Reinforced Concrete Design
Design For Shear

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Introduction

When a simple beam is loaded, as shown in Fig. Below , bending moments and shear forces develop along the beam. To carry the loads safely, the beam must be designed for both types of forces. Flexural design is considered first to establish the dimensions of the beam section and the main reinforcement needed, as explained in the previous chapters.

The beam is then designed for shear. If shear reinforcement is not provided, *shear failure* may occur. Shear failure is characterized by small deflections and lack of ductility, giving little or no warning before failure. On the other hand, flexural failure is characterized by a gradual increase in deflection and cracking, thus giving warning before total failure. This is due to the ACI Code limitation on flexural reinforcement. The design for shear must ensure that shear failure does not occur before flexural failure.

SHEAR STRESSES IN CONCRETE BEAMS

The general formula for the shear stress in a homogeneous beam is

$$\upsilon = \frac{VQ}{I\,b} \ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

Where:

V = total shear at section considered

Q=statical moment about neutral axis of that portion of cross section lying between line through point in question parallel to neutral axis and nearest face, upper or lower, of beam.

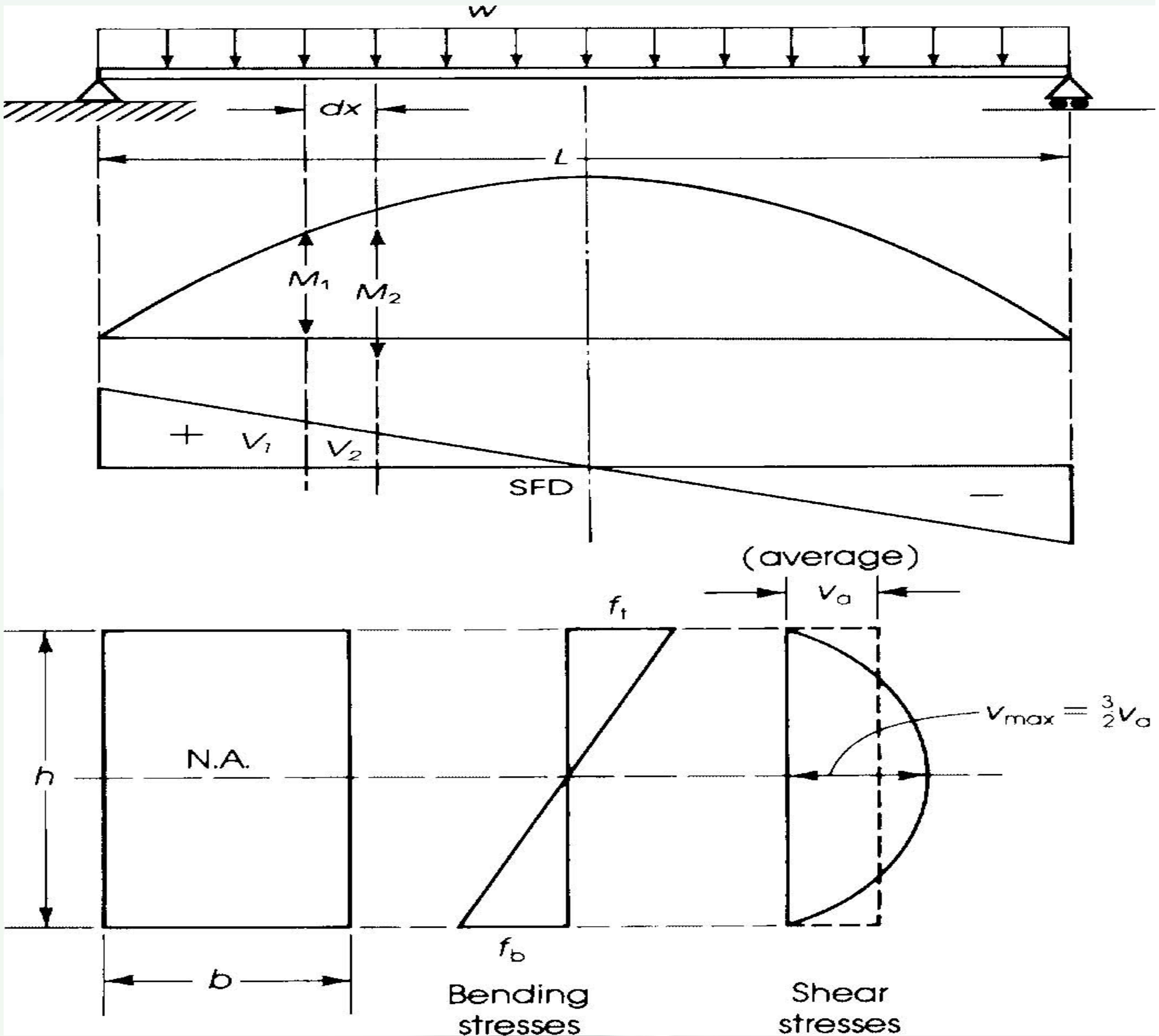
I=moment of inertia of cross section about neutral axis.

b=width of beam at given point.

