Fluids

Definition of Fluids.

Fluids are divided into liquids and gases. A liquid is hard to compress and as in the ancient saying (Water takes the shape of the vessel containing it), it changes its shape according to the shape of its container with an upper free surface. Gas on the other hand is easy to compress, and fully expands to fill its container. There is thus no free surface. Consequently, an important characteristic of a fluid from the viewpoint of fluid mechanics is its compressibility. Another characteristic is its viscosity.

In general, liquids are called incompressible fluids and gases compressible fluids. Nevertheless, for liquids, compressibility must be taken into account whenever they are highly pressurized, and for gases compressibility may be disregarded whenever the change in pressure is small.

1.1 Density, specific gravity, and specific volume.

The mass per unit volume of material is called the density, which is generally expressed by the symbol ρ. The density of a gas changes according to the pressure, but that of a liquid may be considered unchangeable in general. The units of density are kg/m^3 (SI). The density of water at 4°C and 1 atm (101 325 Pa, standard atmospheric pressure) is 1000 kg/m^3. The ratio of the density of a material ρ to the density of water ρ, is called the specific gravity, which is expressed by the symbol s:

$$S = \rho/\rho_w$$

The reciprocal of density, i.e. the volume per unit mass, is called the specific volume, which is generally expressed by the symbol v:

$$v = 1/\rho \qquad (m^3/kg\ \ )$$

$$\rho = \frac{p}{RT} \qquad for\ ideal\ gas$$

Table Density of water and air (standard atmospheric pressure)

Temperature (°C)		0	10	15	20	40	60	80	100
ρ (kg/m^3)	Water	999.8	999.7	999.1	998.2	992.2	983.2	971.8	958.4
	Air	1.293	1.247	1.226	1.205	1.128	1.060	1.000	0.9464

EXAMPLE

Determine the density, specific gravity, and mass of the air in a room whose dimensions are 4 m * 5 m * 6 m at 100 kPa and 25°C.

$$\rho = \frac{P}{RT} = \frac{100}{0,287 * 298} = 1.17\ Kg/m^3$$

$$s = \frac{\rho}{\rho_{H_20}} = \frac{1.17}{1000} = 0.00117$$

Finally, the volume and the mass of air in the room are

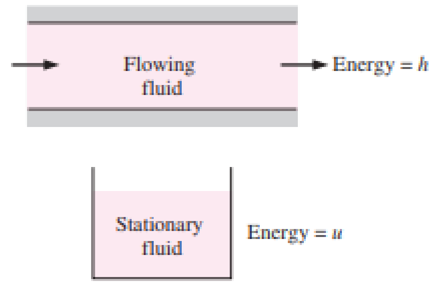
$$V = 4 * 5 * 6 = 120\ m^3$$

$$m = V\rho = 1.17 * 120 = 140kg$$

1.2 ENERGY.

Energy can exist in numerous forms such as thermal, mechanical, kinetic, potential, electrical, magnetic, chemical, and nuclear, and their sum constitutes the total energy E (or e on a unit mass basis) of a system. The sum of all microscopic forms of energy is called the internal energy of a system, and is denoted by U (or u on a unit mass basis). The energy that a system possesses as a result of its motion relative to some reference frame is called kinetic energy, the kinetic energy per unit mass is expressed as $ke=V^2/2$ where V denotes the velocity of the system relative to some fixed reference frame. The energy that a system possesses as a result of its elevation in a gravitational field is called potential energy and is expressed on a per-unit mass basis as $pe = gz$ where g is the gravitational acceleration and z is the elevation of the center of gravity of a system relative to some arbitrarily selected reference plane.

$h = u + pv$



The total energy of a simple compressible system consists of three parts: internal, kinetic, and potential energies. On a unit-mass basis, it is expressed as $e = u + ke + pe$. The fluid entering or leaving a control volume possesses an additional form of energy—the flow energy p/ρ. Then the total energy of a flowing fluid on a unit-mass basis becomes,

$$e_{flowing} = p/\rho + e = h + \frac{V^2}{2} + gz$$

1.3 Coefficient of Compressibility and Coefficient of Volume Expansion

a coefficient of compressibility k (also called the bulk modulus of compressibility or bulk modulus of elasticity) for fluids as

$$k = -\mathrm{V}\left(\frac{\partial p}{\partial \mathrm{V}}\right)_T = \rho\left(\frac{\partial p}{\partial \rho}\right)_T \qquad (p_a)$$

It can also be expressed approximately in terms of finite changes as

$$k \cong -\frac{\Delta p}{\Delta \mathrm{V}/\mathrm{V}} \cong \frac{\Delta p}{\Delta \rho/\rho} \qquad \text{T=constant}$$

Differentiating $\rho = \frac{1}{v}, d\rho = -\frac{dv}{v^2},\ \therefore \frac{d\rho}{\rho} = dv/v$

For an ideal gas,$p = \rho RT\ and\ (\partial p/\partial \rho)_T = RT = p/\rho$

$$k_{ideal\ gas} = p \qquad (p_a)$$

Ideal gas $\quad \frac{\Delta \rho}{\rho} = \frac{\Delta p}{p} \quad (T = constant)$

the variation of the density of a fluid with temperature at constant pressure.

The property that provides that information is the coefficient of volume expansion (or volume expansivity) β, defined as

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p = -1/\rho\left(\frac{\partial \rho}{\partial T}\right)_p \qquad (1/K)$$

$$\beta \approx -\frac{\frac{\Delta v}{v}}{\Delta T} \approx -\frac{\Delta\rho/\rho}{\Delta T} \qquad \text{p=constant}$$

$$\beta_{ideal\ gas} = 1/T \qquad (1/K)$$

where T is the absolute temperature.

The inverse of the coefficient of compressibility is called the isothermal compressibility α and is expressed as,

$$\alpha = \frac{1}{k} = -\mathrm{V}\left(\frac{\partial \mathrm{V}}{\partial p}\right)_T = \rho\left(\frac{\partial \rho}{\partial p}\right)_T \quad (\frac{1}{p_a})$$

The isothermal compressibility of a fluid represents the fractional change in volume or density corresponding to a unit change in pressure.

The combined effects of pressure and temperature changes on the volume change of a fluid can be determined by taking the specific volume to be a function of T and P. Differentiating v = v(T, P) and using the definitions of the compression and expansion coefficients α and β give,

$$dv = \left(\frac{\partial v}{\partial T}\right)_p dT + \left(\frac{\partial v}{\partial p}\right)_T dp = (\beta dT - \alpha dP)v$$

Then the fractional change in volume (or density) due to changes in pressure and temperature can be expressed approximately as

$$\frac{\Delta v}{v} = -\frac{\Delta\rho}{\rho} \cong \beta\Delta T - \alpha\Delta p$$

<u>EXAMPLE</u>

Consider water initially at 20°C and 1 atm. Determine the final density of water (a) if it is heated to 50°C at a constant pressure of 1 atm, and (b) if it is compressed to 100-atm pressure at a constant temperature of 20°C. Take the isothermal compressibility of water to be $\alpha = 4.80 * 10^{-5}$ atm^{-1}. (The density of water at 20°C and 1 atm pressure is $\rho_1 = 998.0$ kg/m^3. The coefficient of volume expansion at the average temperature of (20 + 50)/2 = 35°C is $\beta = 0.337 * 10^{-3}$ K^{-1})

Sol.

$$\Delta\rho \cong \rho(\alpha\Delta p - \beta\Delta T)$$

(a) The change in density due to the change of temperature from 20°C to 50°C at constant pressure is

$$\Delta\rho = -\rho\beta\Delta T = -(0.337 \times 10^{-3}) * 998 * (50 - 20) = -10$$

Noting that $\Delta\rho = \rho_2 - \rho_1$, the density of water at 50°C and 1 atm is

$$\rho_2 = \Delta\rho + \rho_1 = -10 + 998 = 988 kg/m^3$$

(b) The change in density due to a change of pressure from 1 atm to 100 atm at constant temperature is

$$\Delta\rho = \rho\alpha\Delta p = 4.8 \times 10^{-5} * 998(100 - 1) = 4.7 kg/m^3$$

Then the density of water at 100 atm and 20°C becomes

$$\rho_2 = \Delta\rho + \rho_1 = 4.7 + 998 = 1002.7\ kg/m^3$$

1.4 viscosity

When two solid bodies in contact move relative to each other, a friction force develops at the contact surface in the direction opposite to motion. To move a table on the floor, for example, we have to apply a force to the table in the horizontal direction large enough to overcome the friction force. The magnitude of the force needed to move the table depends on the ***friction coefficient*** between the table and the floor.

The situation is similar when a fluid moves relative to a solid or when two fluids move relative to each other. We move with relative ease in air, but not so in water. Moving in oil would be even more difficult, as can be observed by the slower downward motion of a glass ball dropped in a tube filled with oil. It appears that there is a property that represents the internal resistance of a fluid to motion or the "fluidity," and that property is the viscosity. The force a flowing fluid exerts on a body in the flow direction is called the drag force, and the magnitude of this force depends, in part, on viscosity.

To obtain a relation for viscosity, consider a fluid layer between two very large parallel plates (or equivalently, two parallel plates immersed in a large body of a fluid) separated by a distance *l* (Fig. 1–1). Now a constant parallel force F is applied to the upper plate while the lower plate is held fixed. After the initial transients, it is observed that the upper plate moves continuously under the influence of this force at a constant velocity V. The fluid in contact with the upper plate sticks to the plate surface and moves with it at the same velocity, and the shear stress τ acting on this fluid layer is

$$\tau = \frac{F}{A}$$

where A is the contact area between the plate and the fluid. In steady laminar flow, the fluid velocity between the plates varies linearly between 0 and V, and thus the velocity profile and the velocity gradient are

$$u(y) = \frac{y}{l}V \ \ and \ \ \frac{du}{dy} = \frac{V}{l}$$

where y is the vertical distance from the lower plate

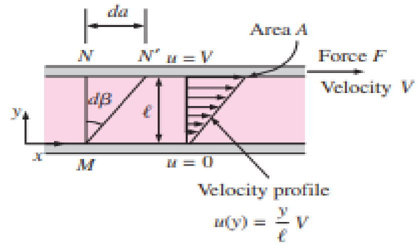


Fig. 1.1

During a differential time interval *dt*, the sides of fluid particles along a vertical line MN rotate through a differential angle dβ while the upper plate moves a differential distance *da* = *V dt*. The angular displacement or deformation (or shear strain) can be expressed as

$$d\beta \approx tan\beta = \frac{da}{l} = \frac{Vdt}{l} = \frac{du}{dy}dt$$

Rearranging, the rate of deformation under the influence of shear stress τ becomes

$$\frac{d\beta}{dt} = \frac{du}{dy}$$

Thus we conclude that the rate of deformation of a fluid element is equivalent to the velocity gradient du/dy. Further, it can be verified experimentally that for most fluids the rate of deformation (and thus the velocity gradient) is directly proportional to the shear stress τ,

$$\tau \propto \frac{d\beta}{dt} \quad or\ \tau \propto \frac{du}{dy}$$

Fluids for which the rate of deformation is proportional to the shear stress are called Newtonian fluids. Most common fluids such as water, air, gasoline, and oils are Newtonian fluids. Blood and liquid plastics are examples of non-Newtonian fluids. In one-dimensional shear flow of Newtonian fluids, shear stress can be expressed by the linear relationship

$$\tau = \mu\frac{du}{dy}$$

where the constant of proportionality μ is called the coefficient of viscosity or the dynamic (or absolute) viscosity of the fluid, whose unit is kg/m · s, or equivalently, N · s/m^2 (or Pa . s where Pa is the pressure unit pascal). A common viscosity unit is poise.

