Engineering Mechanics

Mechanics is the physical science which deals with the effects of forces on objects. No other subject plays a greater role in engineering analysis than mechanics. Although the principles of mechanics are few, they have wide application in engineering. The principles of mechanics are central to research and development in the fields of vibrations, stability and strength of structures and machines, robotics, rocket and spacecraft design, automatic control, engine performance, fluid flow.

Basic Concepts

The following concepts and definitions are basic to the study of mechanics, and they should be understood at the outset.

Space is the geometric region occupied by bodies whose positions are described by linear and angular measurements relative to a coordinate system. For three-dimensional problems, three independent coordinates are needed. For two-dimensional problems, only two coordinates are required.

Time is the measure of the succession of events and is a basic quantity in dynamics. Time is not directly involved in the analysis of statics problems.

Mass is a measure of the inertia of a body, which is its resistance to a change of velocity. Mass can also be thought of as the quantity of matter in a body. The mass of a body affects the gravitational attraction force between it and other bodies. This force appears in many applications in statics.

Rigid body. A body is considered rigid when the change in distance between any two of its points is negligible for the purpose at hand.(A rigid body does not deform under load).

Mechanics: Units

TABLE 1–1 Systems of Units				
Name	Length	Time	Mass	Force
International System of Units SI	meter m	second s	kilogram kg	$\boxed{\text{newton}^*}$ N $\left(\frac{\text{kg} \cdot \text{m}}{\text{s}^2}\right)$
U.S. Customary FPS	foot ft	second s	$\boxed{\text{slug}^*}$ $\left(\frac{\text{lb} \cdot \text{s}^2}{\text{ft}}\right)$	pound lb

*Derived unit.

Mechanics: Units Prefixes

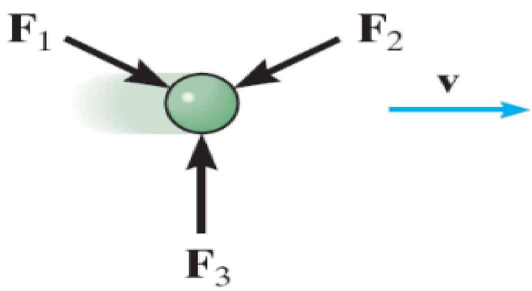
	Exponential Form	Prefix	SI Symbol
Multiple			
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	k
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

Newton's Laws:-

Sir Isaac Newton was the first to state correctly the basic laws governing the motion of a particle and to demonstrate their validity.

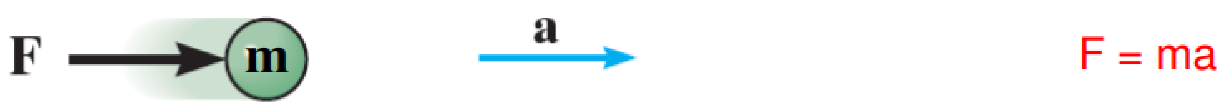
Slightly reworded with modern terminology, these laws are:

Law I. A particle remains at rest or continues to move with uniform velocity (in a straight line with a constant speed) if there is no unbalanced force acting on it.



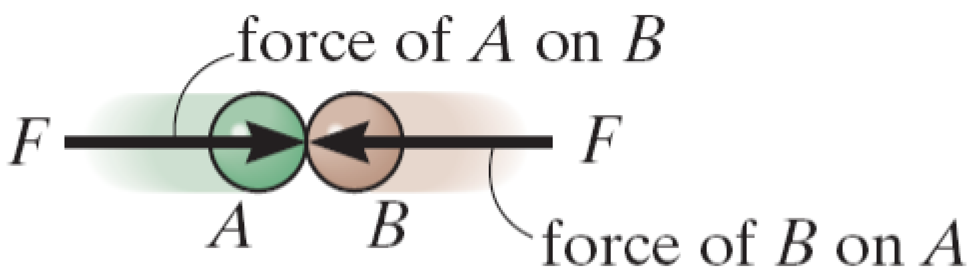
Equilibrium

Law II. The acceleration of a particle is proportional to the vector sum of forces acting on it, and is in the direction of this vector sum.



Acceleration motion

Law III. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear (they lie on the same line).



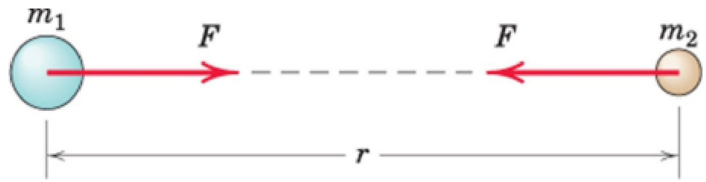
Action- Reaction

Mechanics: Newton's Law of Gravitational Attraction

Weight of a body (gravitational force acting on a body) is required to be computed in Statics as well as Dynamics.

This law governs the gravitational attraction between any two particles.

$$F = G\frac{m_1 m_2}{r^2}$$



F = mutual force of attraction between two particles

G = universal constant of gravitation

Experiments → G = 6.673x10^{-11} $m^3/(kg.s^2)$

Rotation of Earth is not taken into account

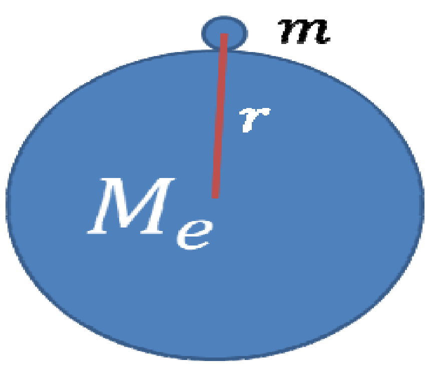
m1, m2 = masses of two particles

r = distance between two particles

Gravitational Attraction of the Earth

Weight of a Body: If a particle is located at or near the surface of the earth, the only significant gravitational force is that between the earth and the particle

Weight of a particle having mass m_1 = m : Assuming earth to be a non-rotating sphere of constant density and having mass m_2 = Me



$$W = G\frac{mM_e}{r^2}$$

r = distance between the earth's center and the particle

Let g = G Me /r^2 = acceleration due to gravity (9.81m/s^2)

Scalars and Vectors

We use two kinds of quantities in mechanics—scalars and vectors. Scalar quantities are those with which only a magnitude is associated. Examples of scalar quantities are time, volume, density, speed, energy and mass. Vector quantities, on the other hand, possess direction as well as magnitude. Examples of vector quantities are displacement, velocity, acceleration, force, moment, and momentum

A Vector V can be written as: V = *V*n

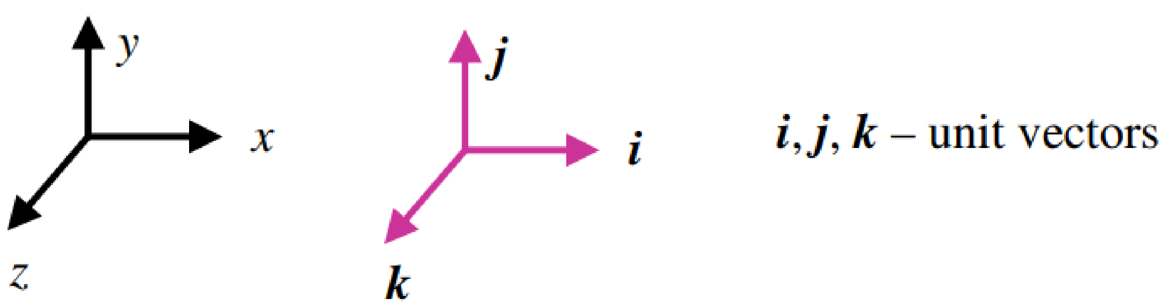
V = magnitude of V

n = unit vector whose magnitude is one and whose direction coincides with that of V.

Unit vector can be formed by dividing any vector, such as the geometric position vector, by its length or magnitude.

Vectors represented by Bold and Non-Italic letters (V).

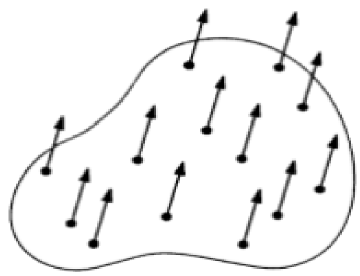
Magnitude of vectors represented by Non-Bold, Italic letters (*V*).



Vectors

Free Vector: whose action is not confined to or associated with a unique line in space

Ex: Movement of a body without rotation.

