Reinforced Concrete Design

Analysis and Design of One Way Concrete Slab

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LOGO

Two way slab behavior

Dimension of Slab $L \times S$

$\frac{L}{S} < 2$ *with Uniform distributed load*

Supported on Four Edges

Considers two strip in two direction

Deflection for assumed simply

supported beam : $\Delta = \frac{5Wl^4}{384\,EI}$

If the two strip have same thickness

then deflection will be :

$\Delta ab = k\,W_{ab}\,S^4$

$\Delta cd = k\,W_{cd}\,l^4$

Where: W_{ab} and W_{cd} *is the transferred load by the strip ab and cd respectively*

$$If\ Wu = W_{ab} + W_{cd}$$

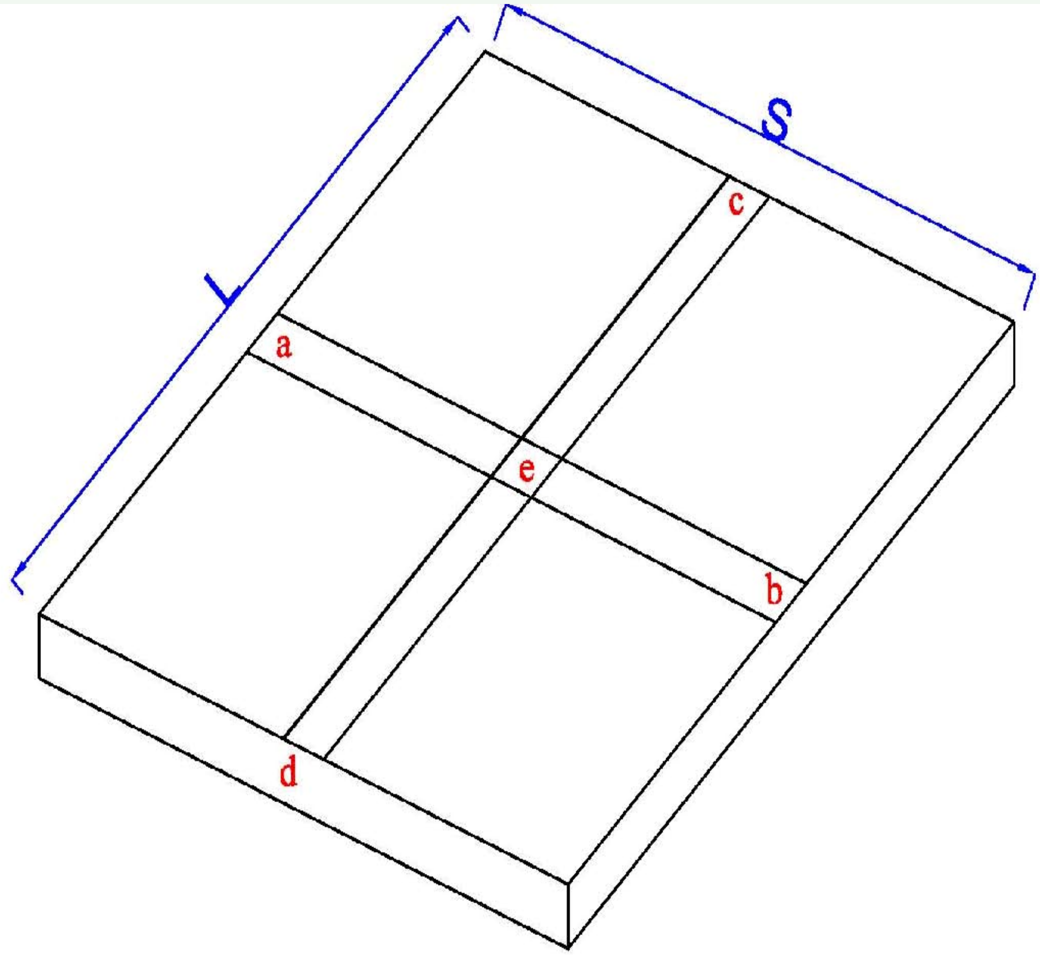
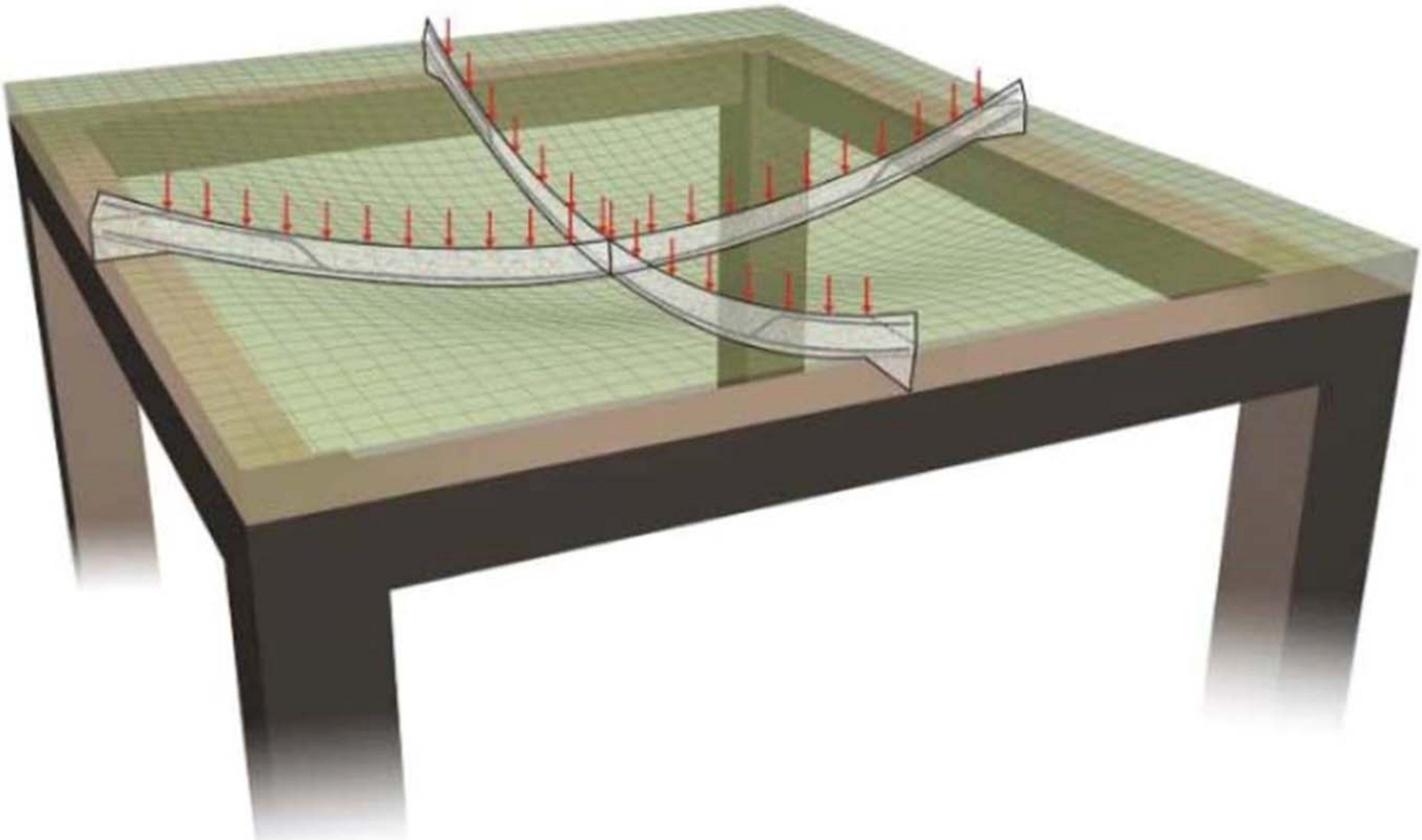


Fig. (1)



Behavior of a two-way slab

The Δ deflection at e are equal for both strip

$k\ Wab\ S^4 = k\ Wcd\ l^4$

$$Wab = \frac{L^4 Wcd}{S^4} = \left(\frac{L}{S}\right)^4 Wcd$$

The transferred load into the short Direction = Load in Long Direction multiply by factor $(L/S)^4$

$If\ \left(\frac{L}{S}\right) = 1.5 \quad then \quad Wcd = 0.165\,W \quad and \quad Wab = 0.835\,W$

$If\ \left(\frac{L}{S}\right) = 2 \quad then \quad Wcd = 0.059\,W \quad and \quad Wab = 0.941\,W$

That's mean the short Direction resist the greater part of total applied load and when (L/S)>2 then the load transferred to the long Direction will be very small and can be neglected.

The analysis method assume :

-Uniform distributed load

-Live Load/ Dead Load ≤ 3 -Thickness of slab

ACI Code 1963 the h_{min} not less then 90 mm according to eq :

$$h_{min} = \frac{2(Ln + Sn)}{180} \geq 90\ mm$$

ACI Code 2014 present equation for slab with beams :
1-Table 8.3.1.1

$$h_{min} = \frac{Ln\left(0.8 + \frac{fy}{1400}\right)}{36 + 9\beta} \geq 90\ mm$$

where:

$$\beta = \frac{Ln}{Sn}$$

Ln , Sn : clear span of long and short direction respectively

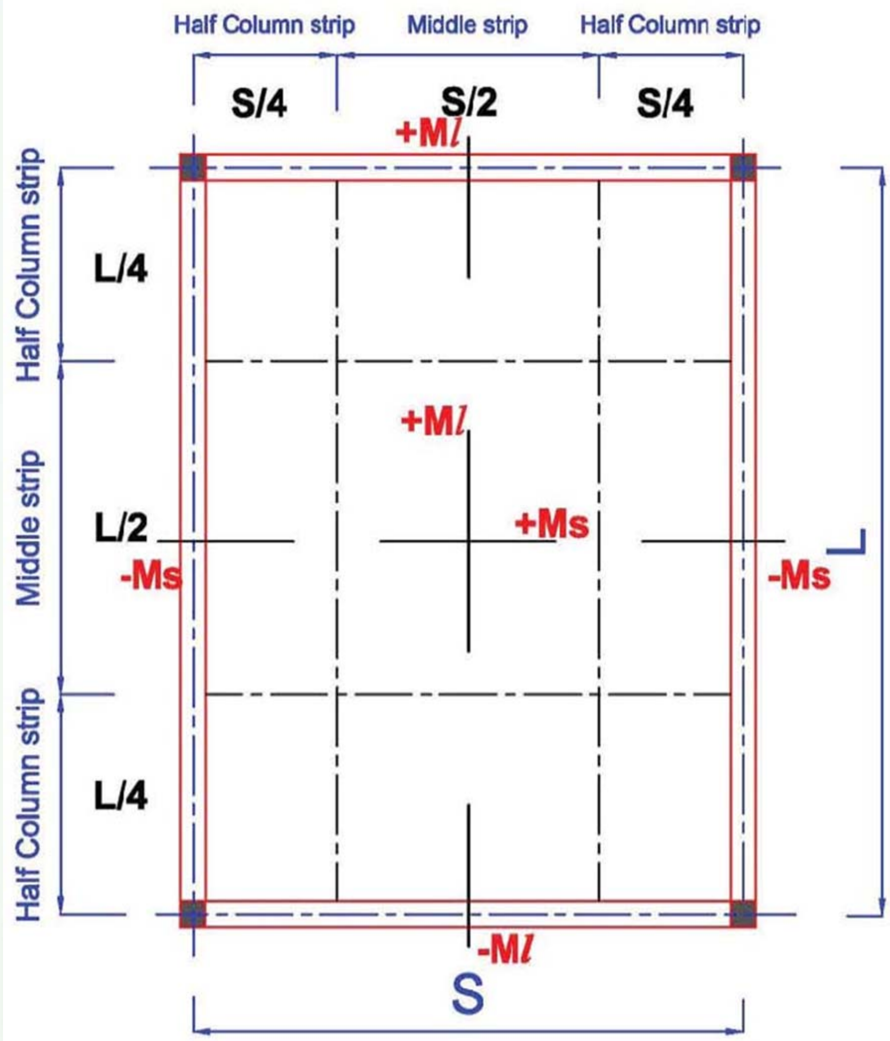


Fig. (2) Column and Middle Strip

Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)[1]

f_y, MPa[2]	Without drop panels[3]			With drop panels[3]		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams[4]		Without edge beams	With edge beams[4]	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

[1] ℓ_n is the clear span in the long direction, measured face-to-face of supports (mm).

[2] For f_y between the values given in the table, minimum thickness shall be calculated by linear interpolation.

[3] Drop panels as given in 8.2.4.

[4] Slabs with beams between columns along exterior edges. Exterior panels shall be considered to be without edge beams if α_f is less than 0.8. The value of α_f for the edge beam shall be calculated in accordance with 8.10.2.7.

Table 8.3.1.2—Minimum thickness of nonprestressed two-way slabs with beams spanning between supports on all sides

α_{fm}[1]	Minimum h, mm		
$\alpha_{fm} \le 0.2$	8.3.1.1 applies		(a)
$0.2 < \alpha_{fm} \le 2.0$	Greater of:	$\dfrac{\ell_n\left(0.8+\dfrac{f_y}{1400}\right)}{36+5\beta(\alpha_{fm}-0.2)}$	(b)[2],[3]
		125	(c)
$\alpha_{fm} > 2.0$	Greater of:	$\dfrac{\ell_n\left(0.8+\dfrac{f_y}{1400}\right)}{36+9\beta}$	(d)[2],[3]
		90	(e)

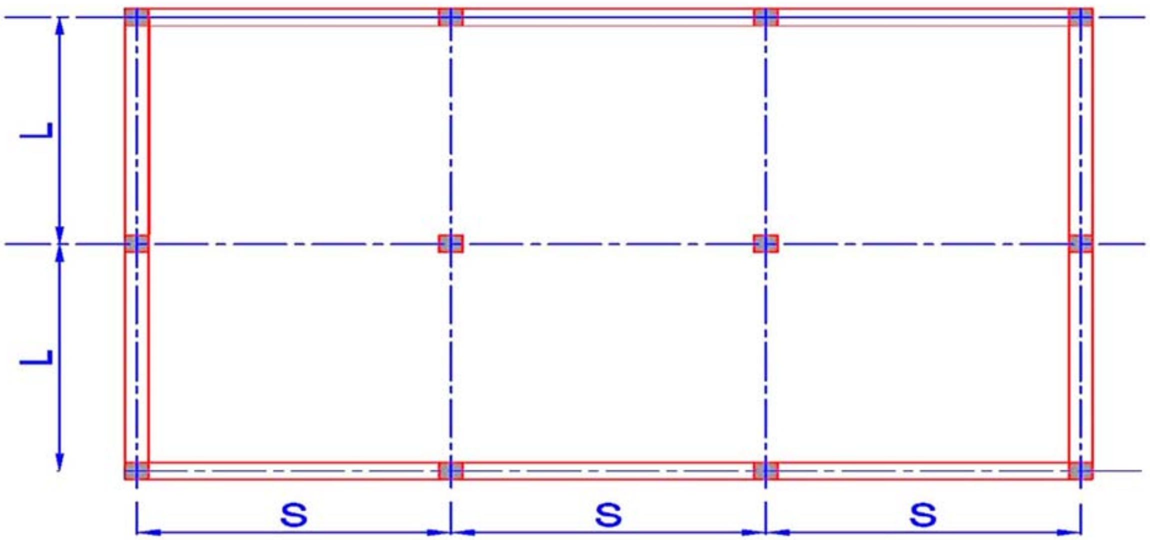
[1]α_{fm} is the average value of α_f for all beams on edges of a panel and α_f shall be calculated in accordance with 8.10.2.7.

