\tau = \frac{P\sin\theta}{A/\cos\theta} = \frac{P}{A}\sin\theta\cos\theta = \frac{P}{2A}\sin 2\theta}$$



Determining the stresses acting on an inclined section of a bar.

Examples

Example (1): Figure below shows a support stand designed to carry downward loads. Compute the stress in the square shaft at the upper part of the stand for a load of 120 kN. The line of action of the applied load is centered on the axis of the shaft, and the load is applied through a thick plate that distributes the force to the entire cross-section of the stand.



$$\text{Stress} = \sigma = \text{Force/Area} = F/A\left(\text{compressive}\right)$$

$$F = 120\ \text{kN}$$

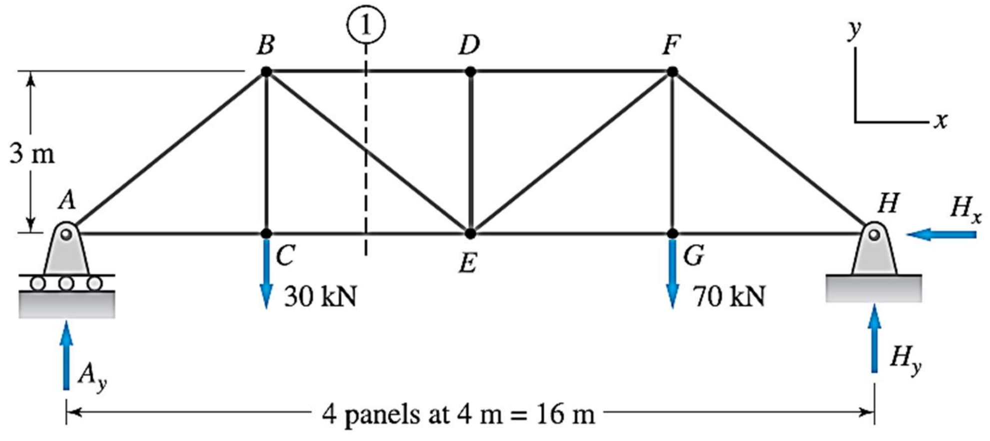
$$A = \left(35\ \text{mm}\right)^2 = 1225\ \text{mm}^2$$

$$\sigma = F/A = 120\,000\ \text{N}/1225\ \text{mm}^2 = 98.0\ \text{N/mm}^2 = 98.0\ \text{MPa}$$

This level of stress would be present at any part of any cross section of the square shaft between its ends.

Example (2):

For the truss shown in Fig. (a), calculate the normal stresses in (1) member AC; and (2) member BD. The cross-sectional area of each member is 900 mm^2.



(a)

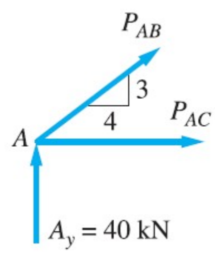
Solution:

Equilibrium analysis using the FBD of the entire truss in Fig. (a) gives the following values for the external reactions:

Ay =40 kN, Hy = 60 kN, and Hx = 0.

$$\sum F_y = 0 \quad +\uparrow \quad 40 + \frac{3}{5} P_{AB} = 0$$

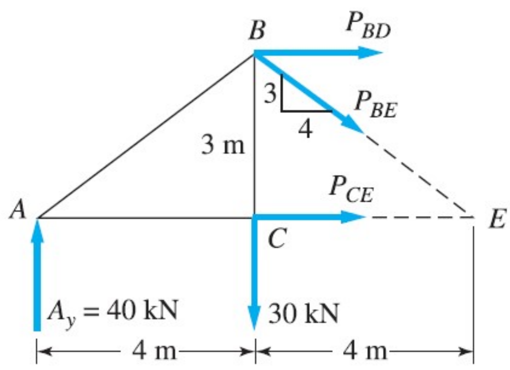
$$\sum F_x = 0 \quad \overset{+}{\rightarrow} \quad P_{AC} + \frac{4}{5} P_{AB} = 0$$



(b) FBD of pin A

Solving the equations gives PAC = 53:33 kN (tension). Thus, the normal stress in member AC is:

$$\sigma_{AC} = \frac{P_{AC}}{A_{AC}} = \frac{53.33 \text{ kN}}{900 \text{ mm}^2} = \frac{53.33 \times 10^3 \text{ N}}{900 \times 10^{-6} \text{ m}^2}$$
$$= 59.3 \times 10^6 \text{ N/m}^2 = 59.3 \text{ MPa (T)}$$



(c) FBD

$$\sum M_E = 0 \quad +\circlearrowleft \quad -40(8) + 30(4) - P_{BD}(3) = 0$$

which yields

$$P_{BD} = -66.67 \text{ kN} = 66.67 \text{ kN (C)}$$

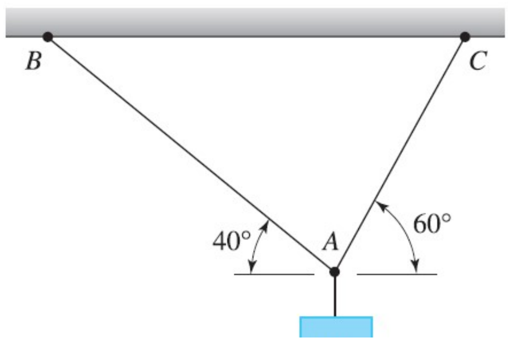
Therefore, the normal stress in member *BD* is

$$\sigma_{BD} = \frac{P_{BD}}{A_{BD}} = \frac{-66.67 \text{ kN}}{900 \text{ mm}^2} = \frac{-66.67 \times 10^3 \text{ N}}{900 \times 10^{-6} \text{ m}^2}$$
$$= -74.1 \times 10^6 \text{ N/m}^2 = 74.1 \text{ MPa (C)}$$

Example (3):

Figure below shows a two-member truss supporting a block of weight W. The cross-sectional areas of the members are 800 mm2 for AB and 400 mm2 for AC.

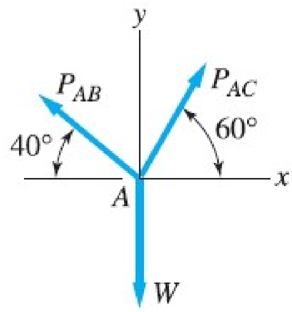
Determine the maximum safe value of W if the working stresses are 110 MPa for AB and 120 MPa for AC.



Solution:

$$\sum F_x = 0 \quad \overset{+}{\rightarrow} \quad P_{AC}\cos 60^\circ - P_{AB}\cos 40^\circ = 0$$

$$\sum F_y = 0 \quad +\uparrow \quad P_{AC}\sin 60^\circ + P_{AB}\sin 40^\circ - W = 0$$



(b) FBD of pin A

Solving simultaneously, we get

$$P_{AB} = 0.5077W \qquad P_{AC} = 0.7779W$$

The value of W that will cause the normal stress in bar AB to equal its working stress is given by

$$P_{AB} = (\sigma_w)_{AB}A_{AB}$$

$$0.5077W = (110 \times 10^6\ \text{N/m}^2)(800 \times 10^{-6}\ \text{m}^2)$$

$$W = 173.3 \times 10^3\ \text{N} = 173.3\ \text{kN}$$

The value of W that will cause the normal stress in bar AC to equal its working stress is found from

$$P_{AC} = (\sigma_w)_{AC} A_{AC}$$
$$0.7779W = (120 \times 10^6 \text{ N/m}^2)(400 \times 10^{-6} \text{ m}^2)$$
$$W = 61.7 \times 10^3 \text{ N} = 61.7 \text{ kN}$$

Choose the Correct Answer

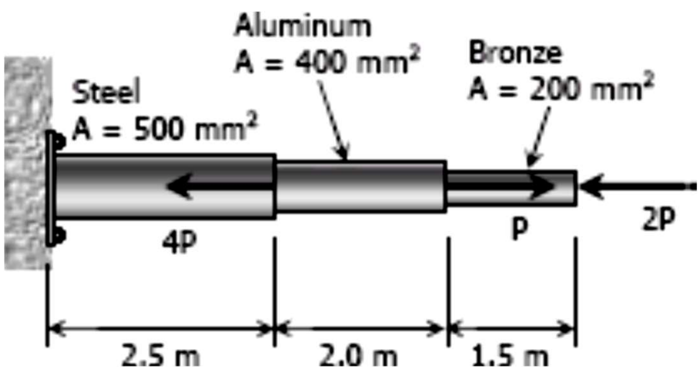
The maximum safe value of W is the smaller of the preceding two values—namely,

$$W = 61.7 \text{ kN} \qquad \textit{Answer}$$

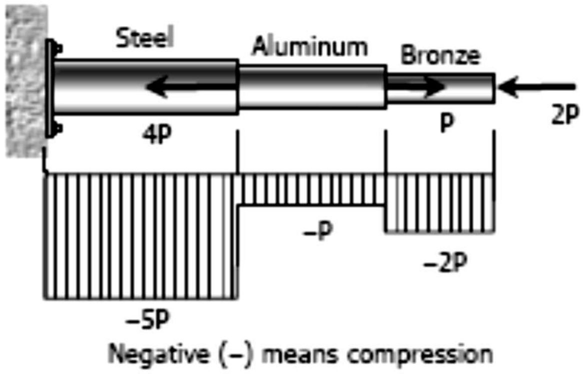
We see that the stress in bar AC determines the safe value of W. The other "solution," $W = 173.3$ kN, must be discarded because it would cause the stress in AC to exceed its working stress of 120 MPa.

Example (4):

An aluminum rod is rigidly attached between a steel rod and a bronze rod as shown in Fig. below. Axial loads are applied at the positions indicated. Find the maximum value of P that will not exceed a stress in steel of 140 MPa, in aluminum of 90 MPa, or in bronze of 100 MPa.



For bronze:
$\sigma_{br} A_{br} = 2P$
$100(200) = 2P$
$P = 10\,000$ N

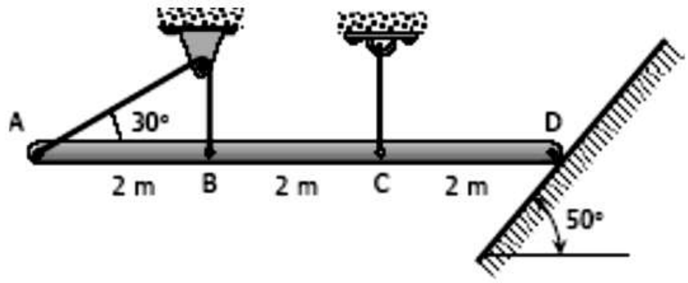


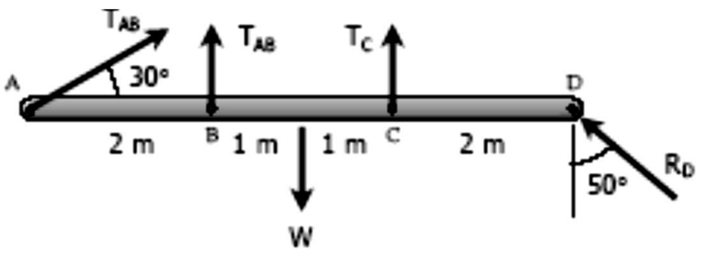
For aluminum:
$\sigma_{al} A_{al} = P$
$90(400) = P$
$P = 36\,000$ N
For Steel:
$\sigma_{st} A_{st} = 5P$
$P = 14\,000$ N

For safe P, use $P = 10\,000$ N = **10 kN**

Example (5):

The homogeneous bar ABCD shown is supported by a cable that runs from A to B around the smooth peg at E, a vertical cable at C, and a smooth inclined surface at D. Determine the mass of the heaviest bar that can be supported if the stress in each cable is limited to 100 MPa. The area of the cable AB is 250 mm^2 and that of the cable at C is 300 mm^2.





$\Sigma F_H = 0$
$T_{AB} \cos 30° = R_D \sin 50°$
$R_D = 1.1305 T_{AB}$

$\Sigma F_V = 0$
$T_{AB} \sin 30° + T_{AB} + T_C + R_D \cos 50° = W$
$T_{AB} \sin 30° + T_{AB} + T_C + (1.1305 T_{AB}) \cos 50° = W$
$2.2267 T_{AB} + T_C = W$
$T_C = W - 2.2267 T_{AB}$

$\Sigma M_D = 0$
$6(T_{AB} \sin 30°) + 4T_{AB} + 2T_C = 3W$
$7T_{AB} + 2(W - 2.2267 T_{AB}) = 3W$
$2.5466 T_{AB} = W$
$T_{AB} = 0.3927W$

$T_C = W - 2.2267 T_{AB}$
$= W - 2.2267(0.3927W)$
$= 0.1256W$

Based on cable AB:
$T_{AB} = \sigma_{AB} A_{AB}$
$0.3927W = 100(250)$
$W = 63\ 661.83$ N

Based on cable at C:
$T_2 = \sigma_C A_C$
$0.1256W = 100(300)$
$W = 238\ 853.50$ N

Safe weight $W = 63\ 669.92$ N
$W = mg$
$63\ 669.92 = m\,(9.81)$
$m = 6\ 490$ kg
$= \mathbf{6.49\ Mg}$

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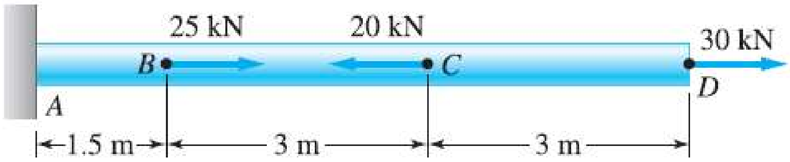
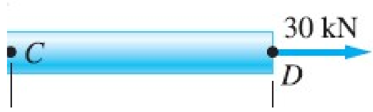
Example (6): The cross-sectional area of bar ABCD is 600 mm2. Determine the maximum normal stress in the bar.