The shear force acting on a Newtonian fluid layer (or, by Newton's third law, the force acting on the plate) is

$$F = \tau A = \mu A \frac{du}{dy}$$

where again A is the contact area between the plate and the fluid. Then the force F required to move the upper plate in Fig. 1–1 at a constant velocity of V while the lower plate remains stationary is

$$F = \mu A \frac{V}{l}$$

In fluid mechanics and heat transfer, the ratio of dynamic viscosity to density appears frequently. For convenience, this ratio is given the name kinematic viscosity n and is expressed as $v = \mu/\rho$.

Consider a fluid layer of thickness *l* within a small gap between two concentric cylinders, such as the thin layer of oil in a journal bearing. The gap between the cylinders can be modeled as two parallel flat plates separated by a fluid. Noting that torque is T = FR (force times the moment arm, which is the radius R of the inner cylinder in this case), the tangential velocity is V = ωR (angular velocity times the radius), and taking the wetted surface area of the inner cylinder to be A = 2πRL by disregarding the shear stress acting on the two ends of the inner cylinder, torque can be expressed as

$$T = FR = \mu \frac{2\pi R^3 \omega L}{l} = \mu \frac{4\pi^2 R^3 \dot{n} L}{l}$$

where L is the length of the cylinder and $\dot{n}$. is the number of revolutions per unit time, which is usually expressed in rpm (revolutions per minute). Note that the angular distance traveled during one rotation is 2π rad, and thus the relation between the angular velocity in rad/min and the rpm is $\omega = 2\pi\dot{n}$.

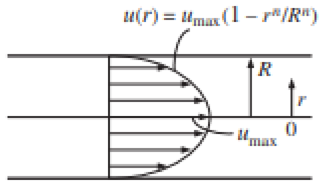
EXAMPLE

The viscosity of a fluid is to be measured by a viscometer constructed of two 40-cm-long concentric cylinders as bellow Fig. The outer diameter of the inner cylinder is 12 cm, and the gap between the two cylinders is 0.15 cm. The inner cylinder is rotated at 300 rpm, and the torque is measured to be 1.8 N . m. Determine the viscosity of the fluid.

$$\mu = T \frac{l}{4\pi^2 R^3 \dot{n} L} = \frac{1.8 * 0.0015}{4\pi^2 0.06^3 \left(\frac{\dot{300}}{60}\right) 0.4} = 0.158\ N.s/m^2$$

Q1

Consider the flow of a fluid with viscosity m through a circular pipe. The velocity profile in the pipe is given as u(r) = $u_{max}(1 - r^n/R^n)$, where umax is the maximum flow velocity, which occurs at the centerline; r is the radial distance from the centerline; and u(r) is the flow velocity at any position r. Develop a relation for the drag force exerted on the pipe wall by the fluid in the flow direction per unit length of the pipe



$$\tau_w = -\mu \frac{du}{dr}\bigg|_{r=R} = -\mu u_{max} \frac{d}{dr}\left(1 - \frac{r^n}{R^n}\right)_{r=R} = -\,\mu u_{max} \frac{-nr^{n-1}}{R^n}\bigg|_{r=R} = \frac{n\mu u_{max}}{R}$$

Q2

As shown in Fig., a cylinder of diameter 122mm and length 200mm is placed inside a concentric long pipe of diameter 125 mm. An oil film is introduced in the gap between the pipe and the cylinder. What force is necessary to move the cylinder at a velocity of lm/s? Assume that the dynamic viscosity of oil is 30 cSt and the specific gravity is 0.9.

Q3

A thin 20-cm × 20-cm flat plate is pulled at 1 m/s horizontally through a 3.6-mm-thick oil layer sandwiched between two plates, one stationary and the other moving at a constant velocity of 0.3 m/s, as shown in below Fig. The dynamic viscosity of oil is 0.027 Pa.s. Assuming the velocity in each oil layer to vary linearly, (a) plot the velocity profile and find the location where the oil velocity is zero and (b) determine the force that needs to be applied on the plate to maintain this motion

1.5 Surface tension

The surface of a liquid is apt to shrink, and its free surface is in such a state where each section pulls another as if an elastic film is being stretched. The tensile strength per unit length of assumed section on the free surface is called the surface tension.

A dewdrop appearing on a plant leaf is spherical in shape. This is also because of the tendency to shrink due to surface tension. Consequently, its internal pressure is higher than its peripheral pressure. Putting d as the diameter of the liquid drop, T as the surface tension, and p as the increase in internal pressure, the following equation is obtained owing to the balance of forces as shown in Fig. 1.3:

Force due to pressure p is

Droplet or Bubble $\quad \frac{\pi d^2}{4}\Delta p$

The opposite force due to surface tension is

Droplet $2\pi RT$ Bubble $2(2\pi R)T$

The balance of forces

$$2\pi RT = \pi R^2 \Delta p_{droplet} \rightarrow \ \Delta p_{droplet} = p_i - p_o = \frac{2T}{R}$$

Or

$$2(2\pi R)T = \pi R^2 \Delta p_{bubble} \rightarrow \ \Delta p_{bubble} = p_i - p_o = \frac{4T}{R}$$

where Pi and Po are the pressures inside and outside the droplet or bubble, respectively. When the droplet or bubble is in the atmosphere, Po is simply atmospheric pressure. The factor 2 in the force balance for the bubble is due to the bubble consisting of a film with two surfaces (inner and outer surfaces) and thus two circumferences in the cross section

The same applies to the case of small bubbles in a liquid.

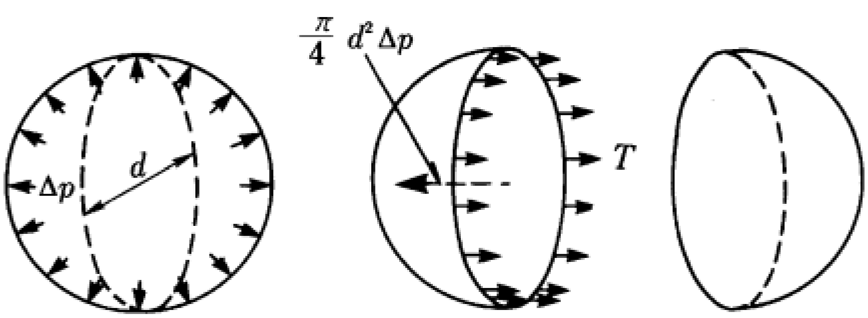


Fig. 1.3 Balance between the pressure increase within a liquid drop and the surface tension.

The excess pressure in a droplet (or bubble) also can be determined by considering a differential increase in the radius of the droplet due to the addition of a differential amount of mass and interpreting the surface tension as the increase in the surface energy per unit area. Then the increase in the surface energy of the droplet during this differential expansion process becomes

$$\delta W_{surface} = TdA = Td(4\pi R^2) = 8\pi RTdR$$

The expansion work done during this differential process is determined by multiplying the force by distance to obtain

$$\delta W_{expansion} = Force \times Distance = FdR = (\Delta PA)dR = 4\pi R^2 \Delta PdR$$

Equating the two expressions above gives $\Delta P_{droplet} = 2T/R$, which is the same relation obtained before and given in above Eq. . Note that the excess pressure in a droplet or bubble is inversely proportional to the radius.

Whenever a fine tube is pushed through the free surface of a liquid, the liquid rises up or falls in the tube as shown in Fig. 1.4 owing to the relation between the surface tension and the adhesive force between the liquid and the solid. This phenomenon is called capillarity. As shown in Fig. 1.5, d is the diameter of the tube, θ the contact angle of the liquid to the wall, ρ the density of liquid, and h the mean height of the liquid surface. The following equation is obtained owing to the balance between the adhesive force of liquid stuck to the wall, trying to pull the liquid up the tube by the surface tension, and the weight of liquid in the tube:

$$\pi dTcos\theta = \frac{\pi d^2}{4}\rho gh$$

Or

$$h = \frac{4Tcos\theta}{\rho gd} \qquad 1.4$$

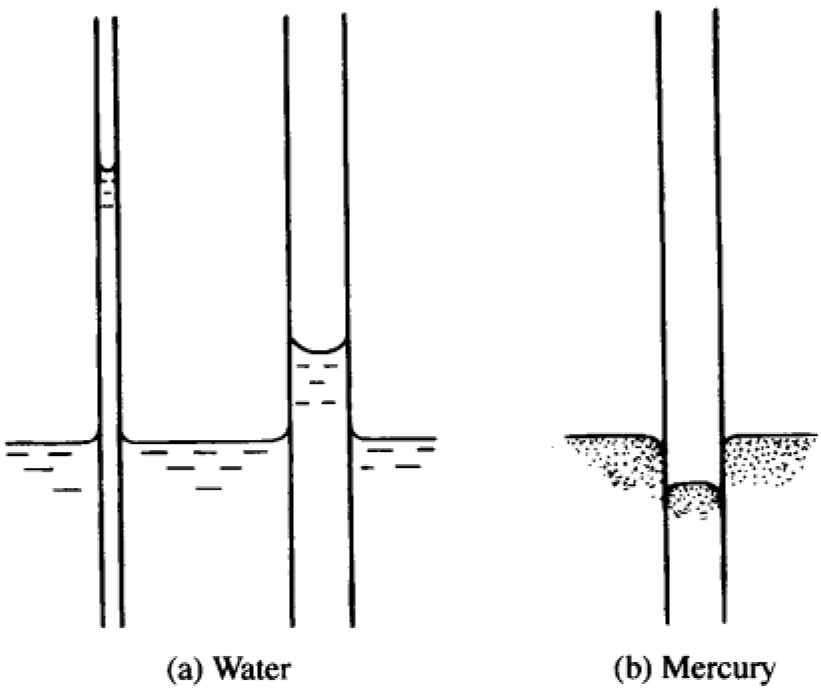


Fig. 1.4 Change of liquid surface due to capillarity

Whenever water or alcohol is in direct contact with a glass tube in air under normal temperature, $\theta \simeq 0$. In the case of mercury, $\theta = 130° - 150°$. In the case where a glass tube is placed in liquid,

$$\text{For water} \quad h = 30/d$$

$$\text{For alcohol} \quad h = 11.6/d \qquad 1.4$$

$$\text{For mercury} \quad h = -10/d$$

(in mm). Whenever pressure is measured using a liquid column, it is necessary to pay attention to the capillarity correction.