[2]ℓ_n is the clear span in the long direction, measured face-to-face of beams (mm)

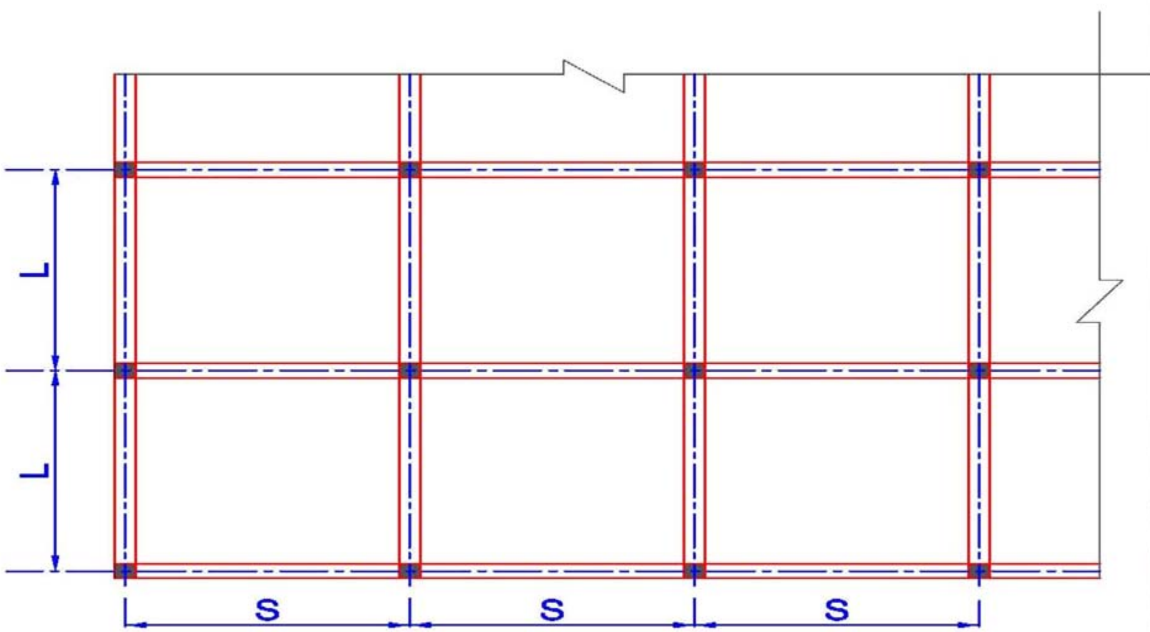
[3]β is the ratio of clear spans in long to short directions of slab

α_m: *is the average of* α_f *of all beams*

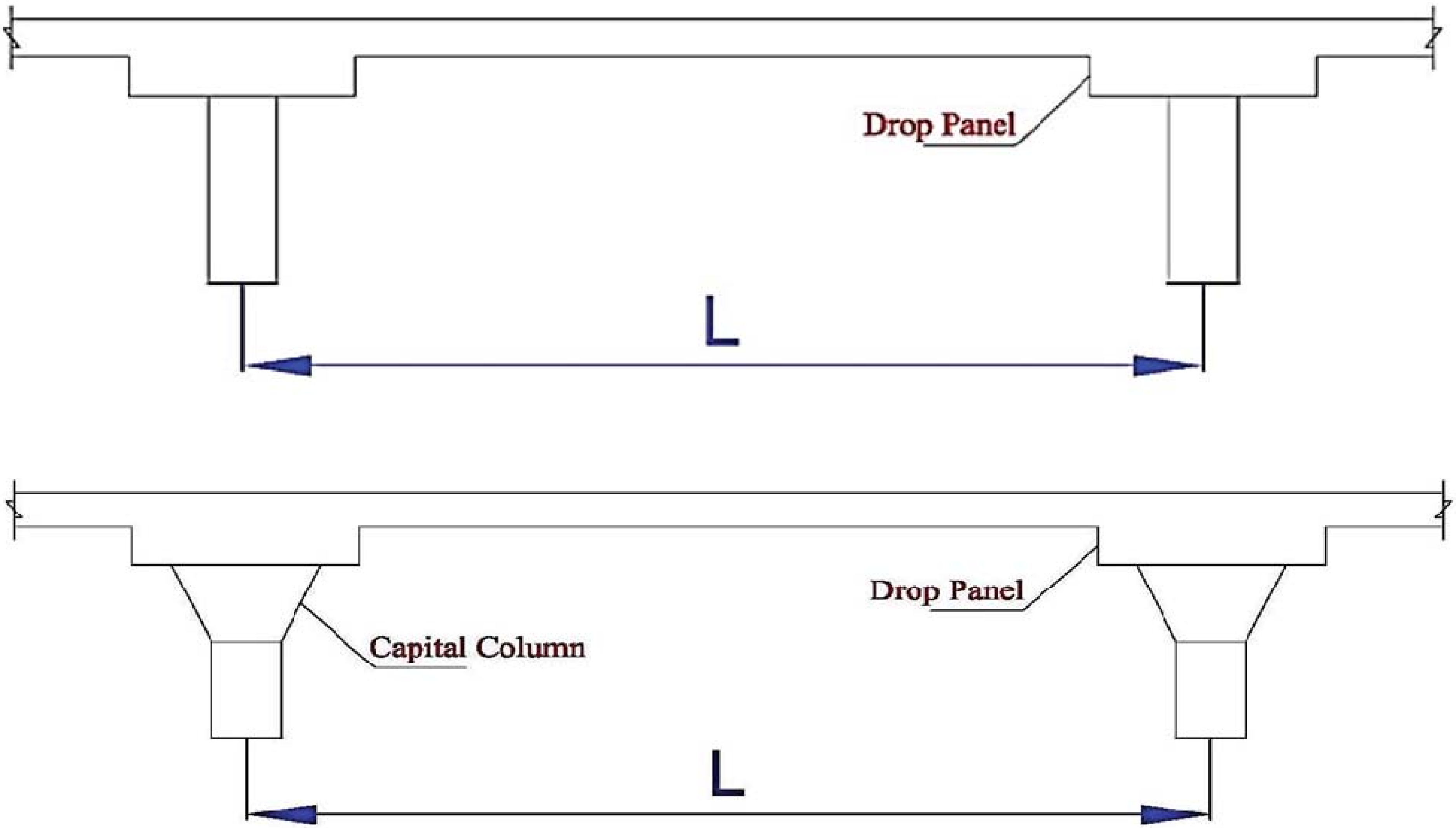
$$\alpha_f: \text{the flexural stiffness of beam/flexural stiffness of slab} = \frac{\dfrac{EbIb}{Lb}}{Es\dfrac{Is}{Ls}}$$



Slab with Edge Beam



Slab with Beam in All Direction



Slabs with Drop Panel

ACI Code suggest 3 methods to analyze the

Two-way slab

ACI Code suggest three methods to analyze the

Two-way slab since 1963

1-method 1

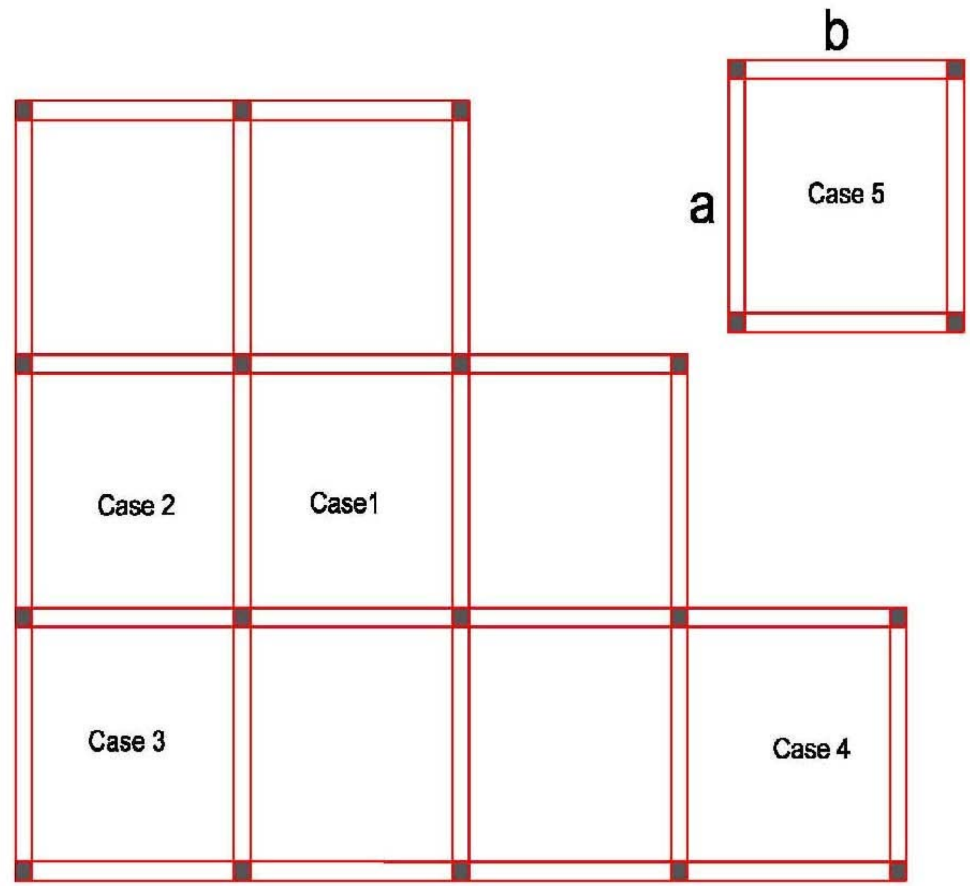
Method 2

The Moment at the middle strip :

$$M = C\,Wu\,S^2$$

C= is a factor can be found from tables

The Moment at the column strip = 2/3 M mid



Slab Cases

For Method 2

Where the negative moment on one side of a support is less than 80 percent of that on the other side, two-thirds of the difference shall be distributed in proportion to the relative stiffness of the slabs.