Solution:

Section CD:



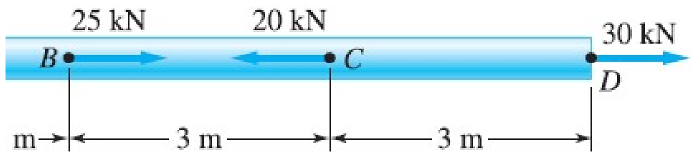
P_{CD}=30 kN

σ_{CD}= 30*1000/600= 50 MPa

Section BC:

P_{BC}=30-20 =10 kN

σ_{CD}= 10*1000/600= 16.67 MPa



Section AB:

P_{BC}=30-20 +25=35 kN

σ_{CD}= 35*1000/600= 58.3 MPa

Ans.: σ_{ma}=58.3 MPa

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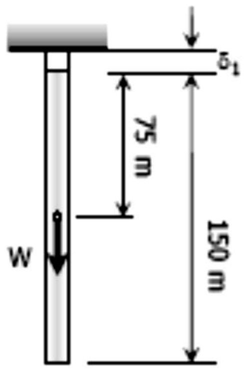
Example (7):

A steel rod having a cross-sectional area of 300 mm^2 and a length of 150mm is suspended vertically from one end. It supports a tensile load of 20 kN at the lower end. If the unit mass of steel is 7850 kg/m^3 and E=200 × 103 MN/m^2, find the total elongation of the rod.

Let δ = total elongation

δ_1 = elongation due to its own weight

δ_2 = elongation due to applied load



$$\delta = \delta_1 + \delta_2$$

$$\delta_1 = \frac{PL}{AE}$$

Where: $P = W = 7850(1/1000)3(9.81)[300(150)(1000)]$

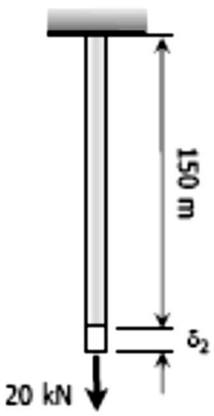
$P = 3465.3825$ N

$L = 75(1000) = 75\ 000$ mm

$A = 300$ mm^2

$E = 200\ 000$ MPa

$$\delta_1 = \frac{3465.3825\,(75000)}{300\,(200\ 000)} = 4.33 \text{ mm}$$



$$\delta_2 = \frac{PL}{AE}$$

Where: $P = 20$ kN $= 20\ 000$ N

$L = 150$ m $= 150\ 000$ mm

$A = 300$ mm^2

$E = 200\ 000$ MPa

$$\delta_2 = \frac{20000(150000)}{300(200000)} = 50 \text{ mm}$$

Total elongation:

$\delta = 4.33 + 50 =$ **54.33 mm**

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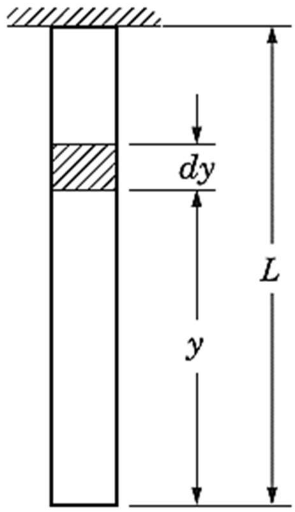
Example (8): Determine the total increase of length of a bar of constant cross section hanging vertically and subject to its own weight as the only load. The bar is initially straight.

SOLUTION: The normal stress (tensile) over any horizontal cross section is caused by the weight of the material below that section. The elongation of the element of thickness dy shown is

$$d\Delta = \frac{\gamma A y \, dy}{AE}$$



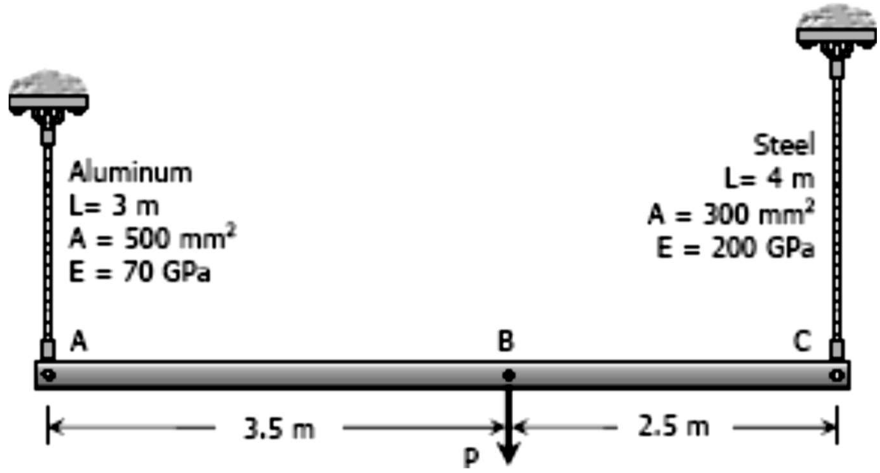
where A denotes the cross-sectional area of the bar and g its specific weight (weight/unit volume). Integrating, the total elongation of the bar is

$$\Delta = \int_0^L \frac{\gamma A y \, dy}{AE} = \frac{\gamma A}{AE}\frac{L^2}{2} = \frac{(\gamma AL)L}{2AE} = \frac{WL}{2AE}$$

where W denotes the total weight of the bar. Note that the total elongation produced by the weight of the bar is equal to that produced by a load of half its weight applied at the end.

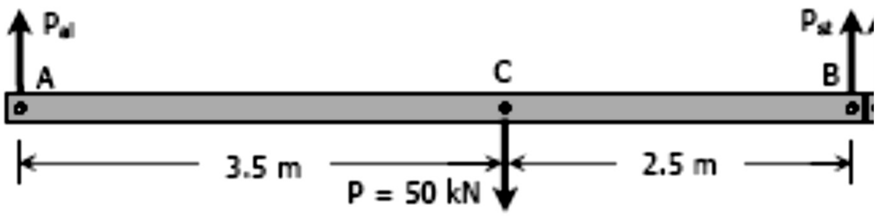
Example (9):

The rigid bar AB, attached to two vertical rods as shown, is horizontal before the load P is applied. Determine the vertical movement of P if its magnitude is 50 kN.



Solution:

Free body diagram:



For aluminum:

$[\Sigma M_B = 0]$ $\qquad 6P_{al} = 2.5(50)$

$$P_{al} = 20.83 \text{ kN}$$

$$\left[\delta = \frac{PL}{AE}\right]_{al} \qquad \delta_{al} = \frac{20.83(3)1000^2}{500(70000)}$$

$$\delta_{al} = 1.78 \text{ mm}$$

For steel:

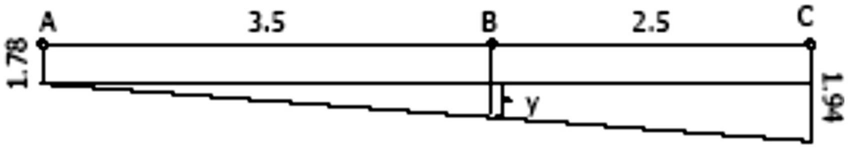
$[\Sigma M_A = 0]$ $\quad 6P_{st} = 3.5(50)$

$P_{st} = 29.17$ kN

$$\left[\delta = \frac{PL}{AE}\right]_{st} \qquad \delta_{st} = \frac{29.17(4)1000^2}{300(200000)}$$

$\delta_{st} = 1.94$ mm

Movement diagram:



$$\frac{y}{3.5} = \frac{1.94 - 1.78}{6}$$

$y = 0.09$ mm

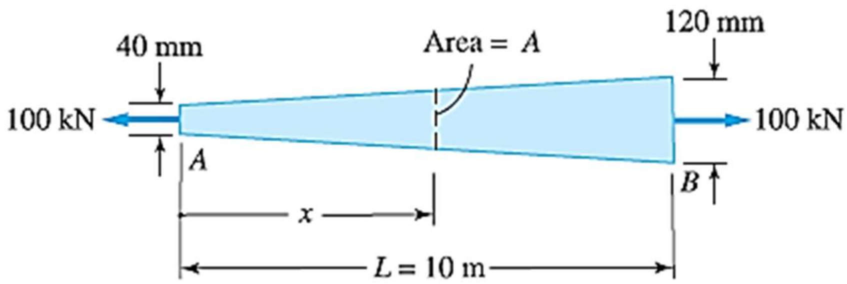
δ_B = vertical movement of P

$\delta_B = 1.78 + y = 1.78 + 0.09$

$\delta_B = \mathbf{1.87}$ **mm**

Example (10):

The cross section of the 10-m-long flat steel bar AB has a constant thickness of 20 mm, but its width varies as shown in the figure. Calculate the elongation of the bar due to the 100-kN axial load. Use E = 200 GPa for steel.



Solution

Equilibrium requires that the internal axial force P = 100 kN is constant along the entire length of the bar. However, the cross-sectional area A of the bar varies with the x-coordinate.

We start by determining A as a function of x. The cross-sectional areas at A and B are $A_A = 20 \times 40 = 800\ mm^2$ and $A_B = 20 \times 120 = 2400\ mm^2$. Between A and B the cross-sectional area is a linear function of x:

$$A = A_A + (A_B - A_A)\frac{x}{L} = 800\ \text{mm}^2 + (1600\ \text{mm}^2)\frac{x}{L}$$

Converting the areas from mm^2 to m^2 and substituting $L = 10$ m, we get

$$A = (800 + 160x) \times 10^{-6}\ \text{m}^2$$

$$\delta = \int_0^L \frac{P}{EA}\,dx = \int_0^{10\ \text{m}} \frac{100 \times 10^3}{(200 \times 10^9)[(800 + 160x) \times 10^{-6}]}\,dx$$

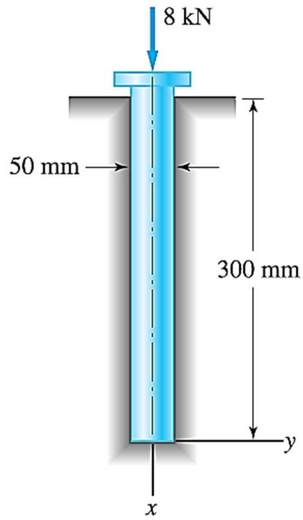
$$= 0.5 \int_0^{10\ \text{m}} \frac{dx}{800 + 160x} = \frac{0.5}{160}[\ln(800 + 160x)]_0^{10}$$

$$= \frac{0.5}{160} \ln\frac{2400}{800} = 3.43 \times 10^{-3}\ \text{m} = 3.43\ \text{mm}$$

Mechanics of Materials

Example (11):

The 50-mm-diameter rubber rod is placed in a hole with rigid, lubricated walls. There is no clearance between the rod and the sides of the hole. Determine the change in the length of the rod when the 8-kN load is applied. Use E = 40 MPa and ν = 0:45 for rubber.



Lubrication allows the rod to contract freely in the axial direction, so that the axial stress throughout the bar is

$$\sigma_x = -\frac{P}{A} = -\frac{8000}{\frac{\pi}{4}(0.05)^2} = -4.074 \times 10^6 \text{ Pa}$$

(the negative sign implies compression). Because the walls of the hole prevent transverse strain in the rod, we have $\epsilon_y = \epsilon_z = 0$. The tendency of the rubber to expand laterally (Poisson's effect) is resisted by the uniform contact pressure p between the walls and the rod, so that $\sigma_y = \sigma_z = -p$.

the condition $\epsilon_y = 0$ becomes

$$\frac{\sigma_y - \nu(\sigma_z + \sigma_x)}{E} = \frac{-p - \nu(-p + \sigma_x)}{E} = 0$$

which yields

$$p = -\frac{\nu\sigma_x}{1-\nu} = -\frac{0.45(-4.074 \times 10^6)}{1-0.45} = 3.333 \times 10^6 \text{ Pa}$$

The axial strain is given by the first of Eqs. (2.12):

$$\begin{aligned}\epsilon_x &= \frac{\sigma_x - \nu(\sigma_y + \sigma_z)}{E} = \frac{\sigma_x - \nu(-2p)}{E} \\ &= \frac{[-4.074 - 0.45(-2 \times 3.333)] \times 10^6}{40 \times 10^6} = -0.026\,86\end{aligned}$$

The corresponding change in the length of the rod is

$$\begin{aligned}\delta &= \epsilon_x L = -0.026\,86(300) \\ &= -8.06 \text{ mm} = 8.06 \text{ mm} \quad \text{(contraction)}\end{aligned}$$ *Answer*

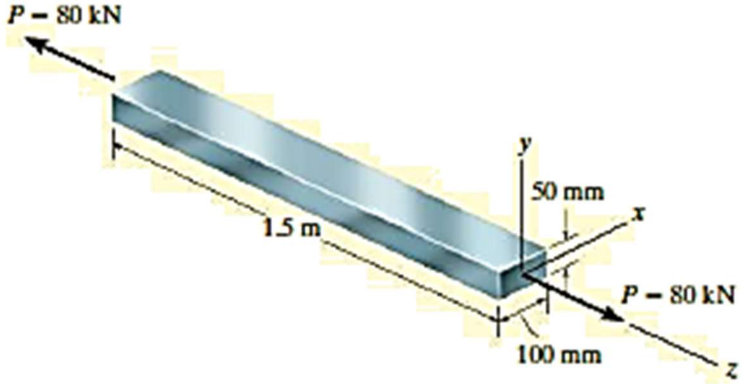
For comparison, note that if the constraining effect of the hole were neglected, the deformation would be

$$\delta = -\frac{PL}{EA} = -\frac{8000(0.3)}{(40 \times 10^6)\left[\frac{\pi}{4}(0.05)^2\right]} = -0.0306 \text{ m} = -30.6 \text{ mm}$$

Mechanics of Materials

Example (12):

A bar made of steel has the dimensions shown . If an axial force of $P = 80$ kN is applied to the bar, determine the change in its length and the change in the dimensions of its cross section after applying the load. The material behaves elastically.