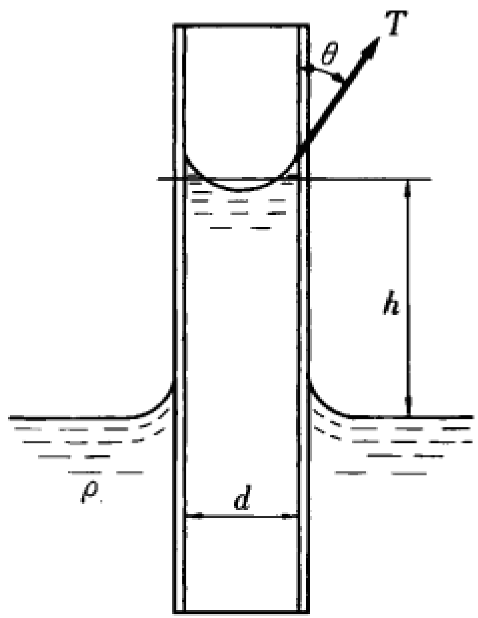


Fig. 1.5 capillarity

EXAMPLE

A 0.6-mm-diameter glass tube is inserted into water at 20°C in a cup. Determine the capillary rise of water in the tube. The surface tension of water at 20°C is 0.073 N/m and the contact angle of water with glass is 0°.

$$h = \frac{4Tcos\theta}{\rho gd} = \frac{4 \times 0.073}{1000 \times 9.81 \times 0.6 \times 10^{-3}} \cos(0) = 0.05m$$

Q1

When two plates are placed vertically on liquid as shown in Fig. 2.1 1, derive the equation showing the increased height of the liquid surface between the plates due to capillarity. Also when flat plates of glass are used with a lmm gap, what is the increased height of the water surface?

Chapter 2

Pressure.

When a uniform pressure acts on a flat plate of area A and a force F pushes the plate, then

$$p = \frac{F}{A} \qquad 2.1$$

In this case, p is the pressure and F is the pressure force. When the pressure is not uniform, the pressure acting on the minute area ΔA is expressed by the following equation:

$$p = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA} \qquad 2.2$$

Units of pressure.

The unit of pressure is the pascal (Pa), but it is also expressed in bars or metres of water column (mH,O). The conversion table of pressure units is given in Table 2.1. In addition, in some cases atmospheric pressure is used:

$$1\ atm = 760mmHg\left(at273.15K, g = 9.80665\frac{m}{s^2}\right) = 101325pa \quad 2.3$$

L atm is standard 1 atmospheric pressure in meteorology and is called the standard atmospheric pressure.

Table 2.1 Conversion of pressure units

Name of unit	Unit	Conversion
Pascal	Pa	1 Pa = 1 N/m^2
Bar	bar	1 bar = 0.1 MPa
Water column metre	mH_2O	1 mH_2O = 9 806.65 Pa
Atmospheric pressure	atm	1 atm = 101 325 Pa
Mercury column metre	mHg	1 mHg = 1/0.76 atm
Torr	torr	1 torr = 1 mm Hg

<u>Absolute pressure and gauge pressure.</u>

There are two methods used to express the pressure: one is based on the perfect vacuum and the other on the atmospheric pressure. The former is called the absolute pressure and the latter is called the gauge pressure. Then,

gauge pressure = absolute pressure - atmospheric pressure

In gauge pressure, a pressure under 1 atmospheric pressure is expressed as a negative pressure. This relation is shown in Fig. 2.1. Most gauges are constructed to indicate the gauge pressure.

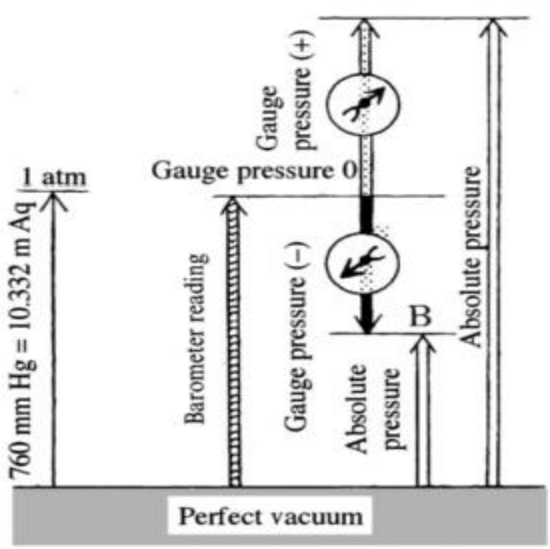


Fig. 2.1 Absolute pressure and gauge pressure.

EXAMPLE.

A vacuum gage connected to a chamber reads 5.8 psi at a location where the atmospheric pressure is 14.5 psi. Determine the absolute pressure in the chamber.

$$P_{abs} = P_{atm} - P_{vac} = 14.5 - 5.8 = 8.7\ psi$$

<u>Characteristics of pressure</u>

The pressure has the following three characteristics.

1. The pressure of a fluid always acts perpendicular to the wall in contact with the fluid.
2. The values of the pressure acting at any point in a fluid at rest are equal regardless of its direction. Imagine a minute triangular prism of unit width in a fluid at rest as shown in Fig. 2.2. Let the pressure acting on the small surfaces dA_1, dA_2 and dA be p_1, p_2 and p respectively. The following equations are obtained from the balance of forces in the horizontal and vertical directions:

$$\sum F_x = 0, \qquad p_1 dA_1 = pdAsin\theta$$

$$\sum F_y = 0, \qquad p_2 dA_2 = pdAcos\theta + \frac{1}{2} dA_1 dA_2 \rho g$$

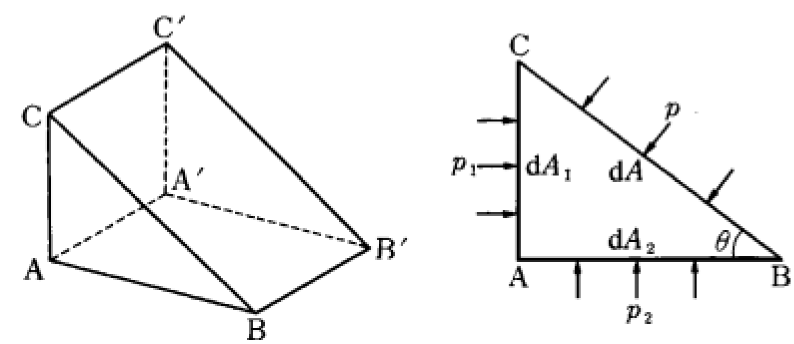


Fig. 2.2 Pressure acting on a minute triangular prism

The weight of the triangle pillar is doubly infinitesimal, so it is omitted. From geometry, the following equations are obtained:

$$dA_1 = dAsin\theta$$

$$dA_2 = dAcos\theta$$

Therefore, the following relation is obtained

$$p_1 = p_2 = p \qquad 2.4$$

Since angle 8 can be given any value, values of the pressure acting at one point in a fluid at rest are equal regardless of its direction.

3. The fluid pressure applied to a fluid in a closed vessel is transmitted to all parts at the same pressure value as that applied (Pascal's law).

In Fig. 2.3, when the small piston of area A_1 is acted upon by the force F_1, the liquid pressure $p = F_1/A_1$ is produced and the large piston is acted upon by the force $F_2 = PA_2$. Thus

$$F_2 = F_1 \frac{A_2}{A_1} \qquad 2.5$$

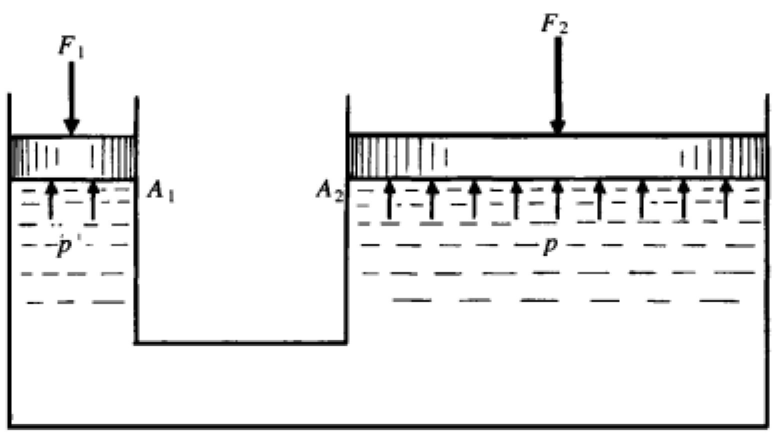


Fig. 2.3 Hydraulic press.

So this device can create the large force F_2 from the small force F_1. This is the principle of the hydraulic press.

Pressure of fluid at rest.