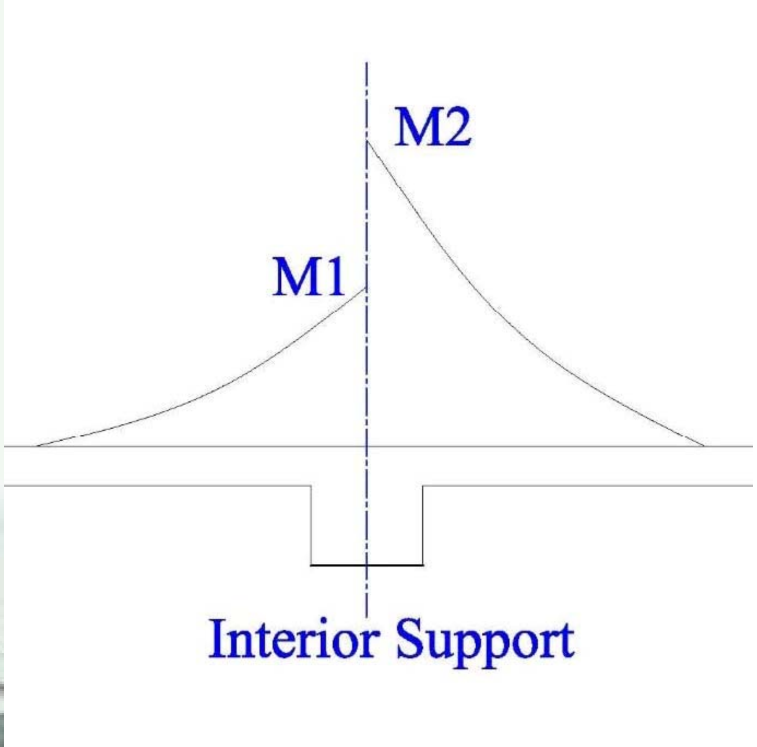
$$\frac{M2}{M1} \leq 0.8$$

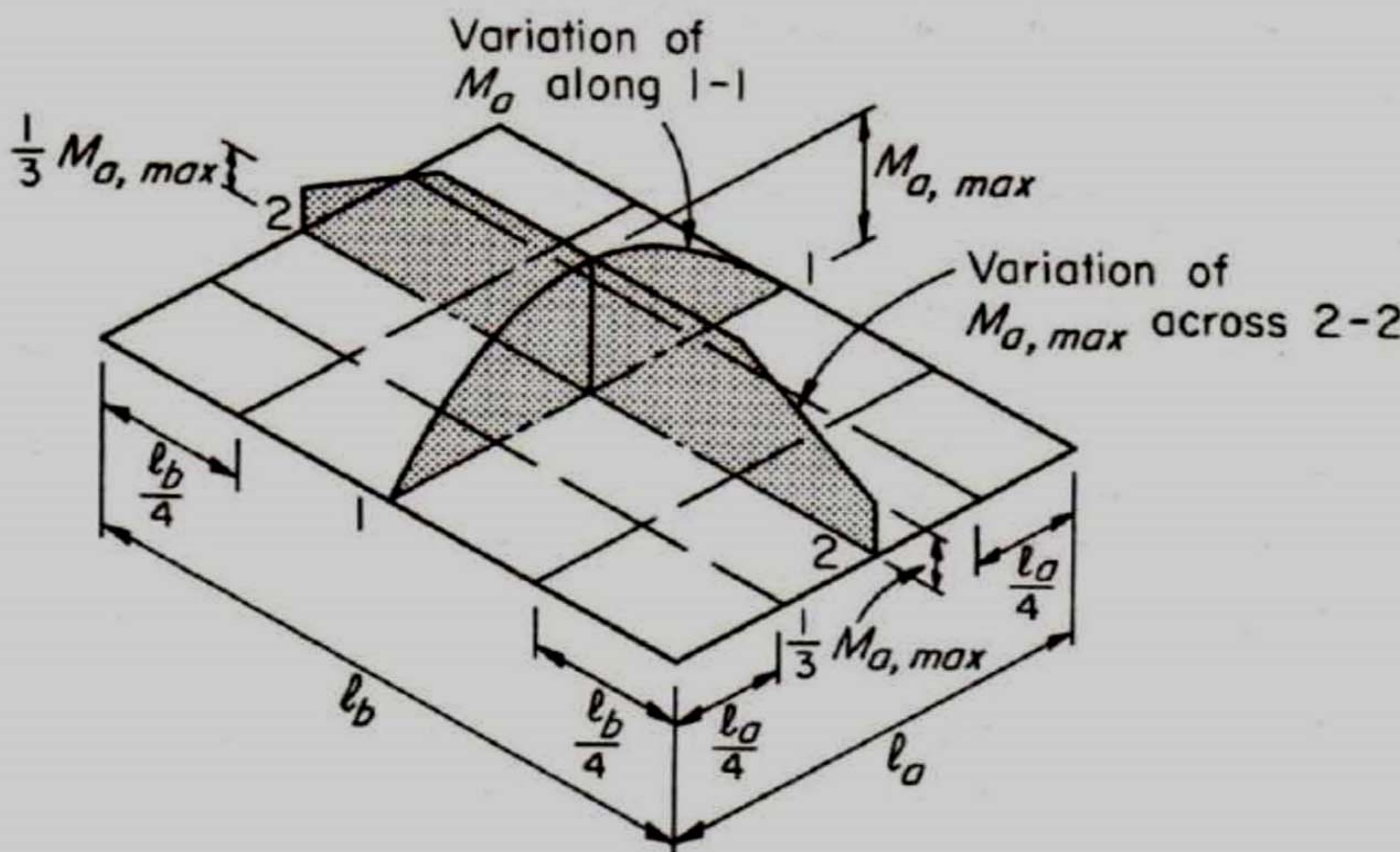
M Difference = M2-M1

2/3 M Difference Distributed for both side according to the slabs stiffness

While In Method 3 if $M1 \neq M2$,

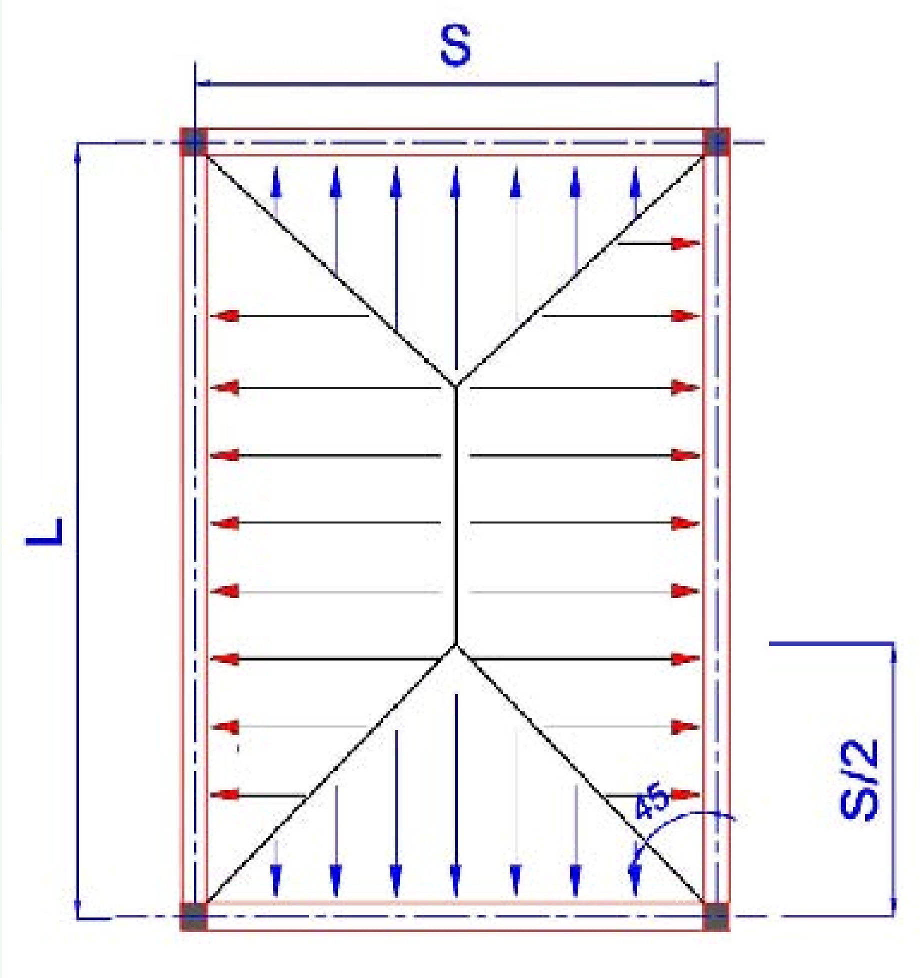
The negative Moments in can be take is the maximum positive moment




Variation of
M_a along 1-1
$\frac{1}{3} M_{a,max}$
2
$M_{a,max}$
1
Variation of
$M_{a,max}$ across 2-2
$\frac{\ell_b}{4}$
1
2
$\frac{\ell_a}{4}$
$\frac{1}{3} M_{a,max}$
ℓ_b
$\frac{\ell_b}{4}$
$\frac{\ell_a}{4}$
ℓ_a

Shear Force

The shear force on slab can be calculated according to the figure shown and transferred the equivalent load to the beams



Short Direction

$$W_{eq} = \frac{Wu\ La}{3} \qquad for\ moment$$

$$W_{eq} = \frac{Wu\ La}{4} \qquad for\ shear$$

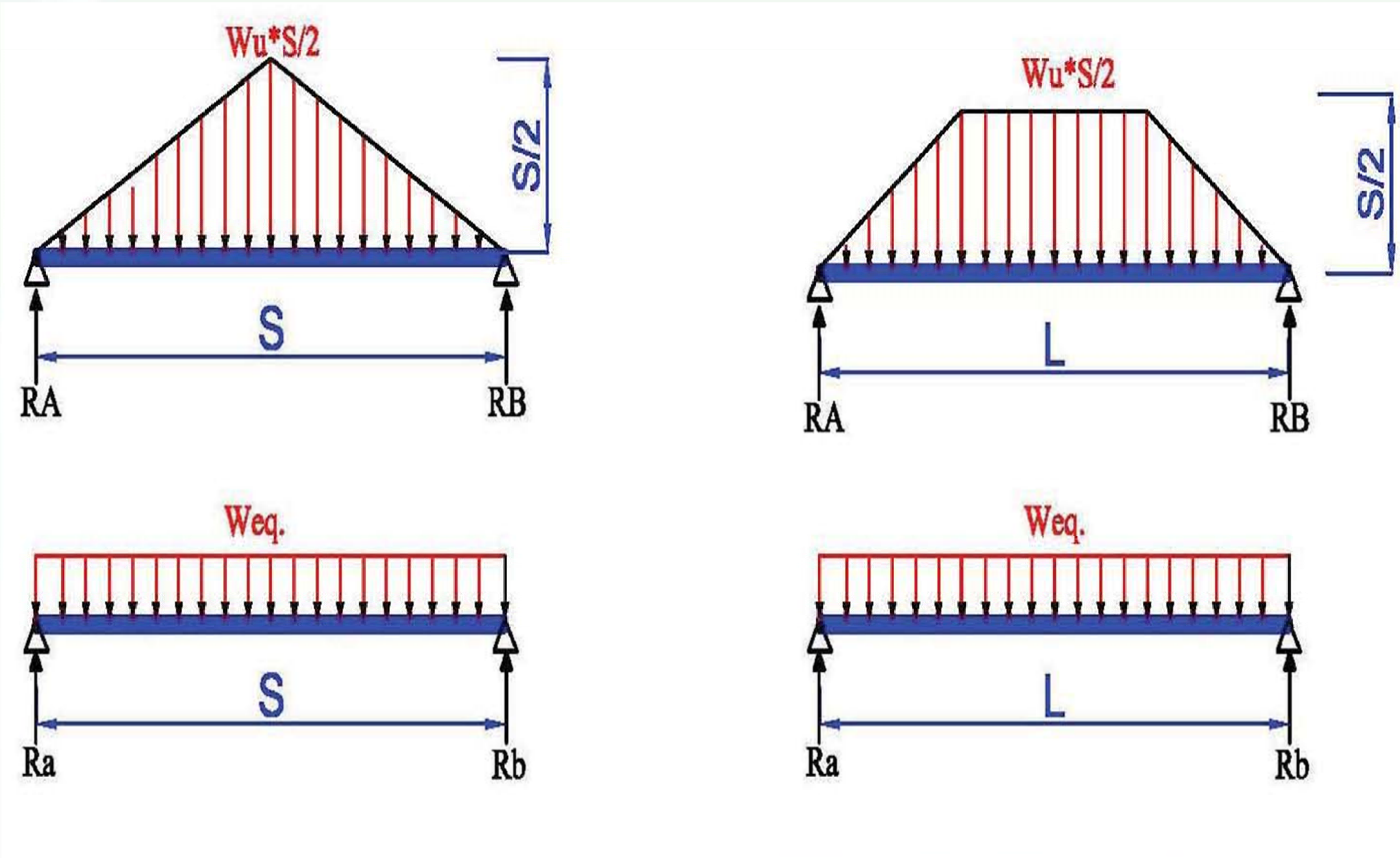
long Direction

$$Weq = \frac{wu\ S}{3}\left(\frac{3-m^2}{2}\right) \qquad for\ Moment$$

$$Weq = \frac{wu\ S}{4}(2-m) \qquad for\ shear$$

$$m = S/L \quad or \quad La/Lb$$

S, L : length of span C/C in both direction



Wu*S/2
S/2
S
RA
RB
Wu*S/2
S/2
L
RA
RB
Weq.
S
Ra
Rb
Weq.
L
Ra
Rb

Example (1) : An Interior Two way slab panel 6.0 m * 7.2m carry a live load 10 KN/m2. The slab thick 200 mm and is supported on beam 300 mm width and 900mm depth. Assume that the super imposed dead load equal to 3 KN/m2 . Determine the principal bending and shear in slab. Fy=420 MPA, fc=21MPa

Solution:

Method (2)

1- Minimum thickness

-ACI code 1963

$$h_{min} = \frac{2 \times (Sn + Ln)}{180} = \frac{2(5700 + 6900)}{180} = 140 \text{ mm}$$

-The ACI code 2014

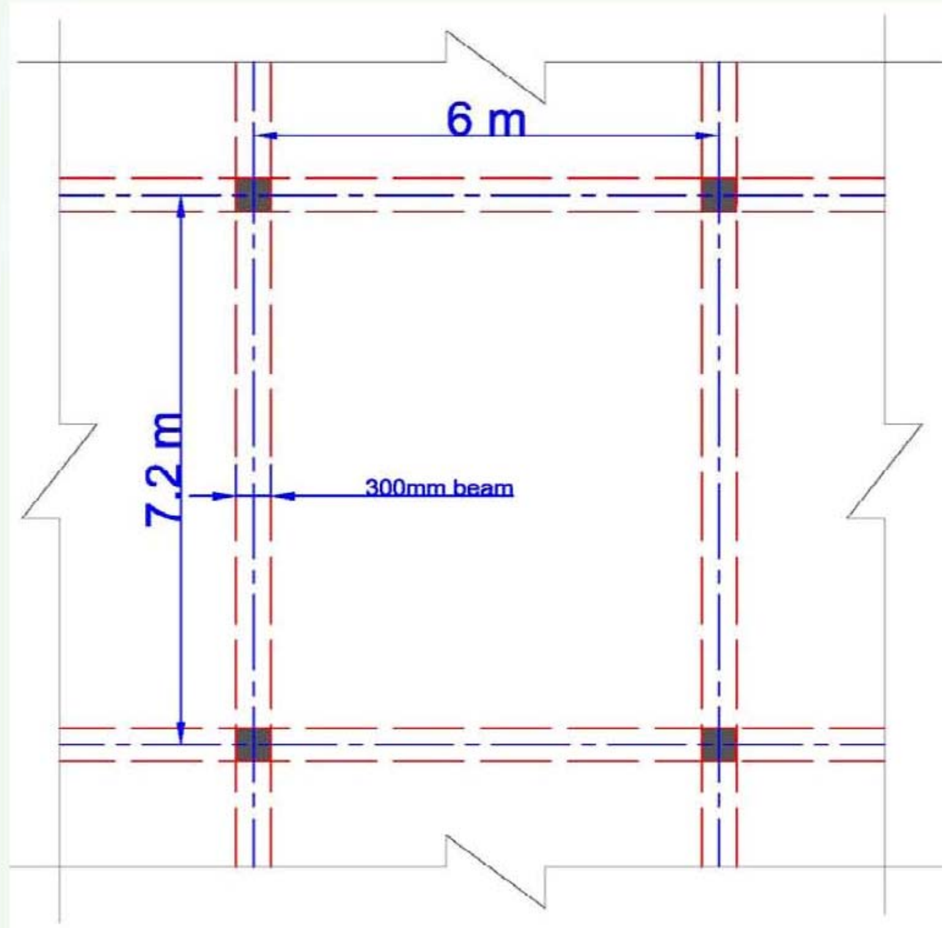
• when the slab does not supported by beams (interior panel) using ACI Table 8.3.1.1

• For slab supported by beams : ($\alpha_m > 2$)

ACI code (table 8.3.1.2):

$$h_{min} = \frac{Ln\left(0.8 + \frac{fy}{1400}\right)}{36 + 9\beta} = \frac{6.9\left(0.8 + \frac{420}{1400}\right)}{36 + 9 \times \left(\frac{6.9}{5.7}\right)} \geq 90\ mm$$

$$= 161.8\ mm \geq 90\ mm$$



We will use h = 200mm (as mention in Example)

$Self\ Wt\ of\ slab\ =\ t\ *1*1*\ \ c = 0.2*1*1*24 = 4.8\ KN/m2$

$Wu =\ 1.2WD + 1.6WL$

$Wu = 1.2(4.8+3) + 1.6\ *\ 10 = 25.36\ KN/m2$

$m = \frac{S}{L} = \frac{6}{7.2} = 0.833\ \ or\ \ m = \left(\frac{Sn}{Ln} = \frac{5.7}{6.9} =\ 0.83\right) no\ big\ difference\ \ \ (for\ method\ 2\ use\ L\ and\ S\ center\ to\ center)$

From Table m lies between 0.8 and 0.9 for interior panel CASE I

Moment factors for Short Direction

Factor	0.8	0.833	0.9	Moment
$-C$	0.048	0.04536*	0.040	Negative moment
$+C$	0.036	0.03402	0.030	Positive moment

$$*C = \frac{(0.9-0.833)\times 0.048 + (0.833-0.8)\times 0.04}{(0.9-0.8)} =\ 0.04536$$

$-Mu = c\ Wu.S^2 = 0.05436 \times 25.36 \times 6^2 = 41.41\ KN.m/m$

$+Mu = c\ Wu.S^2 = 0.03402 \times 25.36 \times 6^2 = 31.06\ KN.m/m$

METHOD 2—TABLE I—MOMENT COEFFICIENTS

Moments	Short span						Long span, all values of m
	Values of m						
	1.0	0.9	0.8	0.7	0.6	0.5 and less	
Case 1—Interior panels							
Negative moment at—							
Continuous edge	0·033	0.040	0.048	0.055	0.063	0.083	0.033
Discontinuous edge	—	—	—	—	—	—	—
Positive moment at midspan	0.025	0.030	0.036	0.041	0.047	0.062	0.025
Case 2—One edge discontinuous							
Negative moment at—							
Continuous edge	0.041	0.048	0.055	0.062	0.069	0.085	0.041
Discontinuous edge	0.021	0.024	0.027	0.031	0.035	0.042	0.021
Positive moment at midspan	0·031	0.036	0.041	0.047	0.052	0.064	0.031
Case 3—Two edges discontinuous							
Negative moment at—							
Continuous edge	0.049	0.057	0.064	0.071	0.078	0.090	0.049
Discontinuous edge	0.025	0.028	0.032	0.036	0.039	0.045	0.025
Positive moment at midspan	0.037	0.043	0.048	0.054	0.059	0.068	0.037
Case 4—Three edges discontinuous							
Negative moment at—							
Continuous edge	0·058	0.066	0.074	0.082	0.090	0.098	0.058
Discontinuous edge	0.029	0.033	0.037	0.041	0.045	0.049	0.029
Positive moment at midspan	0.044	0.050	0.056	0.062	0.068	0.074	0.044
Case 5—Four edges discontinuous							
Negative moment at—							
Continuous edge	—	—	—	—	—	—	—
Discontinuous edge	0.033	0.038	0.043	0.047	0.053	0.055	0.033
Positive moment at midspan	0·050	0.057	0.064	0.072	0.080	0.083	0.050