$E_{st} = 200$ GPa,

SOLUTION

The normal stress in the bar is

$$\sigma_z = \frac{P}{A} = \frac{80(10^3)\text{ N}}{(0.1\text{ m})(0.05\text{ m})} = 16.0(10^6)\text{ Pa}$$

and so the strain in the z direction is

$$\epsilon_z = \frac{\sigma_z}{E_{st}} = \frac{16.0(10^6)\text{ Pa}}{200(10^9)\text{ Pa}} = 80(10^{-6})\text{ mm/mm}$$

The axial elongation of the bar is therefore

$$\delta_z = \epsilon_z L_z = [80(10^{-6})](1.5\text{ m}) = 120\ \mu\text{m}$$

Using Eq. 3–9, where $\nu_{st} = 0.32$ as found from the inside back cover, the lateral contraction strains in *both* the x and y directions are

$$\epsilon_x = \epsilon_y = -\nu_{st}\,\epsilon_z = -0.32[80(10^{-6})] = -25.6\ \mu\text{m/m}$$

Thus the changes in the dimensions of the cross section are

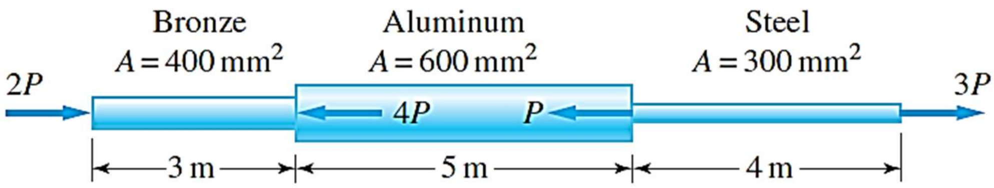
$$\delta_x = \epsilon_x L_x = -[25.6(10^{-6})](0.1\text{ m}) = -2.56\ \mu\text{m}$$

$$\delta_y = \epsilon_y L_y = -[25.6(10^{-6})](0.05\text{ m}) = -1.28\ \mu\text{m}$$

Mechanics of Materials

H.W-1

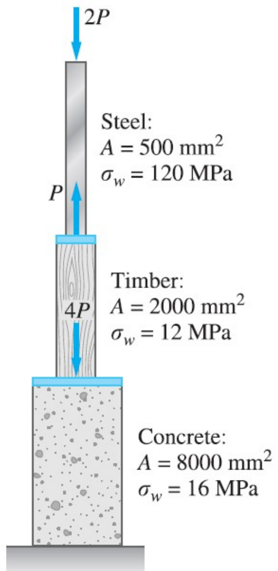
1. Axial loads are applied to the compound rod that is composed of an aluminum segment rigidly connected between steel and bronze segments. What is the stress in each material given that P= 10 kN?



Ans.: $\sigma_{br} = 50$ MPa (C), $\sigma_{al} = 33.3$ MPa (T), $\sigma_{st} = 100$ MPa (T)

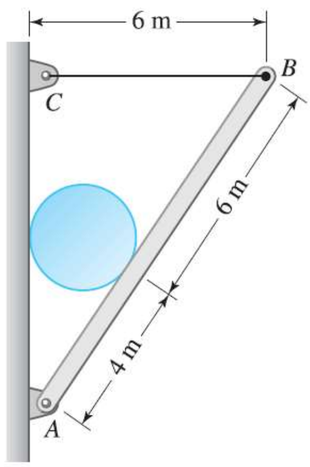
2. Find the maximum allowable value of P for the column. The cross-sectional areas and working stresses (σ_w) are shown in the figure.

Ans.: P=24 kN



3. Determine the weight of the heaviest uniform cylinder that can be supported in the position shown without exceeding a stress of 50 MPa in cable BC. Neglect friction and the weight of bar AB: The cross-sectional area of BC is 100 mm^2

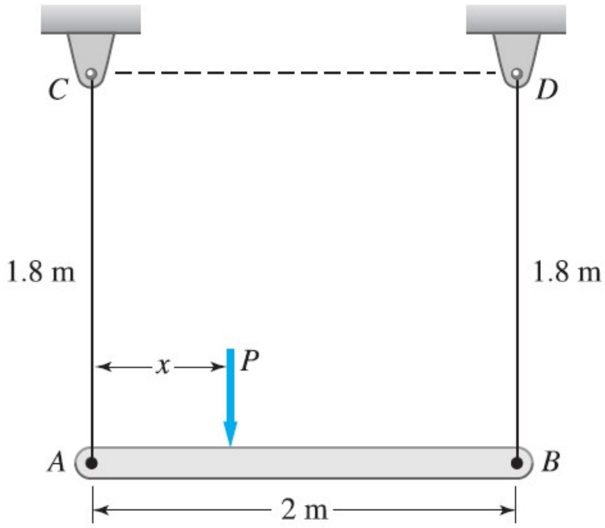
Ans: W= 6 kN



Mechanics of Materials

4. The 1000-kg uniform bar AB is suspended from two cables AC and BD; each with cross-sectional area 400 mm^2. Find the magnitude P and location x of the largest additional vertical force that can be applied to the bar. The stresses in AC and BD are limited to 100 MPa and 50 MPa, respectively.

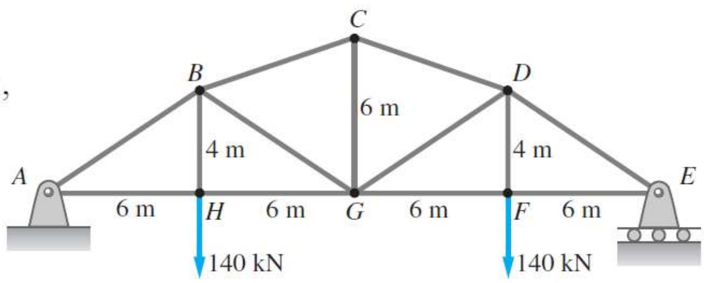
Ans.: $P = 50.2$ kN, $x = 602$ mm



5. Determine the smallest safe cross-sectional areas of members CD, GD, and GF for the truss shown. The working stresses are 140 MPa in tension and 100 MPa in compression. (***The working stress in compression is smaller to reduce the danger of buckling***.)

Ans.:

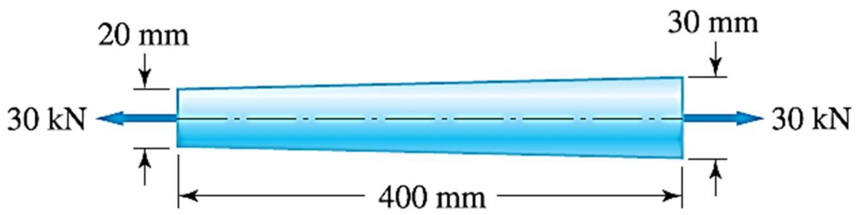
$A_{CD} = 1476$ mm^2, $A_{GD} = 841$ mm^2,
$A_{GF} = 1500$ mm^2



Mechanics of Materials

6.

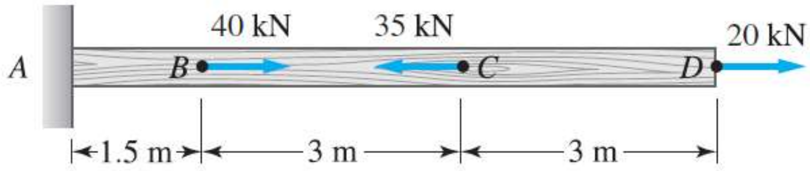
Determine the elongation of the tapered cylindrical aluminum bar caused by the 30-kN axial load. Use $E = 72$ GPa.



Ans.: 0.354 mm

7.

The timber member has a cross-sectional area of 1750 mm^2 and its modulus of elasticity is 12 GPa. Compute the change in the total length of the member after the loads shown are applied.

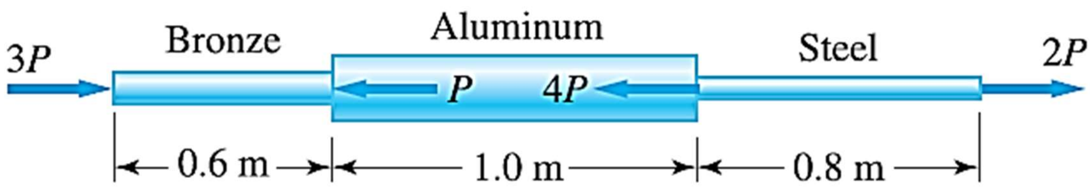


Ans.: 2.50 mm

8.

A compound bar consisting of bronze, aluminum, and steel segments is loaded axially as shown in the figure. Determine the maximum allowable value of P if the change in length of the bar is limited to 2 mm and the working stresses prescribed in the table are not to be exceeded.

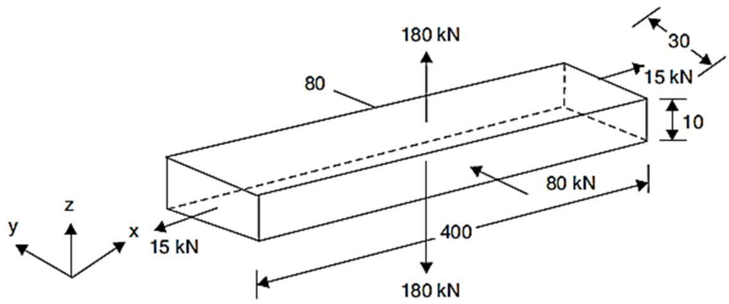
	A (mm^2)	E (GPa)	σ_w (MPa)
Bronze	450	83	120
Aluminum	600	70	80
Steel	300	200	140



Ans.: 18 kN

9. A 400 mm long bar has rectangular cross-section 10 mm × 30 mm. This bar is subjected to (i) 15 kN tensile force on 10 mm × 30 mm faces, (ii) 80 kN compressive force on 10 mm × 400 mm faces, and (iii) 180 kN tensile force on 30 mm × 400 mm faces. Find the change in volume if E = 2 × 105 N/mm^2 and υ = 0.3

.



Mechanics of Materials

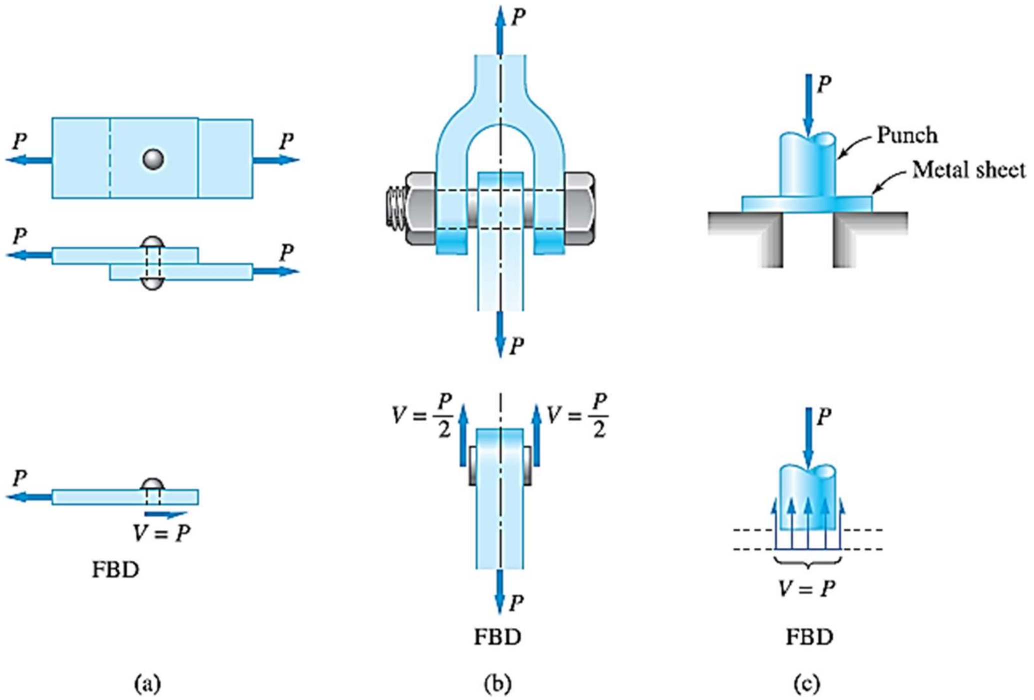
Shear Stresses

By definition, normal stress acting on an interior plane is directed perpendicular to that plane. Shear stress, on the other hand, is tangent to the plane on which it acts. Shear stress arises whenever the applied loads cause one section of a body to slide past its adjacent section. In chapter (1), we examined how shear stress occurs in an axially loaded bar. Three other examples of shear stress are illustrated in below. Figure below shows two plates that are joined by a rivet. As seen in the FBD, the rivet must carry the shear force V = P. Because only one cross section of the rivet resists the shear, the rivet is said to be in single shear. The bolt of the clevis in Fig.(b) carries the load P across two cross-sectional areas, the shear force being V = P=2 on each cross section. Therefore, the bolt is said to be in a state of double shear. In Fig. (c) a circular slug is being punched out of a metal sheet. Here the shear force is P and the shear area is similar to the milled edge of a coin. The loads shown are sometimes referred to as direct shear to distinguish them from the induced shear illustrated.

The distribution of direct shear stress is usually complex and not easily determined. It is common practice to assume that the shear force V is uniformly distributed over the shear area A, so that the shear stress can be computed from

$$\boxed{\tau = \frac{V}{A}}$$

Mechanics of Materials

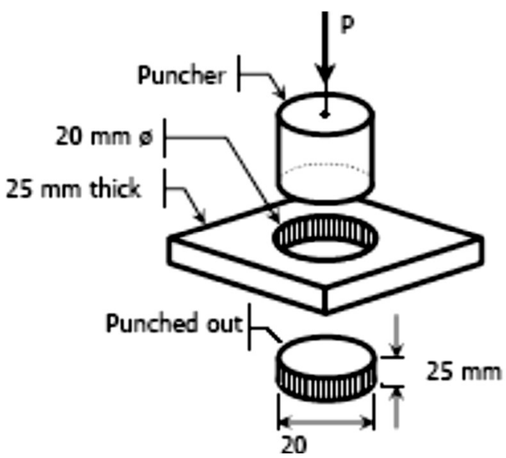


Examples of direct shear: (a) single shear in a rivet; (b) double shear in a bolt; and (c) shear in a metal sheet produced by a punch.

EXAMPLES

EXAMPLE (1):

What force is required to punch a 20-mm-diameter hole in a plate that is 25 mm thick? The shear strength is 350 MN/m^2.



The resisting area is the shaded area along the perimeter and the shear force V is equal to the punching force P.

$$V = \tau A$$

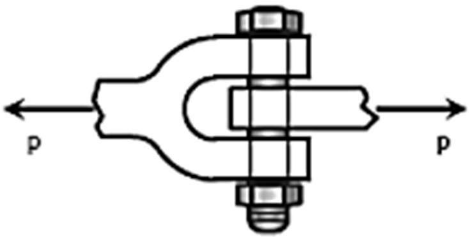
$$P = 350[\pi(20)(25)]$$

$$= 549\,778.7 \text{ N}$$

$$= \mathbf{549.8 \text{ kN}}$$

EXAMPLE (2):

Find the smallest diameter bolt that can be used in the clevis shown if P = 400 kN. The shearing strength of the bolt is 300 MPa.