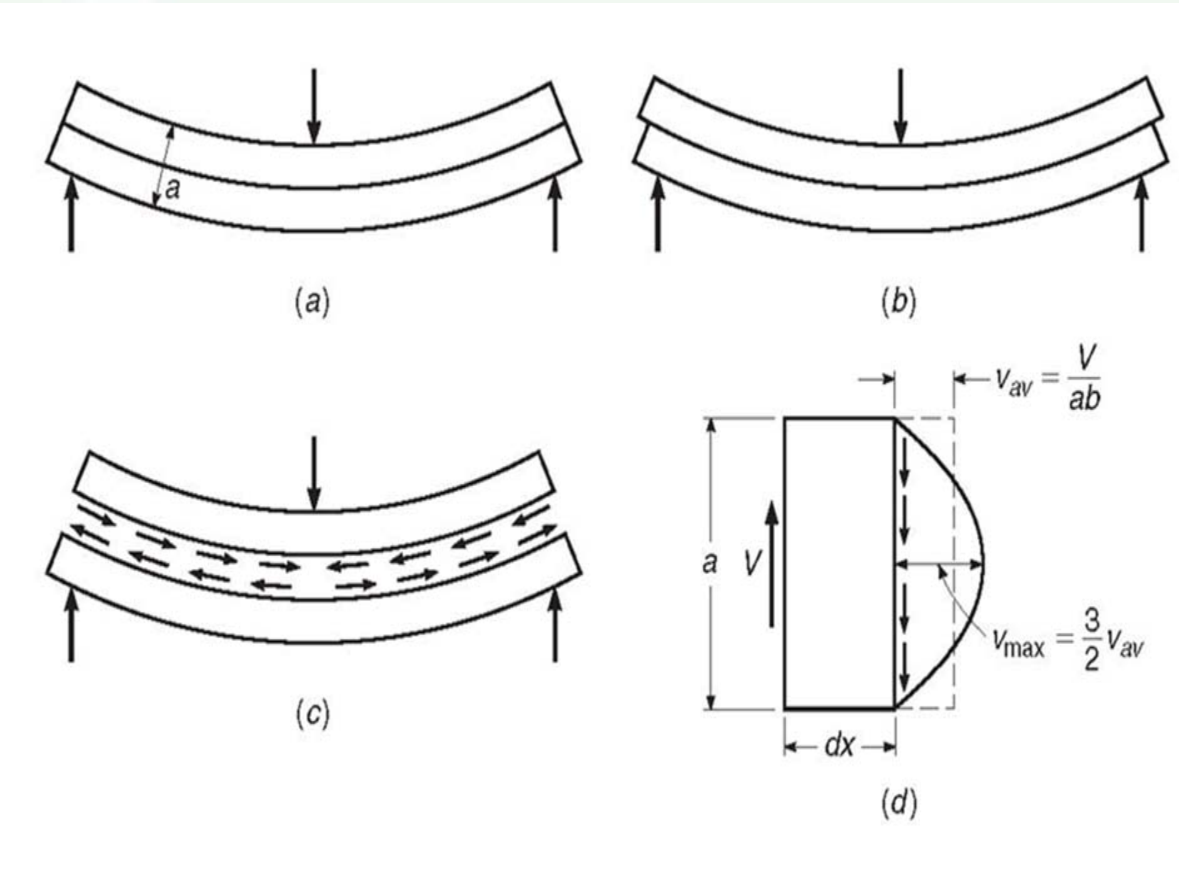
The distribution of bending and shear stresses according to elastic theory for a homogeneous rectangular beam is as shown in Fig. Below. The bending stresses are calculated from the flexural formula $f = \frac{M\,C}{I}$, whereas the shear stress at any point is calculated by the shear formula of Eq.1.

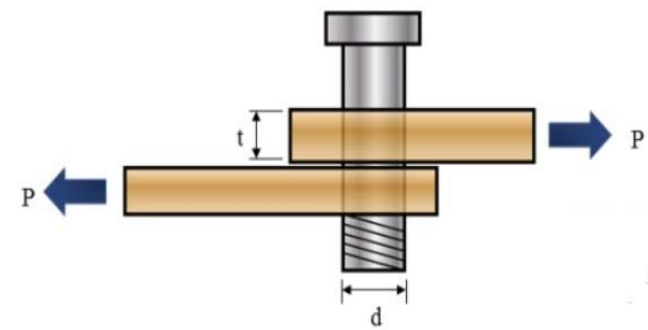
The maximum shear stress is at the neutral axis and is equal to 1.5va (average shear),

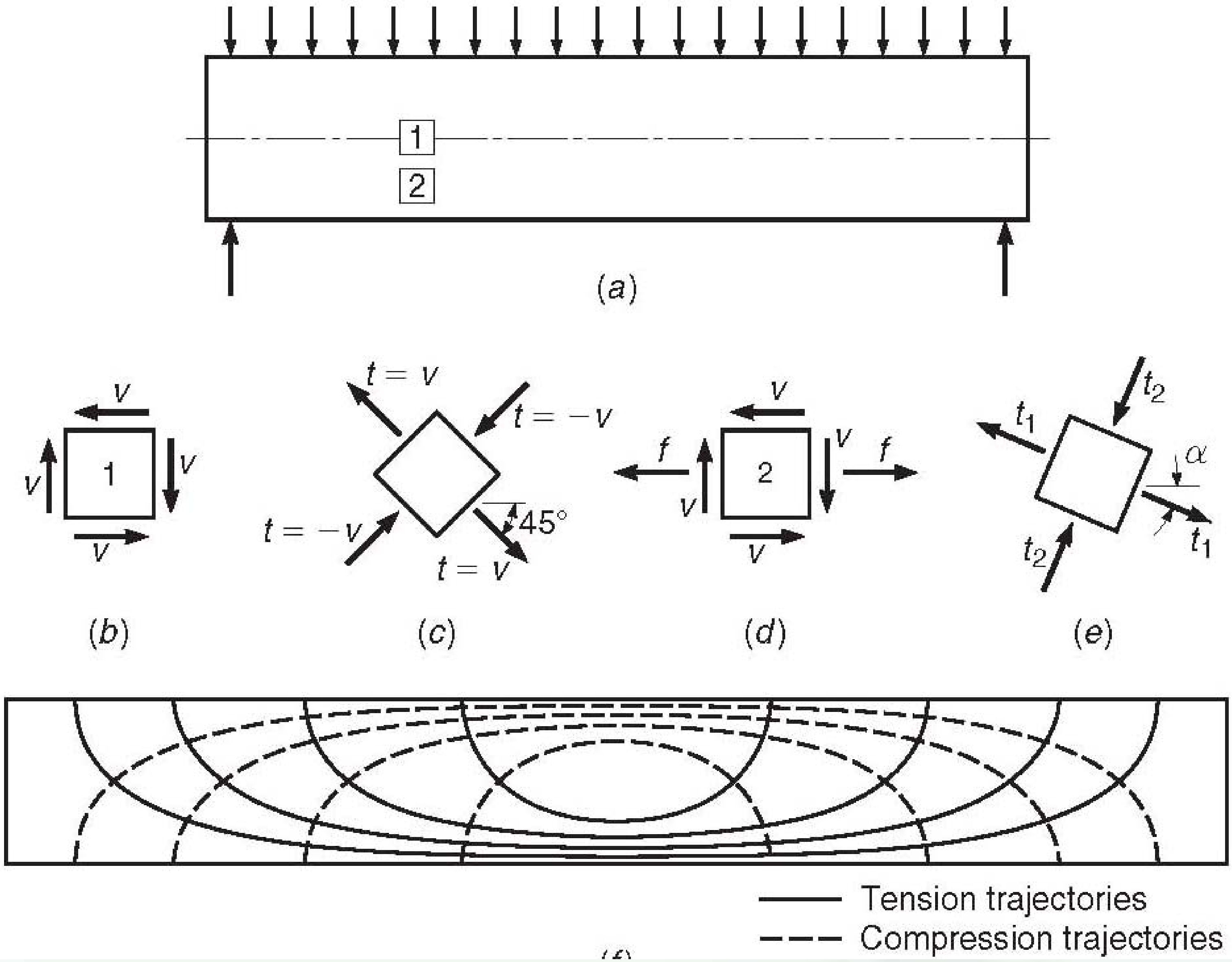
Where: $v_{max} = \frac{3}{2} v_a = \frac{3\,V}{2\,b\,h}$