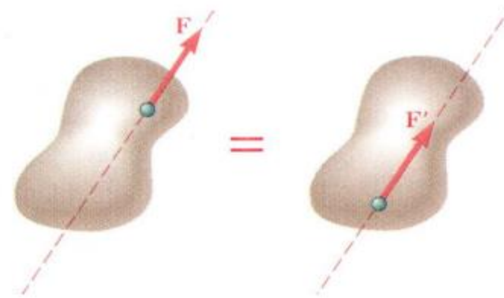


Sliding Vector: has a unique line of action in space but not a unique point of application

Ex: External force on a rigid body

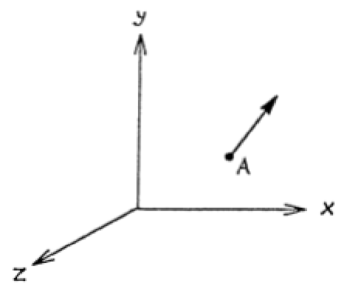
Principle of Transmissibility

Imp in Rigid Body Mechanics



Fixed Vector: for which a unique point of application is specified

Ex: Action of a force on deformable body

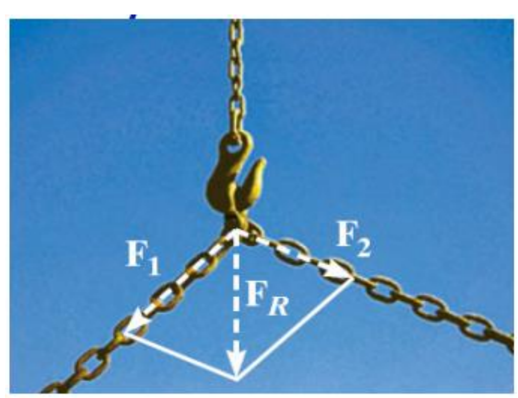


<u>Vector Addition: Procedure for Analysis</u>

1- Parallelogram Law (Graphical)

Resultant Force (diagonal) Components (sides of parallelogram)

Algebraic Solution:- Using the coordinate system



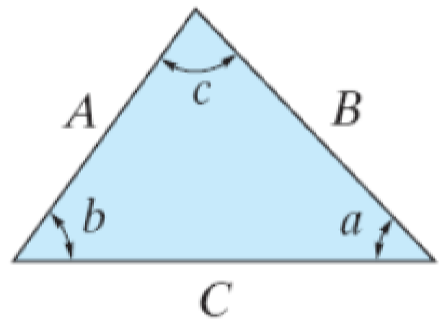
Trigonometry (Geometry):- Resultant Force and Components from Law of Cosines and Law of Sines.

Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$



Force Systems

In this chapter you will study the effect of forces acting on particles. First you will learn how to replace two or more forces acting on a given particle by a single force having the same effect as the original forces. This single equivalent force is the resultant of the original forces acting on the particle.

The first part of the chapter is devoted to the study of forces contained in a single plane, and the second part to the analysis of forces in three-dimensional space.

FORCE ON A PARTICLE. RESULTANT OF TWO FORCES.

A force represents the action of one body on another and is generally characterized by its point of application, its magnitude, and its direction. Each force considered in this chapter will thus be completely defined by its magnitude and direction.

The magnitude of a force is characterized by a certain number of units. As indicated in Chap. 1, the SI units used by engineers to measure the magnitude of a force are the newton (N) and its multiple the kilonewton (kN), equal to 1000 N, while the U.S. customary units used for the same purpose are the pound (lb) and its multiple the kilopound (kip), equal to 1000 lb. The direction of a force is defined by the line of action and the sense of the force. The line of action is the infinite straight line along which the force acts; it is characterized by the angle it forms with some fixed axis (Fig. 2.1).

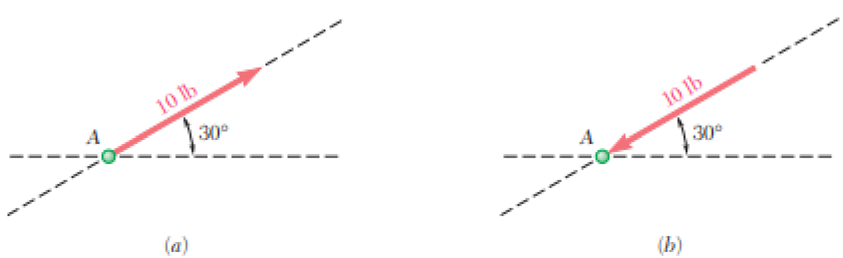
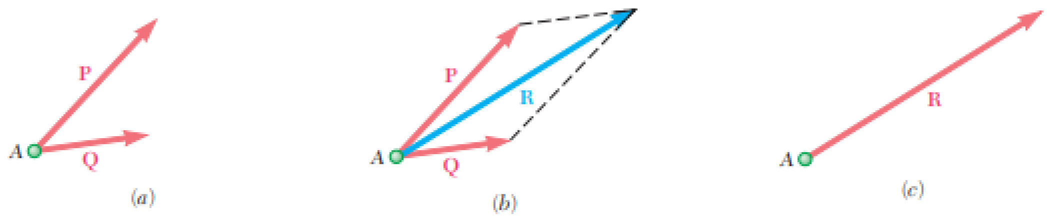


Fig. 2.1

Experimental evidence shows that two forces P and Q acting on a particle A (Fig. 2.2 a) can be replaced by a single force R which has the same effect on the particle (Fig. 2.2 c). This force is called the resultant of the forces P and Q and can be obtained, as shown in Fig. 2.2 b , by constructing a parallelogram, using P and Q as two adjacent sides of the parallelogram. The diagonal that passes through A represents the resultant. This method for finding the resultant is known as the parallelogram law for the addition of two forces. This law is based on experimental



evidence; it cannot be proved or derived mathematically.

Fig. 2.2

From the parallelogram law, we can derive an alternative method for determining the sum of two vectors. This method, known as the triangle rule, is derived as follows. Consider Fig. 2.2, where the sum of the vectors P and Q has been determined by the parallelogram law. Since the side of the parallelogram opposite Q is equal to Q in magnitude and direction, we could draw only half of the parallelogram (Fig. 2.3 a). The sum of the two vectors can thus be found by arranging P and Q in tip-to-tail fashion and then connecting the tail of P with the tip of Q . In Fig. 2.3 b, the other half of the parallelogram is considered, and the same result is obtained. This confirms the fact that vector addition is commutative.

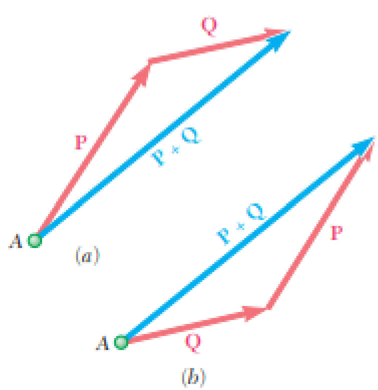


Fig. 2.3

The subtraction of a vector is defined as the addition of the corresponding negative vector. Thus, the vector $P - Q$ representing the difference between the vectors P and Q is obtained by adding to P the negative vector $-Q$ (Fig. 2.4). We write

$$P - Q = P + (-Q)$$

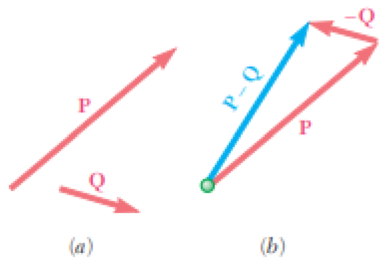


Fig. 2.4

We will now consider the sum of three or more vectors. The sum of three vectors P, Q , and S will, by definition , be obtained by first adding the vectors P and Q and then adding the vector S to the vector$P + Q$. We thus write,

$$P + Q + S = (P + Q) + S$$

as shown in Fig. 2.5.

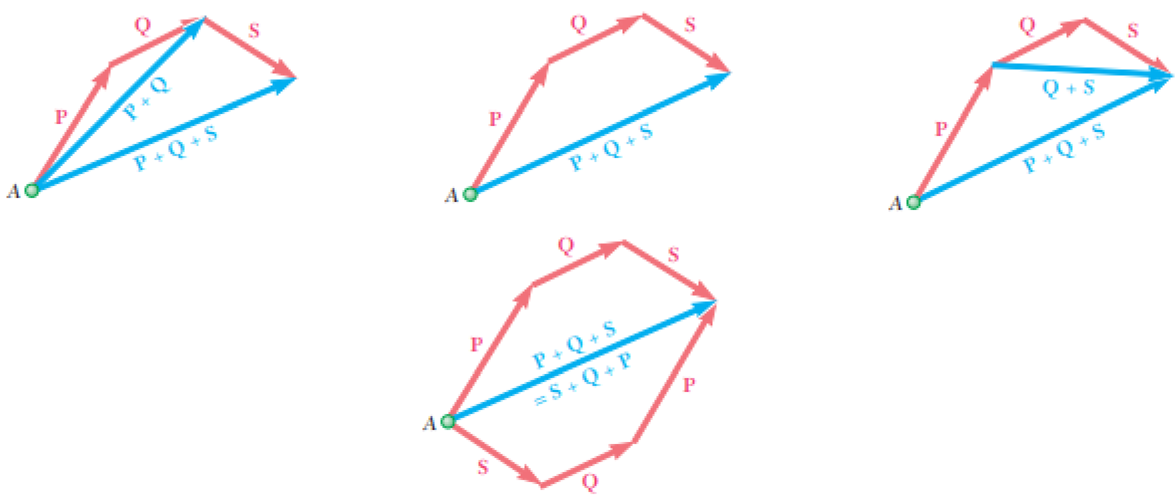
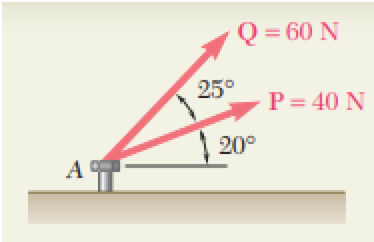


Fig. 2.5

Ex.

The two forces P and Q act on a bolt A . Determine their resultant.

Sol.