The bolt is subject to double shear.

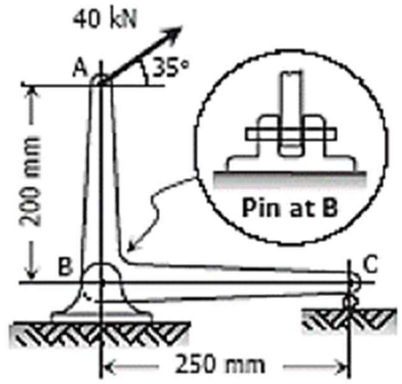
$V = \tau A$

$400(1000) = 300[2(\frac{1}{4}\pi d^2)]$

$d = 29.13$ mm

EXAMPLE (3):

Compute the shearing stress in the pin at B for the member supported as shown. The pin diameter is 20 mm.

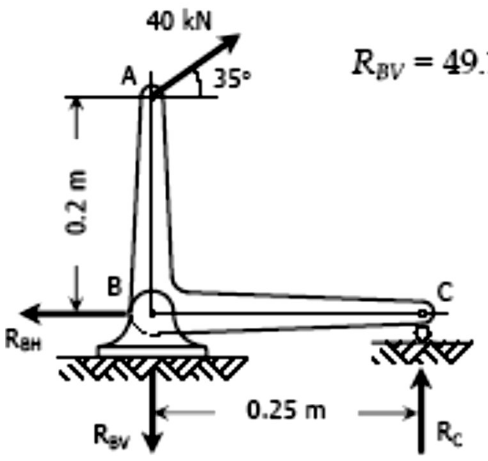


From the FBD:

$\Sigma M_C = 0$

$0.25R_{BV} = 0.25(40 \sin 35°)$
$\quad + 0.2(40 \cos 35°)$

$R_{BV} = 49.156$ kN



Free Body Diagram

$\Sigma F_H = 0$

$R_{BH} = 40 \cos 35°$
$\quad = 32.766$ kN

$R_B = \sqrt{R_{BH}^2 + R_{BV}^2}$

$= \sqrt{32.766^2 + 49.156^2}$

$= 59.076$ kN → shear force of pin at B

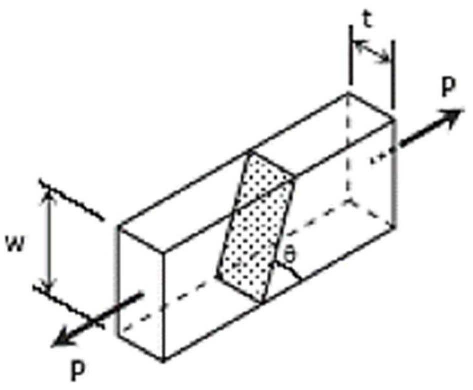
$V_B = \tau_B A$ → double shear

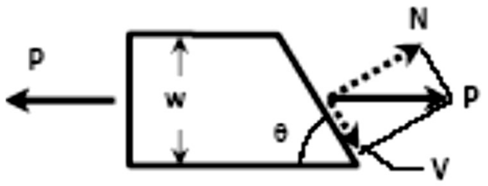
$59.076\,(1000) = \tau_B\{2[\frac{1}{4}\pi(20^2)]\}$

$\tau_B = \mathbf{94.02\ MPa}$

EXAMPLE (4):

Two blocks of wood, width w and thickness t, are glued together along the joint inclined at the angle θ as shown. Using the free-body diagram concept in Fig. 1-4a, show that the shearing stress on the glued joint is $\tau = P \sin 2\theta / 2A$, where A is the cross-sectional area.





Shear area, $A_{shear} = t\,(w \csc\theta)$
$= tw \csc\theta$
$= A \csc\theta$

Shear force, $V = P\cos\theta$

$V = \tau A_{shear}$

$P\cos\theta = \tau(A\csc\theta)$

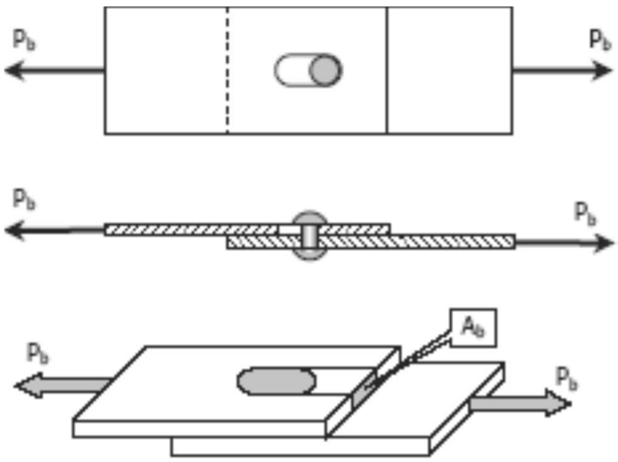
$\tau = P\sin\theta\cos\theta / A$

$= P\,(2\sin\theta\cos\theta) / 2A$

$= P\sin 2\theta / 2A$ *(ok!)*

1- Bearing Stress

Bearing stress is the contact pressure between the separate bodies. It differs from compressive stress, as it is an internal stress caused by compressive forces.

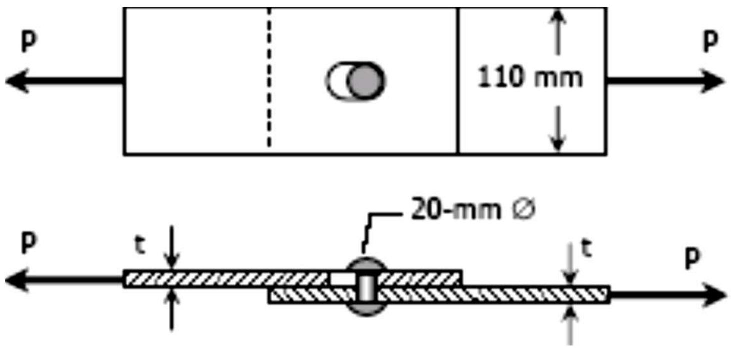


$$\sigma_b = \frac{P_b}{A_b}$$

EXAMPLES

EXAMPLE (1):

In Fig. below, assume that a 20-mm-diameter rivet joins the plates that are each 110 mm wide. The allowable stresses are 120 MPa for bearing in the plate material and 60 MPa for shearing of rivet. Determine (a) the minimum thickness of each plate; and (b) the largest average tensile stress in the plates.



(a) From shearing of rivet:

$P = \tau A_{rivets}$

$= 60[\frac{1}{4}\pi(20^2)]$

$= 6000\pi$ N

From bearing of plate material:

$P = \sigma_b A_b$

$6000\pi = 120(20t)$

$t = 7.85$ **mm**

(b) Largest average tensile stress in the plate:

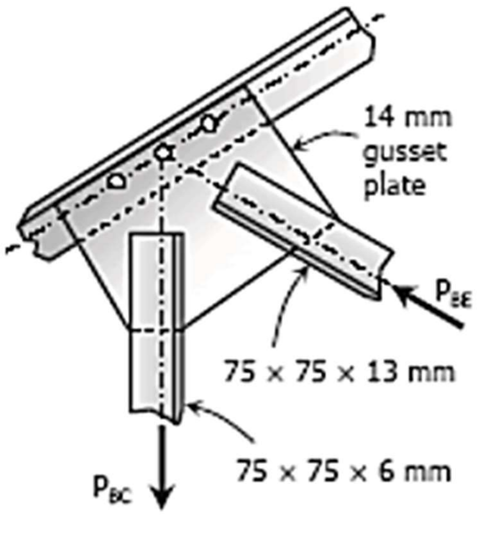
$P = \sigma A$

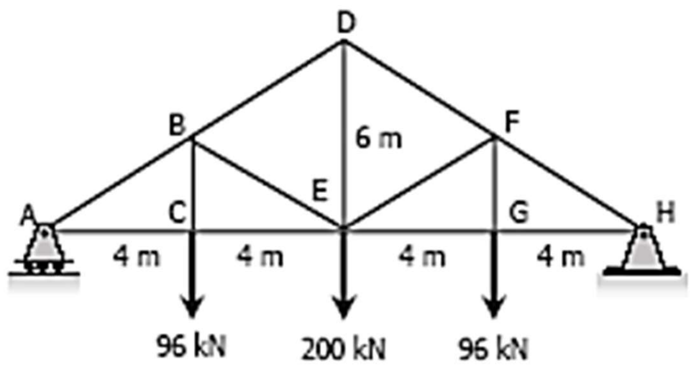
$6000\pi = \sigma[7.85(110 - 20)]$

$\sigma = 26.67$ **MPa**

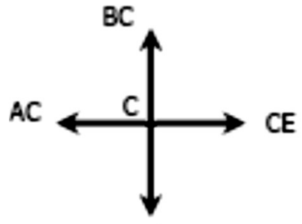
EXAMPLE (2):

Figure below shows a roof truss and the detail of the riveted connection at joint B. Using allowable stresses of $\tau = 70$ MPa and $\sigma b = 140$ MPa, how many 19-mm diameter rivets are required to fasten member BC to the gusset plate? Member BE? What is the largest average tensile or compressive stress in BC and BE?





Mechanics of Materials

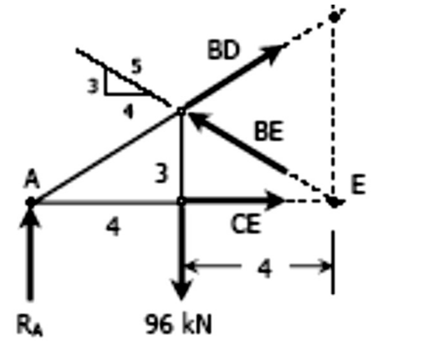


Joint C

At Joint *C*:

$\Sigma F_V = 0$

$BC = 96$ kN (Tension)



Section through BD, BE, and CE

Consider the section through member *BD*, *BE*, and *CE*:

$\Sigma M_A = 0$

$8(\frac{3}{5} BE) = 4(96)$

$BE = 80$ kN (Compression)

For Member *BC*:

Based on shearing of rivets:

$BC = \tau A$

Where A = area of 1 rivet × number of rivets, n

$96\,000 = 70[\frac{1}{4}\pi(19^2)n]$

$n = 4.8$ say 5 rivets

Based on bearing of member:

$BC = \sigma_b A_b$

Where A_b = diameter of rivet × thickness of BC × number of rivets, n

$96\,000 = 140[19(6)n]$

$n = 6.02$ say 7 rivets

use 7 rivets for member *BC*

Mechanics of Materials

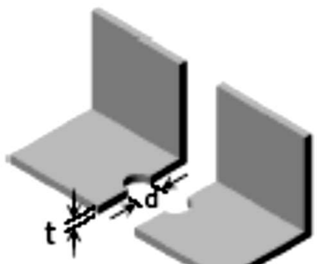
For member *BE*:
Based on shearing of rivets:
$BE = \tau A$
Where A = area of 1 rivet × number of rivets, n
$80\,000 = 70[\frac{1}{4}\pi(19^2)n]$
$n = 4.03$ say 5 rivets

Based on bearing of member:
$BE = \sigma_b A_b$
Where A_b = diameter of rivet × thickness of BE × number of rivets, n
$80\,000 = 140[19(13)n]$
$n = 2.3$ say 3 rivets

use 5 rivets for member *BE*

Relevant data from the table (Appendix B of textbook): *Properties of Equal Angle Sections: SI Units*

Designation	Area
L75 × 75 × 6	864 mm^2
L75 × 75 × 13	1780 mm^2



t = thickness of member
d = diameter of rivet hole

Note:
A = Area – dt

Tensile stress of member *BC* (L75 × 75 × 6):

$$\sigma = \frac{P}{A} = \frac{96(1000)}{864 - 19(6)}$$

σ = 128 Mpa

Compressive stress of member *BE* (L75 × 75 × 13):

$$\sigma = \frac{P}{A} = \frac{80(1000)}{1780}$$

σ = 44.94 Mpa

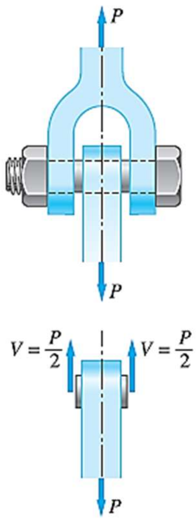
Mechanics of Materials

H.W.2

1- What force is required to punch a 20-mm-diameter hole in a plate that is 25 mm thick? The shear strength of the plate is 350 MN/m^2.

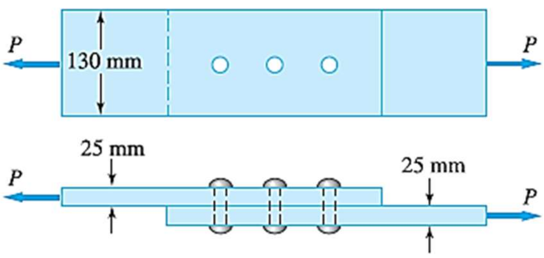
ANS.: 550 kN

2- Find the smallest diameter bolt that can be used in the clevis 11(b) if P = 400 kN. The working shear stress for the bolt is 300 MPa.



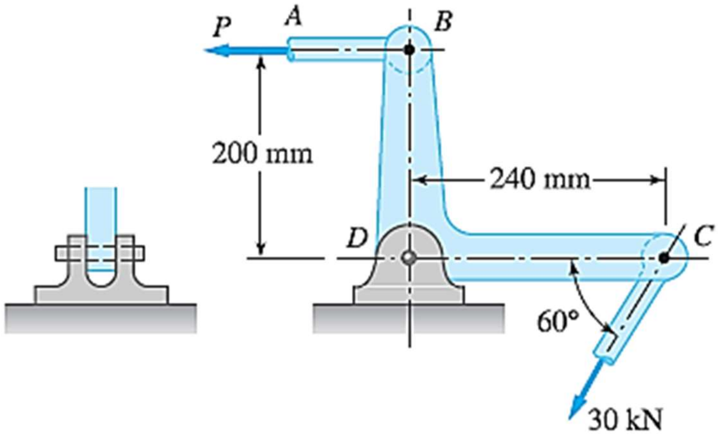
ANS.: 29.1 mm

3- The lap joint is connected by three 20-mm-diameter rivets. Assuming that the axial load P = 50 kN is distributed equally among the three rivets, find (a) the shear stress in a rivet; (b) the bearing stress between a plate and a rivet; and (c) the maximum average tensile stress in each plate.