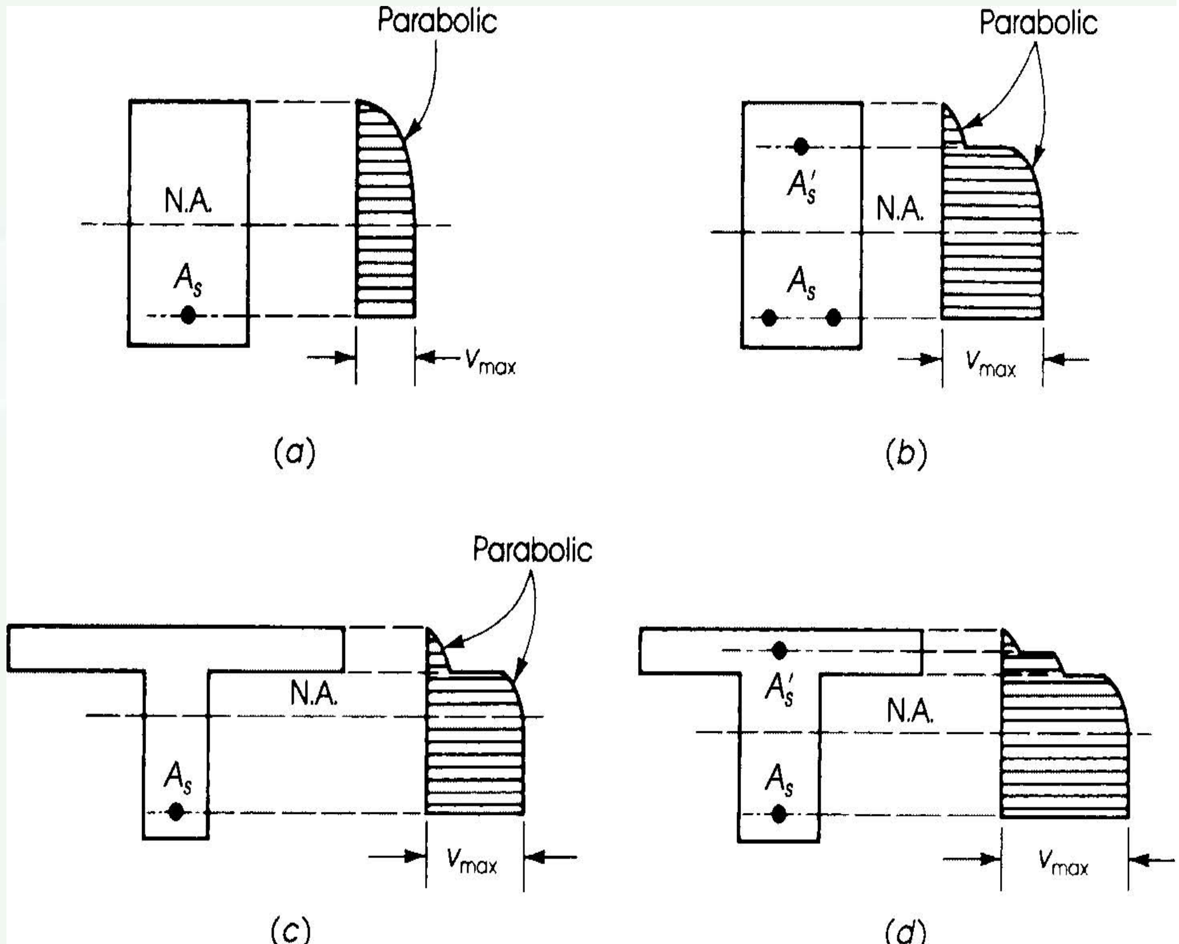
$Fig.$ 5.1)


a
(a)
(b)
(c)
$v_{av} = \frac{V}{ab}$
a
V
$v_{max} = \frac{3}{2} v_{av}$
dx
(d)


t
P
P
d


1
2
(a)
v
v
1
v
v
(b)
t = v
t = −v
t = −v
45°
t = v
(c)
v
f
v
2
v
f
v
(d)
t2
t1
α
t1
t2
(e)
Tension trajectories
Compression trajectories

The shear stress curve is parabolic. For a singly reinforced concrete beam, the distribution of shear stress above the neutral axis is a parabolic curve. Below the neutral axis, the maximum shear stress is maintained down to the level of the tension steel, because there is no change in the tensile force down to this point and the concrete in tension is neglected. The shear stress below the tension steel is zero. For doubly reinforced and T-sections, the distribution of shear stresses is as shown in Fig.