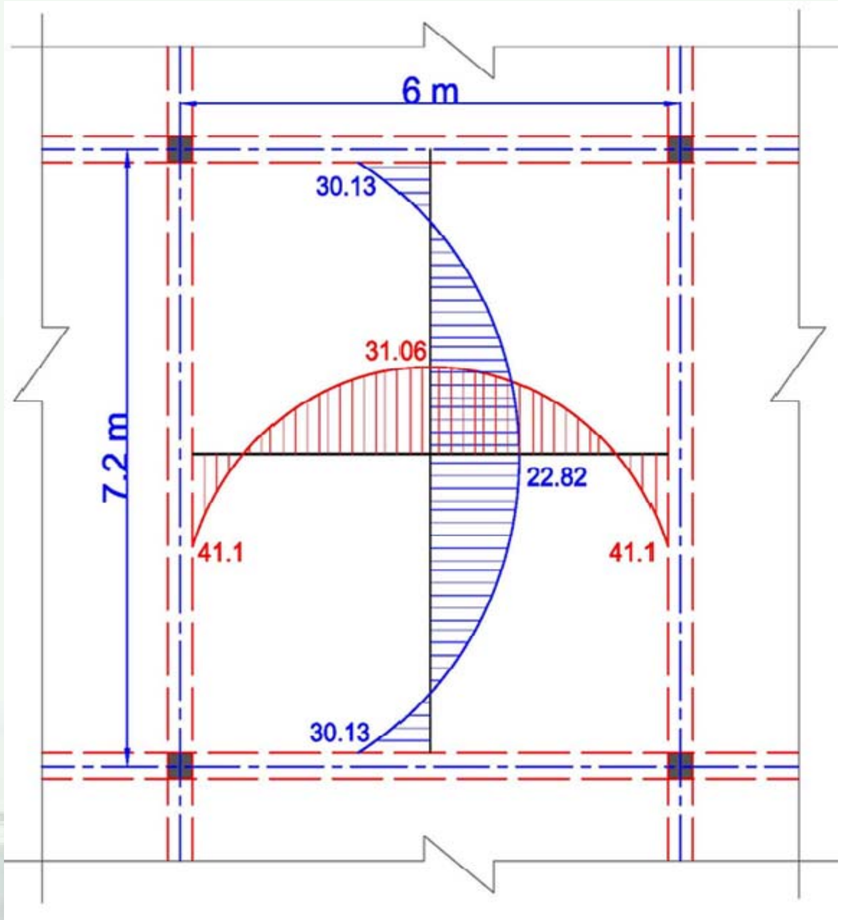
Moment factors for Long Direction

-C= 0.033 negative moment factor

+C=0.025 Positive moment factor

$-Mu = c\ Wu.S^2 = 0.033 \times 25.36 \times\ 62 = 30.13\ KN.m/m$
$+Mu = c\ Wu.S^2 = 0.025 \times 25.36 \times\ 62 = 22.82\ KN.m/m$

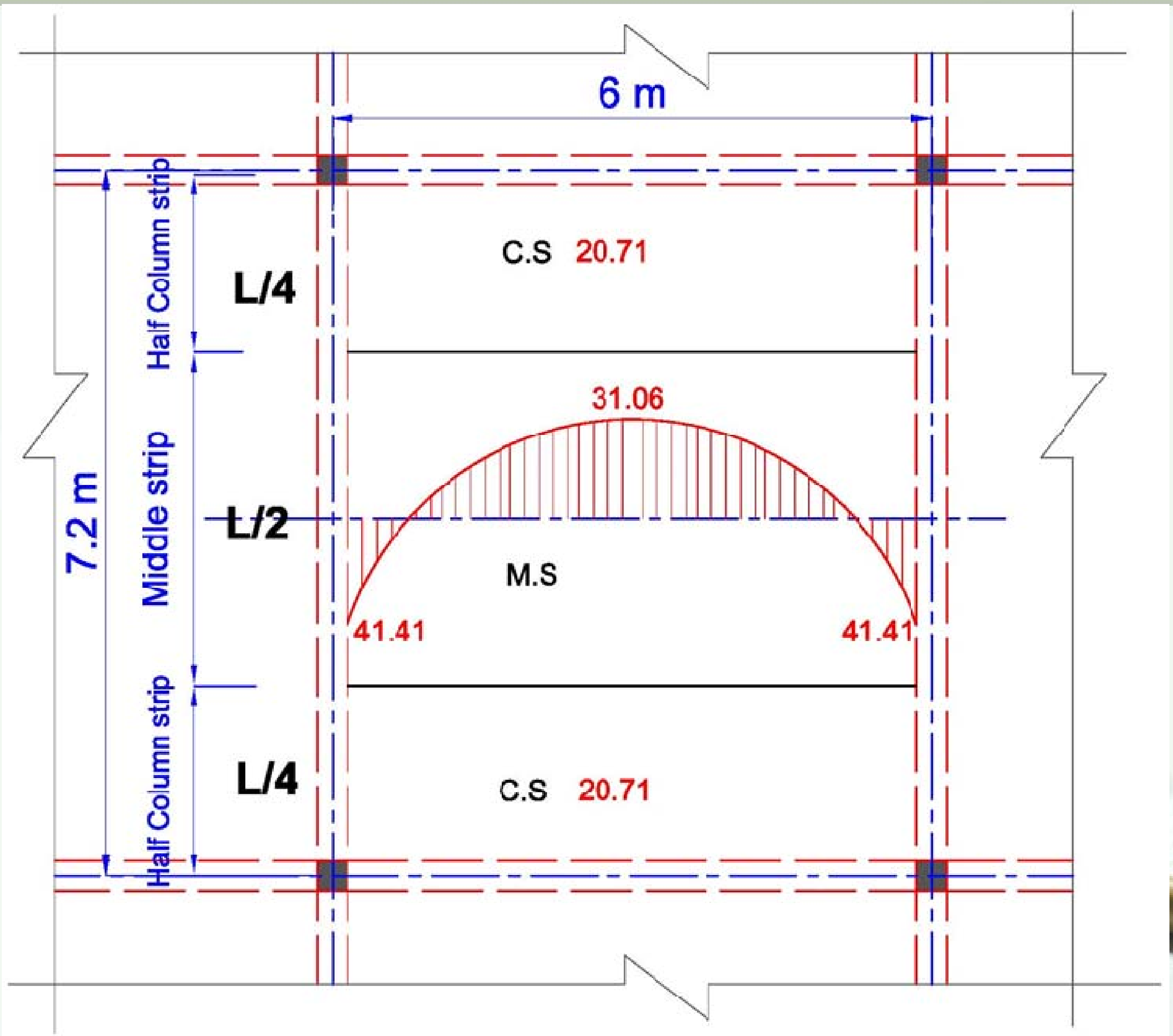
Moment at column strip will be 2/3 from middle strip moment in both direction



Shear in Slab

$$Vu\ = \frac{Wu\ \times S}{2}$$
$$= \frac{25.36\ \times 6}{2}\ =\ 76.08\ KN/m$$

Moment diagram
(KN.m/m)
In Short Direction
Middle and Column Strip



Loads on Beams

Bending Moments

1-long Direction

$$W_{eq} = \frac{wu\,S}{3}\left(\frac{3-m^2}{2}\right) = \frac{25.36 \times 6}{3}\left(\frac{3-0.833^2}{2}\right) = 58.47\ KN/m \quad from\ one\ side$$

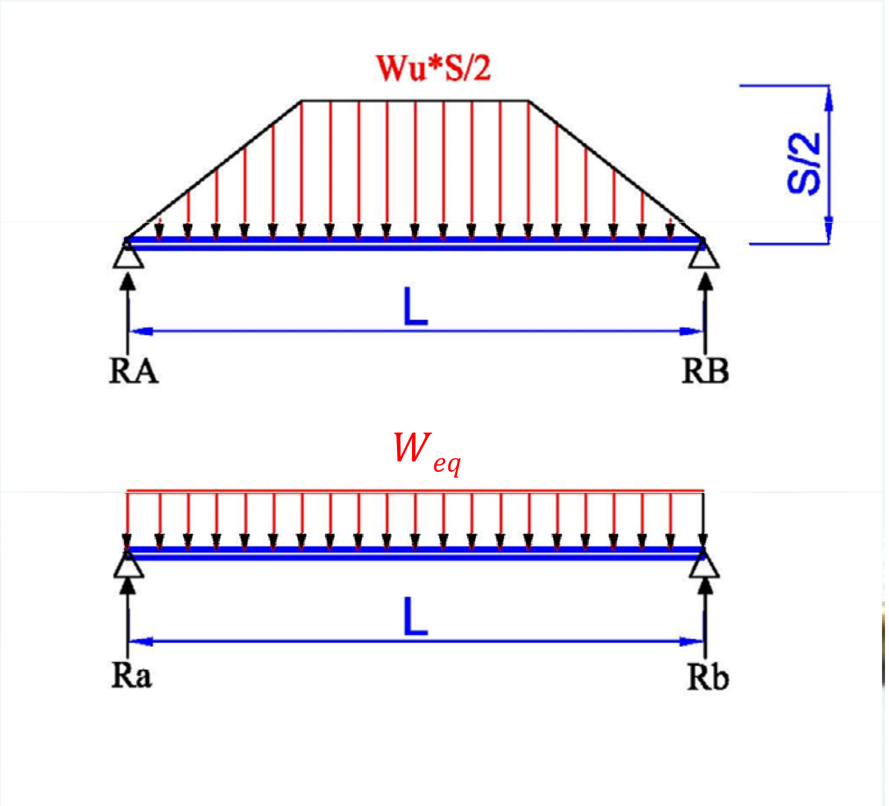
$$W_{eq} = \frac{wu\,S}{4}(2-m) \qquad for\ shear$$

There is two slab transferred load to the beam

$$W_{eq} = 2 \times 58.47 = 116.94\ KN/m \quad (from\ both\ side)$$

$$Self\ weight\ of\ drop\ beam\ part = 1.2 \times (h-t) \times b \times 1 \times \gamma c$$
$$= 1.2 \times (0.9-0.2) \times 1 \times 0.3 \times 24 = 6.05\ KN/m$$

$$Total\ Wu_b = 116.94 + 6.05 = 122.99\ KN/m$$



2- Short Beam

$$Weq = \frac{Wu\,S}{3}$$

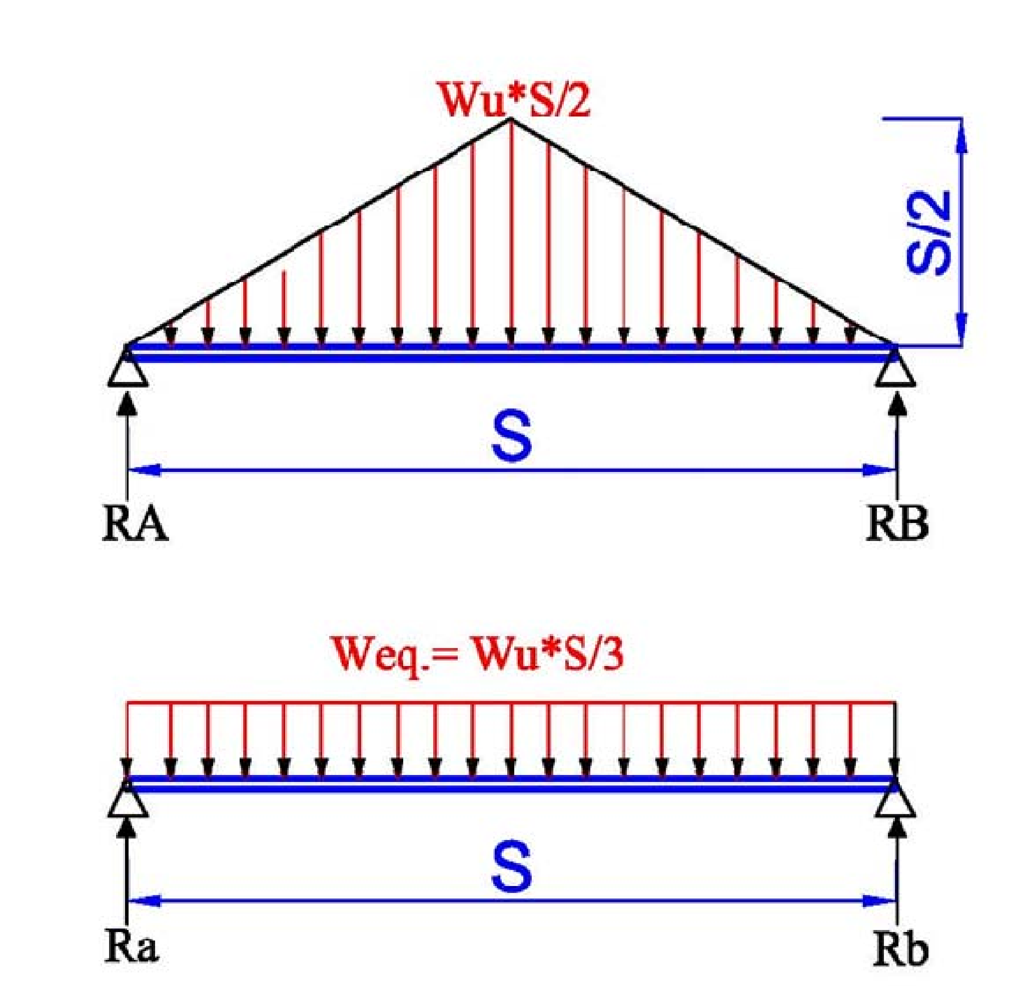
$$= \frac{25.36 \times 6}{3} = 50.6\,KN/m \quad from\ one\ Side$$

There is two slab transferred load to the beam

$$Weq = 2 \times 50.6 = 101.2\,KN/m$$

$$Self\ weight\ of\ drop\ beam\ part = 6.05\,KN/m$$

$$Wua = 101.2 + 6.05 = 107.25\,KN/m$$



Beam Moment Calculation

Using Factored for interior panel for beams

1- Long Direction

$$Wub = \ 122.99\ KN/m$$

$$-M = \frac{1}{11}\ (Wub\ \times\ L^2) = \frac{1}{11} \times (\ 122.99 \times 6.92) = 532.32\ KN.m$$

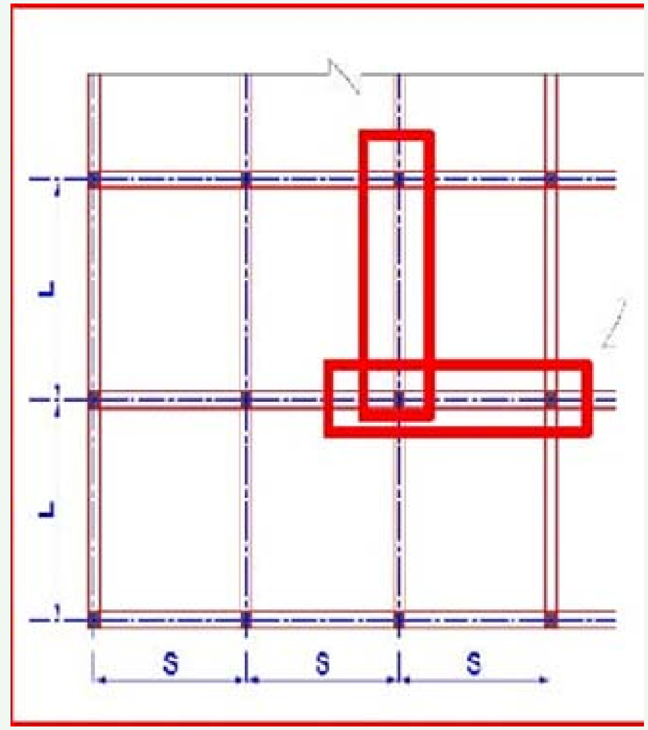
$$+M = \frac{1}{16}\ (Wub \times L^2) = \frac{1}{16} \times (\ 122.99 \times 6.92) = 365.97\ KN.m$$