ANS.: (a) 53.1 MPa; (b) 33.3 MPa; (c) 18.18 MPa

4- The bell crank, which is in equilibrium under the forces shown in the figure, is supported by a 20-mm-diameter pin at D that is in double shear. Determine (a) the required diameter of the connecting rod AB, given that its tensile working stress is 100 MPa; and (b) the shear stress in the pin.



ANS.: (a) 19.92 mm; (b) 84.3 MPa

Mechanics of Materials

Statically Indeterminate Problems

If the equilibrium equations are sufficient to calculate all the forces (including support reactions) that act on a body, these forces are said to be ***statically determinate***. In statically determinate problems, the number of unknown forces is always equal to the number of independent equilibrium equations. If the number of unknown forces exceeds the number of independent equilibrium equations, the problem is said to be ***statically indeterminate***.

Static indeterminacy does not imply that the problem cannot be solved; it simply means that the solution cannot be obtained from the equilibrium equations alone. A statically indeterminate problem always has geometric restrictions imposed on its deformation. The mathematical expressions of these restrictions, known as the compatibility equations, provide us with the additional equations needed to solve the problem (the term compatibility refers to the geometric compatibility between deformation and the imposed constraints). Because the source of the compatibility equations is deformation, these equations contain as unknowns either strains or elongations. We can, however, use Hooke's law to express the deformation measures in terms of stresses or forces. The equations of equilibrium and compatibility can then be solved for the unknown forces.

Procedure for Solving Statically Indeterminate Problems

In summary, the solution of a statically indeterminate problem involves the following steps:

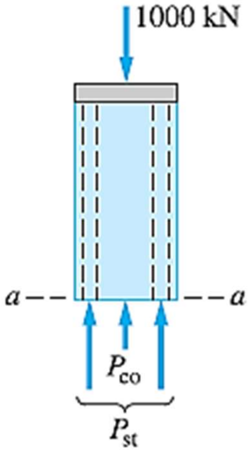
- Draw the required free-body diagrams and derive the equations of
- equilibrium.
- Derive the compatibility equations. To visualize the restrictions on deformation, it is often helpful to draw a sketch that exaggerates the magnitudes of the deformations.
- Use Hooke's law to express the deformations (strains) in the compatibility equations in terms of forces (or stresses).
- Solve the equilibrium and compatibility equations for the unknown forces.

EXAMPLES

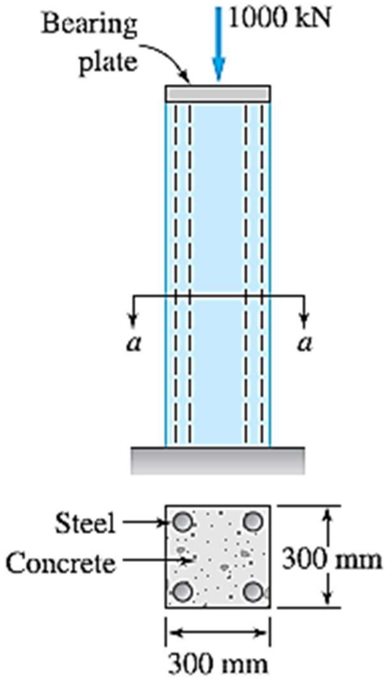
EXAMPLE (1):

The concrete post is reinforced axially with four symmetrically placed steel bars, each of cross-sectional area 900 mm^2. Compute the stress in each material when the 1000-kN axial load is applied. The moduli of elasticity are 200 GPa for steel and 14 GPa for concrete.

SOLUTION:



(b) FBD



Section *a-a*

Equilibrium The FBD in Fig. (b) was drawn by isolating the portion of the post above section *a-a*, where P_{co} is the force in concrete and P_{st} denotes the total force carried by the steel rods. For equilibrium, we must have

$$\Sigma F = 0 \quad +\uparrow \quad P_{st} + P_{co} - 1.0 \times 10^6 = 0$$

which, written in terms of stresses, becomes

$$\sigma_{st} A_{st} + \sigma_{co} A_{co} = 1.0 \times 10^6 \text{ N} \tag{a}$$

Equation (a) is the only independent equation of equilibrium that is available in this problem. Because there are two unknown stresses, we conclude that the problem is statically indeterminate.

Compatibility For the deformations to be compatible, the changes in lengths of the steel rods and the concrete must be equal; that is, $\delta_{st} = \delta_{co}$. Because the lengths of steel and concrete are identical, the compatibility equation, written in terms of strains, is

$$\epsilon_{st} = \epsilon_{co} \tag{b}$$

Hooke's Law From Hooke's law, Eq. (b) becomes

$$\frac{\sigma_{st}}{E_{st}} = \frac{\sigma_{co}}{E_{co}} \tag{c}$$

Equations (a) and (c) can now be solved for the stresses. From Eq. (c) we obtain

$$\sigma_{st} = \frac{E_{st}}{E_{co}}\sigma_{co} = \frac{200}{14}\sigma_{co} = 14.286\sigma_{co} \tag{d}$$

Substituting the cross-sectional areas

$$A_{st} = 4(900 \times 10^{-6}) = 3.6 \times 10^{-3}\ \text{m}^2$$

$$A_{co} = 0.3^2 - 3.6 \times 10^{-3} = 86.4 \times 10^{-3}\ \text{m}^2$$

and Eq. (d) into Eq. (a) yields

$$(14.286\sigma_{co})(3.6 \times 10^{-3}) + \sigma_{co}(86.4 \times 10^{-3}) = 1.0 \times 10^6$$

Solving for the stress in concrete, we get

$$\sigma_{co} = 7.255 \times 10^6\ \text{Pa} = 7.255\ \text{MPa}$$ *Answer*

From Eq. (d), the stress in steel is

$$\sigma_{st} = 14.286(7.255) = 103.6\ \text{MPa}$$ *Answer*

EXAMPLE (2):

Let the allowable stresses in the post described in example (1) be σ_{st}=120MPa and σ_{co} =6 MPa. Compute the maximum safe axial load P that may be applied.

Solution

From Eq. (d) in example (1), we see that equal strains require the following relationship between the stresses:

$$\sigma_{st} = 14.286\sigma_{co}$$

Therefore, if the concrete were stressed to its limit of 6 MPa, the corresponding stress in the steel would be

$$\sigma_{st} = 14.286(6) = 85.72 \text{ MPa}$$

which is below the allowable stress of 120 MPa. The maximum safe axial load is thus found by substituting $\sigma_{co} = 6$ MPa and $\sigma_{st} = 85.72$ MPa into the equilibrium equation:

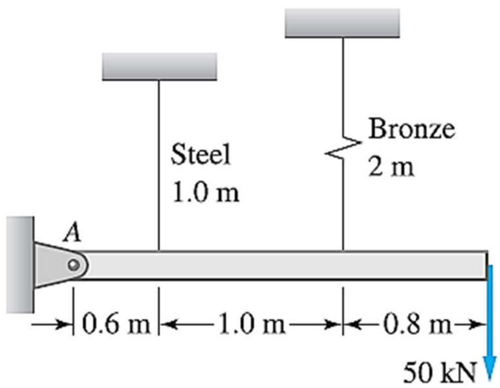
$$\begin{aligned} P &= \sigma_{st}A_{st} + \sigma_{co}A_{co} \\ &= (85.72 \times 10^6)(3.6 \times 10^{-3}) + (6 \times 10^6)(86.4 \times 10^{-3}) \\ &= 827 \times 10^3 \text{ N} = 827 \text{ kN} \end{aligned}$$

Answer

Example (3):

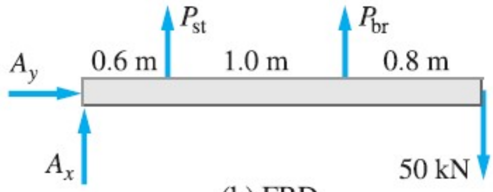
Figure (a) shows a rigid bar that is supported by a pin at A and two rods, one made of steel and the other of bronze. Neglecting the weight of the bar, compute the stress in each rod caused by the 50-kN load, using the following data:

	Steel	Bronze
Area (mm^2)	600	300
E (GPa)	200	83

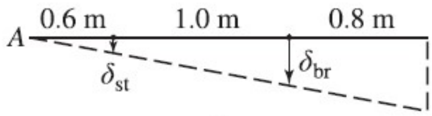


(a)

Solution:



(b) FBD



(c)

Equilibrium The free-body diagram of the bar, shown in Fig. (b), contains four unknown forces. Since there are only three independent equilibrium equations, these forces are statically indeterminate. The equilibrium equation that does not involve the pin reactions at A is

$$\Sigma M_A = 0 \quad +\circlearrowleft \quad 0.6P_{st} + 1.6P_{br} - 2.4(50 \times 10^3) = 0 \tag{a}$$

Compatibility The displacement of the bar, consisting of a rigid-body rotation about A, is shown greatly exaggerated in Fig. (c). From similar triangles, we see that the elongations of the supporting rods must satisfy the compatibility condition

$$\frac{\delta_{st}}{0.6} = \frac{\delta_{br}}{1.6} \tag{b}$$

Hooke's Law When we substitute $\delta = PL/(EA)$ into Eq. (b), the compatibility equation becomes

$$\frac{1}{0.6}\left(\frac{PL}{EA}\right)_{st} = \frac{1}{1.6}\left(\frac{PL}{EA}\right)_{br}$$

Using the given data, we obtain

$$\frac{1}{0.6}\frac{P_{st}(1.0)}{(200)(600)} = \frac{1}{1.6}\frac{P_{br}(2)}{(83)(300)}$$

which simplifies to

$$P_{st} = 3.614P_{br} \tag{c}$$

Note that we did not convert the areas from mm^2 to m^2, and we omitted the factor 10^9 from the moduli of elasticity. Since these conversion factors appear on both sides of the equation, they would cancel out.

Solving Eqs. (a) and (c), we obtain

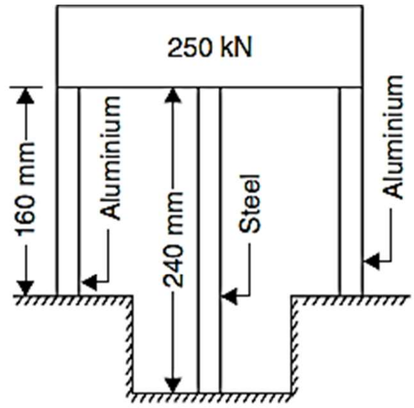
$$P_{st} = 115.08 \times 10^3 \text{ N} \qquad P_{br} = 31.84 \times 10^3 \text{ N}$$

The stresses are

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{115.08 \times 10^3}{600 \times 10^{-6}} = 191.8 \times 10^6 \text{ Pa} = 191.8 \text{ MPa} \qquad \textit{Answer}$$

$$\sigma_{br} = \frac{P_{br}}{A_{br}} = \frac{31.84 \times 10^3}{300 \times 10^{-6}} = 106.1 \times 10^6 \text{ Pa} = 106.1 \text{ MPa} \qquad \textit{Answer}$$

Example (4): Three pillars, two of Aluminum and one of steel support a rigid platform of 250 kN as shown. If area of each Aluminum pillar is 1200 mm^2 and that of steel pillar is 1000 mm^2, find the stresses developed in each pillar.



Solution: Let force shared by each aluminium pillar be P_a and that shared by steel pillar be P_s.

∴ The forces in vertical direction = 0 →

$$P_a + P_s + P_a = 250$$

$$2P_a + P_s = 250 \quad \text{...(1)}$$

From compatibility condition, we get

$$\Delta_s = \Delta_a$$

$$\frac{P_s L_s}{A_s E_s} = \frac{P_a L_a}{A_a E_a}$$

$$\frac{P_s \times 240}{1000 \times 2 \times 10^5} = \frac{P_a \times 160}{1200 \times 1 \times 10^5}$$

$$\therefore \quad P_s = 1.111\ P_a \quad \text{...(2)}$$

From eqns. (1) and (2), we get

$$P_a\ (2 + 1.111) = 250$$

$$\therefore \quad P_a = 80.36 \text{ kN}$$

Hence from eqn. (1),

$$P_s = 250 - 2 \times 80.36 = 89.28 \text{ kN}$$

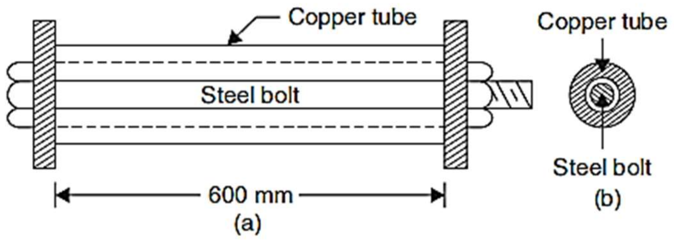
∴ Stresses developed are

$$\sigma_s = \frac{P_s}{A_s} = \frac{89.28 \times 1000}{1000} = \mathbf{89.28\ N/mm^2}$$

$$\sigma_a = \frac{80.36 \times 1000}{1200} = \mathbf{66.97\ N/mm^2}$$

Example (5): A steel bolt of 20 mm diameter passes centrally through a copper tube of internal diameter 28 mm and external diameter 40 mm. The length of whole assembly is 600 mm. After tight fitting of the assembly, the nut is over tightened by quarter of a turn. What are the stresses introduced in the bolt and tube, if pitch of nut is 2 mm?

Es = 2 × 105 N/mm^2 and Ec = 1.2 × 105 N/mm^2.



Solution: Let the force shared by bolt be Ps and that by tube be Pc. Since there is no external force, static equilibrium condition gives Ps + Pc = 0 or Ps = – Pc i.e., the two forces are equal in magnitude but opposite in nature. Obviously bolt is in tension and tube is in compression.

Let the magnitude of force be P. Due to quarter turn of the nut, the nut advances by $\frac{1}{4}$ × pitch $= \frac{1}{4} \times 2 = 0.5$ mm.