It can be observed that almost all the shear force is resisted by the web, whereas the flange resists a very small percentage; in most practical problems, the shear capacity of the flange is neglected.

Referring to Fig. 1 and taking any portion of the beam dx, the bending moments at both ends of the element, M_1 and M_2, are not equal. Because $M_2 > M_1$ and to maintain the equilibrium of the beam portion dx, the compression force C_2 must be greater than C_1 (Fig. 1). Consequently, a shear stress v develops along any horizontal section $a–a_1$ or $b–b_1$ (Fig. 1a). The normal and shear stresses on a small element at levels $a–a_1$ and $b–b_1$ are shown in Fig. 1b. Notice that the normal stress at the level of the neutral axis $b–b_1$ is zero, whereas the shear stress is at maximum.

The horizontal shear stress is equal to the vertical shear stress, as shown in Fig. 1b. When the normal stress f is zero or low, a case of pure shear may occur. In this case, the maximum tensile stress f_t acts at 45∘ (Fig. 1c).

The tensile stresses are equivalent to the principal stresses, as shown in Fig. 5.4d. Such principal stresses are traditionally called diagonal tension stresses. When the diagonal tension stresses reach the tensile strength of concrete, a diagonal crack develops. This brief analysis explains the concept of diagonal tension and diagonal cracking. The actual behavior is more complex, and it is affected by other factors. For the combined action of shear and normal stresses at any point in a beam, the maximum and minimum diagonal tension (principal stresses) f_p are given by the equation

$$f_p = \frac{1}{2}f \mp \sqrt{\left(\frac{1}{2}f\right)^2 + v^2}$$

Where:
f = intensity of normal stress due to bending
v = shear stress

The shear failure in a concrete beam is most likely to occur where shear forces are at maximum, generally near the supports of the member. The first evidence of impending failure is the formation of diagonal cracks.

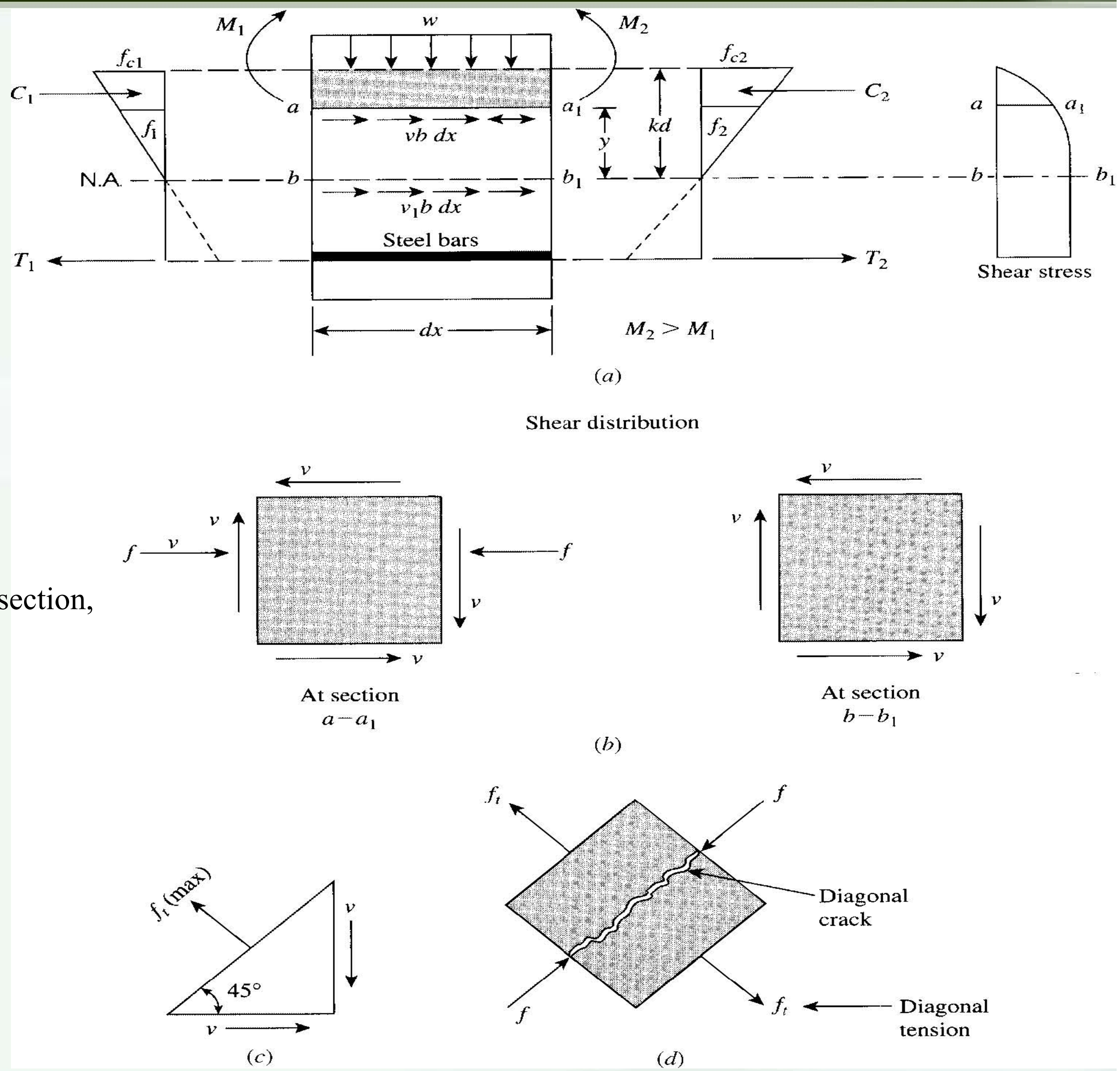


Fig. 1
(a) Forces and stresses along depth of section,
(b) Normal and shear stresses,
(c) Pure shear, and
(d) Diagonal tension.