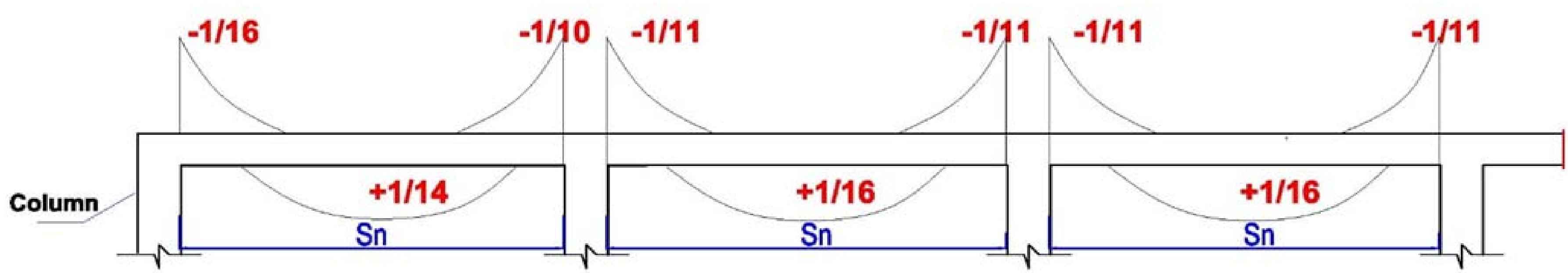
2- Short Direction

$$Wub = \ 107.25\ KN/m$$

$$-M = \frac{1}{11}\ (Wua \times S^2)\ =\ 1/11 \times (\ 107.25 \times 5.72) = 316.78\ KN.m$$

$$+M = \frac{1}{16}\ (Wua \times S^2)\ =\ 1/16 \times (\ 107.25 \times 5.72) = 217.8KN.m$$





B.M Factors

Shear in Beams

1-Long direction

$W_{ub} = Wu \times S/4 \times (2 - m)$

$= 25.36 \times 6/4 \times (2 - 0.833) = 44.38\ KN/m$

From both side have load

$2 \times 44.38 = 88.76\ KN/m$

$Self\ weight\ of\ Beam = 6.05\ KN/m$

W_{ub}= 88.76+6.05= 94.81 KN/m

Shear force at support

$$Vu = \frac{Wu \times L}{2} = \frac{94.81 \times 7.2}{2} = 341.22\ KN$$

1-Short direction

$$W_{ua} = \frac{Wu\ S}{4} \qquad for\ shear$$

$$= \frac{25.36 \times 6}{4} = 38.04\ KN/m$$

From both side have load and adding self weight of beam

$Wu\ a = 2 \times 38.04 + 6.05 = 82.13\ KN/m$

Shear force at support

$$Vu = \frac{Wu \times S}{2} = \frac{82.81 \times 6}{2} = 246.4\ KN$$

Method 3

ACI code using method 3 and denoted to long direction as b and short direction with a and considering the live load effect.

- Negative Moment

1- Short direction (a)

$$-M_a = C_{a\ neg}\ Wu\ L_a^2$$

2- Long direction (b)

$$-M_b = C_{b\ neg}\ Wu\ L_b^2$$

Where:

Wu : total uniform factored load (D.L + L.L)

C_a: *Moment coefficient from table*

C_b: *Moment coefficient from table*

L_a: *clear span for short direction*

L_b: *clear span for short direction*

Positive Moment

1 − *Short direction* (*a*)

$$+Ma_{D.L} = C_{a\,DL} \times Wu_{DL} \times La^2$$

$$+Ma_{L.L} = C_{a\,LL} \times Wu_{LL} \times La^2$$

$$+Ma = +Ma_{D.L} + Ma_{L.L}$$

2 − *Long direction* (*b*)

$$+Mb_{D.L} = C_{b\,DL} \times Wu_{DL} \times Lb^2$$

$$+Mb_{L.L} = C_{b\,LL} \times Wu_{LL} \times Lb^2$$

$$+Mb = +Mb_{D.L} + Mb_{L.L}$$

Note:

When two negative moment at support are different for continuous slab, can take average Moment:

$$-M = \frac{M\,left + Mright}{2}$$

Item	Moment Direction	
	Short Direction S or (a)	Long Direction L or (b)
Negative Moment (-M)	$-M_a = C_{a\ neg}\ Wu\ La^2$	$-M_b = C_{b\ neg}\ Wu\ Lb^2$
Positive Moment (+M)	$+Ma_{\ D.L} = C_{a\ DL} \times Wu_{\ DL} \times La^2$ $+Ma_{\ L.L} = C_{a\ LL} \times Wu_{\ LL} \times La^2$	$+Mb_{D.L} = C_{b\ DL} \times Wu_{DL} \times Lb^2$ $+Mb_{L.L} = C_{b\ LL} \times Wu_{LL} \times Lb^2$
	$+Ma = +Ma\ D.L + Ma\ L.L$	$+Mb = +Mb\ D.L + Mb\ L.L$

Example (2) : (as in Ex. 1) An Interior Two way slab panel 6.0 m * 7.2m carry a live load 10 KN/m^2. The slab thick 200 mm and is supported on beam 300 mm width and 900mm depth. Assume that the super imposed dead load equal to 3 KN/m^2 . Determine the principal bending and shear in slab. Fy=280 MPa, fc=21MPa

Sol.

Wu= 25.36 KN/m^2 (exa. 1)

Interior panel continues from all side (Case 2) Table 1

$$\frac{La}{Lb} = \frac{(6-0.3)}{(7.2-0.3)} = 0.826 \quad (or\ a/b\ )$$

1- Negative Moment Factors

- ***Short Direction (by interpolation)***

0.8	0.065
0.826	
0.85	0.06

$$Ca\ neg. = \frac{0.06 \times (0.826-0.8) + 0.065 \times (0.85-0.826)}{(0.85-0.8)} = 0.0624$$

-Long Direction (by interpolation)

0.8	0.027
0.826	
0.85	0.031

$$Cb\ neg. = \frac{0.031 \times (0.826-0.8) + 0.027 \times (0.85-0.826)}{(0.85-0.8)} = 0.02908$$

METHOD 3—TABLE I—COEFFICIENTS FOR NEGATIVE MOMENTS IN SLABS*

$$M_{A\,neg} = C_{A\,neg} \times w \times A^2$$
$$M_{B\,neg} = C_{B\,neg} \times w \times B^2$$
where w = total uniform dead plus live load

Ratio $m = \frac{A}{B}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{A\,neg}$		0.045		0.050	0.075	0.071		0.033	0.061
	$C_{B\,neg}$		0.045	0.076	0.050			0.071	0.061	0.033
0.95	$C_{A\,neg}$		0.050		0.055	0.079	0.075		0.038	0.065
	$C_{B\,neg}$		0.041	0.072	0.045			0.067	0.056	0.029
0.90	$C_{A\,neg}$		0.055		0.060	0.080	0.079		0.043	0.068
	$C_{B\,neg}$		0.037	0.070	0.040			0.062	0.052	0.025
0.85	$C_{A\,neg}$		0.060		0.066	0.082	0.083		0.049	0.072
	$C_{B\,neg}$		0.031	0.065	0.034			0.057	0.046	0.021
0.80	$C_{A\,neg}$		0.065		0.071	0.083	0.086		0.055	0.075
	$C_{B\,neg}$		0.027	0.061	0.029			0.051	0.041	0.017
0.75	$C_{A\,neg}$		0.069		0.076	0.085	0.088		0.061	0.078
	$C_{B\,neg}$		0.022	0.056	0.024			0.044	0.036	0.014
0.70	$C_{A\,neg}$		0.074		0.081	0.086	0.091		0.068	0.081
	$C_{B\,neg}$		0.017	0.050	0.019			0.038	0.029	0.011
0.65	$C_{A\,neg}$		0.077		0.085	0.087	0.093		0.074	0.083
	$C_{B\,neg}$		0.014	0.043	0.015			0.031	0.024	0.008
0.60	$C_{A\,neg}$		0.081		0.089	0.088	0.095		0.080	0.085
	$C_{B\,neg}$		0.010	0.035	0.011			0.024	0.018	0.006
0.55	$C_{A\,neg}$		0.084		0.092	0.089	0.096		0.085	0.086
	$C_{B\,neg}$		0.007	0.028	0.008			0.019	0.014	0.005
0.50	$C_{A\,neg}$		0.086		0.094	0.090	0.097		0.089	0.088
	$C_{B\,neg}$		0.006	0.022	0.006			0.014	0.010	0.003

*A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.

METHOD 3—TABLE 2—COEFFICIENTS FOR DEAD LOAD POSITIVE MOMENTS IN SLABS*

$$\left.\begin{aligned} M_{A\,pos\,DL} &= C_{A\,DL} \times w \times A^2 \\ M_{B\,pos\,DL} &= C_{B\,DL} \times w \times B^2 \end{aligned}\right\} \text{ where } w = \text{total uniform dead load}$$

Ratio $m = \frac{A}{B}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{A\,DL}$	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023
	$C_{B\,DL}$	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020
0.95	$C_{A\,DL}$	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.022	0.024
	$C_{B\,DL}$	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017
0.90	$C_{A\,DL}$	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026
	$C_{B\,DL}$	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015
0.85	$C_{A\,DL}$	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028
	$C_{B\,DL}$	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013
0.80	$C_{A\,DL}$	0.056	0.026	0.034	0.039	0.032	0.045	0.045	0.032	0.029
	$C_{B\,DL}$	0.023	0.011	0.020	0.016	0.009	0.015	0.022	0.015	0.010
0.75	$C_{A\,DL}$	0.061	0.028	0.040	0.043	0.033	0.048	0.051	0.036	0.031
	$C_{B\,DL}$	0.019	0.009	0.018	0.013	0.007	0.012	0.020	0.013	0.007
0.70	$C_{A\,DL}$	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.033
	$C_{B\,DL}$	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006
0.65	$C_{A\,DL}$	0.074	0.032	0.054	0.050	0.036	0.054	0.065	0.044	0.034
	$C_{B\,DL}$	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005
0.60	$C_{A\,DL}$	0.081	0.034	0.062	0.053	0.037	0.056	0.073	0.048	0.036
	$C_{B\,DL}$	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004
0.55	$C_{A\,DL}$	0.088	0.035	0.071	0.056	0.038	0.058	0.081	0.052	0.037
	$C_{B\,DL}$	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003
0.50	$C_{A\,DL}$	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.038
	$C_{B\,DL}$	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002

*A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.

METHOD 3—TABLE 3—COEFFICIENTS FOR LIVE LOAD POSITIVE MOMENTS IN SLABS*

$$M_{A\,pos\,LL} = C_{A\,LL} \times w \times A^2$$
$$M_{B\,pos\,LL} = C_{B\,LL} \times w \times B^2$$

where w = total uniform live load

Ratio $m = \frac{A}{B}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{A\,LL}$	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030
	$C_{B\,LL}$	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.028
0.95	$C_{A\,LL}$	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032
	$C_{B\,LL}$	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025
0.90	$C_{A\,LL}$	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036
	$C_{B\,LL}$	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022
0.85	$C_{A\,LL}$	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039
	$C_{B\,LL}$	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020
0.80	$C_{A\,LL}$	0.056	0.041	0.045	0.048	0.044	0.051	0.051	0.044	0.042
	$C_{B\,LL}$	0.023	0.017	0.022	0.020	0.016	0.019	0.023	0.019	0.017
0.75	$C_{A\,LL}$	0.061	0.045	0.051	0.052	0.047	0.055	0.056	0.049	0.046
	$C_{B\,LL}$	0.019	0.014	0.019	0.016	0.013	0.016	0.020	0.016	0.013
0.70	$C_{A\,LL}$	0.068	0.049	0.057	0.057	0.051	0.060	0.063	0.054	0.050
	$C_{B\,LL}$	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011
0.65	$C_{A\,LL}$	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054
	$C_{B\,LL}$	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009
0.60	$C_{A\,LL}$	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059
	$C_{B\,LL}$	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007
0.55	$C_{A\,LL}$	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	0.063
	$C_{B\,LL}$	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006
0.50	$C_{A\,LL}$	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067
	$C_{B\,LL}$	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.004

*A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.

$Ma = Ca.neg.Wu.la^2 = 0.0624 \times 25.36 \times (5.7)^2 = 51.41\ KN.m/m$

$-Mb = Cb.neg.Wu.lb^2 = 0.02908 \times 25.36 \times (6.9)^2 = 35.03\ KN.m/m$

2- Positive Moment

Short Direction

-Factors of Dead Load *(from Table 2)*

0.8 0.026

0.826 $Ca.DL = \dfrac{0.024 \times (0.826 - 0.8) + 0.026 \times (0.85 - 0.826)}{(0.85 - 0.8)} = 0.02496$

0.85 0.024

$Self\ Wt\ of\ slab = t \times 1 \times 1 \times \gamma c = 0.2 \times 1 \times 1 \times 24 = 4.8 KN/m2$

$Wu_D = 1.2\ (4.8 + 3) = 9.36\ KN/m2$

$+Ma_{DL} = 0.02496 \times 9.36 \times 5.72 = 7.6\ KN.m/m$

-Factors of Live Load (from Table 2)

0.8 0.041

0.826 $Ca.LL = \dfrac{0.037 \times (0.826 - 0.8) + 0.041 \times (0.85 - 0.826)}{(0.85 - 0.8)} = 0.03892$

0.85 0.037

$Wu\ LL = 1.6 \times 10 = 16\ KN/m2$

$+Ma\ LL = 0.03892 \times 16 \times 5.72 = 20.23\ KN.m/m$

$+Ma = Ma\ DL + Ma\ LL = 7.6 + 20.23 = 27.83\ KN.m/m$

Long Direction

-*Factors of Dead Load (from Table 3)*

0.8 0.011

0.826 $Cb.DL = \dfrac{0.012 \times (0.826 - 0.8) + 0.011 \times (0.85 - 0.826)}{(0.85 - 0.8)} = 0.01148$

0.85 0.012

$Wu_D = 9.36\ KN/m2$

$+Mb_{DL} = 0.01148 \times 9.36 \times 6.92 = 5.12\ KN.m/m$

$-$*Factors of Live Load (from Table 3)*

0.8 0.017

0.826 $Cb.LL = \dfrac{0.024 \times (0.826 - 0.8) + 0.026 \times (0.85 - 0.826)}{(0.85 - 0.8)} = 0.01804$

0.85 0.019

$Wu_{LL} = 16\ KN/m2$

$+Mb_{LL} = 0.01804 \times 16 \times 6.92 = 13.81\ KN.m/m$

$+Mb = Mb_{DL} + Mb_{LL} = 5.12 + 13.78 = 18.93\ KN.m/m$

Shear On Slab

-Short Direction (from Table 4)

0.8 0.71

0.826 $C_{wa} = \frac{0.66 \times (0.826 - 0.8) + 0.71 \times (0.85 - 0.826)}{(0.85 - 0.8)} = 0.684$

0.85 0.66

$Wa = 0.684 * 25.36 = 17.35\ KN/m2$

$Vu = Wa \times La/2 = 17.35 \times \frac{5.7}{2} = 49.43\ KN/m$

Long Direction (*from Table* 4)