[**Note.** Pitch means advancement of nut in one full turn]

During this process bolt is extended and copper tube is shortened due to force P developed. Let Δ_s be extension of bolt and Δ_c shortening of copper tube. Final position of assembly be Δ, then

$$\Delta_s + \Delta_c = \Delta$$

i.e.
$$\frac{P_s L_s}{A_s E_s} + \frac{P_c L_c}{A_c E_c} = 0.5$$

$$\frac{P \times 600}{(\pi/4) \times 20^2 \times 2 \times 10^5} + \frac{P \times 600}{(\pi/4)(40^2 - 28^2) \times 1.2 \times 10^5} = 0.5$$

$$\frac{P \times 600}{(\pi/4) \times 10^5}\left[\frac{1}{20^2 \times 2} + \frac{1}{(40^2 - 28^2) \times 1.2}\right] = 0.5$$

$\therefore$
$$P = 28816.8 \text{ N}$$

$\therefore$
$$p_s = \frac{P_s}{A_s} = \frac{28816.8}{(\pi/4) \times 20^2} = \mathbf{91.72\ N/mm^2}$$

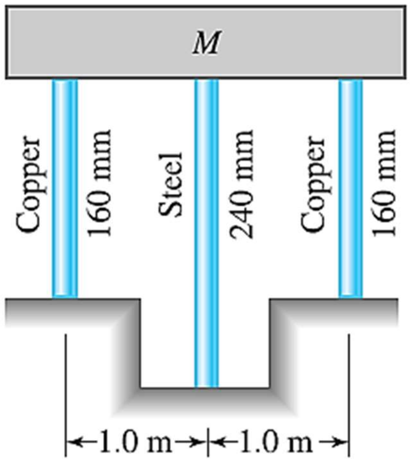
$$p_c = \frac{P_c}{A_c} = \frac{28816.8}{(\pi/4)(40^2 - 28^2)} = \mathbf{44.96\ N/mm^2}$$

Mechanics of Materials

H.W.3

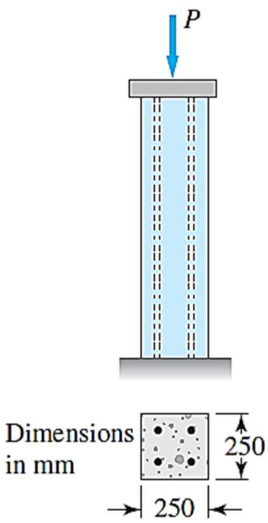
1. The rigid block of mass M is supported by the three symmetrically placed rods. The ends of the rods were level before the block was attached. Determine the largest allowable value of M if the properties of the rods are as listed (σ_w is the working stress):

	E (GPa)	A (mm^2)	σ_w (MPa)
Copper	120	900	70
Steel	200	1200	140

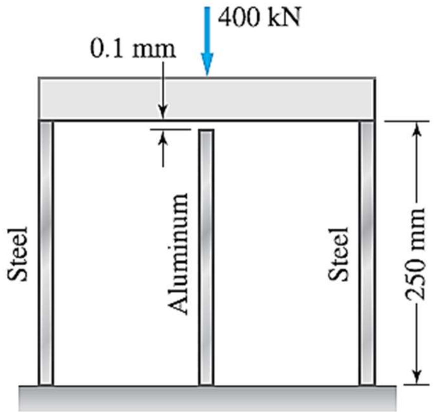


2. The concrete column is reinforced by four steel bars of total cross-sectional area 1250 mm^2. If the working stresses for steel and concrete are 180 MPa and 15 MPa, respectively, determine the largest axial force P that can be safely applied to the column. E_{st}=200 GPa and E_{co}=24 GPa.

ANS.: P=1.075 MN

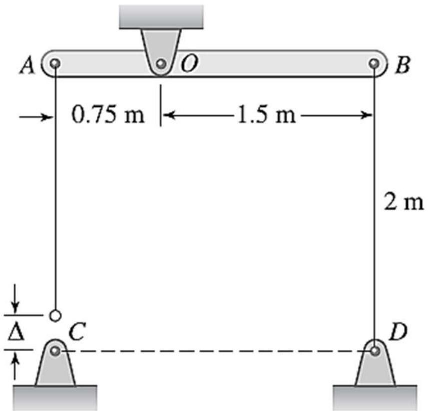


3. Before the 400-kN load is applied, the rigid platform rests on two steel bars, each of cross-sectional area 1400 mm2, as shown in the figure. The cross-sectional area of the aluminum bar is 2800 mm2. Compute the stress in the aluminum bar after the 400-kN load is applied. Use E = 200 GPa for steel and E = 70 GPa for aluminum. Neglect the weight of the platform.



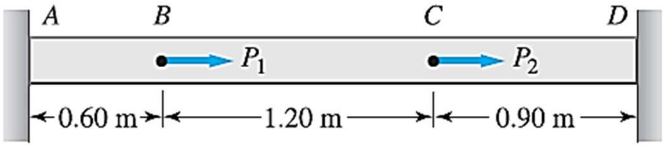
ANS.: 16.30 MPa (C)

4. The rigid bar AB of negligible weight is supported by a pin at O. When the two steel rods are attached to the ends of the bar, there is a gap Δ between the lower end of the left rod and its pin support at C. After attachment, the strain in the left rod is 1.5×10^{-3}. What is the length of the gap Δ? The cross-sectional areas are 300 mm^2 for rod AC and 250 mm^2 for rod BD. Use E = 200 GPa for steel.



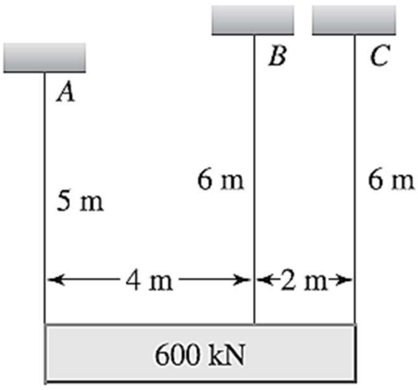
ANS.: 3.9mm

5. The homogeneous bar with a cross-sectional area of 600 mm2 is attached to rigid supports. The bar carries the axial loads P1 = 20 kN and P2 = 60 kN, as shown. Determine the stress in segment BC.



ANS.: 25.9 MPa (T)

6. The rigid, homogeneous slab weighing 600 kN is supported by three rods of identical material and cross section. Before the slab was attached, the lower ends of the rods were at the same level. Compute the axial force in each rod.



ANS.: PA= 239 kN, PB = 184.2 kN, PC = 177.2 kN

Thermal Stresses

It is well known that changes in temperature cause dimensional changes in a body: An increase in temperature results in expansion, whereas a temperature decrease produces contraction. This deformation is isotropic (the same in every direction) and proportional to the temperature change. It follows that the associated strain, called thermal strain, is

$$\boxed{\epsilon_T = \alpha(\Delta T)}$$

where the constant α is a material property known as the coefficient of thermal expansion, and ΔT is the temperature change. The coefficient of thermal expansion represents the normal strain caused by a one-degree change in temperature. By convention, ΔT is taken to be positive when the temperature increases, and negative when the temperature decreases. Thus, positive ΔT produces positive strain (elongation) and negative ΔT produces negative strain (contraction). The units of α are 1/°C .

> Typical values of α are $23 \times 10^{-6}/°C$ ($13 \times 10^{-6}/°F$) for aluminum and $12 \times 10^{-6}/°C$ ($6.5 \times 10^{-6}/°F$) for steel.

If the temperature change is uniform throughout the body, the thermal strain is also uniform. Consequently, the change in any dimension L of the body is given by

$$\boxed{\delta_T = \epsilon_T L = \alpha(\Delta T)L}$$

If thermal deformation is permitted to occur freely (by using expansion joints or roller supports, for example), no internal forces will be induced in the body—there will be strain, but no stress. In cases where the deformation of a body is restricted, either totally or partially, internal forces will develop that oppose the thermal expansion or contraction. The stresses caused by these internal forces are known as thermal stresses.

The forces that result from temperature changes cannot be determined by equilibrium analysis alone; that is, these forces are statically indeterminate.

Consequently, the analysis of thermal stresses follows the same principles that previously used: equilibrium, compatibility, and Hooke's law. The only deference here is that we must now include thermal expansion in the analysis of deformation.

Procedure for Deriving Compatibility Equations

The following procedure for deriving the equations of compatibility are:

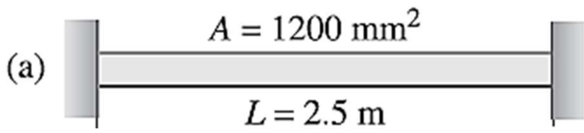
- Remove the constraints that prevent the thermal deformation to occur freely (this procedure is sometimes referred to as ''relaxing the supports'').
- Show the thermal deformation on a sketch using an exaggerated scale.
- Apply the forces that are necessary to restore the specified conditions of constraint. Add the deformations caused by these forces to the sketch that was drawn in the previous step. (Draw the magnitudes of the deformations so that they are compatible with the geometric constraints.).
- By inspection of the sketch, write the relationships between the thermal deformations and the deformations due to the constraint forces.

Mechanics of Materials

EXAMPLES

Example (1): The horizontal steel rod, 2.5 m long and 1200 mm^2 in cross-sectional area, is secured between two walls as shown. If the rod is stress-free at 20/°C, compute the stress when the temperature has dropped to -20/°C. Assume that (1) the walls do not move and (2) the walls move together a distance Δ= 0.5 mm. Use $\alpha = 11.7 \times 10^{-6}$/°C and E=200 GPa.

Solution:



Part 1

Compatibility We begin by assuming that the rod has been disconnected from the right wall, as shown in Fig. (b), so that the contraction δ_T caused by the temperature drop ΔT can occur freely. To reattach the rod to the wall, we must stretch the rod to its original length by applying the tensile force P. Compatibility of deformations requires that the resulting elongation δ_P, shown in Fig. (c), must be equal to δ_T; that is,

$$\delta_T = \delta_P$$

Hooke's Law If we substitute $\delta_T = \alpha(\Delta T)L$ and $\delta_P = PL/(EA) = \sigma L/E$, the compatibility equation becomes

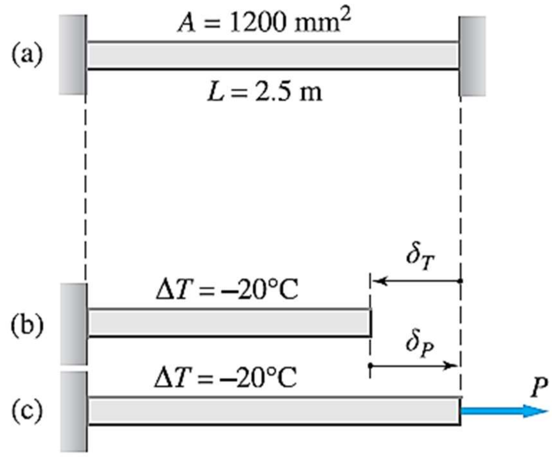
$$\frac{\sigma L}{E} = \alpha(\Delta T)L$$

Therefore, the stress in the rod is

$$\sigma = \alpha(\Delta T)E = (11.7 \times 10^{-6})(40)(200 \times 10^9)$$

$$= 93.6 \times 10^6 \text{ Pa} = 93.6 \text{ MPa}$$ *Answer*

Note that L canceled out in the preceding equation, which indicates that the stress is independent of the length of the rod.



Mechanics of Materials

Part 2

Compatibility When the walls move together a distance Δ, we see from Figs. (d) and (e) that the free thermal contraction δ_T is related to Δ and the elongation δ_P caused by the axial force P by

$$\delta_T = \delta_P + \Delta$$

Hooke's Law Substituting for δ_T and δ_P as in Part 1, we obtain

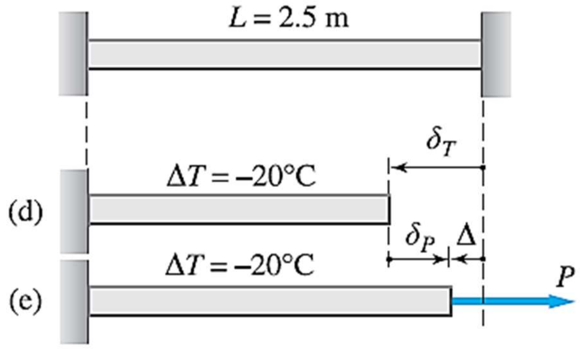
$$\alpha(\Delta T)L = \frac{\sigma L}{E} + \Delta$$

The solution for the stress σ is

$$\begin{aligned}\sigma &= E\left[\alpha(\Delta T) - \frac{\Delta}{L}\right]\\ &= (200 \times 10^9)\left[(11.7 \times 10^{-6})(40) - \frac{0.5 \times 10^{-3}}{2.5}\right]\\ &= 53.6 \times 10^6 \text{ Pa} = 53.6 \text{ MPa}\end{aligned}$$

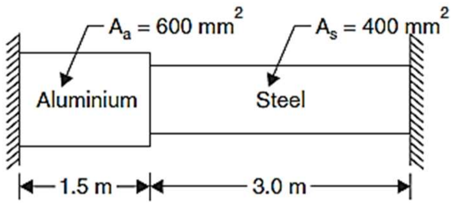
Answer

We see that the movement of the walls reduces the stress considerably. Also observe that the length of the rod does not cancel out as in Part 1.



Example 2: The composite bar shown is rigidly fixed at the ends A and B. Determine the reaction developed at ends when the temperature is raised by 18°C. Given

$$E_a = 70\ kN/mm^2$$
$$E_s = 200\ kN/mm^2$$
$$\alpha_a = 11 \times 10^{-6}/°C$$
$$\alpha_s = 12 \times 10^{-6}/°C$$



(a)

Solution: Free expansion $= \alpha_a tL_a + \alpha_s tL_s$

$$= 11 \times 10^{-6} \times 18 \times 1500 + 12 \times 10^{-6} \times 18 \times 3000$$
$$= 0.945\ \text{mm}$$

Since this is prevented

$$\Delta = 0.945\ \text{mm}.$$
$$E_a = 70\ \text{kN/mm}^2 = 70000\ \text{N/mm}^2\ ;$$
$$E_s = 200\ \text{kN/mm}^2 = 200 \times 1000\ \text{N/mm}^2$$

If P is the support reaction,

$$\Delta = \frac{PL_a}{A_a E_a} + \frac{PL_s}{A_s E_s}$$

i.e.

$$0.945 = P\left[\frac{1500}{600 \times 70000} + \frac{3000}{400 \times 200 \times 1000}\right]$$
$$0.945 = 73.214 \times 10^{-6}\ P$$

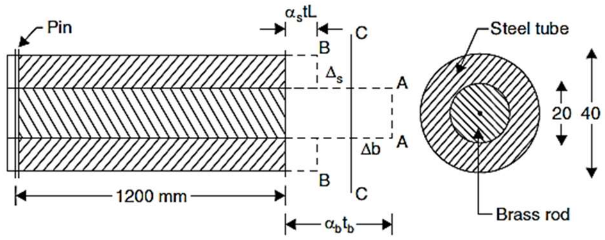
or

$$\mathbf{P = 12907.3\ N}$$

EXAMPLE (3): A bar of brass 20 mm is enclosed in a steel tube of 40 mm external diameter and 20 mm internal diameter. The bar and the tubes are initially 1.2 m long and are rigidly fastened at both ends using 20 mm diameter pins. If the temperature is raised by 60°C, find the stresses induced in the bar, tube and pins.