3. BEHAVIOR OF BEAMS WITHOUT SHEAR REINFORCEMENT

Concrete is weak in tension, and the beam will collapse if proper reinforcement is not provided. The tensile stresses develop in beams due to axial tension, bending, shear, torsion, or a combination of these forces. The location of cracks in the concrete beam depends on the direction of principal stresses. For the combined action of normal stresses and shear stresses, maximum diagonal tension may occur at about a distance d from the face of the support.

The behavior of reinforced concrete beams with and without shear reinforcement tested under increasing load was discussed in chapter of analysis of beam under flexural. In the tested beams, vertical flexural cracks developed at the section of maximum bending moment when the tensile stresses in concrete exceeded the modulus of rupture of concrete, or $f_r = 7.5\,\lambda\sqrt{f_c'}$. Inclined cracks in the web developed at a later stage at a location very close to the support.

An inclined crack occurring in a beam that was previously uncracked is generally referred to as a web-shear crack. If the inclined crack starts at the top of an existing flexural crack and propagates into the beam, the crack is referred to as flexural-shear crack (Fig. 2). Web-shear cracks occur in beams with thin webs in regions with high shear and low moment. They are relatively uncommon cracks and may occur near the inflection points of continuous beams or adjacent to the supports of simple beams.

Flexural-shear cracks are the most common type found in reinforced concrete beams. A flexural crack extends vertically into the beam; then the inclined crack forms, starting from the top of the beam when shear stresses develop in that region. In regions of high shear stresses, beams must be reinforced by stirrups or by bent bars to produce ductile beams that do not rupture at a failure.

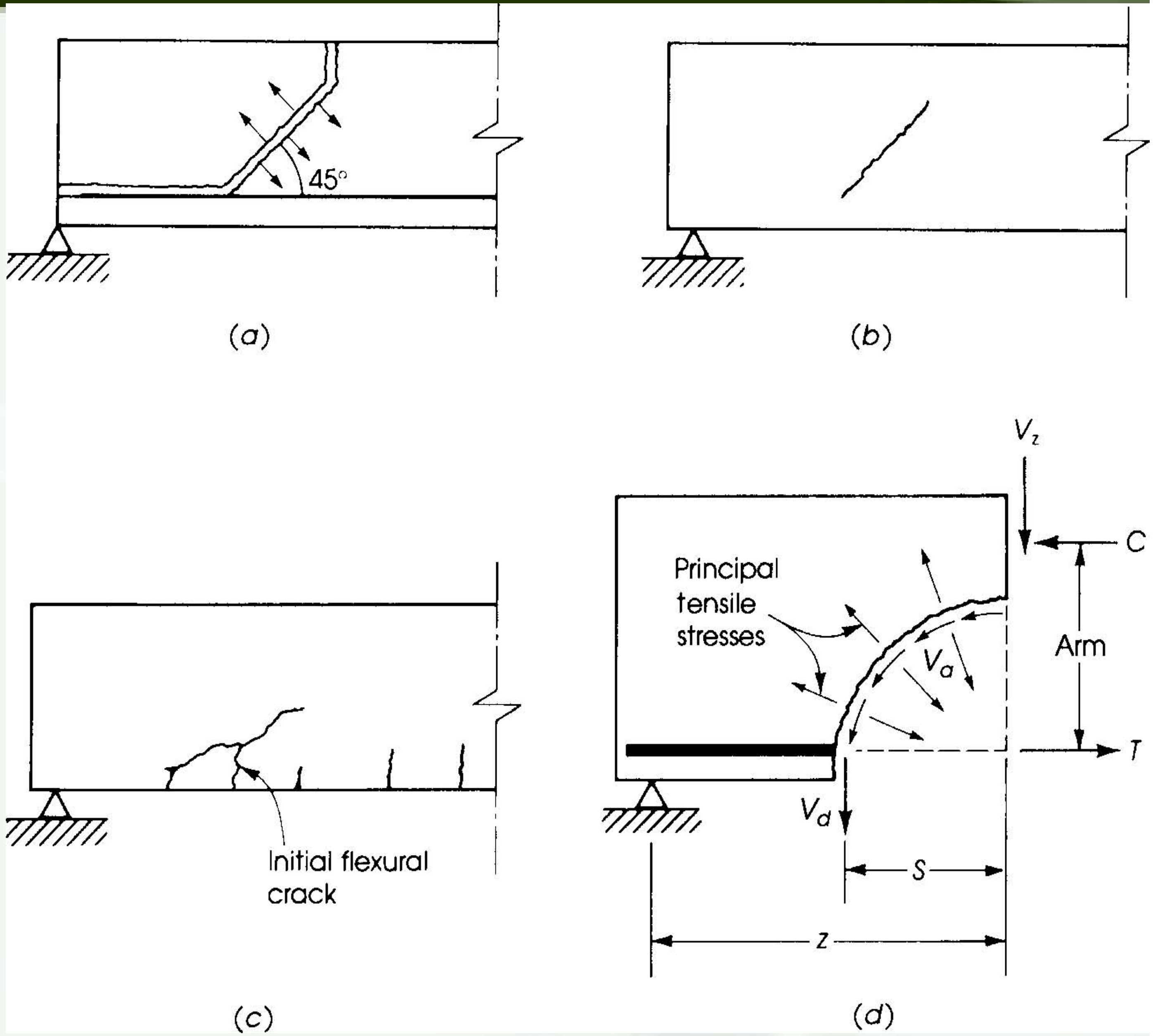


Fig. 2 Shear failure:
(a) General form,
(b) web-shear crack,
(c) Flexural-shear crack,
(d) Analysis of forces involved in shear
V_z = shear resistance,
V_a = interface shear,
V_d = dowel force.