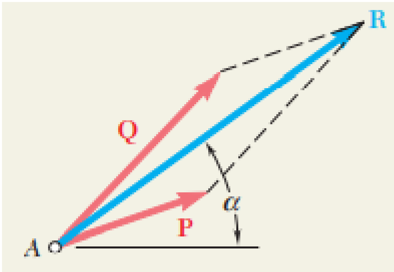


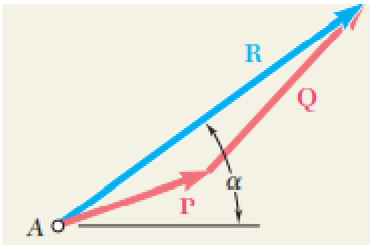
Graphical Solution. A parallelogram with sides equal to P and Q is drawn to scale. The magnitude and direction of the resultant are measured and found to be

$$R = 98\ N,\quad \propto = 35°, R = 98\ N \quad \measuredangle 35°$$

The triangle rule may also be used. Forces P and Q are drawn in tip-totail fashion. Again the magnitude and direction of the resultant are measured.

$$R = 98\ N,\quad \propto = 35°, R = 98\ N \quad \measuredangle 35°$$





Trigonometric Solution. The triangle rule is again used; two sides and the included angle are known. We apply the law of cosines.

$$R^2 = P^2 + Q^2 - 2PQ\ cos\beta$$

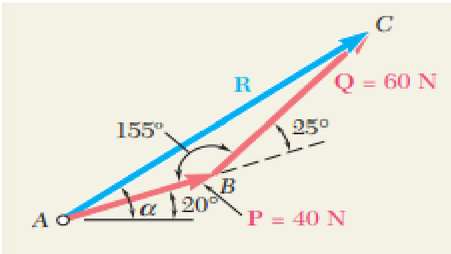
$$R^2 = (40)^2 + (60)^2 - 2 * 40 * 60 * \cos 155°, R = 97.73\ N$$

Now, applying the law of sines, we write

$$\frac{sinA}{Q} = \frac{sinB}{R}$$

$$\frac{sinA}{60} = \frac{sin155}{97.73}$$

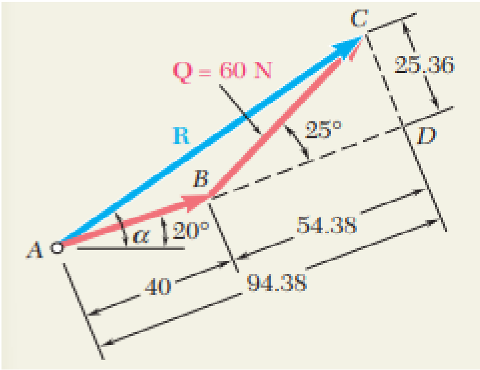
$$A = 15.04°, \alpha = 20° + A = 35.04°$$



Alternative Trigonometric Solution. We construct the right triangle BCD and compute

CD = (60 N) sin 25° = 25.36 N

BD = (60 N) cos 25° = 54.38 N



Then, using triangle ACD , we obtain

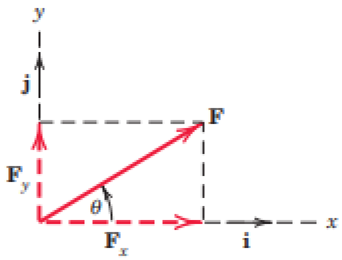
$$tanA = \frac{25.36}{94.38}, \quad A = 15.04°$$

$$R = \frac{25.36}{sinA}, \quad R = 97.73\ N$$

$$\alpha = 20° + A = 35.04°$$

<u>Rectangular Components</u>

$$F = F_x + F_y$$



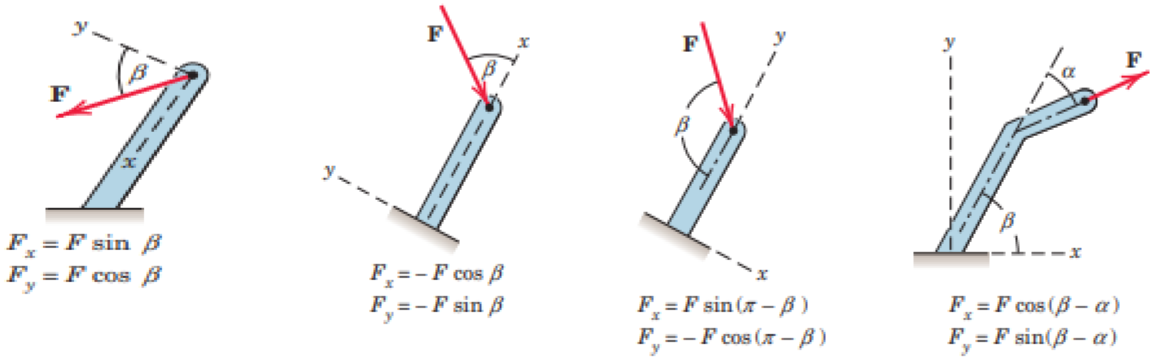
where F_x and F_y are vector components of F in the x- and y-directions. Each of the two vector components may be written as a scalar times the appropriate unit vector. In terms of the unit vectors i and j of Fig. 2/5, $F_x = F_x i$ and $F_y = F_y j$, and thus we may write

$$F = F_x i + F_y j$$

where the scalars F_x and F_y are the x and y scalar components of the vector F

$$F_x = Fcos\theta \ , \quad F = \sqrt{F_x^2 + F_y^2}$$

$$F_y = Fsin\theta \ , \quad \theta = tan^{-1}\frac{F_y}{F_x}$$



Rectangular components are convenient for finding the sum or resultant R of two forces which are concurrent. Consider two forces F_1 and F_2 which are originally concurrent at a point O.

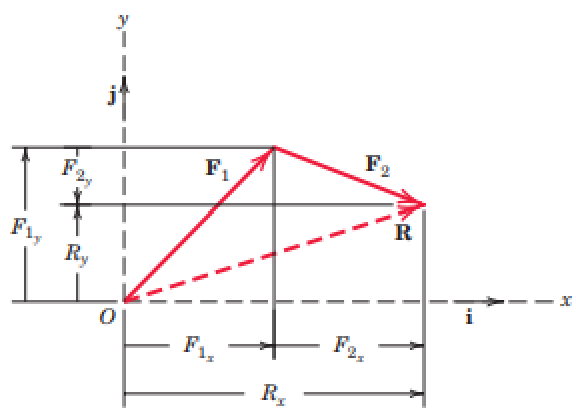
$$R = F_1 + F_2 = \left(F_{1_x} i + F_{1_y} j\right) + \left(F_{2_x} i + F_{2_y} j\right)$$

$$R_x i + R_y j = (F_{1_x} + F_{2_x}) i + \left(F_{1_y} + F_{2_y}\right) j$$

from which we conclude that

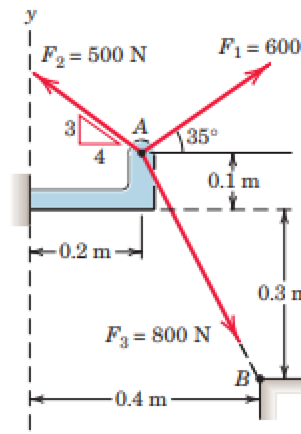
$$R_x = F_{1_x} + F_{2_x} = \sum F_x$$

$$R_y = F_{1_y} + F_{2_y} = \sum F_y$$



Ex.

The forces F_1, F_2, and F_3, all of which act on point A of the bracket, are specified in three different ways. Determine the x and y scalar components of each of the three forces.



Solution. The scalar components of F_1, from Fig.

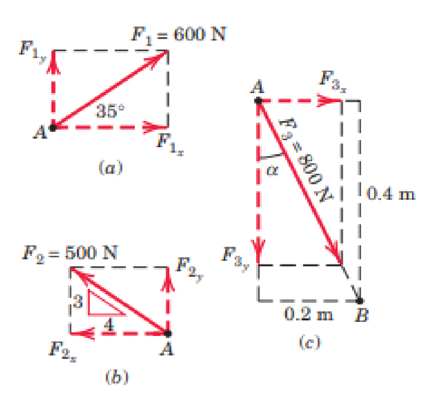
$$F_{1x} = 600\cos 35° = 491\ N$$

$$F_{1y} = 600\sin 35° = 344\ N$$

The scalar components of F2, from Fig.

$$F_{2x} = -500\frac{4}{5} = -400\ N$$

$F_{2y} = 500\frac{3}{5} = 300\ N$



The scalar components of F_3 can be obtained by first computing the angle ∝ of Fig.

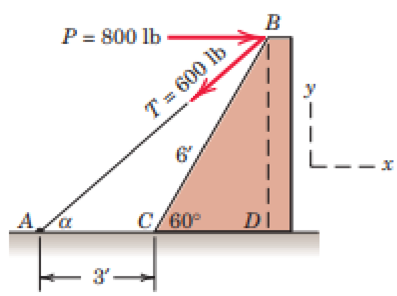
$$\propto = \tan^{-1}\frac{0.2}{0.4} = 26.6°$$

$$F_{3x} = 800\sin 26.6° = 358\ N$$

$$F_{3y} = 800\cos 26.6° = -716\ N$$

Ex.

Combine the two forces P and T, which act on the fixed structure at B, into a single equivalent force R.