Given:

$$E_s = 2 \times 10^5 \ N/mm^2$$
$$E_b = 1 \times 10^5 \ N/mm^2$$
$$\alpha_s = 11.6 \times 10^{-6}/°C$$
$$\alpha_b = 18.7 \times 10^{-6}/°C.$$



$$t = 60° \quad E_s = 2 \times 10^5 \text{ N/mm}^2 \quad E_b = 1 \times 10^5 \text{ N/mm}^2$$
$$\alpha_s = 11.6 \times 10^{-6}/°\text{C} \quad \alpha_b = 18.7 \times 10^{-6}/°\text{C}$$

$$A_s = \frac{\pi}{4}(40^2 - 20^2) \quad A_b = \frac{\pi}{4} \times 20^2$$
$$= 942.48 \text{ mm}^2 \qquad = 314.16 \text{ mm}^2$$

Since free expansion of brass ($\alpha_b \, tL$) is more than free expansion of steel ($\alpha_s \, tL$), compressive force P_b develops in brass and tensile force P_s develops in steel to keep the final position at CC

Horizontal equilibrium condition gives $P_b = P_s$, say P. From the figure, it is clear that

$$\Delta_s + \Delta_b = \alpha_b \, tL - \alpha_s tL = (\alpha_b - \alpha_s)tL$$

where Δ_s and Δ_b are the changes in length of steels and brass bars.

$$\therefore \quad \frac{PL}{A_s E_s} + \frac{PL}{A_b E_b} = (18.7 - 11.6) \times 10^{-6} \times 60 \times 1200.$$

$$P \times 1200 \left[\frac{1}{942.48 \times 2 \times 10^5} + \frac{1}{314.16 \times 1 \times 10^5}\right] = 7.1 \times 10^{-6} \times 60 \times 1200$$

$$\therefore \quad P = 11471.3 \text{ N}$$

$$\therefore \quad \textbf{Stress in steel} = \frac{P}{A_s} = \frac{11471.3}{942.48} = \mathbf{12.17 \ N/mm^2}$$

and

$$\textbf{Stress in brass} = \frac{P}{A_b} = \frac{11471.3}{314.16} = \mathbf{36.51 \ N/mm^2}$$

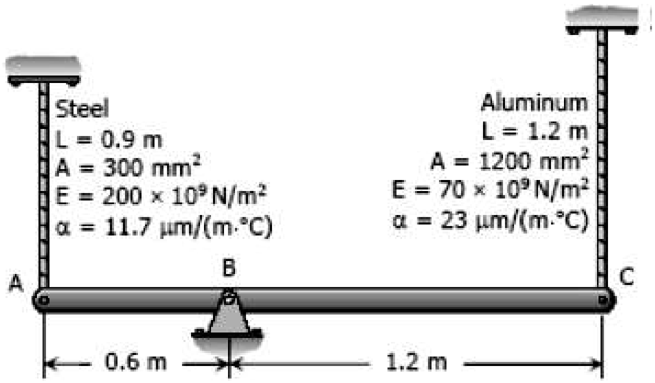
The pin resist the force P at the two cross-sections at junction of two bars.

$$\therefore \quad \textbf{Shear stress in pin} = \frac{P}{2 \times \text{Area of pin}}$$

$$= \frac{11471.3}{2 \times \pi/4 \times 20^2} = \mathbf{18.26 \ N/mm^2}$$

EXAMPLE (4):

The rigid bar ABC is pinned at B and attached to the two vertical rods. Initially, the bar is horizontal and the vertical rods are stress-free. Determine the stress in the aluminum rod if the temperature of the steel rod is decreased by 40°C. Neglect the weight of bar ABC.



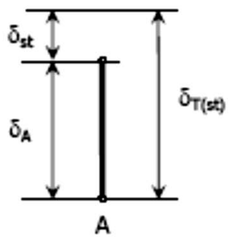
SOLUTION:

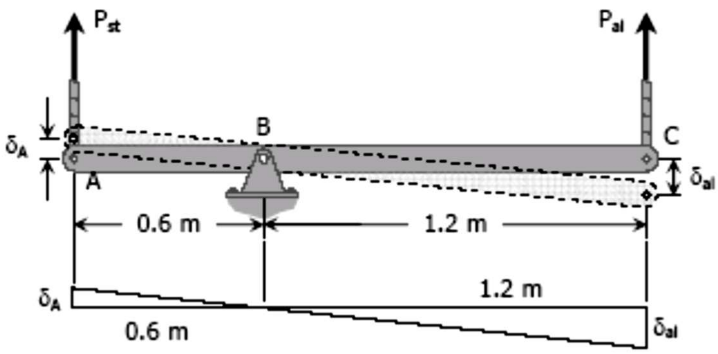
Contraction of steel rod, assuming complete freedom:

$$\delta_{T(st)} = \alpha L\,\Delta T$$
$$= (11.7 \times 10^{-6})(900)(40)$$
$$= 0.4212 \text{ mm}$$

The steel rod cannot freely contract because of the resistance of aluminum rod. The movement of A (referred to as δ_A), therefore, is less than 0.4212 mm. In terms of aluminum, this movement is (by ratio and proportion):

$$\frac{\delta_A}{0.6} = \frac{\delta_{al}}{1.2}$$
$$\delta_A = 0.5\delta_{al}$$





$\delta_{T(st)} - \delta_{st} = 0.5\ \delta_{al}$

$$0.4212 - \left(\frac{PL}{AE}\right)_{st} = 0.5\left(\frac{PL}{AE}\right)_{al}$$

$$0.4212 - \frac{P_{st}(900)}{300(200000)} = 0.5\left[\frac{P_{al}(1200)}{1200(70000)}\right]$$

$28080 - P_{st} = 0.4762P_{al}$ → Equation (1)

$\sum M_B = 0$

$0.6P_{st} = 1.2P_{al}$

$P_{st} = 2P_{al}$ → Equation (2)

Equations (1) and (2)

$28080 - 2P_{al} = 0.4762P_{al}$

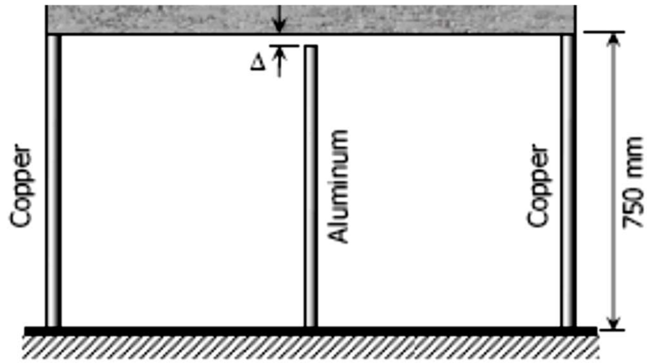
$P_{al} = 11340\ \text{N}$

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{11340}{1200}$$

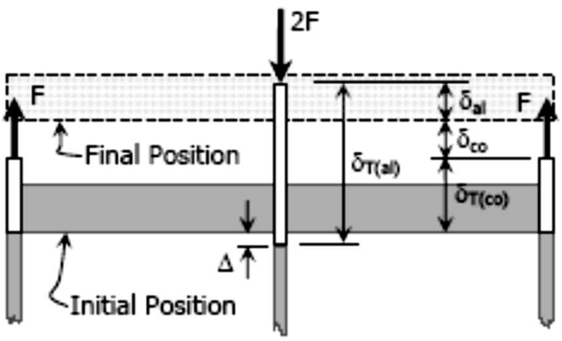
$\sigma_{al} = \mathbf{9.45\ MPa}$

Example (5):

As shown , there is a gap between the aluminum bar and the rigid slab that is supported by two copper bars. At 10°C, Δ = 0.18 mm. Neglecting the mass of the slab, calculate the stress in each rod when the temperature in the assembly is increased to 95°C. For each copper bar, A= 500 mm², E = 120 GPa, and α = 16.8 μm/(m·°C). For the aluminum bar, A = 400 mm², E = 70 GPa, and α = 23.1 μm/(m·°C).



Solution:



Assuming complete freedom:

$$\delta_T = \alpha L\,\Delta T$$

$$\delta_{T(co)} = (16.8 \times 10^{-6})(750)(95 - 10) = 1.071 \text{ mm}$$

$$\delta_{T(al)} = (23.1 \times 10^{-6})(750 - 0.18)(95 - 10) = 1.472 \text{ mm}$$

From the figure:

$$\delta_{T(al)} - \delta_{al} = \delta_{T(co)} + \delta_{co}$$

$$1.472 - \left(\frac{PL}{AE}\right)_{al} = 1.071 + \left(\frac{PL}{AE}\right)_{co}$$

$$1.472 - \frac{2F(750 - 0.18)}{400(70000)} = 1.071 + \frac{F(750)}{500(120000)}$$

$$0.401 = (6.606 \times 10^{-5})\,F$$

$$F = 6070.37 \text{ N}$$

$$P_{co} = F = 6070.37 \text{ N}$$

$$P_{al} = 2F = 12\,140.74 \text{ N}$$

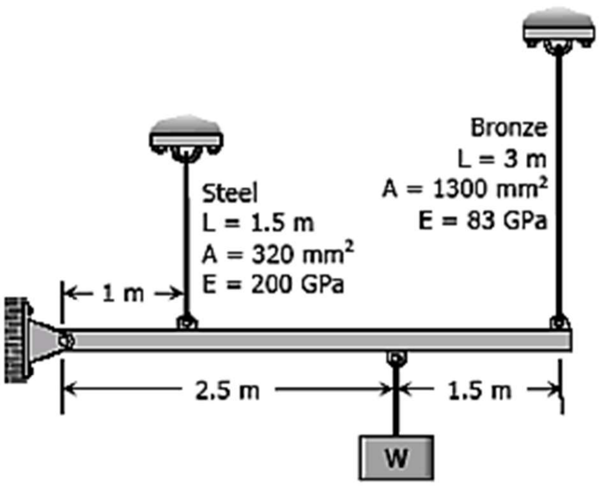
$$\sigma = P/A$$

$$\sigma_{co} = \frac{6070.37}{500} = \mathbf{12.14\ MPa}$$

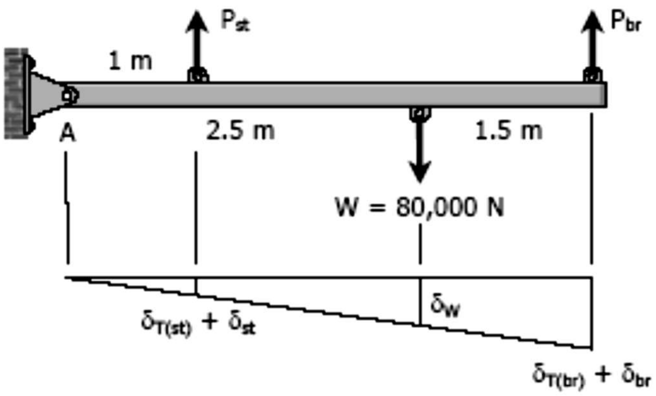
$$\sigma_{al} = \frac{12\,140.74}{400} = \mathbf{30.35\ MPa}$$

Example (6):

A rigid bar of negligible weight is supported as shown . If W = 80 kN, compute the temperature change that will cause the stress in the steel rod to be 55 MPa. Assume the coefficients of linear expansion are 11.7 μm/(m·°C) for steel and 18.9 μm / (m·°C) for bronze.



Solution:



$$\Sigma M_A = 0$$
$$4P_{br} + P_{st} = 2.5(80000)$$
$$4\sigma_{br}(1300) + 55(320) = 2.5(80000)$$
$$\sigma_{br} = 35.08 \text{ MPa}$$

By ratio and proportion:

$$\frac{\delta_{T(st)} + \delta_{st}}{1} = \frac{\delta_{T(br)} + \delta_{br}}{4}$$
$$\delta_{T(st)} + \delta_{st} = 0.25[\delta_{T(br)} + \delta_{br}]$$
$$(\alpha L\,\Delta T)_{st} + \left(\frac{\sigma L}{E}\right)_{st} = 0.25\left[(\alpha L\,\Delta T)_{br} + \left(\frac{\sigma L}{E}\right)_{br}\right]$$
$$(11.7\times10^{-6})(1500)\,\Delta T + \frac{55(1500)}{2000} = 0.25\left[(18.9\times10^{-6})(3000)\,\Delta T + \frac{35.08(3000)}{83000}\right]$$
$$0.017\,55\,\Delta T + 0.4125 = 0.014\,175\,\Delta T + 0.317$$
$$\Delta T = -28.3\ °C$$

A temperature drop of 28.3 °C is needed to stress the steel to 55 MPa.

Example (7):

For the assembly shown , find the stress in each rod if the temperature rises 30°C after a load W = 120 kN is applied.