In general, in a fluid at rest the pressure varies according to the depth. Consider a minute column in the fluid as shown in Fig. 2.4. Assume that the sectional area is dA and the pressure acting upward on the bottom surface is p and the pressure acting downward on the upper surface (dz above the bottom surface) is $p + (dp/dz)dz$. Then, from the balance of forces acting on the column, the following equation is obtained:

$$pdA - \left(p + \frac{dp}{dz}dz\right)dA - \rho g dAdz = 0$$

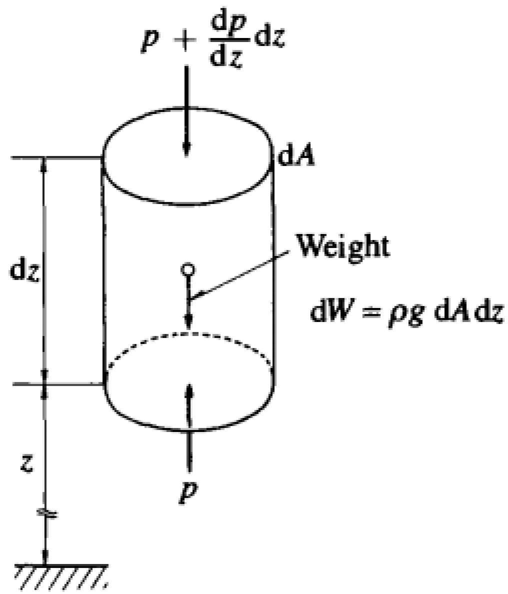


Fig. 2.4 Balance of vertical minute cylinder

$$\frac{dp}{dz} = -\rho g \qquad 2.6$$

Since p is constant for liquid, the following equation ensues:

$$p = -\rho g \int dz = -\rho g z + c \qquad 2.7$$

When the base point is set at z_o below the upper surface of liquid as shown in Fig. 2.5, and p_o is the pressure acting on that surface, then $p = p_o$ when $z = z_o$, so

$$c = p_\circ + \rho g z_\circ$$

Substituting this equation into eqn (2.7),

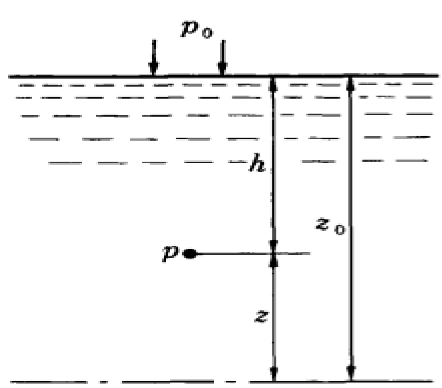


Fig. 2.5 pressure in liquid

$$p = p_\circ + (z_\circ - z)\rho g = p_\circ + \rho g h \qquad 2.8$$

Thus it is found that the pressure inside a liquid increases in proportion to the depth.

For the case of a gas, let us study the relation between the pressure and the height of the atmosphere surrounding the earth. In this case, since the density of gas changes with pressure, it is not possible to integrate simply as in the case of a liquid. As the altitude increases, the temperature decreases. Assuming this temperature change to be polytropic, then $pv^n = C$. is the defining -relationship.

Putting the pressure and density at $z = 0$ (sea level) as p_o and ρ_o respectively, then

$$\frac{p}{\rho^n} = \frac{p_o}{\rho_o^n} \qquad 2.9$$

Substituting ρ into eqn (2.6),

$$dz = -\frac{dp}{\rho g} = -\frac{1}{g}\frac{p_o^{\frac{1}{n}}}{\rho_o}\, p^{-\frac{1}{n}} dp = -\frac{1}{g}\frac{p_o}{\rho_o}\left(\frac{p_o}{p}\right)^{\frac{1}{n}} d\left(\frac{p}{p_o}\right) \qquad 2.10$$

Integrating this equation from z = 0 (sea level),

$$z = \int_0^z dz = \frac{1}{g}\frac{n}{n-1}\frac{p_o}{\rho_o}\left[1 - \left(\frac{p}{p_o}\right)^{(n-1)/n}\right] \qquad 2.11$$

The relation between the height and the atmospheric pressure develops into the following equation by eqn (2.1 1):

$$\frac{p(z)}{p_o} = \left(1 - \frac{n-1}{n}\frac{\rho_o g}{p_o} z\right)^{n/(n-1)} \qquad 2.12$$

Also, the density is obtained as follows from eqs (2.9) and (2.12):

$$\frac{\rho(z)}{\rho_o} = \left(1 - \frac{n-1}{n}\frac{\rho_o g}{p_o} z\right)^{1/(n-1)} \qquad 2.13$$

When the absolute temperatures at sea level and at the point of height z are T_o, and T respectively, the following equation is obtained from eqn ($pv = RT$):

$$\frac{p}{\rho T} = \frac{p_o}{\rho_o T_o} = R \qquad 2.14$$

From eqs (2.12)-(2.14)

$$\frac{T(z)}{T_o} = 1 - \frac{n-1}{n}\frac{\rho_o g}{p_o} z \qquad 2.15$$

From eqn (2.15)

$$\frac{dT}{dz} = -\frac{n-1}{n}\frac{\rho_o g}{p_o} T_o \qquad 2.16$$

In aeronautics, it has been agreed to make the combined values of p_o = 101.325 kPa, T_o= 288.15 K and ρ_o = 1.225 kg/m^3 the standard atmospheric condition at sea level. The temperature decreases by 0.65^{o}C every l00m of height in the troposphere up to approximately 1 km high, but is constant at -50.5 ^{0}C from 1 km to l0km high. For the troposphere, from the above values for p_o, T_o and ρ_o in eqn (2.10), n = 1.235 is obtained as the polytropic index.

Measurement of pressure: -

Manometer A device which measures the fluid pressure by the height of a liquid column is called a manometer. For example, in the case of measuring the pressure of liquid flowing inside a pipe, the pressure *p* can be obtained by measuring the height of liquid *H* coming upwards into a manometer made to stand upright as shown in Fig. 2.6 (a). When p_o is the atmospheric pressure and ρ is the density, the following equation is obtained:

$$p = p_o + \rho g H \qquad 2.17$$

When the pressure p is large, this is inconvenient because H is too high. So a U-tube manometer, as shown in Fig. 2.6 (b), containing a high-density liquid such as mercury is used. In this case, when the density is ρ',

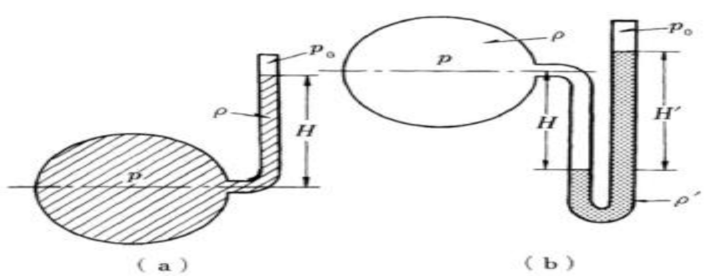


Fig. 2.5 Manometer

$$p + \rho g H = p_o + \rho' g H'$$

$$or \quad p + \rho g H = p_o + \rho' g H' \qquad 2.18$$

In the case of measuring the air pressure, ρ' >> ρ, so pgH in eqn (2.18) may be omitted. In the case of measuring the pressure difference between two pipes in both of which a liquid of density ρ flows, a differential manometer as shown in Fig. 2.7 is used. In the case of Fig. 2.7(a), where the differential pressure of the liquid is small, measurements are made by filling the upper section of the meter with a liquid whose density is less than that of the liquid to be measured, or with a gas. Thus

$$p_1 - p_2 = (\rho - \rho')gH \qquad 2.19$$

and in the case where ρ' is a gas,

$$p_1 - p_2 = \rho g H \qquad 2.20$$

Figure 2.7(b) shows the case when the differential pressure is large. This time, a liquid column of a larger density than the measuring fluid is used.

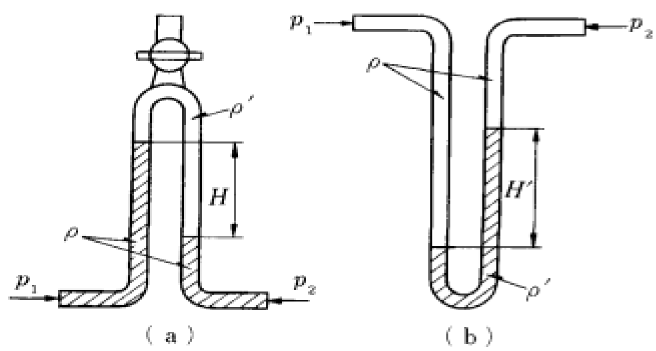


Fig. 2.7 Differential manometer (1)

Thus

$$p_1 - p_2 = (\rho' - \rho)gH' \qquad 2.21$$

and in the case where ρ is a gas,

$$p_1 - p_2 = \rho' g H' \qquad 2.22$$

A U-tube manometer as shown in Fig. 2.7 is inconvenient for measuring fluctuating pressure, because it is necessary to read both the right and left water levels simultaneously to measure the different pressure. For measuring the differential pressure, if the sectional area of one tube is made large enough, as shown in Fig. 2.8, the water column of height H could be measured by just reading the liquid surface level in the other tube because the surface fluctuation of liquid in the tank can be ignored. To measure a minute pressure, a glass tube inclined at an appropriate angle as shown in Fig. 2.9 is used as an inclined manometer. When the angle of inclination is α and the movement of the liquid surface level is L, the differential pressure H is as shown in the following equation:

$$H = L\, sin\alpha \quad 2.23$$

Accordingly, if a is made smaller, the reading of the pressure is magnified. Besides this, GÖttingen-type micromanometer, Chattock tilting micromanometer, etc., are used.