0.8 0.29

0.826 $C_{wb} = \frac{0.34 \times (0.826 - 0.8) + 0.29 \times (0.85 - 0.826)}{(0.85 - 0.8)} = 0.316$

0.85 0.34

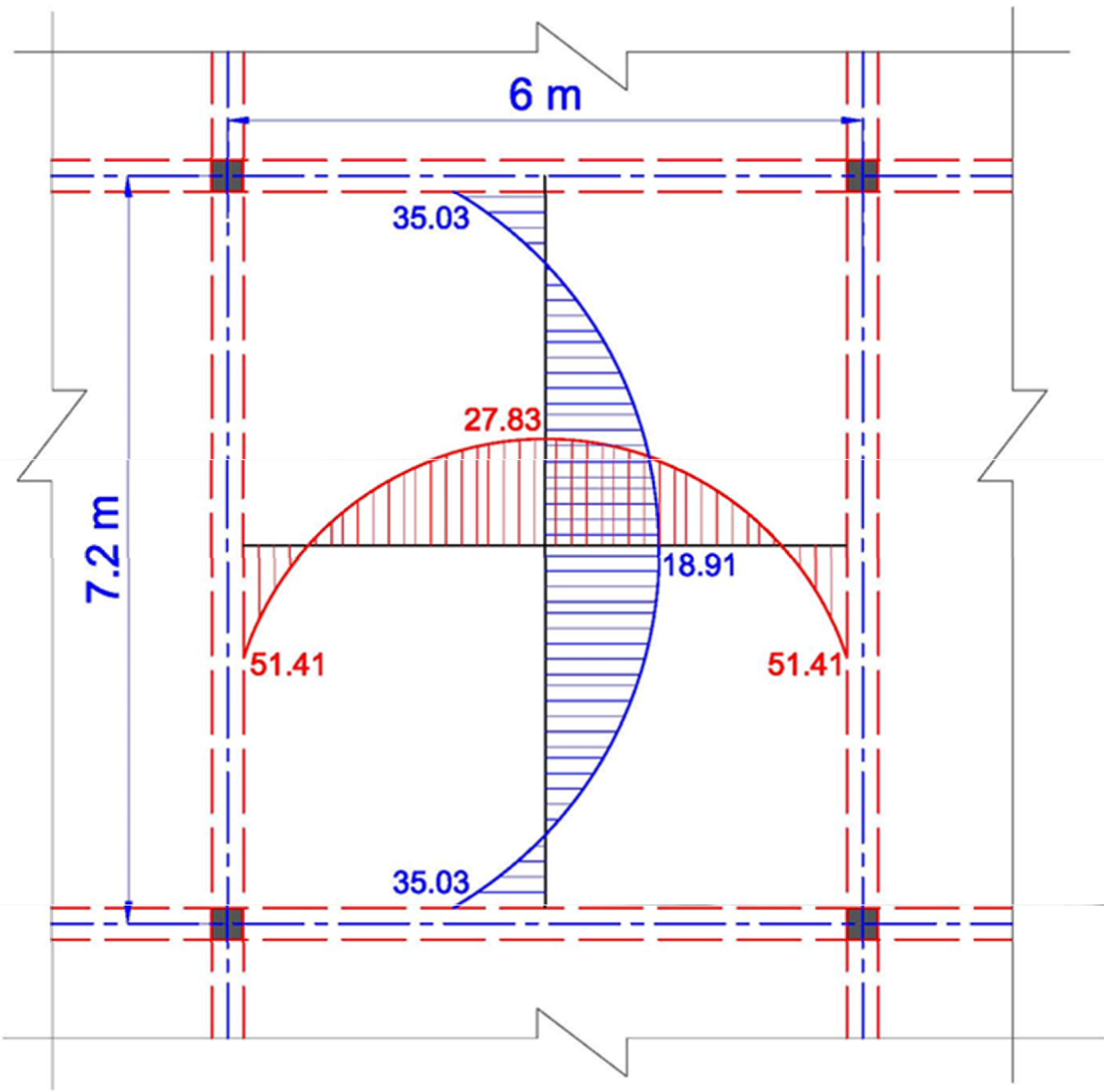
$Wb == 0.316 \times 25.36 = 8.01\ KN/m2$

$Vu = Wb \times \frac{Lb}{2} = 8.01 \times 6.9/2 = 27.65 KN/m$

METHOD 3—TABLE 4—RATIO OF LOAD *w* IN *A* and *B* DIRECTIONS FOR SHEAR IN SLAB AND LOAD ON SUPPORTS*

Ratio $m = \frac{A}{B}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	W_A	0.50	0.50	0.17	0.50	0.83	0.71	0.29	0.33	0.67
	W_B	0.50	0.50	0.83	0.50	0.17	0.29	0.71	0.67	0.33
0.95	W_A	0.55	0.55	0.20	0.55	0.86	0.75	0.33	0.38	0.71
	W_B	0.45	0.45	0.80	0.45	0.14	0.25	0.67	0.62	0.29
0.90	W_A	0.60	0.60	0.23	0.60	0.88	0.79	0.38	0.43	0.75
	W_B	0.40	0.40	0.77	0.40	0.12	0.21	0.62	0.57	0.25
0.85	W_A	0.66	0.66	0.28	0.66	0.90	0.83	0.43	0.49	0.79
	W_B	0.34	0.34	0.72	0.34	0.10	0.17	0.57	0.51	0.21
0.80	W_A	0.71	0.71	0.33	0.71	0.92	0.86	0.49	0.55	0.83
	W_B	0.29	0.29	0.67	0.29	0.08	0.14	0.51	0.45	0.17
0.75	W_A	0.76	0.76	0.39	0.76	0.94	0.88	0.56	0.61	0.86
	W_B	0.24	0.24	0.61	0.24	0.06	0.12	0.44	0.39	0.14
0.70	W_A	0.81	0.81	0.45	0.81	0.95	0.91	0.62	0.68	0.89
	W_B	0.19	0.19	0.55	0.19	0.05	0.09	0.38	0.32	0.11
0.65	W_A	0.85	0.85	0.53	0.85	0.96	0.93	0.69	0.74	0.92
	W_B	0.15	0.15	0.47	0.15	0.04	0.07	0.31	0.26	0.08
0.60	W_A	0.89	0.89	0.61	0.89	0.97	0.95	0.76	0.80	0.94
	W_B	0.11	0.11	0.39	0.11	0.03	0.05	0.24	0.20	0.06
0.55	W_A	0.92	0.92	0.69	0.92	0.98	0.96	0.81	0.85	0.95
	W_B	0.08	0.08	0.31	0.08	0.02	0.04	0.19	0.15	0.05
0.50	W_A	0.94	0.94	0.76	0.94	0.99	0.97	0.86	0.89	0.97
	W_B	0.06	0.06	0.24	0.06	0.01	0.03	0.14	0.11	0.03

*A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.


6 m
7.2 m
35.03
27.83
18.91
51.41
51.41
35.03

Shear On Beams

-Short Direction

the load transfer from slab with long direction (on short beam)

$= 27.65\ KN/m$

$and\ there\ is\ two\ slab\ from\ both\ side$

$= 2 \times 27.65 = 55.3\ KN$

$Selfwt.of\ beam\ = 6.05\ KN/m$

$Total\ Wua = 55.3 + 6.05 = 61.35\ KN/m$

$$Vu = \frac{Wua \times La}{2} = \frac{61.35 \times 5.7}{2} = 174.85KN$$

-Long Direction

the load transfer from slab with short direction (on long beam)

$= 49.43\ KN/m$

$and\ there\ is\ two\ slab\ from\ both\ side$

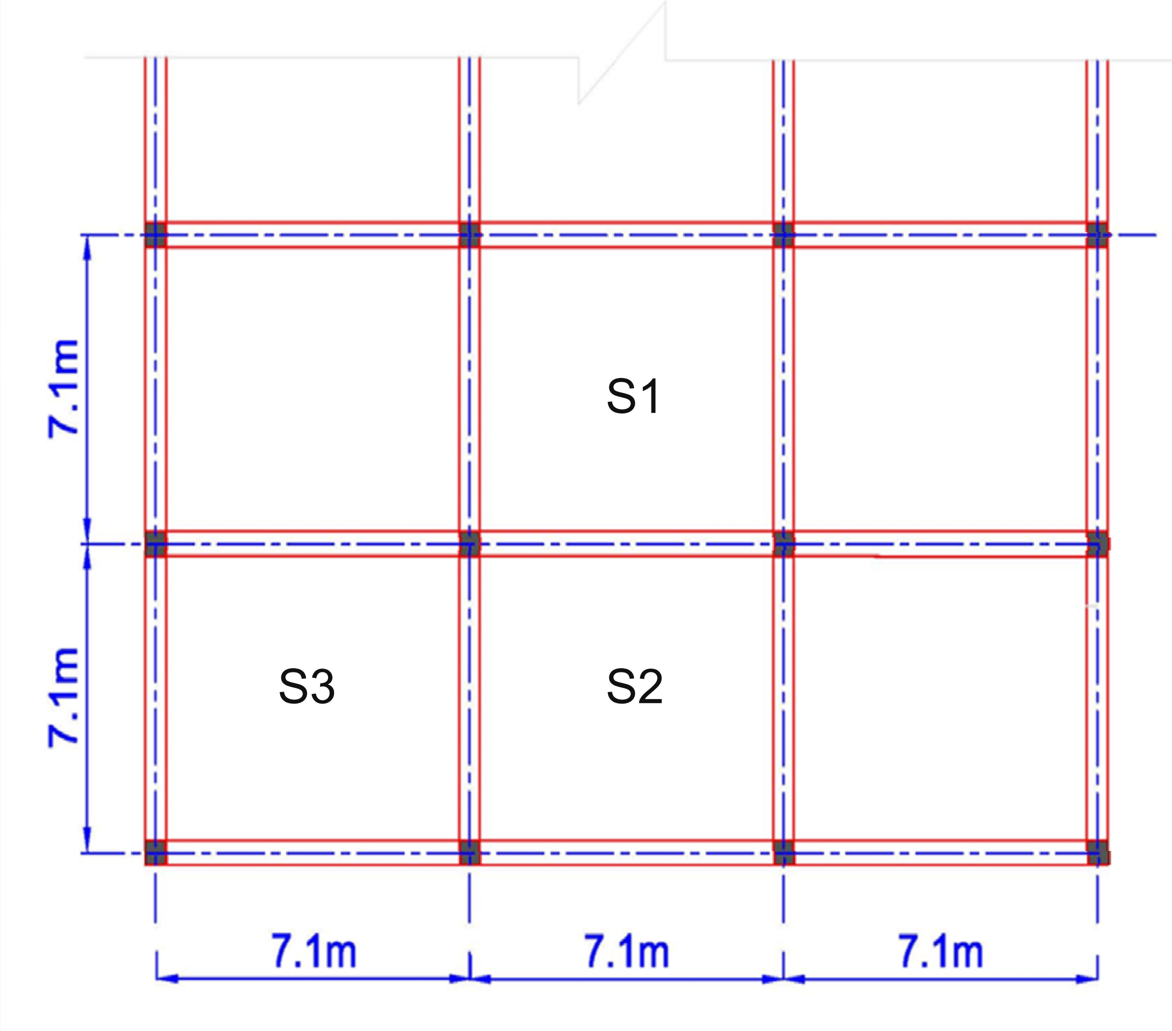
$= 2 \times 49.43\ = 98.86\ KN$

$Selfwt.of\ beam\ = 6.05\ KN/m$

$Total\ Wub = 98.86 + 6.05 = 104.91KN/m$

$$Vu = \frac{Wub \times Lb}{2} = \frac{104.91 \times 6.9}{2} = 361.93KN$$

Example (3) : An Apartment building is designed using 6.1*6.1 m Two way slabs system. The live load 2 KN/m2 , the superimposed load (partition loads) is 1.5 KN/m2 and the floor finish load is 2 KN/m2. Design a typical panels. Assume f'c=21MPa, fy =280 Mpa. The column dimension 300* 300 mm and the supporting beams are 300 mm width . Also Design the interior beam.



Sol.

-Slab Thickness

ACI Code 1963 allowed slab thickness

not less than 90 mm

$$t_{min} = \frac{2\,(L+S)}{180} \geq 90\ mm$$

$$= \frac{2(5.8+5.8)}{180} = 128.9\ mm$$

ACI Code 2014 *allowed using equation*

where $\alpha\ m \geq 2$

$$\beta = \frac{Ln}{Sn} = 1.0$$

$$t_{min} = \frac{5.8 \times \left[0.8 + \left(\frac{280}{1400}\right)\right]}{36 + 9 \times 1}$$

$= 141.5mm\ Use\ t\ or\ h = 150\ mm$

Load On Slab

$D.L\ of\ slab\ =\ 0.15 \times 1 \times 1 \times 24 = 3.6\ KN/m^2$

$Floor\ Finishing\ =\ 2\ KN/m^2$

$Partitions\ =\ 1.5\ KN/m^2$

$Total\ DL\ =\ 7.1\ KN/m^2$

$L.L\ =\ 2\ KN/m^2$

$Wu\ =\ 1.2\ DL +\ 1.6\ LL$

$=\ 1.2 \times 7.1\ +\ 1.6 \times 2 =\ 11.72\ KN/m^2$

Using Method 2

$M =\ ceof. \times Wu \times S$

$From\ table\ 1\ of\ Method\ 2$

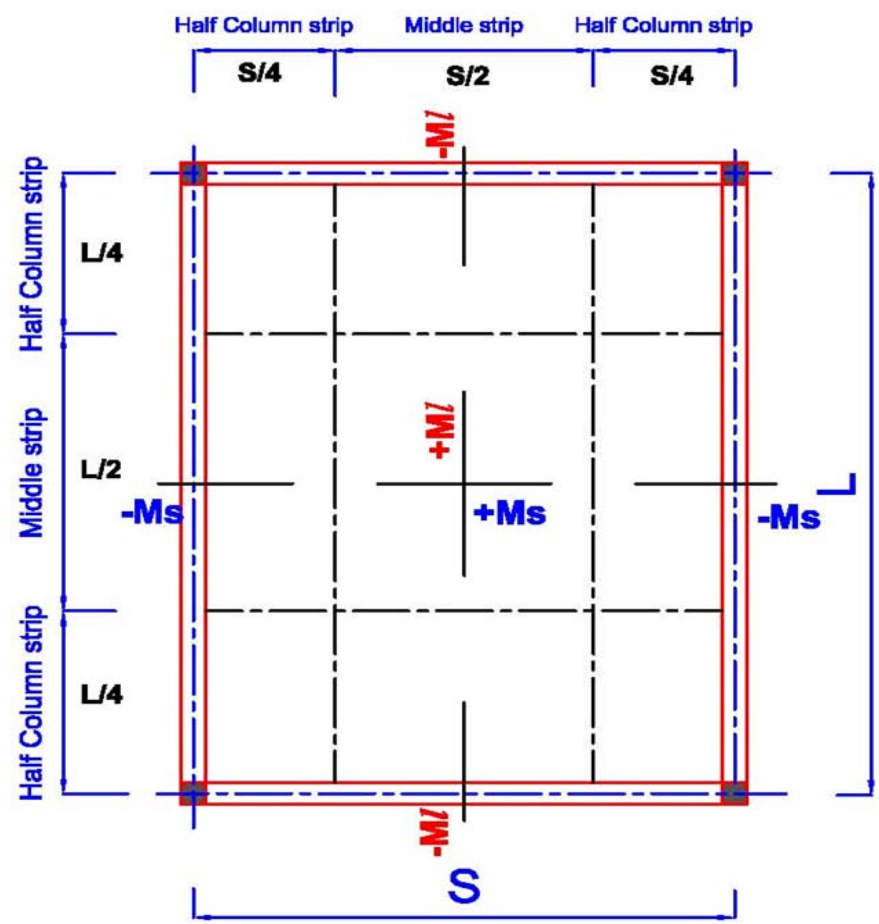
$dav. = h - cover\ - \phi =\ 150 - 20 - 10 \qquad (use\ \phi 10\ mm)$