Solution:

$$\Sigma M_A = 0$$
$$4P_{br} + P_{st} = 2.5(80000)$$
$$4\sigma_{br}(1300) + \sigma_{st}(320) = 2.5(80000)$$
$$16.25\sigma_{br} + \sigma_{st} = 625$$
$$\sigma_{st} = 625 - 16.25\sigma_{br} \qquad \rightarrow \text{Equation (1)}$$

$$\frac{\delta_{T(st)} + \delta_{st}}{1} = \frac{\delta_{T(br)} + \delta_{br}}{4}$$
$$\delta_{T(st)} + \delta_{st} = 0.25[\delta_{T(br)} + \delta_{br}]$$

$$(\alpha L\, \Delta T)_{st} + \left(\frac{\sigma L}{E}\right)_{st} = 0.25\left[(\alpha L\, \Delta T)_{br} + \left(\frac{\sigma L}{E}\right)_{br}\right]$$

$$(11.7 \times 10^{-6})(1500)(30) + \frac{\sigma_{st}(1500)}{200000}$$
$$= 0.25\left[(18.9 \times 10^{-6})(3000)(30) + \frac{\sigma_{br}(3000)}{83000}\right]$$

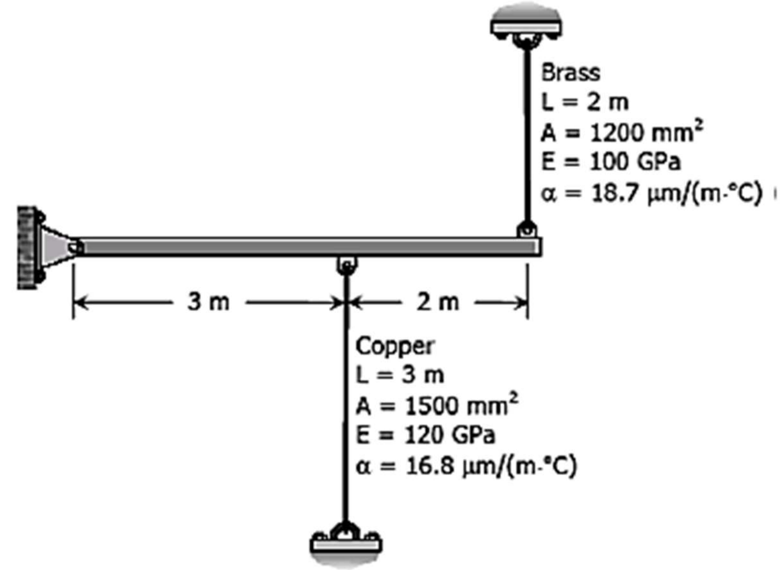
$$0.5265 + 0.0075\sigma_{st} = 0.425\,25 + 0.00904\sigma_{br}$$
$$0.0075\sigma_{st} - 0.00904\sigma_{br} = -0.10125$$
$$0.0075(625 - 16.25\sigma_{br}) - 0.00904\sigma_{br} = -0.10125$$
$$4.6875 - 0.121\,875\sigma_{br} - 0.009\,04\sigma_{br} = -0.101\,25$$
$$4.788\,75 = 0.130\,915\sigma_{br}$$
$$\sigma_{br} = \mathbf{36.58\ °C}$$

$$\sigma_{st} = 625 - 16.25(36.58)$$
$$\sigma_{st} = \mathbf{30.58\ °C}$$

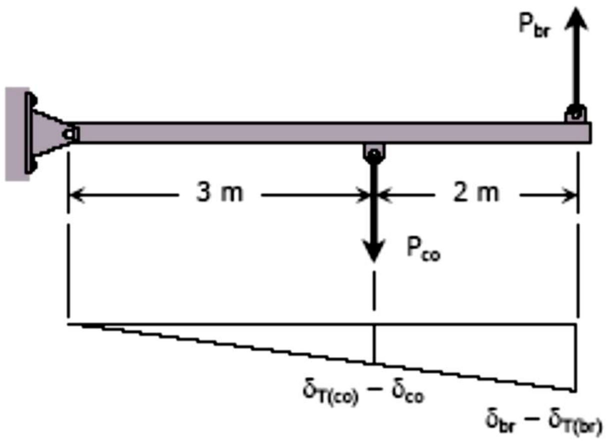
Example (8):

A rigid horizontal bar of negligible mass is connected to two rods as shown

If the system is initially stress-free. Calculate the temperature change that will cause a tensile stress of 90 MPa in the brass rod. Assume that both rods are subjected to the change in temperature.



Solution:



$\Sigma M_{\text{hinge support}} = 0$

$5P_{br} - 3P_{co} = 0$

$5\sigma_{br} A_{br} - 3\sigma_{co} A_{co} = 0$

$5(90)(1200) - 3\sigma_{co}(1500) = 0$

$\sigma_{co} = 120 \text{ MPa}$

$\delta = \sigma L / E$

$\delta_{br} = 90(2000) / 100\,000 = 1.8 \text{ mm}$

$\delta_{co} = 120(3000) / 120\,000 = 3 \text{ mm}$

$$\frac{\delta_{T(co)} - \delta_{co}}{3} = \frac{\delta_{br} - \delta_{T(br)}}{5}$$

$5\delta_{T(co)} - 5\delta_{co} = 3\delta_{br} - 3\delta_{T(br)}$

$5(16.8\times10^{-6})(3000)\,\Delta T - 5(3)$

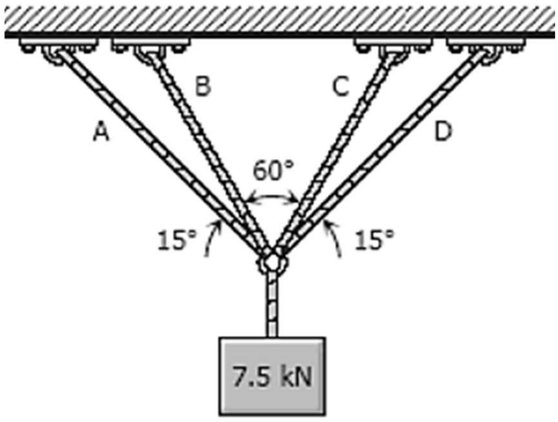
$= 3(1.8) - 3(18.7\times10^{-6})(2000)\,\Delta T$

$0.3642\,\Delta T = 20.4$

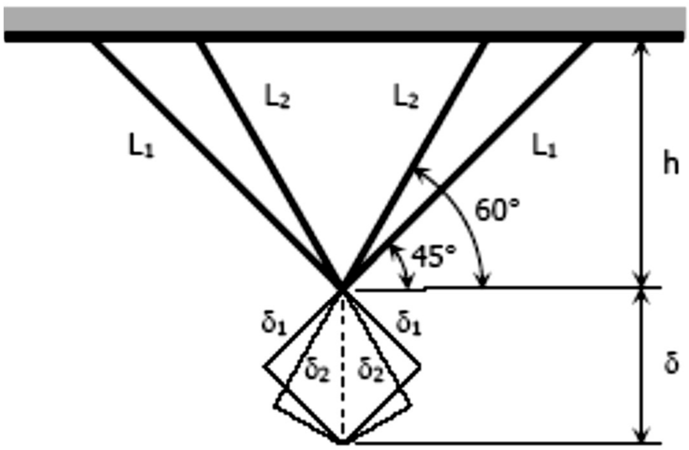
$\Delta T = 56.01°C$ drop in temperature

Example (9):

Four steel bars jointly support a mass of 15 Mg as shown . Each bar has a cross-sectional area of 600 mm2. Find the load carried by each bar after a temperature rise of 50°C. Assume α = 11.7 μm/(m·°C) and E = 200 GPa.



Solution:



$$h = L_1 \sin 45^\circ$$
$$h = L_2 \sin 60^\circ$$

$$h = h$$
$$L_1 \sin 45^\circ = L_2 \sin 60^\circ$$
$$L_1 = 1.2247 L_2$$

$$\delta_1 = \delta \sin 45^\circ$$
$$\delta_2 = \delta \sin 60^\circ$$
$$\frac{\delta_1}{\delta_2} = \frac{\delta \sin 45^\circ}{\delta \sin 60^\circ}$$
$$\delta_1 = 0.8165 \delta_2$$

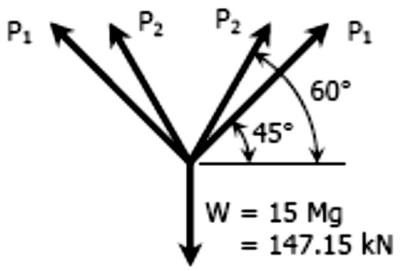
Mechanics of Materials

$$\alpha L_1 \, \Delta T + \frac{P_1 L_1}{AE} = 0.8165\left[\alpha L_2 \, \Delta T + \frac{P_2 L_2}{AE}\right]$$

$$(11.7 \times 10^{-6}) L_1 \,(50) + \frac{P_1 L_1}{600(200000)}$$

$$= 0.8165\left[(11.7 \times 10^{-6}) L_2 \,(50) + \frac{P_2 L_2}{600(200000)}\right]$$

$$70{,}200 L_1 + P_1 L_1 = 0.8165(70{,}200 L_2 + P_2 L_2)$$
$$(70{,}200 + P_1) L_1 = 0.8165(70{,}200 + P_2) L_2$$
$$(70{,}200 + P_1) 1.2247 \cancel{L_2} = 0.8165(70{,}200 + P_2) \cancel{L_2}$$
$$1.5(70{,}200 + P_1) = 70{,}200 + P_2$$
$$P_2 = 1.5 P_1 + 35{,}100 \qquad \rightarrow \text{Equation (1)}$$



$$\Sigma F_V = 0$$
$$2(P_1 \sin 45°) + 2(P_2 \sin 60°) = 147.15(1000)$$
$$P_1 \sin 45° + P_2 \sin 60° = 72{,}575$$
$$P_1 \sin 45° + (1.5 P_1 + 35{,}100) \sin 60° = 72{,}575$$
$$0.7071 P_1 + 1.299 P_1 + 30{,}397.49 = 72{,}575$$
$$2.0061 P_1 = 42{,}177.51$$
$$P_1 = 21{,}024.63 \text{ N}$$

$$P_2 = 1.5(21{,}024.63) + 35{,}100$$
$$P_2 = 66{,}636.94 \text{ N}$$

$$P_A = P_D = P_1 = \mathbf{21.02 \text{ kN}}$$
$$P_B = P_C = P_2 = \mathbf{66.64 \text{ kN}}$$

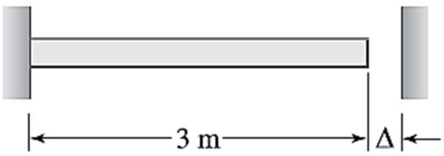
Mechanics of Materials

H.W.4

1-

The bronze bar 3 m long with a cross-sectional area of 350 mm^2 is placed between two rigid walls. At a temperature of $-20°C$, there is a gap $\Delta = 2.2$ mm, as shown in the figure. Find the temperature at which the compressive stress in the bar will be 30 MPa. Use $\alpha = 18.0 \times 10^{-6}/°C$ and $E = 80$ GPa.

ANS.: 41.6/^{O}C



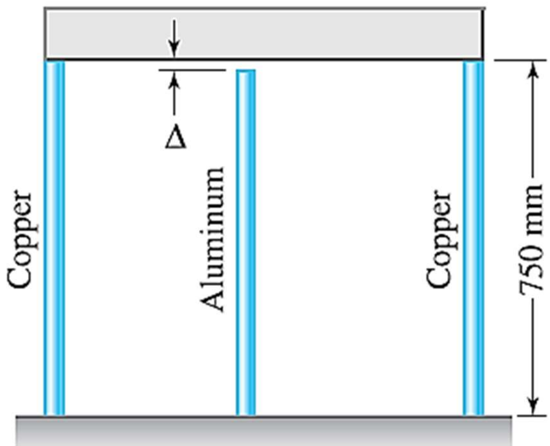
2-

The rigid, horizontal slab is attached to two identical copper rods. There is a gap $\Delta = 0.18$ mm between the middle bar, which is made of aluminum, and the slab. Neglecting the mass of the slab, calculate the stress in each rod when the temperature in the assembly is increased by 85°C. Use the following data:

	A (mm^2)	α (/°C)	E (GPa)
Each copper rod	500	16.8×10^{-6}	120
Aluminum rod	400	23.1×10^{-6}	70

ANS.:

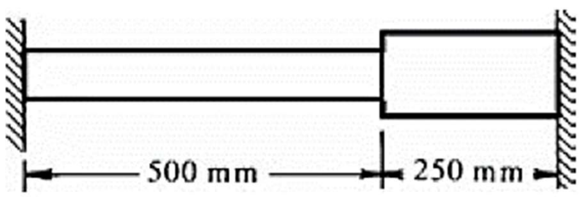
$\sigma_{cu} = 6.71$ MPa (T), $\sigma_{al} = 16.77$ MPa (C)



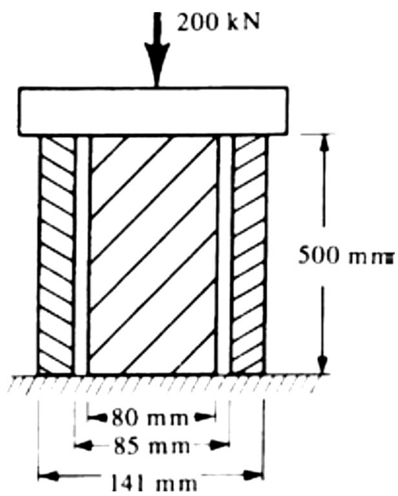
3- Two bars are joined together and attached to supports as shown. The left bar is brass for which $E = 90$ GPa, $\alpha = 20\times10^{-6}/^{O}C$, and the right bar is aluminum for which $E = 70$ GPa, $\alpha = 25\times10^{-6}/^{O}C$. The cross-sectional area of the brass bar is 500 mm^2, and that of the aluminum bar is 750 mm^2. Let us suppose that the system is initially stress free and that the temperature then drops $20^{O}C$.

(*a*) If the supports are unyielding, find the normal stress in each bar.
(*b*) If the right support yields 0.1 mm, find the normal stress in each bar.

Ans. **(*a*) $\sigma_{br} = 41$ MPa, $\sigma_{al} = 27.33$ MPa; (*b*) $\sigma_{br} = 28.4$ MPa, $\sigma_{al} = 19$ MPa**



4- An aluminum right-circular cylinder surrounds a steel cylinder as shown. The axial compressive load of 200 kN is applied through the rigid cover plate shown. If the aluminum cylinder is originally 0.25 mm longer than the steel before any load is applied, find the normal stress in each when the temperature has dropped 20 K and the entire load is acting. For steel take E = 200 GPa, a = 12 × 10–6/° C, and for aluminum assume E = 70 GPa, a = 25 × 10–6/° C.



Ans. σ_{st} = 9 MPa, σ_{al} = 15.5 MPa