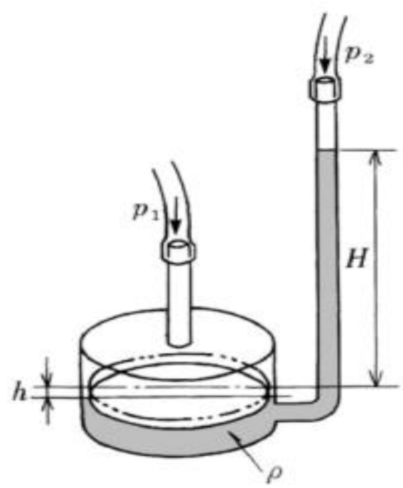


Fig. 2.8 Differential manometer (2)

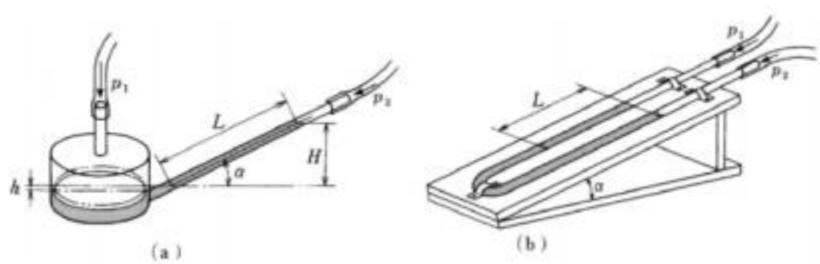
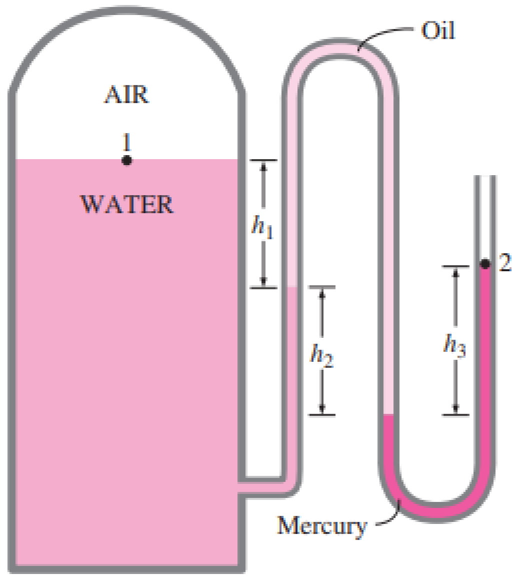


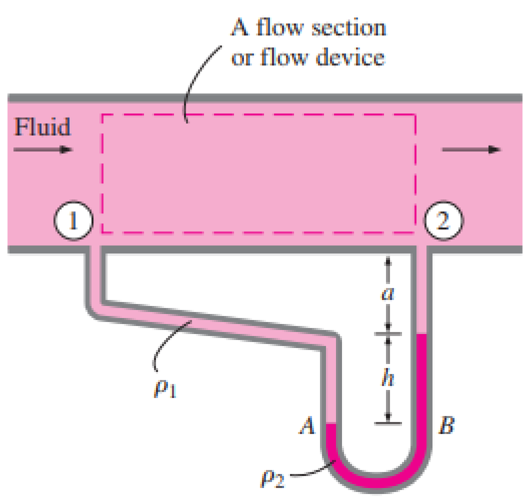
Fig. 3.9 inclined manometer

EXAMPLE.

The water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer as shown in below Figure. The tank is located on a mountain at an altitude of 1400 m where the atmospheric pressure is 85.6 kPa.

Determine the air pressure in the tank if h_1= 0.1 m, h_2= 0.2 m, and h_3= 0.35 m. Take the densities of water, oil, and mercury to be 1000 kg/m^3, 850 kg/m^3, and 13,600 kg/m^3, respectively.





Elastic-type pressure gauge

An elastic-type pressure gauge is a type of pressure gauge which measures the pressure by balancing the pressure of the fluid with the force of deformation of an elastic solid. The Bourdon tube (invented by Eugene Bourdon, 1808-84) (Fig. 2.10), the diaphragm (Fig. 2.1 l), the bellows, etc., are widely employed for this type of pressure gauge. Of these, the Bourdon tube pressure gauge (Bourdon gauge) of Fig. 2.10 is the most widely used in industry. A curved metallic tube of elliptical cross section (Bourdon tube) is closed at one end which is free to move, but the other end is rigidly fixed to the frame. When the pressure enters from the fixed end, the cross-section tends to become circular so the free end moves outward. By amplifying this movement, the pressure values can be read. When the pressure becomes less than the atmospheric pressure (vacuum), the free end moves inward, so this gauge can be used as a vacuum gauge.

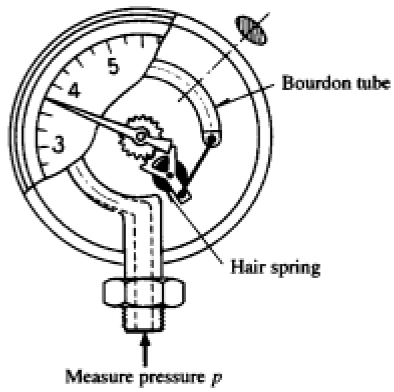


Fig. 2.10 Bourdon tube pressure gauge

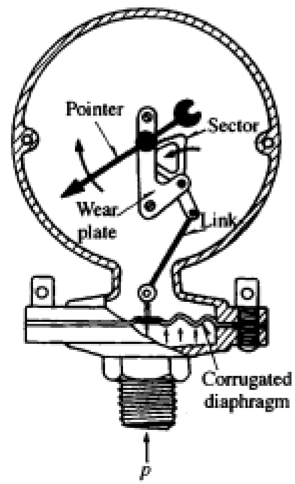
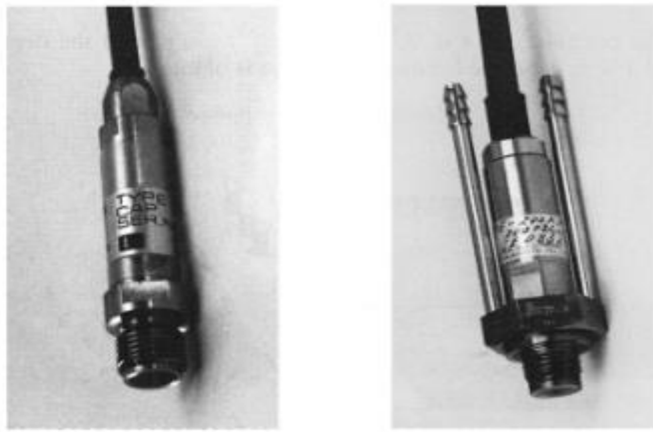


Fig. 2.11 Diaphragm pressure gauge

<u>Electric-type pressure gauge</u>

The pressure is converted to the force or displacement passing through the diaphragm, Bourdon tube bellows, etc., and is detected as a change in an electrical property using a wire strain gauge, a semiconductor strain gauge (applied piezo resistance effect), etc. These types of pressure gauge are useful for measuring fluctuating pressures. Two examples of pressure gauges utilizing the wire strain gauge are shown in Fig. 2.12.



	PGM-C (a) Small size	PE-J, P (b) For indicating pressure of an engine
Pressure range (MPa)	0.2–1.0	5–20
Natural frequency (kHz)	25–40	25–40
Response frequency (kHz)	5–8	5–8

Fig 2.13 Wire strain gauge type of pressure transducer

<u>2.2 forces acting on the vessel of liquid:-</u>

How large is the force acting on the whole face of a solid wall subject to water pressure, such as the bank of a dam, the sluice gate of a dam or the wall of a water tank? How large must the torque be to open the sluice gate of a dam? What is the force required to tear open a cylindrical vessel subject to inside pressure? Here, we will study forces like these.

<u>2.3 Force acting on the vessel of liquid</u>

How large is the force acting on the whole face of a solid wall subject to water pressure, such as the bank of a dam, the sluice gate of a dam or the wall of a water tank? How large must the torque be to open the sluice gate of a dam? What is the force required to tear open a cylindrical vessel subject to inside pressure? Here, we will study forces like these.

3.2.1 <u>Water pressure acting on a bank or a sluice gate</u>

How large is the total force due to the water pressure acting on a bank built at an angle Θ to the water surface (Fig. 2.14)? Here, disregarding the atmospheric pressure, the pressure acting on the surface is zero. The total pressure dP acting on a minute area dA is $\rho gh\, dA = \rho gy \sin \Theta\, dA$. So, the total pressure P acting on the underwater area of the bank wall A is:

$$F = \int_A dp = \rho g sin\theta \int_A ydA$$

When the centroid (The centre of mass when the mass is distributed uniformly on the plane of some figure, namely the point applied to the centre of gravity, is called a centroid.) of A is G, its y coordinate is y_G and the depth to G is h_G, $\int_A ydA = y_G A$. So the following equation is obtained:

$$F = \rho g sin\theta y_G A = \rho g h_G A \qquad 2.24$$

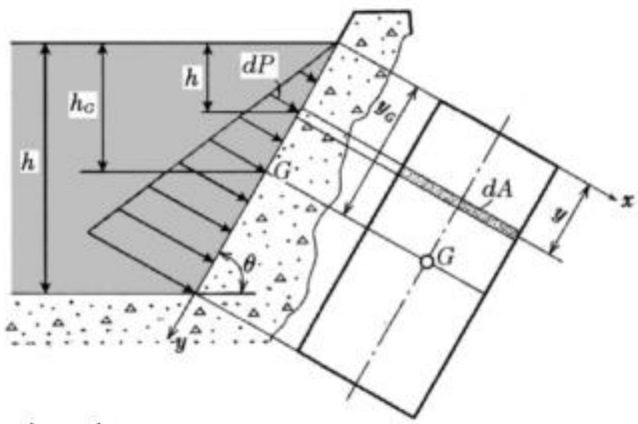


Fig 2.14 force acting on dam

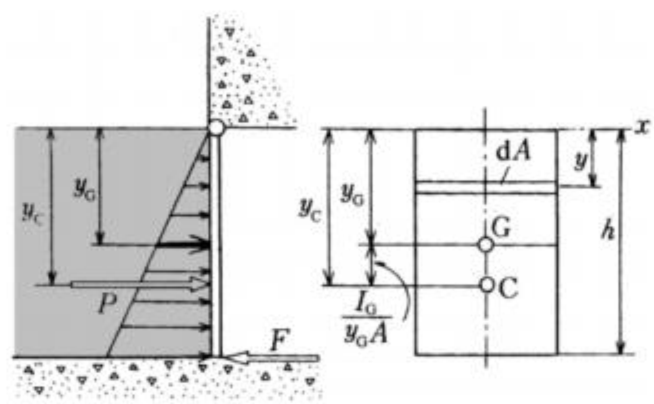


Fig. 2.15 revolving power acting on water gate (1) (case where revolving axis of water gate is just on the water level)

So the total force F equals the product of the pressure at the centroid G and the underwater area of the bank wall. Next, let us study a rectangular sluice gate as shown in Fig. 2.15. How large is the torque acting on its turning axis (the x axis)? The force F acting on the whole plane of the gate is $\rho g y_G A$ by eqn

(2.24). The force acting on a minute area dA (a horizontal strip of the gate face) is $\rho g y dA$, the moment of this force around the x axis is $\rho g y dA * y$ and the total moment on the gate is $\int \rho g y^2 dA = \rho g \int y^2 dA = \rho g I_x$, $\int y^2 dA$ is called the geometrical moment of inertia I_x, for the x axis.