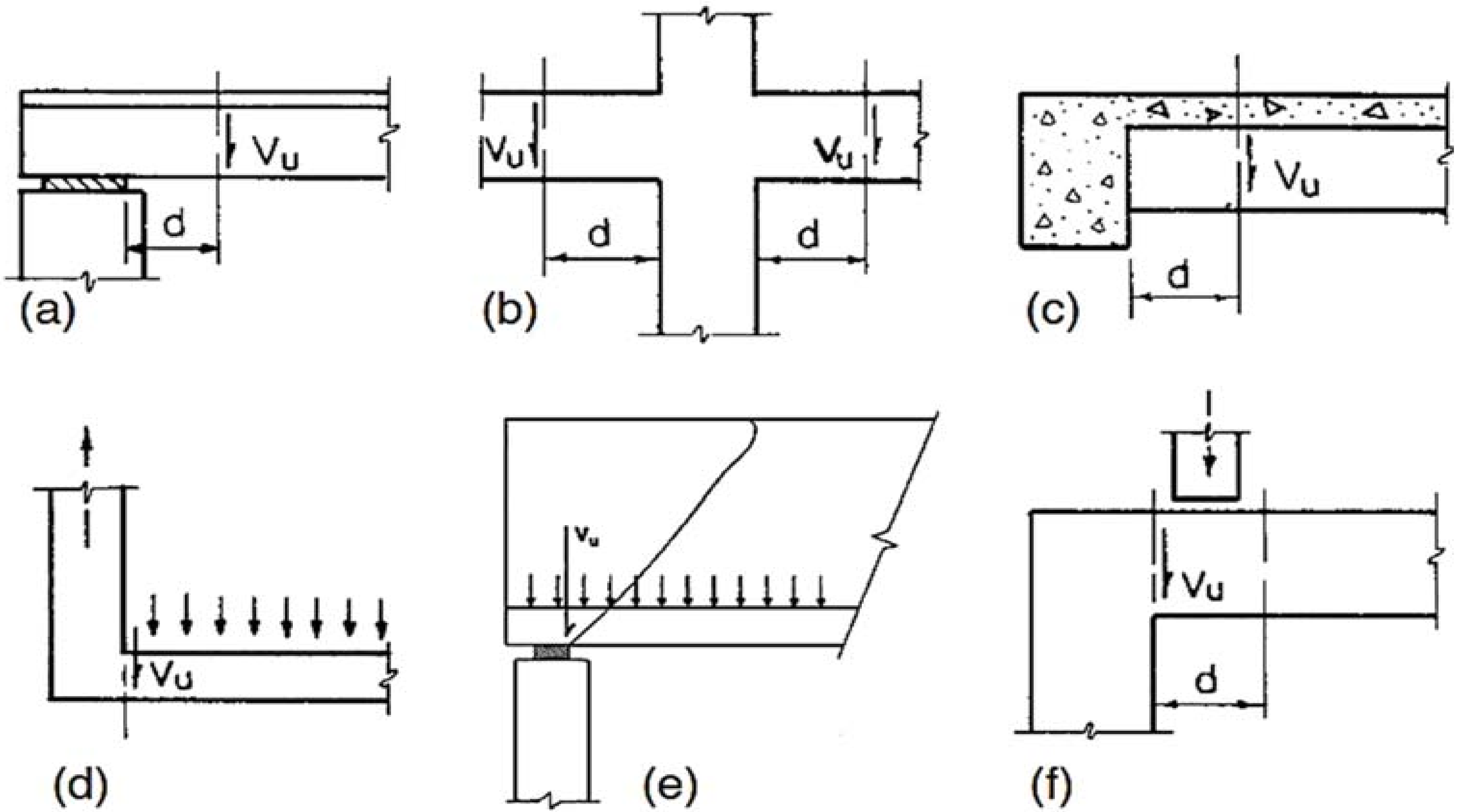


Figure 3 Typical Support Conditions for Locating Factored Shear Force V_u

4. MOMENT EFFECT ON SHEAR STRENGTH

In simply supported beams under uniformly distributed load, the midspan section is subjected to a large bending moment and zero or small shear, whereas sections near the ends are subjected to large shear and small bending moments . The shear and moment values are both high near the intermediate supports of a continuous beam. At a location of large shear force and small bending moment, there will be little flexural cracking, and an average stress v is equal to V/bd.

The diagonal tensile stresses are inclined at about 45∘ (Fig. 1c). Diagonal cracks can be expected when the diagonal tensile stress in the vicinity of the neutral axis reaches or exceeds the tensile strength of concrete. In general, the factored shear strength varies between $3.5\sqrt{f_c'}$ and $5\sqrt{f_c'}$. After completing a large number of beam tests on shear and diagonal tension, it was found that in regions with large shear and small moment, diagonal tension cracks were formed at an average shear force of:

$$V_c = 3.5\sqrt{f_c'}\,b_w d$$

where b_w is the width of the web in a T-section or the width of a rectangular section and d is the effective depth of the beam.