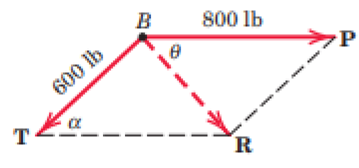


Graphical solution.

$$\tan \propto = \frac{BD}{AD} = \frac{6\sin 60°}{3 + 6\cos 60°} = 0.866, \propto = 40.9°$$

Measurement of the length R and direction θ of the resultant force R yields the approximate results.

$$R = 525\ Ib, \quad \theta = 49°$$



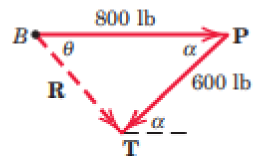
(a)

Geometric solution.

$$R^2 = (600)^2 + (800)^2 - 2 * 600 * 800\ cos40.9° = 274.3$$

$R = 524\ Ib$

$$\frac{600}{sin\theta} = \frac{524}{sin40.9}, \quad \theta = 48.6°$$



(b)

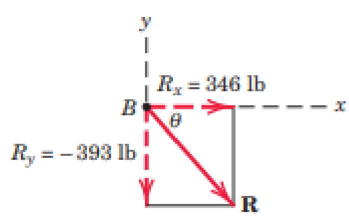
Algebraic solution. By using the x-y coordinate system on the given figure.

$$R_x = \sum F_x = 800 - 600\ cos40.9 = 346\ Ib$$

$$R_y = \sum F_y = -600\ sin\ 40.9 = -393\ Ib$$

$$R = \sqrt{R_x^2 + R_y^2} = 524\ Ib$$

$\theta = \tan^{-1}\frac{393}{346} = 48.6°$



(c)

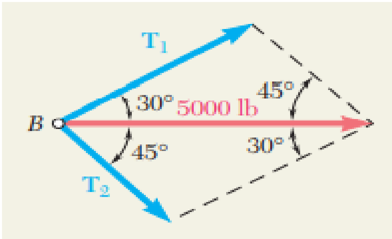
Ex.

A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is a 5000-lb force directed along the axis of the barge, determine (a) the tension in each of the ropes knowing that a 5 45°, (b) the value of a for which the tension in rope 2 is minimum.

Sol.

a. Tension for a 5 45°. Graphical Solution.

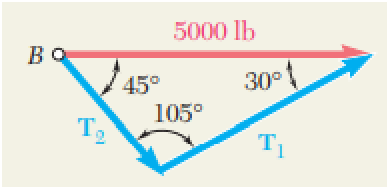
$T_1 = 3700\ Ib, T_2 = 2600\ Ib$



Trigonometric Solution. The triangle rule can be used.

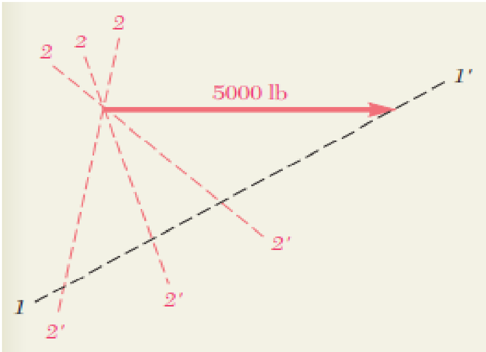
$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{5000\ Ib}{\sin 105°}$$

$T_1 = 3660\ Ib, T_2 = 2590\ Ib$

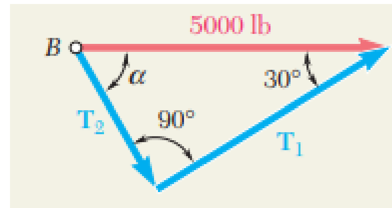


b. Value of a for Minimum T_2. To determine the value of a for which the tension in rope 2 is minimum, the triangle rule is again used. In the sketch shown, line 1-1' is the known direction of T_1 . Several possible directions of T_2 are shown by the lines 2-2'. We note that the minimum value of T_2 occurs when T_1 and T_2 are perpendicular. The minimum value of T_2 is

$T_2 = 5000 \sin 30° = 2500\ Ib$



Corresponding values of T1 and a are



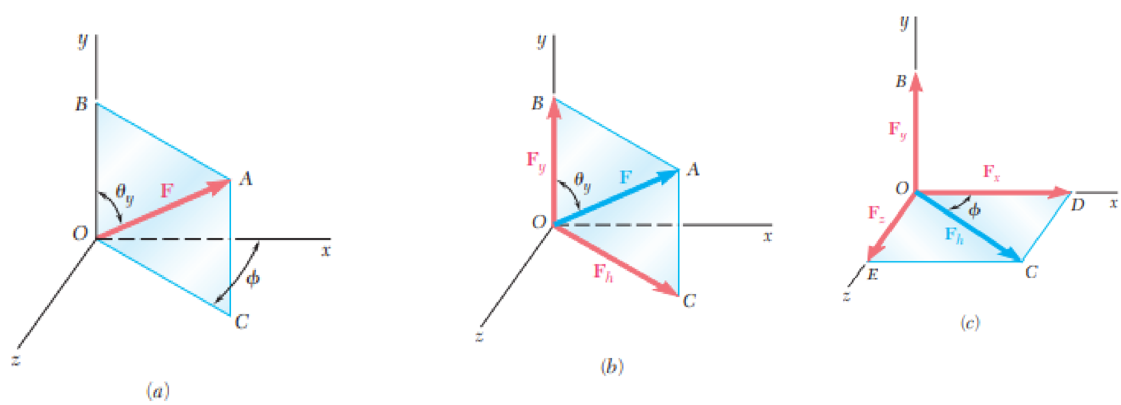
$$T_1 = 5000 \cos 30° = 4330\ Ib$$

$$\propto = 90 - 30 = 60°$$

FORCES IN SPACE

RECTANGULAR COMPONENTS OF A FORCE IN SPACE

Consider a force *F* acting at the origin O of the system of rectangular coordinates *x*, *y*, *z*. To define the direction of *F* , we draw the vertical plane OBAC containing *F* (Fig. 2.6 a). This plane passes through the vertical *y* axis; its orientation is defined by the angle $\emptyset$ it forms with the *xy* plane. The direction of *F* within the plane is defined by the angle $\theta_y$that *F* forms with the *y* axis. The force *F* may be resolved into a vertical component F_y and a horizontal component F_h; this operation, shown in Fig. 2.6 b , is carried out in plane OBAC according to the rules developed in the first part of the chapter. The corresponding scalar components are,



$$F_y = F cos\theta, \qquad F_h = F sin\theta$$

But F_h may be resolved into two rectangular components F_x and F_z along the *x* and *z* axes, respectively. This operation, shown in Fig. 2.6 c , is carried out in the *xz* plane. We obtain the following expressions for the corresponding scalar components:

$$F_x = F_h cos\emptyset = Fsin\theta \, cos\emptyset$$

$$F_z = F_h sin\emptyset = Fsin\theta \, sin\emptyset$$

The given force F has thus been resolved into three rectangular vector components F_x, F_y, F_z, which are directed along the three coordinate axes.

Applying the Pythagorean theorem to the triangles OAB and OCD of Fig. 2.6, we write

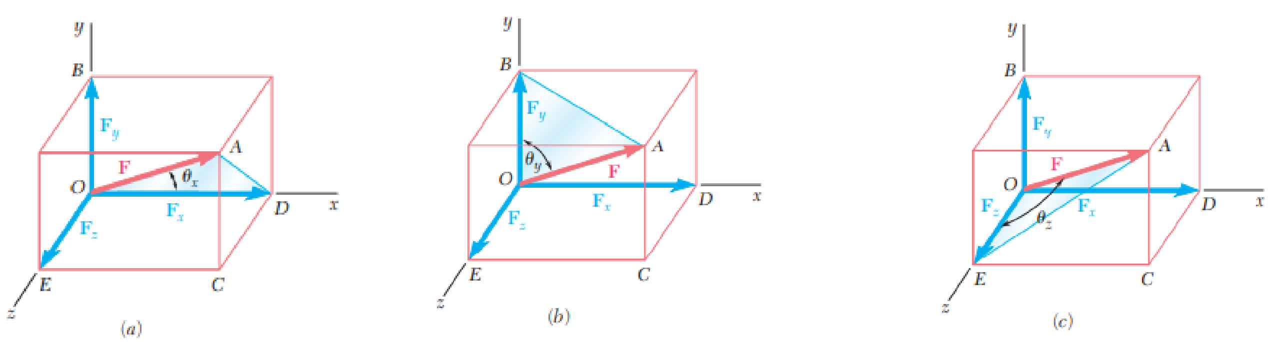
$$F^2 = (OA)^2 = (OB)^2 + (BA)^2 = F_y^2 + F_h^2$$

$$F_h^2 = (OC)^2 = (OD)^2 + (DC)^2 = F_x^2 + F_z^2$$

Eliminating F_h^2 from these two equations and solving for F , we obtain the following relation between the magnitude of F and its rectangular scalar components:

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

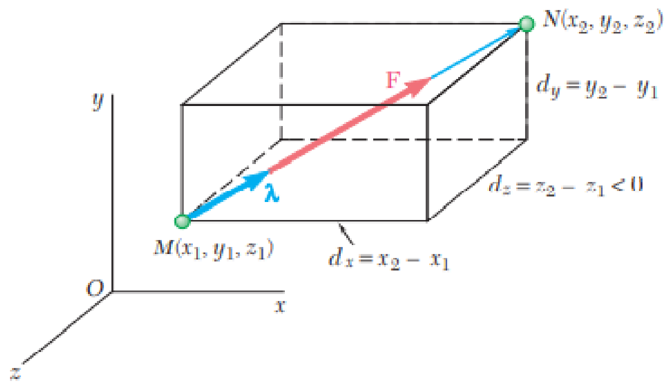
The relationship existing between the force F and its three components F_x , F_y , F_z is more easily visualized if a "box" having F_x , F_y , F_z for edges is drawn as shown in Fig. 2.7.



$$F_x = Fcos\theta_x, F_y = Fcos\theta_y, F_z = Fcos\theta_z$$

FORCE DEFINED BY ITS MAGNITUDE AND TWO POINTS ON ITS LINE OF ACTION.