Now let us locate the action point of F (i.e. the centre of pressure C) at which a single force F produces a moment equal to the total sum of the moments around the turning axis (x axis) of the sluice gate produced by the total water pressure acting on all points of the gate. When the location of C is y_c,

$$F y_c = \rho g I_x \qquad 2.25$$

Now, when I_G, is the geometrical moment of inertia of area for the axis which is parallel to the x axis and passes through the centroid G, the following relation exist (Parallel axb theorem: The moment of inertia with respect to any axis equals the sum of the moment of inertia with respect to the axis parallel to this axis which passes through the centroid and the product of the sectional area and the square of the distance to the centroid from the former axis.):

$$I_x = I_G + A y_G^2 \qquad 2.26$$

Values of I_G, for a rectangular plate and for a circular plate are shown in Fig. 2.15.

Substitute eqn (2.26) into (2.25) to calculate y_c,

$$y_c = y_G + \frac{I_G}{A y_G} \qquad 2.27$$

$$= y_G + \frac{h^2}{12 y_G}$$

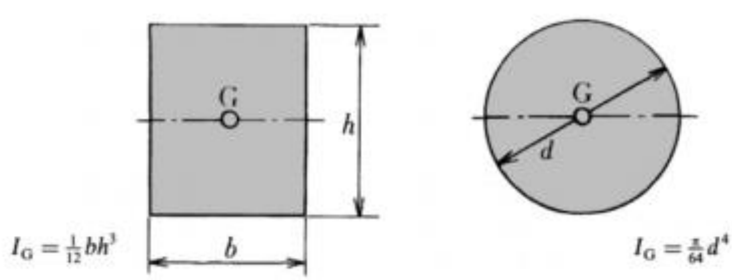


Fig. 3.15 Geometrical moment of inertia for axis passing centroid G

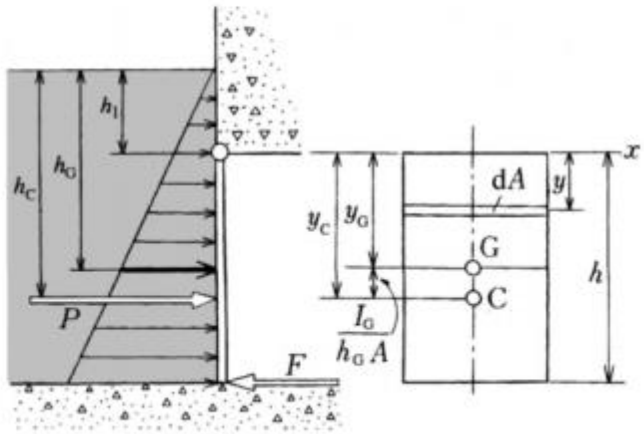


Fig 3.16 rotational force acting on water gate (2) (case where water gate is under water)

From eqn (2.27), it is clear that the action point C of the total pressure P is located deeper than the centroid G by $h^2/12y_G$.

The position of y_c in such a case where the sluice gate is located under the water surface as shown in Fig. 2.16 is given by eqn (2.28) where h_G is substituted for y_G in the second term on the right of eqn (2.27)

$$y_c = y_G + \frac{I_G}{Ah_G} \qquad 2.28$$

$$y_c = y_G + \frac{h^2}{12h_G}$$

A heavy car plunges into a lake during an accident and lands at the bottom of the lake on its wheels (Fig. 3–17). The door is 1.2 m high and 1 m wide, and the top edge of the door is 8 m below the free surface of the water. Determine the hydrostatic force on the door and the location of the pressure center, and discuss if the driver can open the door.

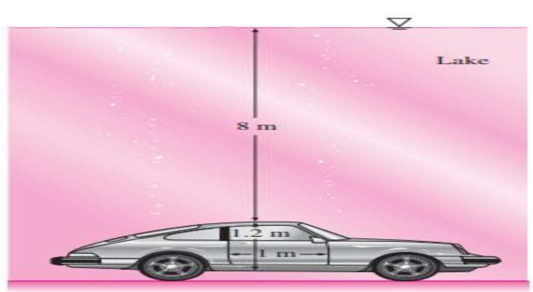


Fig. 3.17

$$P_c = \rho g h_c = 1000 * 9.81 * \left(8 + \frac{1.2}{2}\right) = 84.4\ kN/m^2$$

$$F = P_c A = 84.4 * 1 * 1.2 = 101.3\ kN$$

$$y_c = 8 + \frac{b}{2} + \frac{b^2}{12(8 + b/2)} = 8.61$$

2.2.2. ***HYDROSTATIC FORCES ON SUBMERGED CURVED SURFACES***

The easiest way to determine the resultant hydrostatic force FR acting on a two-dimensional curved surface is to determine the horizontal and vertical components F_H and F_V separately. This is done by considering the free-body diagram of the liquid block enclosed by the curved surface and the two plane surfaces (one horizontal and one vertical) passing through the two ends of the curved surface, as shown in Fig. 3–19. Note that the vertical surface of the liquid block considered is simply the projection of the curved surface on a vertical plane, and the horizontal surface is the projection of the curved surface on a horizontal plane.

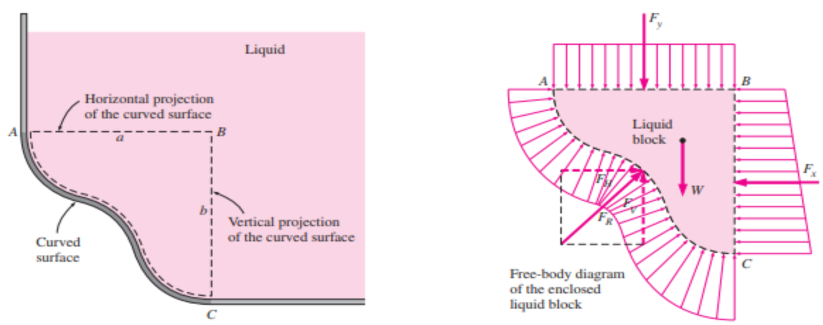


Fig. 3.19

Horizontal force component on curved surface: $F_H = F_x$

Vertical force component on curved surface: $F_V = F_y + W$

$$W = \rho g V$$

$$F_R = \sqrt{F_H^2 + F_V^2}$$

$$tan \propto = \frac{F_V}{F_H}$$

The magnitude of the resultant hydrostatic force acting on the curved surface is F_R, and the tangent of the angle it makes with the horizontal is $tan \propto = \frac{F_V}{F_H}$.

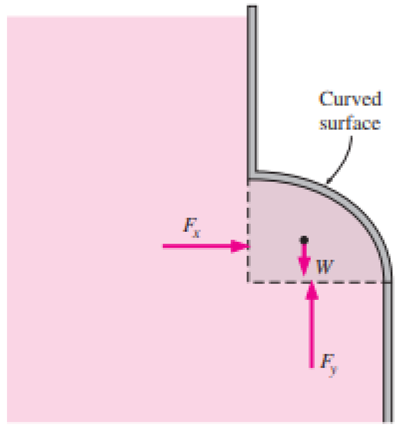


Fig. 3–20

EXAMPLE

A long solid cylinder of radius 0.8 m hinged at point A is used as an automatic gate, as shown in Fig. 3–21. When the water level reaches 5 m, the gate opens by turning about the hinge at point A. Determine (a) the hydrostatic force acting on the cylinder and its line of action when the gate opens and (b) the weight of the cylinder per m length of the cylinder.

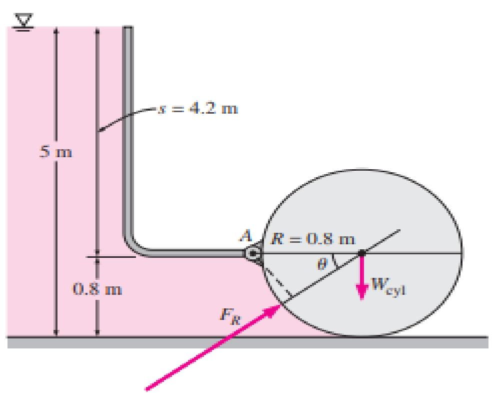


Fig. 3–21

$$F_H = F_x = P_c A = \rho g h_c A = \rho g (s + R/2) A$$
$$= 1000 * 9.81 * (4.2 + 0.4) * (0.8 * 1) = 36.1\ kN$$

$$F_y = PA = \rho g h A = \rho g h_{bottom} A = 1000 * 9.81 * 5 * (0.8 * 1) = 39.2 kN$$

$$W = mg = \rho V g = \rho g \left(R^2 - \frac{\pi R^2}{4} \right) * 1 = 1.3\ kN$$

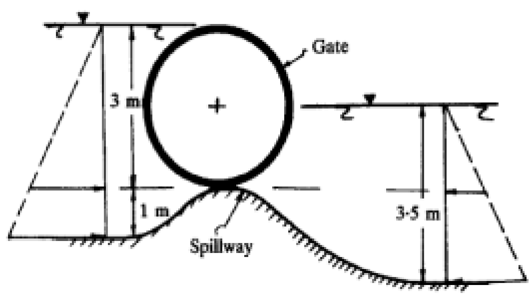
$$F_V = F_y - W = 37.9\ kN$$

$$F_R = 52.3\ kN$$

$$tan \propto = 1.05, \propto = 46.4°$$

Example

A 3 m diameter roller gate retains water on both sides of a spillway crest as shown in the figure below. Determine (i) the magnitude, direction and location of the resultant hydrostatic thrust acting on the gate per unit length, and (ii) the horizontal water thrust on the spillway per unit length.