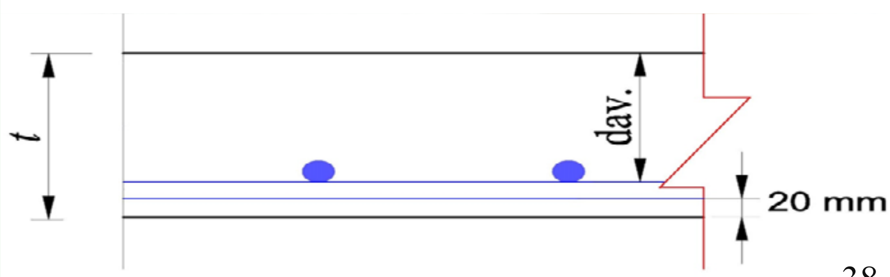
$=\ 120\ mm$

Notes

1- For square panel use d average

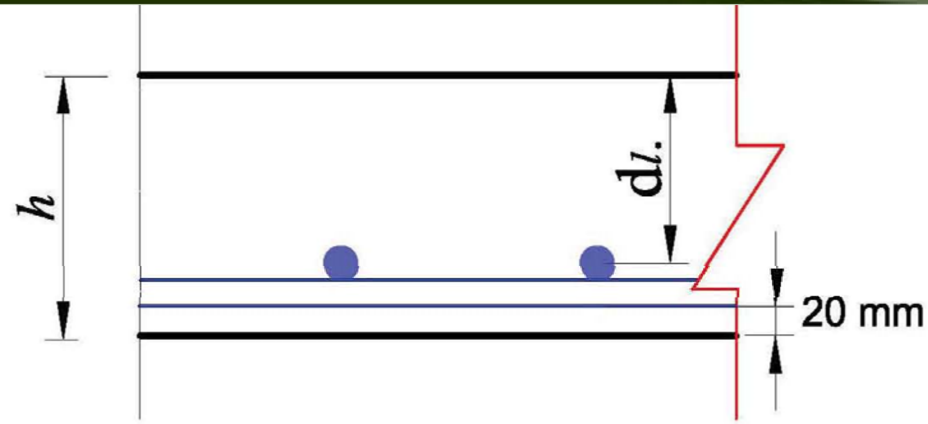
2- rectangular panel the steel in short direction at bottom layer (large M, d the greater) and for long direction the steel at top layer (d shorter)





Long span

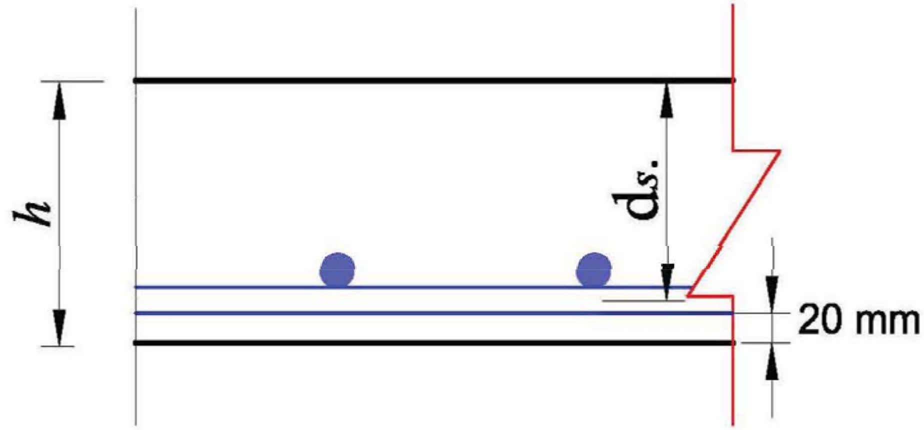
$$d_L = h - 20 - \phi_s - \frac{\phi_L}{2}$$



(a) Effective depth in long direction

short span

$$d_S = h - 20 - \frac{\phi s}{2}$$



(b) Effective depth in short direction

$Choose\ (S3)\ = CASE\ 3$

$From\ Table\ (1) m\ = \frac{S}{L} =\ 1$

$Moment\ factors\ for\ both\ Direction\ (\ square\ panel\ )$

$-C\ =\ 0.049$ $\quad Negative\ moment\ Factor\ Discontinuous\ edge$

$-C\ =\ 0.025$ $\quad Negative\ moment\ Factor\ Continuous\ edge$

$+C\ =\ 0.037$ $\quad Positive\ moment\ Factor\ Midspan$

$-Mu\ =\ c\ Wu.S^2 = 0.049 \times 11.72 \times 6.12 =\ 21.37\ KN.m/m$ $\quad Cont.$

$-Mu\ =\ c\ Wu.S^2 = 0.025 \times 11.72 \times 6.12 =\ 10.9\ KN.m/m$ $\quad Discont.$

$+Mu\ =\ c\ Wu.S^2 = 0.037 \times 11.72 \times 6.12 = 16.14\ KN.m/m$ $\quad Mid\ span$

$Mid\ Span$

$$R = \frac{Mu}{\rho b d^2}$$

$$R = \frac{16.14 \times 10^6}{(\ 0.9 \times 1000 \times 120^2)} = 1.245$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right)$$

$$m\ = \frac{fy}{0.85 * fc} =\ 15.68$$

$$\rho\ = \left(\frac{1}{15.68}\right) \times \left(\ 1 - \sqrt{1 - \frac{(2 \times 15.68 \times 1.245)}{280}}\right) = 0.00461$$

Method 2
Table 1

METHOD 2—TABLE 1—MOMENT COEFFICIENTS

Moments	Short span, Values of *m*: 1.0	0.9	0.8	0.7	0.6	0.5 and less	Long span, all values of *m*
Case 1—Interior panels							
Negative moment at—							
Continuous edge	0.033	0.040	0.048	0.055	0.063	0.083	0.033
Discontinuous edge	—	—	—	—	—	—	—
Positive moment at midspan	0.025	0.030	0.036	0.041	0.047	0.062	0.025
Case 2—One edge discontinuous							
Negative moment at—							
Continuous edge	0.041	0.048	0.055	0.062	0.069	0.085	0.041
Discontinuous edge	0.021	0.024	0.027	0.031	0.035	0.042	0.021
Positive moment at midspan	0.031	0.036	0.041	0.047	0.052	0.064	0.031
Case 3—Two edges discontinuous							
Negative moment at—							
Continuous edge	0.049	0.057	0.064	0.071	0.078	0.090	0.049
Discontinuous edge	0.025	0.028	0.032	0.036	0.039	0.045	0.025
Positive moment at midspan	0.037	0.043	0.048	0.054	0.059	0.068	0.037
Case 4—Three edges discontinuous							
Negative moment at—							
Continuous edge	0.058	0.066	0.074	0.082	0.090	0.098	0.058
Discontinuous edge	0.029	0.033	0.037	0.041	0.045	0.049	0.029
Positive moment at midspan	0.044	0.050	0.056	0.062	0.068	0.074	0.044
Case 5—Four edges discontinuous							
Negative moment at—							
Continuous edge	—	—	—	—	—	—	—
Discontinuous edge	0.033	0.038	0.043	0.047	0.053	0.055	0.033
Positive moment at midspan	0.050	0.057	0.064	0.072	0.080	0.083	0.050

$As = \rho . b . d = 0.00461 \times 1000 \times 120 = 553\ mm^2/m$

$Use\ \phi\ 10\ mm$

$$S = \frac{78 \times 1000}{553} = 142\ mm$$

$Use\ \phi\ 10\ mm\ at\ 140\ mm\ c/c$

$As\ \min = \rho . b . h\ (mm^2)$

$\rho min = 0.0018$

$As\ \min. = 0.0018 \times 1000 \times 150 = 270\ mm\ 2/\ m < As\ Provide\ (OK)$

$S\ \max = 2 \times h = 300\ or\ 450\ mm \quad at\ critical\ section \qquad ACI\ (8.7.2.2)$

$Use\ \phi\ 10\ mm\ @\ 140\ mm$

$$Column\ Strip\ Moment = \frac{2}{3}\ M\ mid = 16.14 \times \frac{2}{3} = 10.76\ KN.m/m$$

$Or\ can\ use\ the\ spacing\ of$

$1.5 * middle\ strip\ spacing = 213\ mm\ C/C < 2\ h = 300\ mm$

$Use\ Use\ \phi\ 10\ mm\ @\ 210\ mm$

Negative Moment

$-Discontinues\ edge$

- M= 10.9 KN.m/m

$$R = \frac{Mu}{\phi b d^2}$$

$$R = \frac{10.9 \times 10^6}{0.9 \times 1000 \times 120^2} = 0.841$$

$$m = \frac{fy}{0.85 \times fc} = 15.68$$

$$\rho = \frac{1}{15.68}\left(1 - \sqrt{1 - \frac{(2 \times 15.68 \times 0.841}{280}}\right) = 0.00308$$

$As = \rho.b.d = 0.00308 \times 1000 \times 120 = 370\ mm^2/m$

$Use\ \phi\ 10\ mm$

$S = \frac{78 \times 1000}{370} = 211mm$, $\quad Use\ \phi\ 10\ mm\ at\ 210\ mm\ c/c$

−Continuous Edge

$$R = \frac{Mu}{\phi b d2}$$

$$= \frac{21.37 \times 10^6}{(0.9 \times 1000 \times 120^2)} = 1.649$$

$$\rho = \left(\frac{1}{15.68}\right) \times \left(1 - \sqrt{1 - \frac{2 \times 15.68 \times 1.649}{280}}\right)$$

$= 0.006189$

$As = \rho.b.d = 0.006189 \times 1000 \times 120 = 742mm^2/m$

$Use\ \phi\ 10\ mm$

$S = 78 \times 1000/\ 742 = 105mm\ Use\ \phi\ 10\ mm\ at\ 100\ mm\ c/c$

Note: The reinforcement detail for long Direction same as of short direction cause the panel is square (L = S)

Check for Shear

The shear force on slab can be calculated according to (same in both direction):

$$V = \frac{Wu.S}{2} \qquad at\ center\ of\ support$$

$$= \frac{11.72 \times 6.1}{2} = 35.75\ KN/m$$

$$Vud = Vu - Wu \times \frac{0.3}{2} - Wu \times d$$

$$= 35.75 - 11.72 \times \frac{0.3}{2} - 11.72 \times 0.12 = 32.59\ KN/m$$

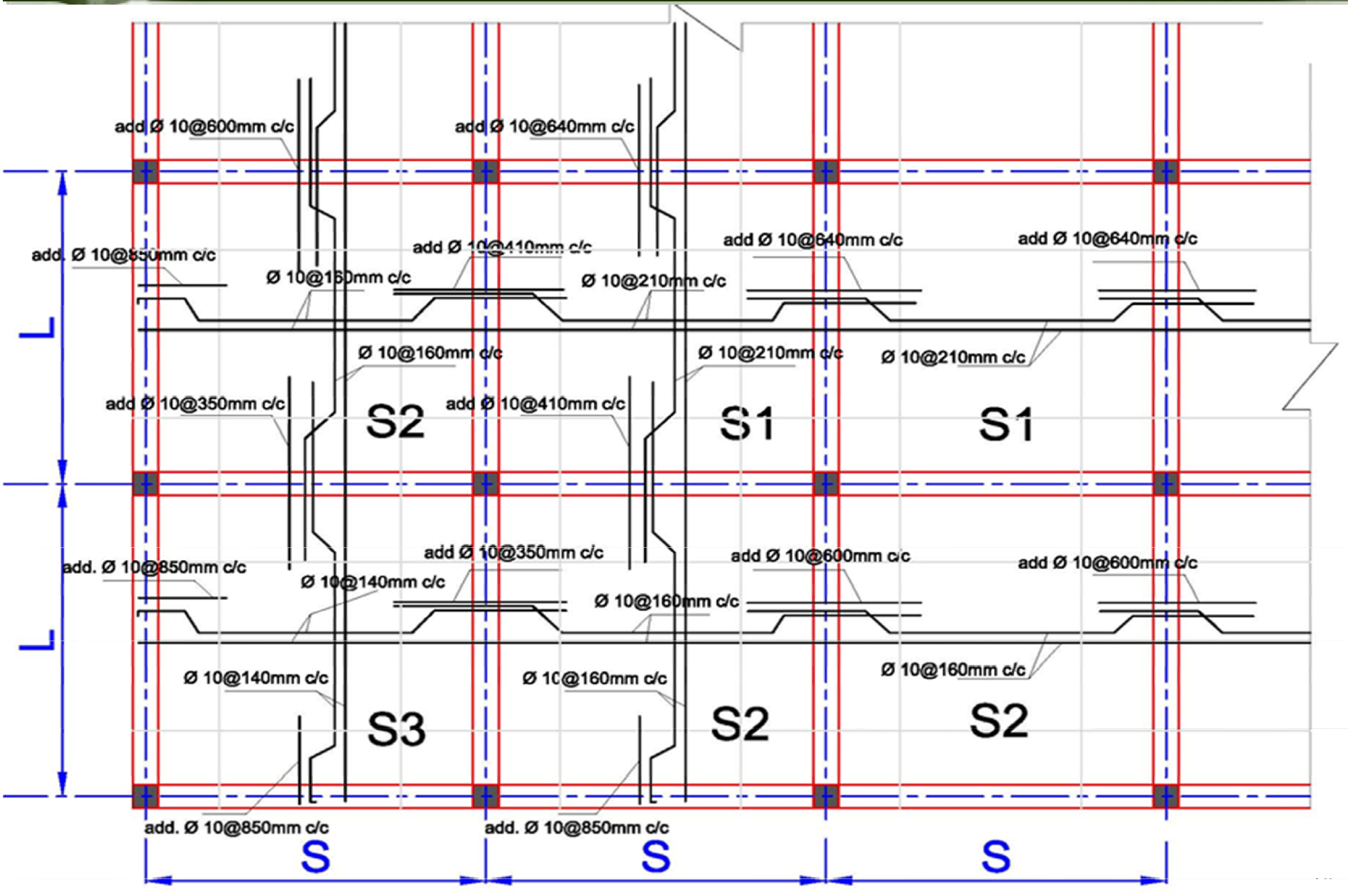
$$\phi\ Vc = \phi \times 0.17\sqrt{f'c} \times b \times d = 0.75 \times 0.17 \times \sqrt{21} \times 1000 \times 120 = 70.11\ KN/m$$