In locations where shear forces and bending moments are high, flexural cracks are formed first. At a later stage, some cracks bend in a diagonal direction when the diagonal tension stress at the upper end of such cracks exceeds the tensile strength of concrete. Given the presence of large moments on a beam, for which adequate reinforcement is provided, the nominal shear force at which diagonal tension cracks develop is given by:

$$V_c = 1.9\,\lambda\sqrt{f_c'}\,b_w d$$

This value is a little more than half the value in last Eq. when bending moment is very small. This means that large bending moments reduce the value of shear stress for which cracking occurs. The following equation has been suggested to predict the nominal shear stress at which a diagonal crack is expected:

$$v_c = \frac{V}{b_w d} = (1.9\,\lambda\sqrt{f_c'} + 2500\,\rho\frac{Vd}{M}) \le 3.5\lambda\sqrt{f_c'}$$

5. BEAMS WITH SHEAR REINFORCEMENT

Different types of shear reinforcement may be used:

1. Stirrups, which can be placed either perpendicular to the longitudinal reinforcement or inclined, usually making a 45∘ angle and welded to the main longitudinal reinforcement. Vertical stirrups, using no. 3 (10 mm) or no. 4 (12 mm) U-shaped bars, are the most commonly used shear reinforcement in beams (Fig. 4a).
2. Bent bars, which are part of the longitudinal reinforcement, bent up (where they are no longer needed) at an angle of 30∘ to 60∘, usually at 45∘.
3. Combinations of stirrups and bent bars.
4. Welded wire fabric with wires perpendicular to the axis of the member.
5. Spirals, circular ties, or hoops in circular sections, as columns.

The shear strength of a reinforced concrete beam is increased by the use of shear reinforcement. Prior to the formation of diagonal tension cracks, shear reinforcement contributes very little to the shear resistance. After diagonal cracks have developed, shear reinforcement augments the shear resistance of a beam, and a redistribution of internal forces occurs at the cracked section. When the amount of shear reinforcement provided is small, failure due to yielding of web steel may be expected, but if the amount of shear reinforcement is too high, a shear–compression failure may be expected, which should be avoided.

Concrete, stirrups, and bent bars act together to resist transverse shear. The concrete, by virtue of its high compressive strength, acts as the diagonal compression member of a lattice girder system, where the stirrups act as vertical tension members. The diagonal compression force is such that its vertical component is equal to the tension force in the stirrup. Bent-up reinforcement acts also as tension members in a truss, as shown in Fig. 4.

In general, the contribution of shear reinforcement to the shear strength of a reinforced concrete beam can be described as follows:

1. It resists part of the shear, V_s.
2. It increases the magnitude of the interface shear, V_a , by resisting the growth of the inclined crack.
3. It increases the dowel force, V_d (Fig. 2), in the longitudinal bars.
4. The confining action of the stirrups on the compression concrete may increase its strength.
5. The confining action of stirrups on the concrete increases the rotation capacity of plastic hinges that develop in indeterminate structures at maximum load and increases the length over which yielding takes place.

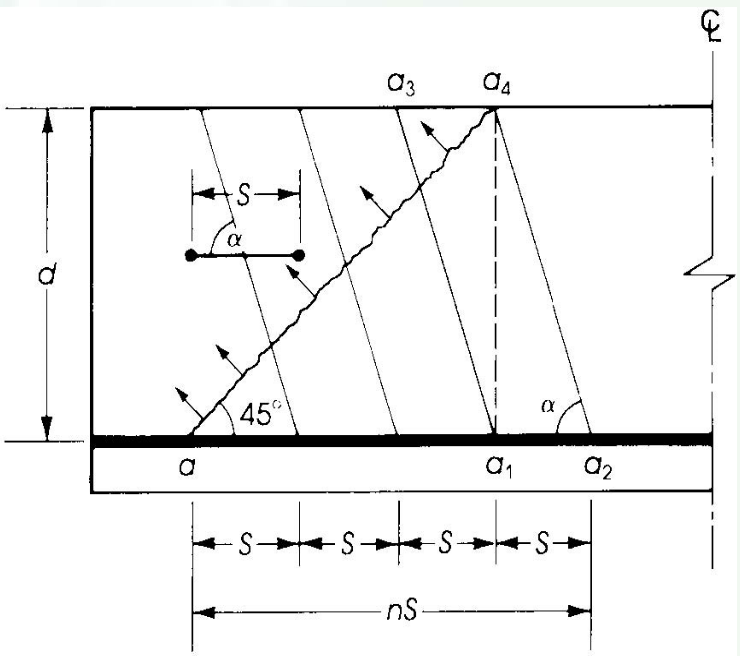
The total nominal shear strength of beams with shear reinforcement V_n is due partly to the shear strength attributed to the concrete, V_c, and partly to the shear strength contributed by the shear reinforcement, V_s:

$$V_n = V_c + V_s$$

The shear force Vu produced by factored loads must be less than or equal to the total nominal shear strength V_n, or

$$V_u \le \phi V_n = \phi\,(V_c + V_s)$$

where $V_u = 1.2\ V_D + 1.6\ V_L$ and $\phi = 0.75$.