Left side

$$F_x = \frac{1}{2} \rho g\ 3^2 = 44.14\ kN$$

$$F_V = \rho g\ \frac{1}{2} \frac{\pi}{4}\ 3^2 = 34.67 kN$$

Right side

$$F_x = \frac{1}{2} \rho g\ 1.5^2 = 11.03\ kN$$

$$F_V = \rho g\ \frac{1}{4} \frac{\pi}{4}\ 3^2 = 17.34 kN$$

Net F_x

$$F_x = 44.14 - 11.03 = 33.11 kN$$

Net F_y

$$F_y = 34.67 + 17.34 = 50.01\ kN$$

$$R = \sqrt{F_x^2 + F_y^2} = 60\ kN$$

2.2.2 force to tear a cylinder

In the case of a thin cylinder where the inside pressure is acting outward, as shown in Fig. 2.17(a), what kind of force is required to tear this cylinder in the longitudinal direction? Now, consider the cylinder longitudinally half sectioned as shown in Fig. 2.17(b), with diameter d, length 1 and inside press p. the force acting on the assumed vertical center wall ABCD is pdl which balances the force in the direction acting outward on the cylinder wall. In other words, the force generated by the pressure in the x direction on a curved surface equals the pressure pdl, since the same pressure acts on the projected area of the curved surface. Furthermore, this force is the force 2Tl (T is the force acting per unit length of wall which tears this cylinder longitudinally in halves along the lines BC and AD):

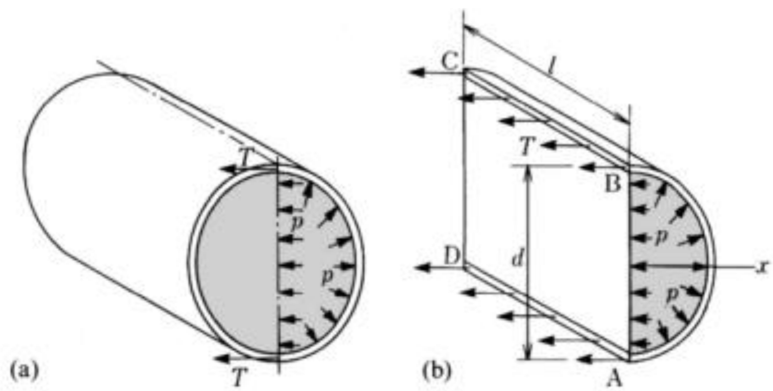


Fig. 2.17 cylinder acted on by inertial pressure

$$2Tl = pdl$$

Or

$$T = \frac{pd}{2} \qquad 2.29$$

If the tensile stress due to T is lower than the allowable stress, safety is assured. By utilizing this principle, a thin-walled pressure tank can be designed.

2.3 BUOYANCY AND STABILITY

Fluid pressure acts all over the wetted surface of a body floating in a fluid, and the resultant pressure acts in a vertical upward direction. This force is called buoyancy. The buoyancy of air is small compared with the gravitational force of the immersed body, so it is normally ignored. Suppose that a cube is located in a liquid of density ρ as shown in Fig. 2.18. The pressure acting on the cube due to the liquid in the horizontal direction is balanced right and left. For the vertical direction, where the atmospheric pressure is p_o, the force F_l acting on the upper surface A is expressed by the following equation:

$$F_1 = (p_o + \rho g h_1)A \qquad 2.30$$

The force F_2 acting on the lower surface is

$$F_2 = (p_o + \rho g h_2)A \qquad 2.31$$

So, when the volume of the body in the liquid is V, the resultant force F from the pressure acting on the whole surface of the body is

$$F_B = F_2 - F_1 = \rho g(h_2 - h_1)A = \rho g h_{sub} A = \rho g V_{sub} \qquad 2.32$$

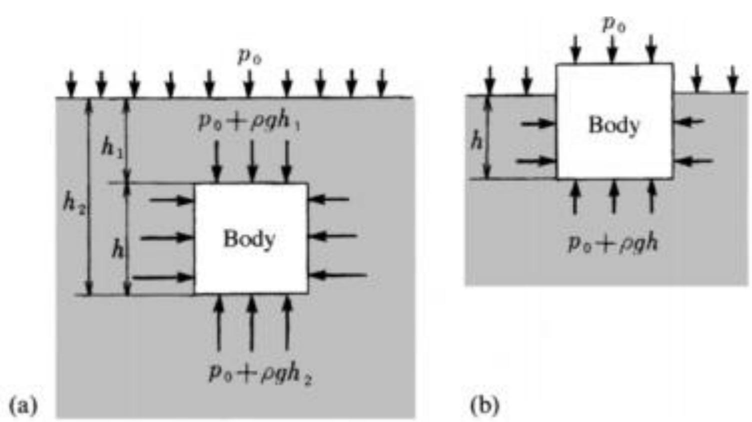


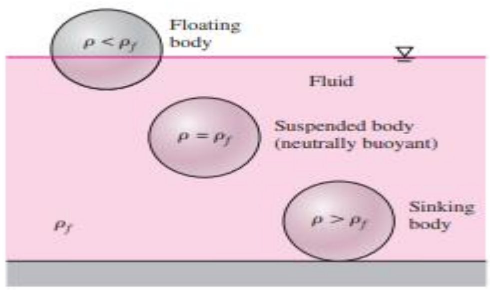
Fig. 2.18 Cube in liquid

The same applies to the case where a cube is floating as shown in Fig. 2.18(b). From this equation, the body in the liquid experiences a buoyancy equal to the weight of the liquid displaced by the body. This result is known as Archimedes' principle.

(The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume.)

For floating bodies, the weight of the entire body must be equal to the buoyant force, which is the weight of the fluid whose volume is equal to the volume of the submerged portion of the floating body. That is,

$$F_B = W \rightarrow \rho_f g V_{sub} = \rho_b g V_{total} \rightarrow \frac{V_{sub}}{V_{total}} = \frac{\rho_b}{\rho_f}$$



EXAMPLE 3–1

A crane is used to lower weights into the sea (density " 1025 kg/m3) for an underwater construction project (Fig.). Determine the tension in the rope of the crane due to a rectangular 0.4-m (0.4-m (3-m concrete block (density " 2300 kg/m3) when it is (a) suspended in the air and (b) completely immersed in water.

a)

$$V = 0.4 \times 0.4 \times 3 = 0.48\ m^3$$

$$F_{T.air} = W = \rho g V = 10.8\ kN$$

b)

$$F_B = \rho_f g V_{sub} = 1025 * 9.81 * 0.48 = 4.8 kN$$

$$F_{T.water} = W - F_B = 10.8 - 4.8 = 6\ kN$$

The center of gravity of the displaced liquid is called ‘center of buoyancy’ and is the point of action of the buoyancy force. Next, let us study the stability of a ship. Figure 2.19 shows a ship of weight W floating in the water with an inclination of small angle Θ. The location of the centroid G does not change with the inclination of the ship. But since the center of buoyancy C moves to the new point C′, a couple of forces *Ws* = *Fs* is produced and this couple restores the ship's position to stability.

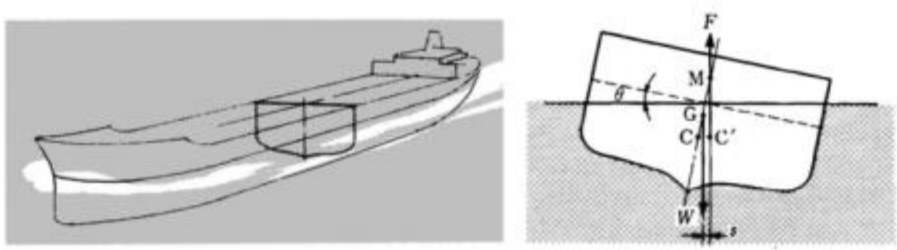


Fig. 2.19 stability of a ship

The forces of the couple *Ws* are called restoring forces. The intersecting point M on the vertical line passing through the center of buoyancy C' (action line of the buoyancy *F*) and the center line of the ship is called the metacenter, and GM is called the metacentric height (How high is the metacenter of a real ship? It is said that the height of metacenter of a warship is about 0.8-1.2 m, a sailing ship 1 .O-1.4 m and a large passenger ship 0.3-0.7 m. When these ships go out to sea the wave cycle is 12-13 seconds.). As shown in the figure, if M is located higher than G, the restoring force acts to stabilize the ship, but if M is located lower than G, the couple of forces acts to increase the roll of the ship and so make the ship unstable.

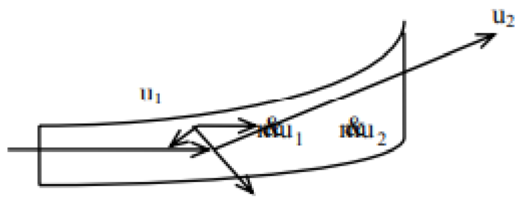
A pipe bend tapers from a diameter of d1 of 500mm at inlet to a diameter d2 of 250mm at outlet and turns the flow trough an angle u of 45o . Measurements of pressure at inlet and outlet show that p1 = 40 kPa and p2 = 23 kPa. If the pipe is conveying oil (r = 850 kg/m3). Calculate the magnitude and direction of the resultant force on the bend when the oil is flowing at the rate of 0.45m3 /s.

Fluid Momentum

$$F = ma = m\frac{dv}{dt} = \frac{d}{dt}(mv)$$

m –mass

a- Acceleration



Where

$$dm\ v_1 is\ the\ mass\ flow\ in\ input$$

$$dm\ v_2 is\ the\ mass\ flow\ in\ output$$

$$dm\ is\ the\ mass\ flow\ in\ the\ input\ and\ output\ in\ control\ volume\ with\ dt$$

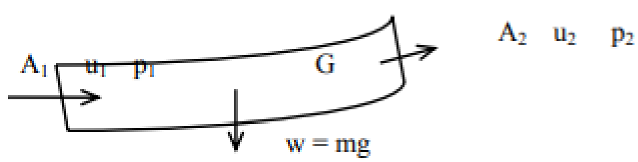
Where the $\frac{dm}{dt} = \dot{m}\ is\ the\ mass\ flow\ rate\ in\ the\ input\ output$

$$F = \dot{m}v_1 - \dot{m}v_2$$

$$\dot{m} = \rho_1 A_1 v_1 = \rho_2 A_2 v_2,$$

$$F = \dot{m}(v_1 - v_2)$$

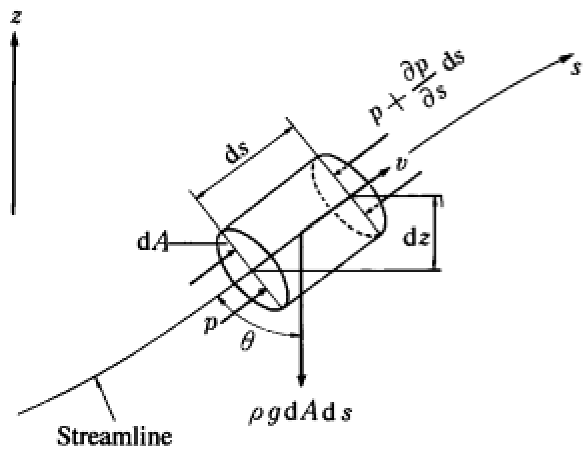
$$R = -F = \dot{m}(v_1 - v_2)$$



$$R = -F = (p_1 A_1 + \dot{m}v_1) - (p_2 A_2 + \dot{m}v_2) - W$$

$$R = -F = A_1(p_1 + \rho v_1^2) - A_2(p_2 + \rho v_2^2)$$

Bernoulli's equation



Force acting on fluid on streamline

$$\rho dAds\frac{dv}{dt} = -dA\frac{\partial p}{\partial s}ds - \rho gdAdscos\theta$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial s} - gcos\theta$$

v = v(s, t)

$$dv = \frac{\partial v}{\partial t}dt + \frac{\partial v}{\partial s}ds$$

The acceleration is then

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial s}\frac{ds}{dt} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial s}$$