In many applications, the direction of a force F is defined by the coordinates of two points, $M\ (x_1, y_1, z_1)$ and $N\ (x_2, y_2, z_2)$, located on its line of action (Fig. 2.8). Consider the vector $\overrightarrow{MN}$ joining M and N



$$\overrightarrow{MN} = d_x i + d_y j + d_z k$$

The unit vector λ

$$\lambda = \frac{\overrightarrow{MN}}{MN} = \frac{1}{d}\left(d_x i + d_y j + d_z k\right)$$

$$d = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

$$\boldsymbol{F} = F\lambda = \frac{F}{d}\left(d_x i + d_y j + d_z k\right)$$

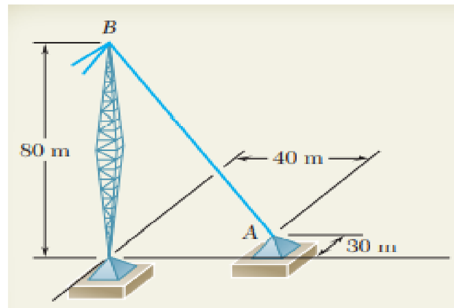
$$F_x = F\frac{d_x}{d}, F_y = F\frac{d_y}{d}, F_z = F\frac{d_z}{d}$$

$$cos\theta_x = \frac{d_x}{d}, cos\theta_y = \frac{d_y}{d}, cos\theta_z = \frac{d_z}{d}$$

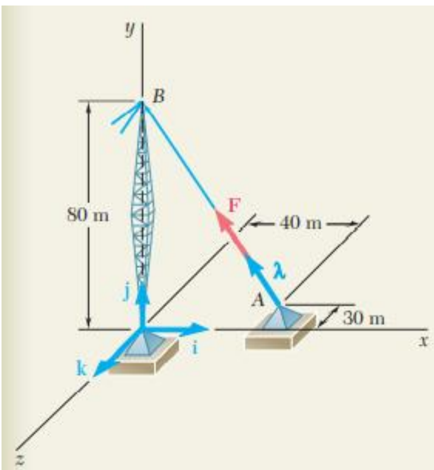
Ex.

A tower guy wire is anchored by means of a bolt at A. The tension in the wire is 2500 N. Determine (a) the components F_x, F_y, F_z of the force acting on the bolt, (b) the angles $\theta_x, \theta_y, \theta_z$ defining the direction of the force.

Sol.



$$d_x = -40, d_y = 80, d_z = 30$$



$$AB = d = \sqrt{d_x^2 + d_y^2 + d_z^2} = \sqrt{-40^2 + 80^2 + 30^2 = 94.3\, m}$$

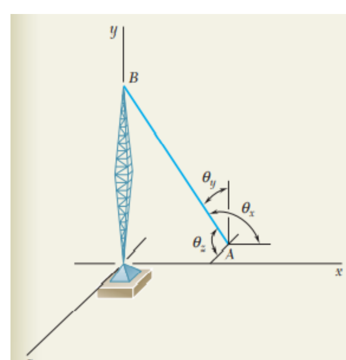
$$\overrightarrow{AB} = -40i + 80j + 30k$$

$$\lambda = \frac{\overrightarrow{AB}}{AB}, \boldsymbol{F} = F\lambda = \frac{2500}{94.3}(-40i + 80j + 30k)$$

$$\boldsymbol{F} = -1060i + 2120j + 795k$$

$F_x = -1060N, F_y = 2120\ N, F_z = 795\ N$

b. Direction of the Force.



$$cos\theta_x = \frac{F_x}{F}, \theta_x = 115.1°, cos\theta_y = \frac{F_y}{F}, \theta_y = 32°$$

$$cos\theta_z = \frac{F_z}{F}, \theta_z = 71°$$

Rigid Bodies

Equivalent Systems of Forces

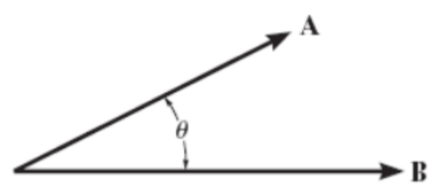
In this chapter you will study the effect of forces exerted on a rigid body, and you will learn how to replace a given system of forces by a simpler equivalent system. This analysis will rest on the fundamental assumption that the effect of a given force on a rigid body remains unchanged if that force is moved along its line of action (principle of transmissibility). It follows that forces acting on a rigid body can be represented by sliding vectors.

Two important concepts associated with the effect of a force on a rigid body are the moment of a force about a point, and the moment of a force about an axis. Since the determination of these quantities involves the computation of vector products and scalar products of two vectors, the fundamentals of vector algebra will be introduced in this chapter and applied to the solution of problems involving forces acting on rigid bodies.

VECTOR PRODUCT OF TWO VECTORS

Dot product

$$A.B = AB\cos\theta$$



Applications:

to determine the angle between two vectors, to determine the projection of a vector in a specified direction

$$A.B = B.A$$

$$A.(B + C) = A.B + A.C$$

$$A.B = (A_x i + A_y j + A_z k).(B_x i + B_y j + B_z k)$$

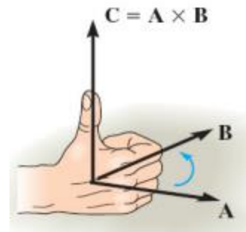
$$A.B = A_x B_x + A_y B_y + A_z B_z, \quad i.i = 1, \quad i.j = 0$$

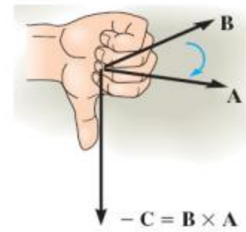
Cross Product:

$$A \times B = C = ABsin\theta$$

$$A \times B = -B \times A$$

$$A.B = (A_x i + A_y j + A_z k).(B_x i + B_y j + B_z k)$$

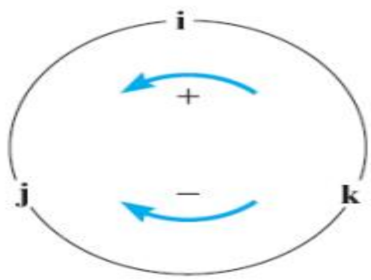


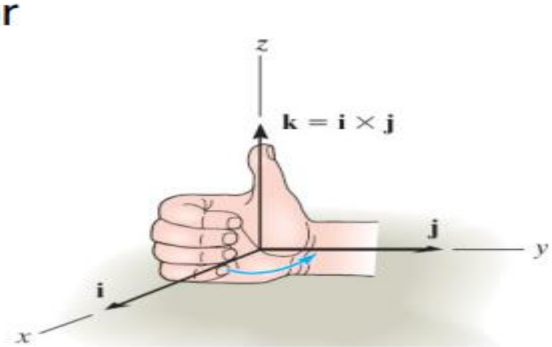


$$\left|\begin{array}{ccc|cc} i & j & k & i & j \\ A_x & A_y & A_z & A_x & A_y \\ B_x & B_y & B_z & B_x & B_y \end{array}\right.$$

$$= (A_y B_z - A_z B_y)i + (A_z B_x - A_x B_z)j + (A_x B_y - A_y B_x)K$$

Cartesian Vector





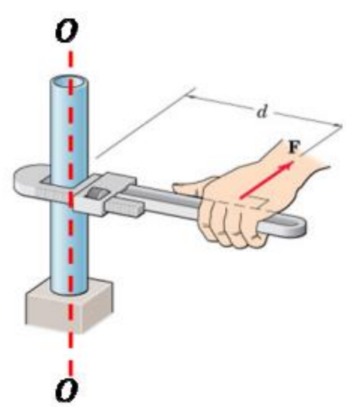
$$i \times j = k, \quad i \times k = -j, \quad i \times i = 0$$

$$j \times k = i, \quad j \times i = -k, \quad j \times j = 0$$

$$k \times i = j, \quad k \times j = -i, \quad k \times k = 0$$

Moment of a Force (Torque)

Moment about axis o-o is $M_o = Fd$, magnitude of M_o measures tendency of F to cause rotation of the body about an axis along M_o.