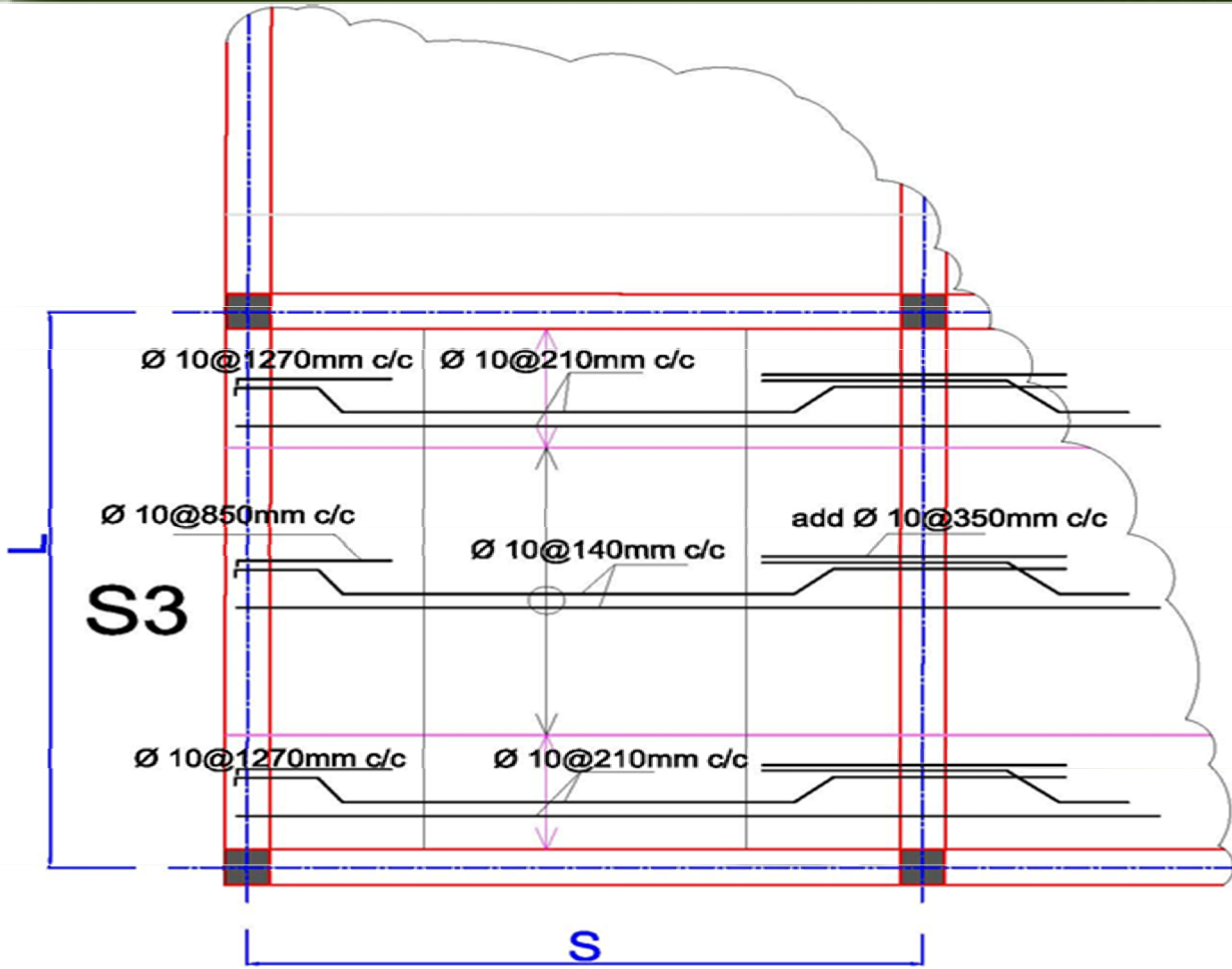
$$\phi\ Vc > Vud \qquad (\ OK\ section\ is\ safe\ )$$

No.	Detail	Interior Panal (S1)					
		Short Span			Long Span		
		(-M) Cont.	(+M) Mid	(-M) Discont.	(-M) Discont.	(+M) Mid	(-M) Discont.
1	Mu x 106 (N.mm/m)	14.40	10.90	14.40	14.40	10.90	14.40
2	d (mm)	120	120	120	120	120	120
3	m=	15.69	15.69	15.69	15.69	15.69	15.69
4	Rn=	1.111	.841	1.111	1.111	0.841	1.111
5	ρ=r.b.h (mm2)	0.0041	0.0031	0.0041	0.0041	0.0031	0.0041
6	As (calculated)	492.0	369.4	492.0	492.0	369.4	492.0
7	As(min)= 0.0018 b.h	270	270	270	270	270	270
8	As(choosed)=	492	369	492	492	369	492
9	S=1000*Ab/As (mm)	160	213	160	160	213	160
10	S(max)= 2*h=300 or 450 mm	300	300	300	300	300	300
11	S(choosed)=	160	212.6	159.6	160	213	160
12	Use S=	150	210	150	150	210	150

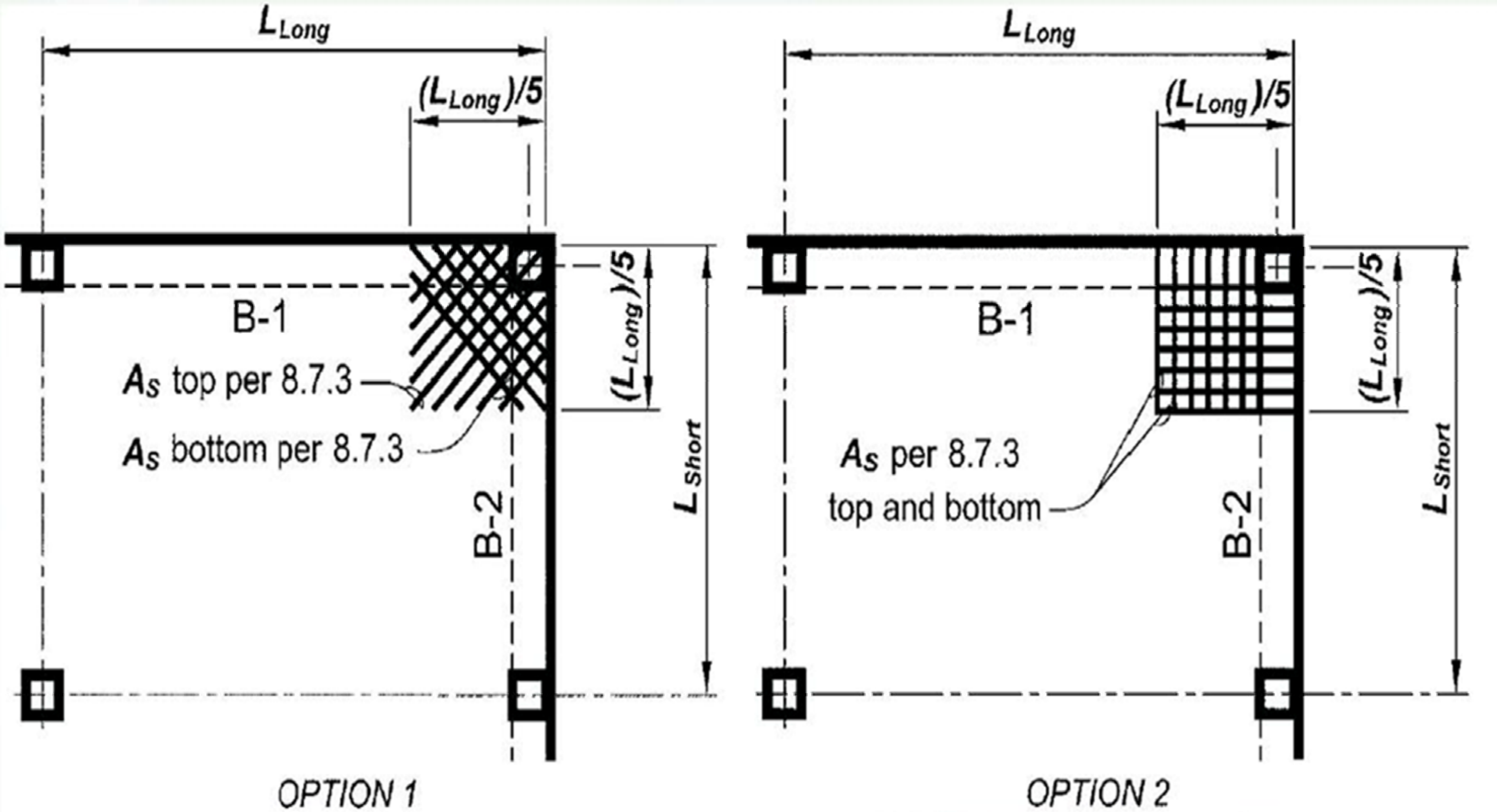
No.	Detail	Interior Panal (S2)					
		Short Span			Long Span		
		(-M) Cont.	(+M) Mid	(-M) Discont.	(-M) Discont.	(+M) Mid	(-M) Discont.
1	Mu x 106 (N.mm/m)	9.16	13.52	17.88	17.88	13.52	17.88
2	d (mm)	120	120	120	120	120	120
3	m=	15.69	15.69	15.69	15.69	15.69	15.69
4	Rn=	0.707	1.043	1.380	1.380	1.043	1.380
5	As =ρ.b.h (mm2)	0.0026	0.0038	0.0051	0.0051	0.0038	0.0051
6	As (calculated)	309	461	616	616	461	616
7	As(min)= 0.0018 b.h	270	270	270	270	270	270
8	As(choosed)=	309	461	616	616	461	616
9	S=1000*Ab/As (mm)	254	170	127	127	170	127
10	S(max)= 2*h=300 or 450 mm	300	300	300	300	300	300
11	S(choosed)=	254.0	170.4	127.5	127	170.4	127.5
12	Use S=	250	160	120	120	160	120

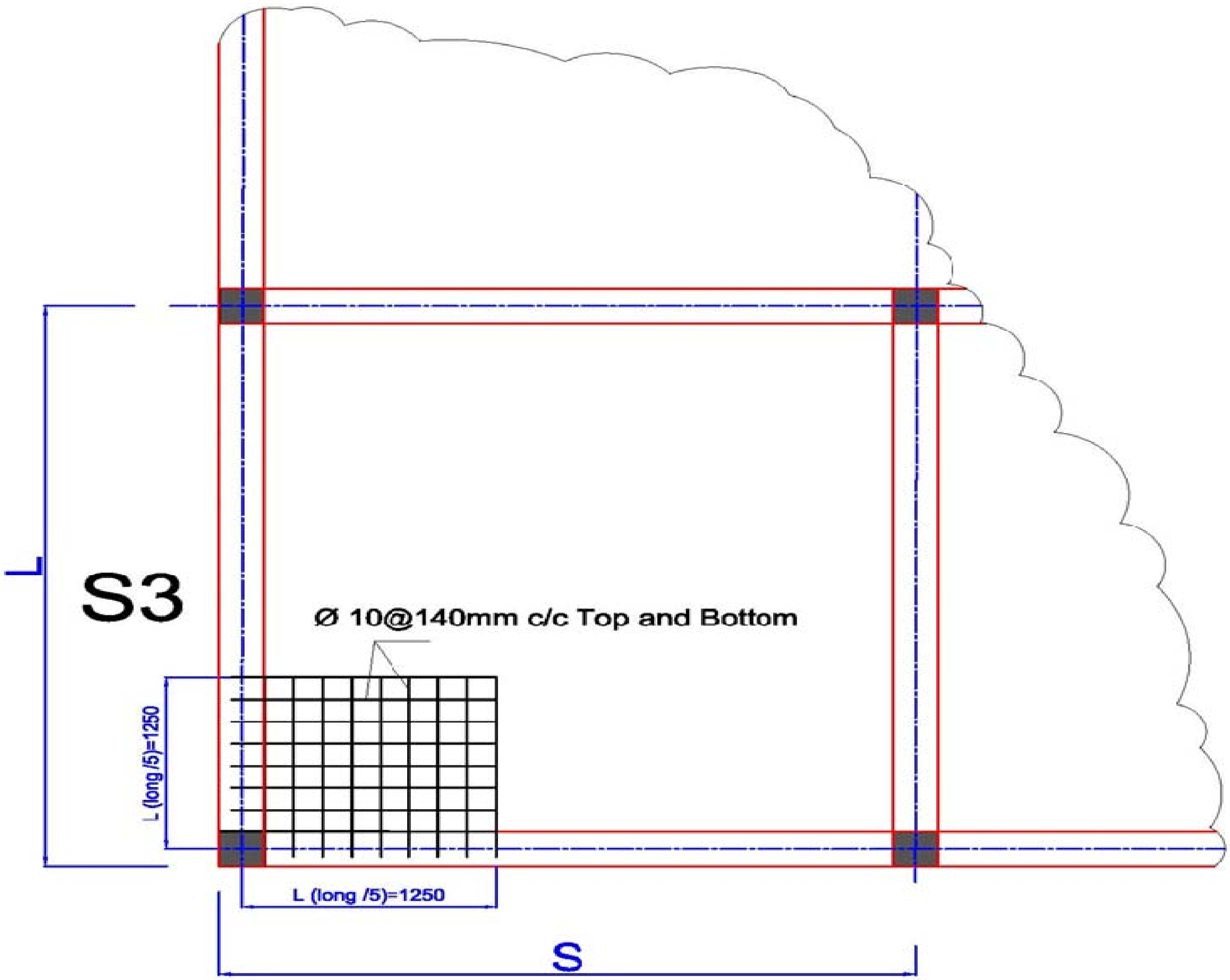
No.	Detail	Interior Panal (S3)					
		Short Span			Long Span		
		(-M) Cont.	(+M) Mid	(-M) Discont.	(-M) Discont.	(+M) Mid	(-M) Discont.
1	Mu x 106 (N.mm/m)	10.90	16.14	21.37	10.90	16.14	21.37
2	d (mm)	120	120	120	120	120	120
3	m=	15.69	15.69	15.69	15.69	15.69	15.69
4	Rn=	0.841	1.245	1.649	0.841	1.245	1.649
5	As =ρ.b.h (mm2)	0.0031	0.0046	0.0062	0.0031	0.0046	0.0062
6	As (calculated)	369	554	743	369	554	743
7	As(min)= 0.0018 b.h	270	270	270	270	270	270
8	As(choosed)=	369	554	743	369	554	743
9	S=1000*Ab/As (mm)	213	142	106	213	142	106
10	S(max)= 2*h=300 or 450 mm	300	300	300	300	300	300
11	S(choosed)=	212.6	141.8	105.7	213	141.8	105.7
12	Use S=	210	140	100	210	140	100


add Ø 10@600mm c/c
add Ø 10@640mm c/c
add. Ø 10@850mm c/c
Ø 10@160mm c/c
add Ø 10@410mm c/c
Ø 10@210mm c/c
add Ø 10@640mm c/c
add Ø 10@640mm c/c
Ø 10@160mm c/c
Ø 10@210mm c/c
Ø 10@210mm c/c
add Ø 10@350mm c/c
S2
add Ø 10@410mm c/c
S1
S1
L
add. Ø 10@850mm c/c
add Ø 10@350mm c/c
Ø 10@140mm c/c
add Ø 10@600mm c/c
add Ø 10@600mm c/c
Ø 10@160mm c/c
L
Ø 10@160mm c/c
Ø 10@140mm c/c
Ø 10@160mm c/c
S3
S2
S2
add. Ø 10@850mm c/c
add. Ø 10@850mm c/c
S
S
S


Ø 10@1270mm c/c
Ø 10@210mm c/c
Ø 10@850mm c/c
add Ø 10@350mm c/c
Ø 10@140mm c/c
L
S3
Ø 10@1270mm c/c
Ø 10@210mm c/c
S

Corner slab reinforcement detail




S3
L
Ø 10@140mm c/c Top and Bottom
L (long /5)=1250
L (long /5)=1250
S

Thank You

………….To be Continued