If the z axis is the vertical direction

$$cos\theta = dz/ds$$

So $\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial s} - gcos\theta$ becomes

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial s} = -\frac{1}{\rho}\frac{\partial p}{\partial s} - g\frac{dz}{ds}$$

In the steady state, $\frac{\partial v}{\partial t} = 0$

$$v\frac{\partial v}{\partial s} = -\frac{1}{\rho}\frac{\partial p}{\partial s} - g\frac{dz}{ds}$$

is called equation of motion for one dimensional non-viscous fluid flow

Euler's equation is integrated with respect to s to obtain a relationship between points a finite distance apart along the streamline. This gives

$$\frac{v^2}{2} + \int \frac{dp}{\rho} + gz = constant$$

and for an incompressible fluid

$$\frac{v^2}{2} + \frac{p}{\rho} + gz = constant \quad 5.7$$

between arbitrary points, and therefore at all points, along a streamline. Dividing each term in eqn $\frac{v^2}{2} + \frac{p}{\rho} + gz = constant$ by g,

$$\frac{v^2}{2g} + \frac{p}{\rho g} + z = H = constant \quad 5.8$$

Multiplying each term of eqn (5.7) by ρ,

$$\frac{\rho v^2}{2} + p + \rho gz = constant \quad 5.9$$

The units of the terms in eqn (5.7) are m^2/s^2, which can be expressed as $kgm^2/(s^2$ kg). Since kgm^2/s^2 = J (for energy), then $v^2/2$, p/ρ and gz in eqn (5.7) represent the kinetic energy, energy due to pressure and potential energy respectively, per unit mass.

The terms of eqn (5.8) represent energy per unit weight, and they have the units of length (m) so they are commonly termed heads.

$u^2/2g$ - : velocity head

p/ρg pressure head

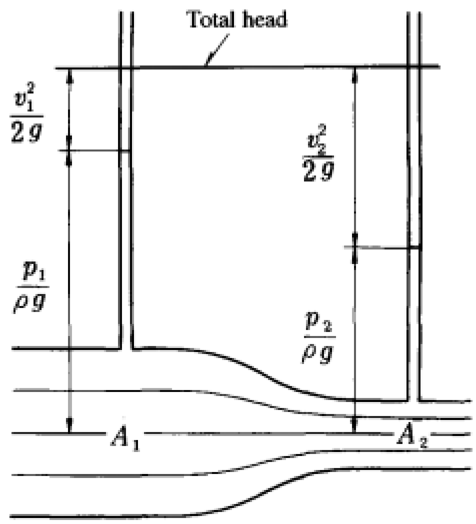
z : potential head

H : total head

The units of the terms of eqn (5.9) are kg/(s^2m) expressing energy per unit volume. Thus, eqns (5.7) to (5.9) express the law of conservation of energy in that the sum of the kinetic energy, energy due to pressure and potential energy (Le. the total energy) is always constant. This is Bernoulli's equation. If the streamline is horizontal, then the term ρgh can be omitted giving the following:

$$\frac{\rho v^2}{2} + p_s = p_t$$

where $\rho v^2/2$ is called the dynamic pressure, p_s the static pressure, and p_t the total pressure or stagnation pressure.



Exchange between pressure head and velocity head

Bernoulli's equation in such system is simply an expression of the work-energy theorem. What is the energy of the fluid at some position in the pipe? It is a sum of kinetic energy, potential gravitational energy, and internal energy, put by external force. For a mass element dm=ρAdx, we can write this as

$$E = E_K + E_P + E_i$$

$$E_K = \frac{dmv^2}{2}$$

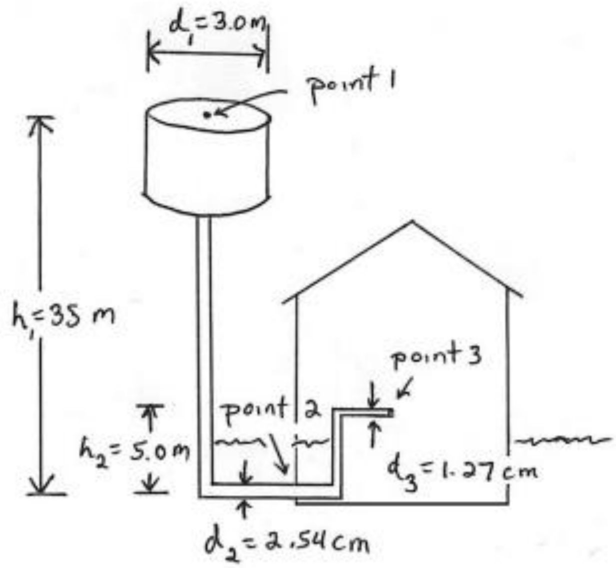
$$E_P = dmgh$$

$$E_i = pdV$$

$$P + \frac{1}{2}\rho v^2 + \rho gy = \text{constant}$$

Ex

A cylindrical water tower of diameter 3.0 m supplies water to a house. The level of water in the water tower is 35 m above the point where the water enters the house through a pipe that has an inside diameter 5.1cm. The intake pipe delivers water at a maximum rate of $2.0 \times 10^{-3}\ \mathrm{m}^3 \cdot \mathrm{s}^{-1}$. The pipe is connected to a narrower pipe leading to the second floor that has an inside diameter 2.5 cm. What is the pressure and speed of the water in the narrower pipe at a point that is a height 5.0 m above the level where the pipe enters the house?



$$\frac{\rho v_1^2}{2} + p_1 + \rho g z_1 = \frac{\rho v_2^2}{2} + p_2 + \rho g z_2$$

We assume that the speed of the water at the top of the tower is negligibly small due to the fact that the water level in the tower is maintained at the same height and so we set $v_1 = 0$. The pressure at point 2 is then

$$P_2 = P_1 + \rho g(y_1 - y_2) - \frac{1}{2}\rho v_2^{\,2} \, .$$

we use the value for the density of water ρ = 1.0 ×103 kg ·m−3 , the change in height is ($y_1 - y_2$) = 35 m , and the pressure at the top of the water tower is P_1 = 1atm . The rate R that the water flows at point 1 satisfies $R = A_1 v_1 = \pi(d_1 / 2)^2 v_1$. Therefore, the speed of the water at point 1 is

$$v_1 = \frac{R}{\pi(d_1/2)^2} = \frac{2.0\times10^{-3}\ \mathrm{m^3\cdot s^{-1}}}{\pi(1.5\,\mathrm{m})^2} = 2.8\times10^{-4}\ \mathrm{m\cdot s^{-1}}$$

$$v_2 = \frac{R}{\pi(d_2/2)^2} = \frac{2.0\times10^{-3}\ \mathrm{m^3\cdot s^{-1}}}{\pi(2.5\times10^{-2}\ \mathrm{m})^2} = 1.0\,\mathrm{m\cdot s^{-1}}$$

$$v_2 = (d_1^{\,2} / d_2^{\,2})v_1$$

$$P_2 = 1.01\times10^5\ \text{Pa} + (1.0\times10^3\ \text{kg}\cdot\text{m}^{-3})(9.8\ \text{m}\cdot\text{s}^{-2})(35\ \text{m}) - \frac{1}{2}(1.0\times10^3\ \text{kg}\cdot\text{m}^{-3})(1.0\ \text{m}\cdot\text{s}^{-1})^2$$

$$P_2 = 1.01\times10^5\ \text{Pa} + 3.43\times10^5\ \text{Pa} - 5.1\times10^2\ \text{Pa} = 4.4\times10^5\ \text{Pa}$$

We now apply Bernoulli's Equation to the points 2 and 3,

$$P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 = P_3 + \frac{1}{2}\rho v_3^2 + \rho g y_3$$

$$P_3 = P_2 + \frac{1}{2}\rho(v_2^2 - v_3^2) + \rho g(y_2 - y_3)$$

The change in height $y_2 - y_3 = -5.0\ \text{m}$. The speed of the water at point 3 is

$$v_3 = \frac{R}{\pi(d_3/2)^2} = \frac{2.0\times10^{-3}\ \text{m}^3\cdot\text{s}^{-1}}{\pi(1.27\times10^{-2}\ \text{m})^2} = 3.9\ \text{m}\cdot\text{s}^{-1}$$

$$P_3 = (4.4\times10^5\ \text{Pa}) + \frac{1}{2}(1.0\times10^3\ \text{kg}\cdot\text{m}^{-3})((1.0\ \text{m}\cdot\text{s}^{-1})^2 - (3.9\ \text{m}\cdot\text{s}^{-1})^2)$$
$$-(1.0\times10^3\ \text{kg}\cdot\text{m}^{-3})(9.8\ \text{m}\cdot\text{s}^{-2})(5.0\ \text{m})$$

$$= (4.4\times10^5\ \text{Pa}) - (7.1\times10^3\ \text{Pa}) - 4.9\times10^4\ \text{Pa}$$
$$= 3.8\times10^5\ \text{Pa}$$

A pipe bend tapers from a diameter of d_1 of 500 mm at inlet to a diameter d_2 of 250 mm at outlet and turns the flow through an angle Θ of 45^o. Measurements of pressure at inlet and outlet show that p_1 = 40 kPa and p_2 = 23 kPa. If the pipe is conveying oil (ρ = 850 kg/m^3). Calculate the magnitude and direction of the resultant force on the bend when the oil is flowing at the rate of 0.45m3 /s.