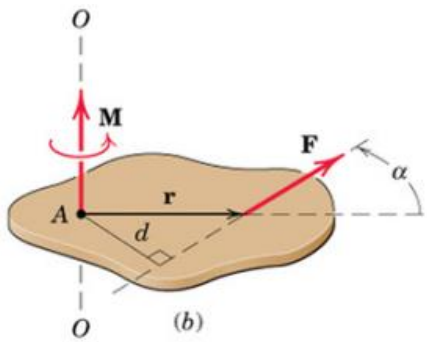


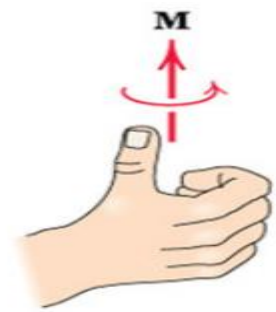
Moment about axis O-O is

$$M_o = Fr\ sin \propto$$

$$M_o = r\ \times F$$

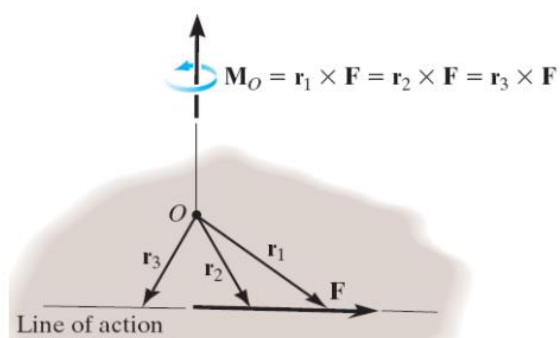
Sense of the moment may be determined by the right-hand rule





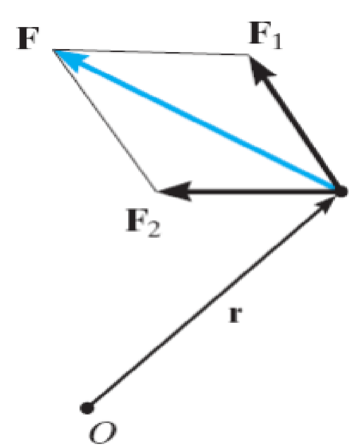
Principle of Transmissibility

Any force that has the same magnitude and direction as F, is equivalent if it also has the same line of action and therefore, produces the same moment.



Varignon's Theorem (Principle of Moments)

Moment of a Force about a point is equal to the sum of the moments of the force's components about the point.



$$M_o = r \times F = r \times (F_1 + F_2) = r \times F_1 + r \times F_2$$

Rectangular Components of a Moment

The moment of F about O,

$$M_o = r \times F$$

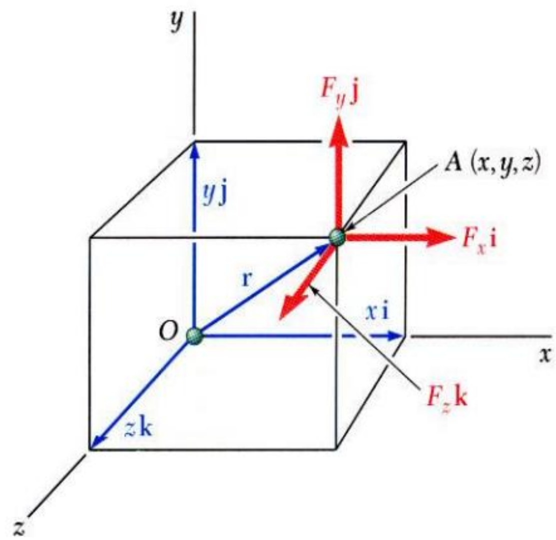
$$F = F_x i + F_y j + F_z k$$

$$r = xi + yj + zk$$

$$M_o = M_x i + M_y j + M_z k$$

$$= \begin{vmatrix} i & j & k \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \left(yF_z - zF_y\right)i + (zF_x - xF_z)j$$
$$+ \left(xF_y + yF_x\right)k$$



The moment of F about B,

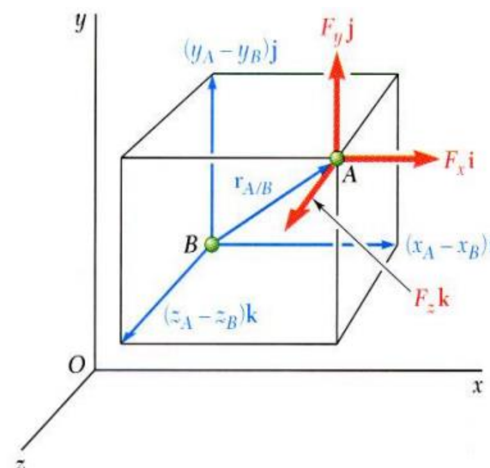
$$M_B = r_{AB} \times F$$

$$r_{AB} = (x_A - x_B)i + (y_A - y_B)j$$
$$+ (z_A - z_B)k$$

$$F = F_x i + F_y j + F_z k$$

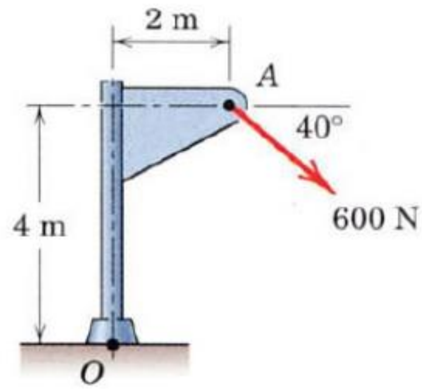
$$M_B = M_x i + M_y j + M_z k$$

$$= \begin{vmatrix} i & j & k \\ x_A - x_B & y_A - y_B & z_A - z_B \\ F_x & F_y & F_z \end{vmatrix}$$



Ex.

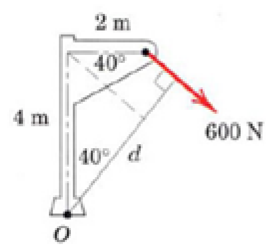
Calculate the magnitude of the moment about the base point O of the 600 N force in different ways.



Solution 1.

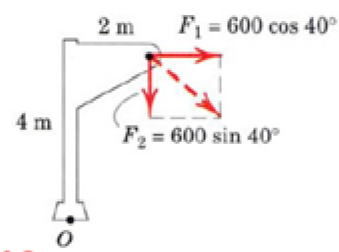
Moment about O is

$$M_o = dF \qquad d = 4cos40^o + 2sin40^o = 4.35m$$



$$M_o = 600(4.35) = 2610\, N.m \ \textbf{Ans}$$

Solution 2.



$$F_x = 600cos40^o = 460\, N$$

$$F_x = 600sin40^o = 386\, N$$

$$M_o = 460(4.00) + 386(2.00) = 2610\, N.m \ \textbf{Ans}$$

Solution 3.

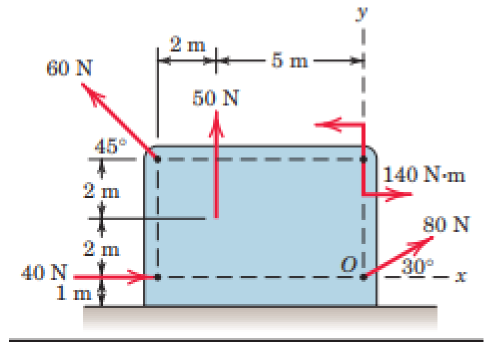
$$\boldsymbol{M_0} = \boldsymbol{r} \times \boldsymbol{F} = (2\boldsymbol{i} + 4\boldsymbol{j}\,) \times 600(cos40^o\boldsymbol{i} - sin40^o\boldsymbol{j})$$

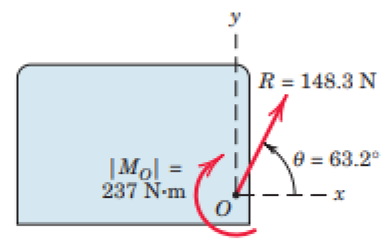
$$\boldsymbol{M_0} = -2610\, N.m \ \textbf{Ans}$$

Ex.

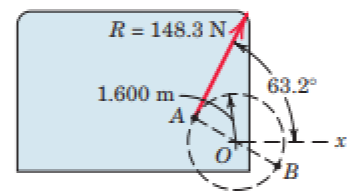
Determine the resultant of the four forces and one couple which act on the plate shown.

