

THE STRONG DEFORMATION RETRACTION

Prof. Dr. Hana' M. Ali



THE RETRACTION

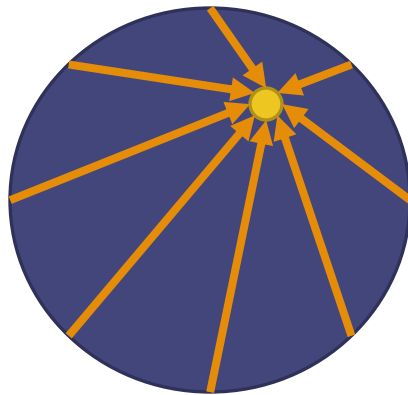
Prof. Dr. Hana' M. ALi

DEFINITION:

A subset A of a topological space X is called a retract of X , if there exists a continuous map $r: X \rightarrow A$ such that $r \circ i = \mathcal{I}_A: A \rightarrow A$, i.e. $r(a) = a$, for all $a \in A$. We call r a retraction.

Example:

Recall, $D^n \simeq \{x_0\}$. The constant map $C: D^n \rightarrow \{x_0\}$ form a retraction and $\{x_0\}$ is a retract of D^n .



THE STRONG DEFORMATION RETRACT

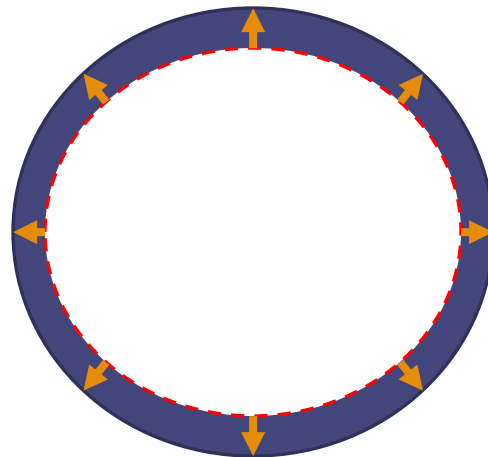
Prof. Dr. Hana' M. ALi

DEFINITION:

A subset A of a topological space X is called a strong deformation retract of X , if there exists a retract $r: X \rightarrow A$ such that $i \circ r \simeq \mathcal{I}_X: X \rightarrow X$. We call r a strong deformation retraction.

Example:

Recall, $D^n - \{0\} \simeq S^{n-1}$. the map $r: D^n - \{0\} \rightarrow S^{n-1}$ that defined as; $r(x) = \frac{x}{\|x\|}$, for all $x \in D^n - \{0\}$ form a strong deformation retract



THE STRONG DEFORMATION RETRACT

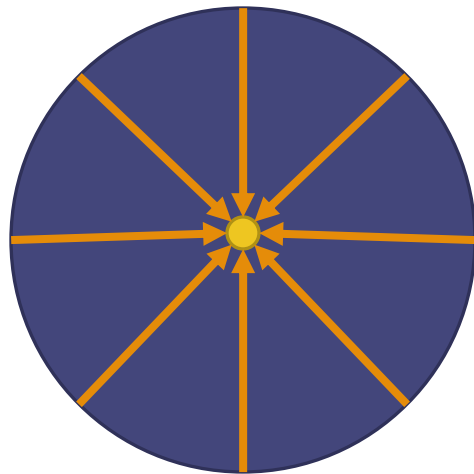
Prof. Dr. Hana' M. ALi

DEFINITION:

A topological space X is called contractible to a point, if there exists a point $x_0 \in X$ such that $\{x_0\}$ is a strong deformation retract of X , i.e. the constant map $C: X \rightarrow \{x_0\}$ is a strong deformation retraction ($i \circ C \simeq I_X: X \rightarrow X$).

Example:

Recall, $\{x_0\}$ is a strong deformation retract of D^n , that is, D^n



EXERCISES:

Prof. Dr. Hana' M. ALi

Exercise 1:

Prove that, The circle $S = \{(x_1, x_2, 0) \in \mathbb{R}^3 \mid x_1^2 + x_2^2 = 1\}$ is a strong deformation retract of the cylinder $C = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1^2 + x_2^2 = 1 \text{ and } -1 \leq x_3 \leq 1\}$

Exercise 2:

A space X is contractible if, and only if, the identity map $J_X: X \rightarrow X$ is null homotopic.