$[R_x = \Sigma F_x]$	$R_x = 40 + 80 \cos 30° - 60 \cos 45° = 66.9\text{ N}$
$[R_y = \Sigma F_y]$	$R_y = 50 + 80 \sin 30° + 60 \cos 45° = 132.4\text{ N}$
$[R = \sqrt{R_x^2 + R_y^2}]$	$R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3\text{ N}$
$\left[\theta = \tan^{-1}\dfrac{R_y}{R_x}\right]$	$\theta = \tan^{-1}\dfrac{132.4}{66.9} = 63.2°$
$[M_O = \Sigma(Fd)]$	$M_O = 140 - 50(5) + 60 \cos 45°(4) - 60 \sin 45°(7)$
	$= -237\text{ N}\cdot\text{m}$





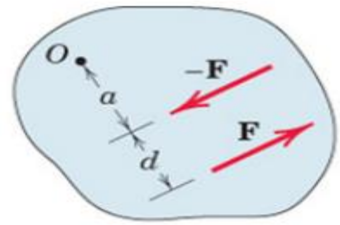
$[Rd = \lvert M_O \rvert]$	$148.3d = 237$	$d = 1.600\text{ m}$



Moment of a Couple

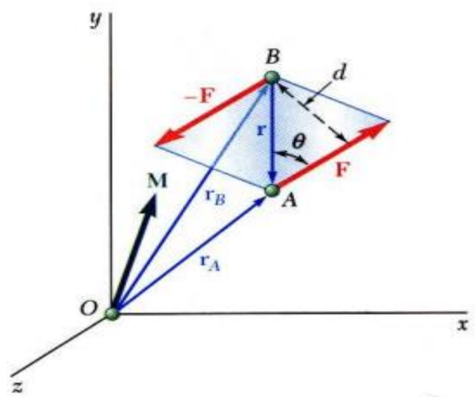
Moment produced by two equal, opposite and non-collinear forces is called a couple. Magnitude of the combined moment of the two forces about O:

$M = F(a + d) - Fa = Fd$

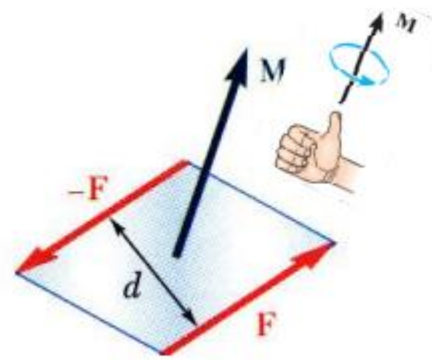


$$M = r_A \times F + r_B \times (-F) = (r_A - r_B) \times F$$
$$= r \times F$$

$$M = rFsin\theta = Fd$$



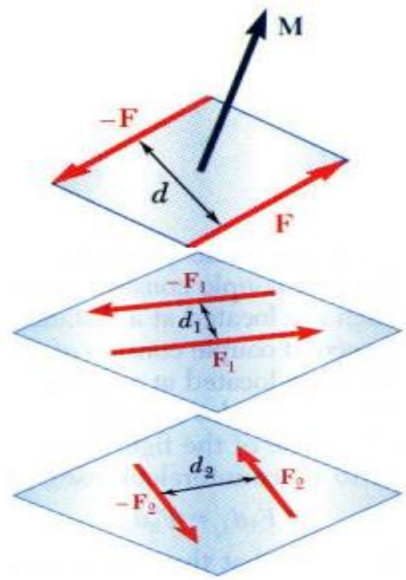
The moment vector of the couple is independent of the choice of the origin of the coordinate axes, i.e., it is a free vector that can be applied at any point with the same effect.



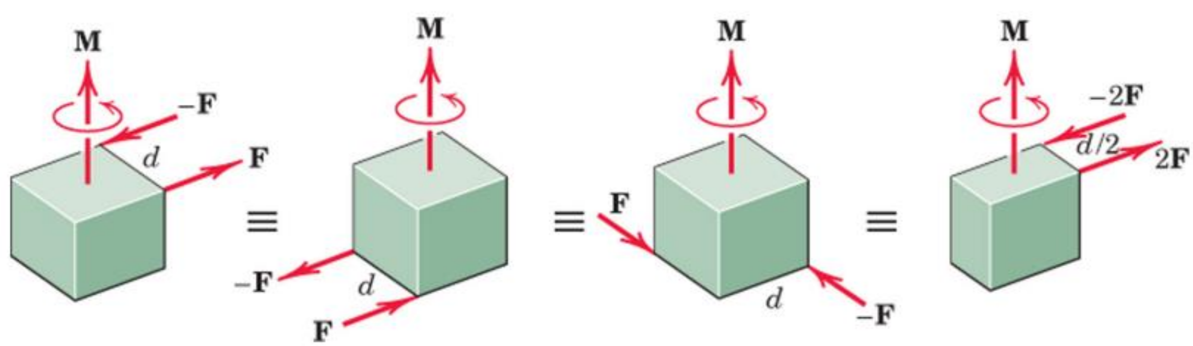
Two couples will have equal moments if

$$F_1 d_1 = F_2 d_2$$

The two couples lie in parallel planes The two couples have the same sense or the tendency to cause rotation in the same direction.



Examples:

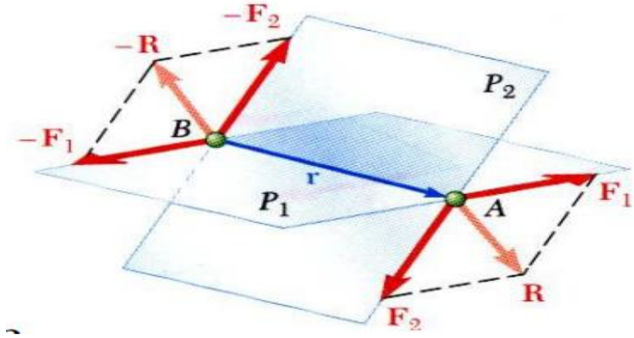


Addition of Couples

Consider two intersecting planes P 1 and P2 with each containing a couple

$M_1 = r \times F_1$ in plane P_1

$M_2 = r \times F_2$ in plane P_2

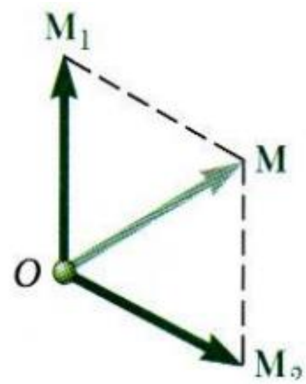


Resultants of the vectors also form a couple

$M = r \times R = r \times (F_1 + F_2)$

By Varigon's theorem

$$M = r \times F_1 + r \times F_2 = M_1 + M_2$$

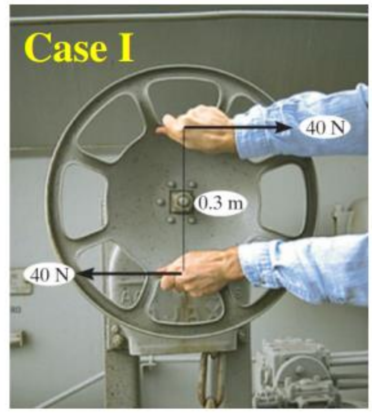


Couple: Example

Moment required to turn the shaft connected at center of the wheel = 12 Nm

Case I: Couple Moment produced by 40 N forces = 12 Nm

Case II: Couple Moment produced by 30 N forces = 12 Nm



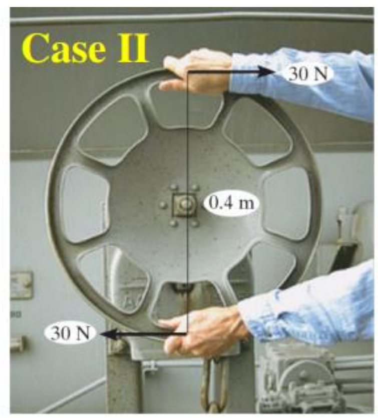
If only one hand is used?

Force required for case I is 80N

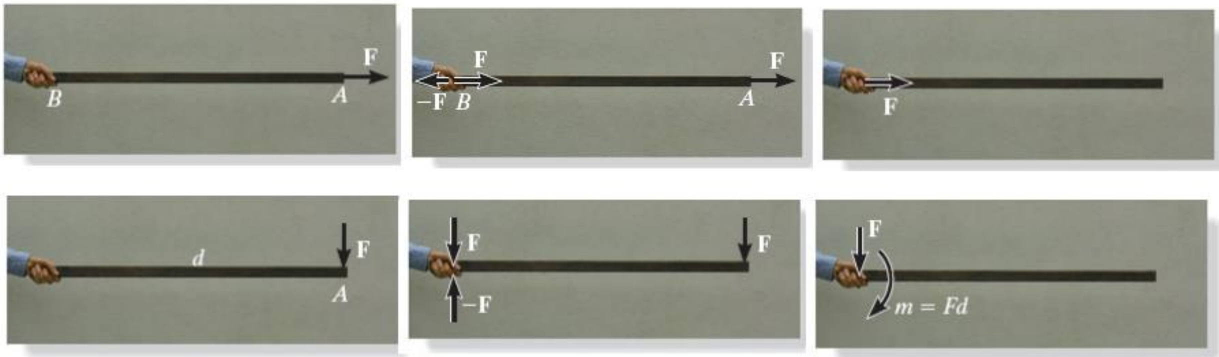
Force required for case II is 60N

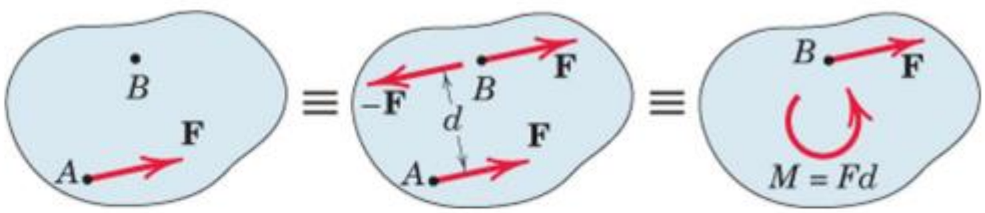
What if the shaft is not connected at the center of the wheel?

Is it a Free Vector?