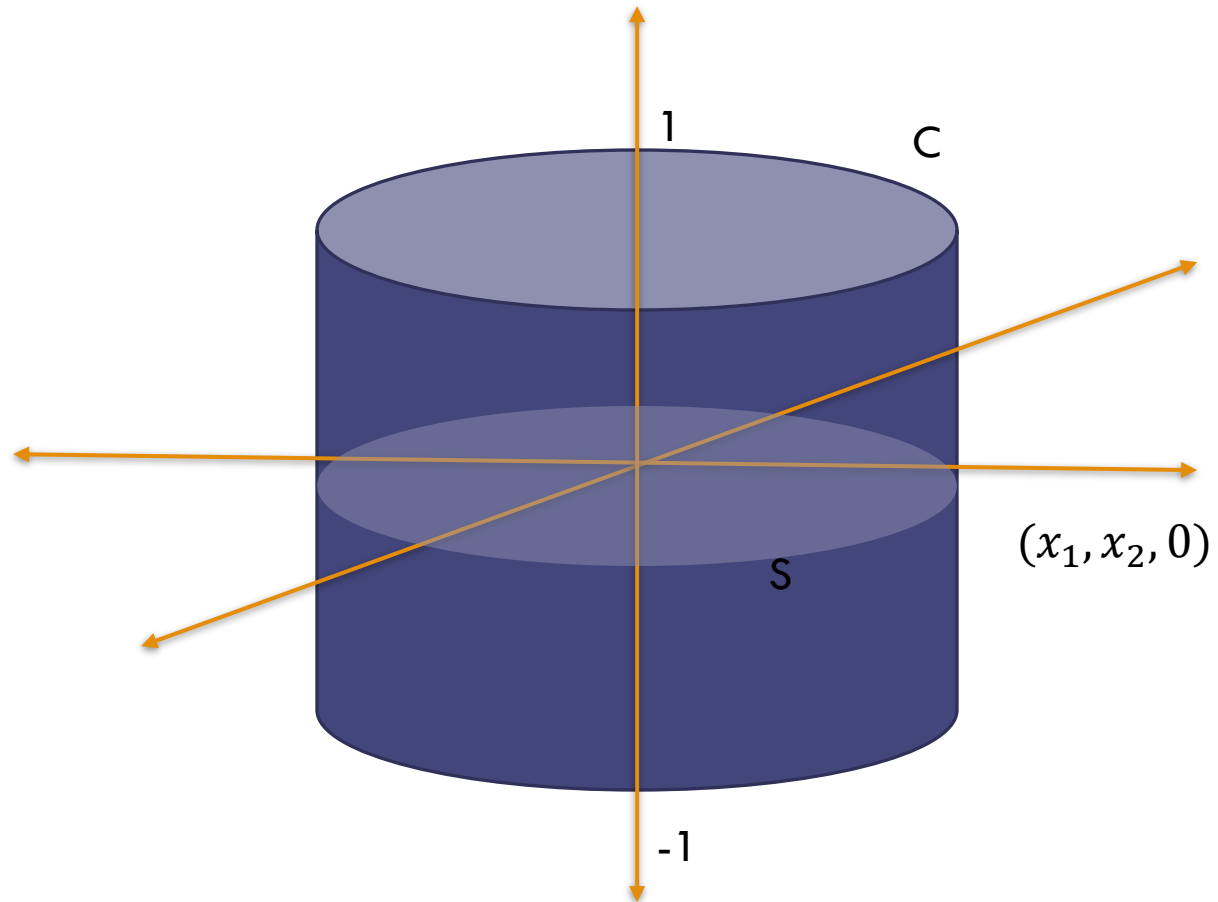
Exercise 3:

Every contractible space is path connected.

EXERCISES:

Prof. Dr. Hana' M. ALi

Exercise 1:



THANK YOU FOR YOUR ATTENTION