Equivalent Systems



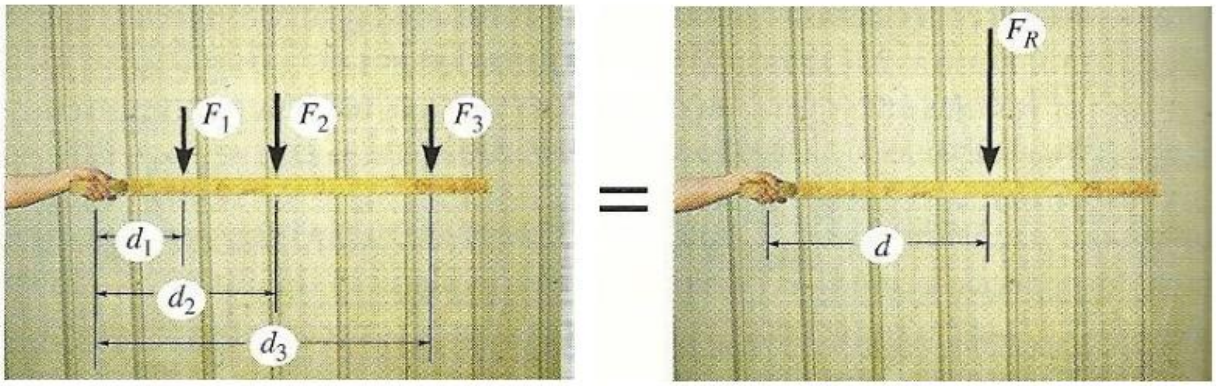


At support O

$$W_r = W_1 + W_2$$

$$M_o = W_1 d_1 + W_2 d_2$$

Equivalent Systems: Resultants



$$F_R = F_1 + F_2 + F_3$$

What is the value of d?

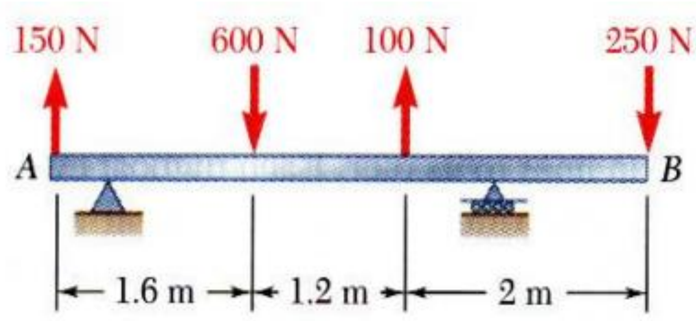
Moment of the Resultant force about the grip must be equal to the moment of the forces about the grip

$$F_R d = F_1 d_1 + F_2 d_2 + F_3 d_3$$

Equilibrium Conditions

Example.

For the beam, reduce the system of forces shown to (a) an equivalent force-couple system at A, (b) an equivalent force couple system at B, and (c) a single force or resultant. Note: Since the support reactions are not included, the given system will not maintain the beam in equilibrium.

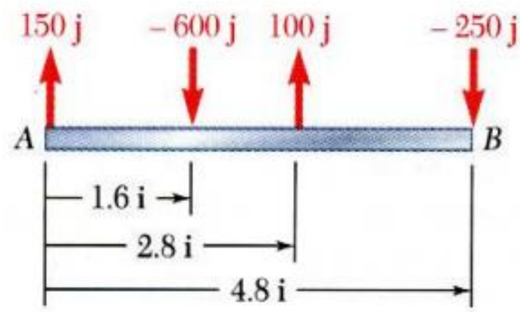


Solution: a) Compute the resultant force for the forces shown and the resultant couple for the moments of the forces about A. b) Find an equivalent force-couple system at B based on the force couple system at A. c) Determine the point of application for the resultant force such that its moment about A is equal to the resultant couple at A.

(a) Compute the resultant force and the resultant couple at A.

$$R = \sum F = 150j - 600j + 100j - 250j$$
$$= -600Nj$$

$M_A^R = \sum r \times F = 1.6i \times -600j + 2.8i \times 100j + 4.8i \times -250j = -1880N.m\ (k)$

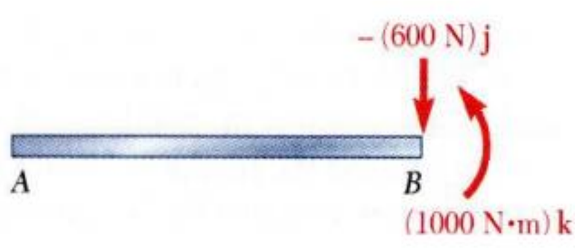


b) Find an equivalent force-couple system at B based on the force-couple system at A. The force is unchanged by the movement of the force-couple system from A to B.

$$R = \sum F = 150j - 600j + 100j - 250j = -600Nj$$

The couple at B is equal to the moment about B of the force-couple system found at A.

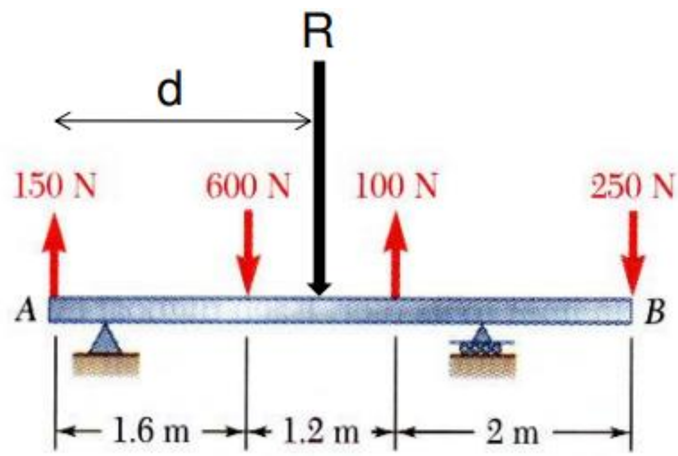
$$M_B^R = M_A^R + r_{BA} \times R = -1800k + (-4.8i \times -600j) = 1000\ N.m\ (k)$$



C)

$$R = F_1 + F_2 + F_3 + F_4 = 150 - 600 + 100 - 250 = -600\ N$$

$$Rd = F_1 d_1 + F_2 d_2 + F_3 d_3 + F_4 d_4, \quad d = 3.13\ m$$



Rigid Body Equilibrium

A rigid body will remain in equilibrium provided

• sum of all the external forces acting on the body is equal to zero, and

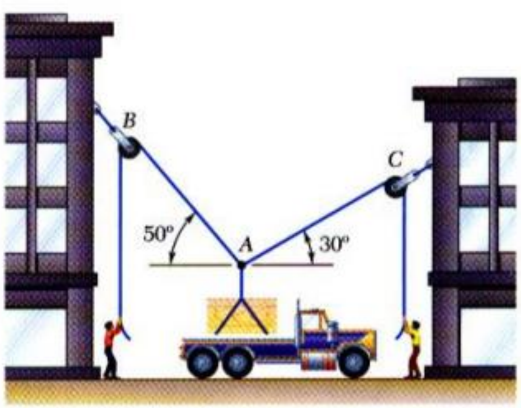
• Sum of the moments of the external forces about a point is equal to zero

$$\boxed{\begin{aligned}\Sigma F_x &= 0\\ \Sigma F_y &= 0\\ \Sigma F_z &= 0\end{aligned}} \qquad \boxed{\begin{aligned}\Sigma M_x &= 0\\ \Sigma M_y &= 0\\ \Sigma M_z &= 0\end{aligned}}$$

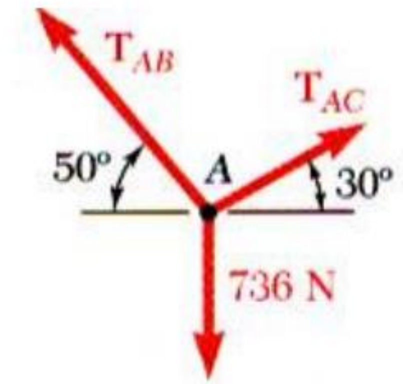
Rigid Body Equilibrium

Free-Body Diagrams

Space Diagram: A sketch showing the physical conditions of the problem.



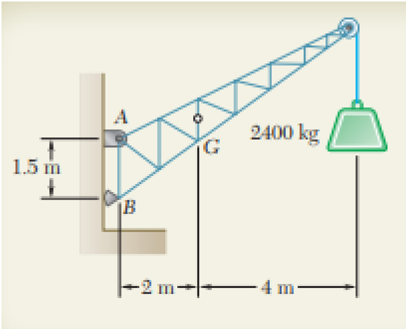
Free-Body Diagram: A sketch showing only the forces on the selected particle. Free-Body Diagrams



Support or Connection	Reaction	Number of Unknowns
Rollers; Rocker; Frictionless surface	Force with known line of action	1
Short cable; Short link	Force with known line of action	1
Collar on frictionless rod; Frictionless pin in slot	90° Force with known line of action	1
Frictionless pin or hinge; Rough surface	or α Force of unknown direction	2
Fixed support	or α Force and couple	3

Example.

A fixed crane has a mass of 1000 kg and is used to lift a 2400-kg crate. It is held in place by a pin at A and a rocker at B. The center of gravity of the crane is located at G. Determine the components of the reactions at A and B.



Example

Three loads are applied to a beam as shown. The beam is supported by a roller at A and by a pin at B. Neglecting the weight of the beam, determine the reactions at A and B when P 5 15 kips.