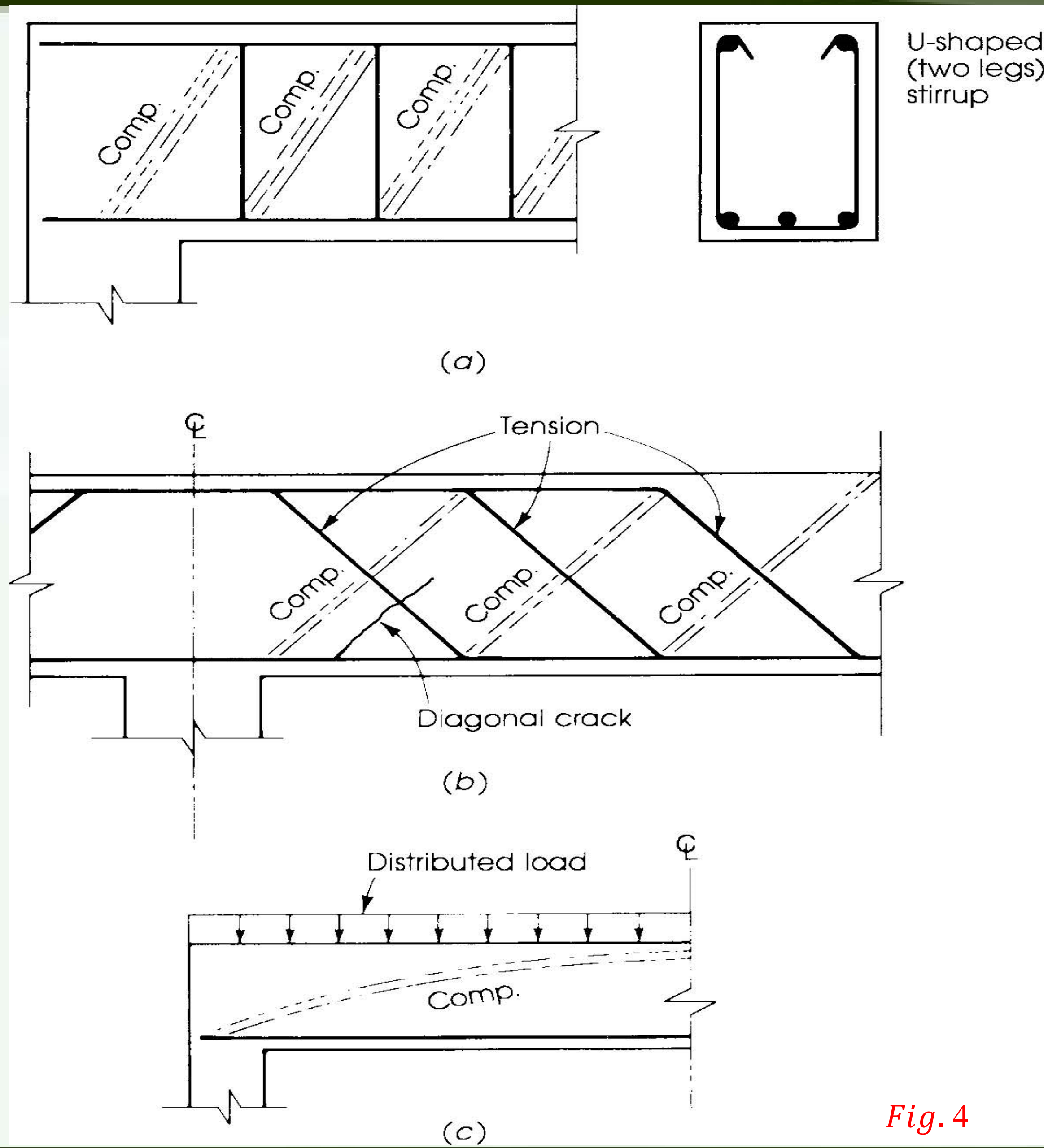


Fig. 4

An expression for V_s may be developed from the truss analogy (Fig. 4). For a 45∘ crack and a series of inclined stirrups or bent bars, the vertical shear force V_s resisted by shear reinforcement is equal to the sum of the vertical components of the tensile forces developed in the inclined bars.

Therefore,

$$V_s = n\, A_v\, f_{yt} \sin\alpha \qquad Eq.2$$

where A_v is the area of shear reinforcement with a spacing s and f_{yt} is the yield strength of shear reinforcement; ns is defined as the distance aa_1a_2:

$$d = \begin{cases} a_1a_4 = aa_1 \tan 45\circ \ (from\ triangle\ a\ a_1a_4) \\ a_1a_4 = a_1a_2 \tan\alpha \ (from\ triangle\ a_1a_2a_4) \end{cases}$$

$$n * s = aa_1 + a_1a_2$$

$$= d(\cot 45^\circ + \cot\alpha) = d(1 + \cot\alpha)$$

$$n = \frac{d}{S}(1 + \cot\alpha)$$

Substituting this value in Eq.2 gives

$$V_s = \frac{A_v\, f_{yt} d}{S} \sin\alpha(1 + \cot\alpha) = \frac{A_v\, f_{yt} d}{S}(\sin\alpha + \cos\alpha)$$

For the case of vertical stirrups, α =90∘ and

$$\boxed{V_s = \frac{A_v f_{yt} d}{S} \quad or \quad S = \frac{A_v f_{yt} d}{V_s} \qquad Eq.3}$$

In the case of T-sections, b is replaced by the width of web bw in all shear equations. When α =45∘, Eq.3 becomes

$$V_s = 1.4\left(\frac{A_v f_{yt} d}{S}\right) or \ S = 1.4\left(\frac{A_v f_{yt} d}{V_s}\right)$$

For a single bent bar or group of parallel bars in one position, the shearing force resisted by steel is

$$V_s = A_v \ f_{yt} \sin\alpha \qquad or \ Av = \frac{V_s}{f_{yt} \sin\alpha}$$

For α =45°,

$$Av = 1.4\left(\frac{V_s}{f_{yt}}\right)$$

For circular sections, mainly in columns, V_s will be computed from Eq.3 using (d = 0.8 × diameter), and (A_v =two times the area of the bar in a circular tie, hoop, or spiral).

6. ACI CODE SHEAR DESIGN REQUIREMENTS

6.1 Critical Section for Nominal Shear Strength Calculation

The ACI Code, Section 9.4.3.2, permits taking the critical section for nominal shear strength calculation at a distance d from the face of the support. This recommendation is based on the fact that the first inclined crack is likely to form within the shear span of the beam at some distance d away from the support. This critical section is permitted on the condition that the support reaction introduces compression into the end region, loads are applied at or near the top of the member, and no concentrated load occurs between the face of the support and the location of the critical section.

The Code also specifies that shear reinforcement must be provided between the face of the support and the distance d using the same reinforcement adopted for the critical section.

6.2 Minimum Area of Shear Reinforcement

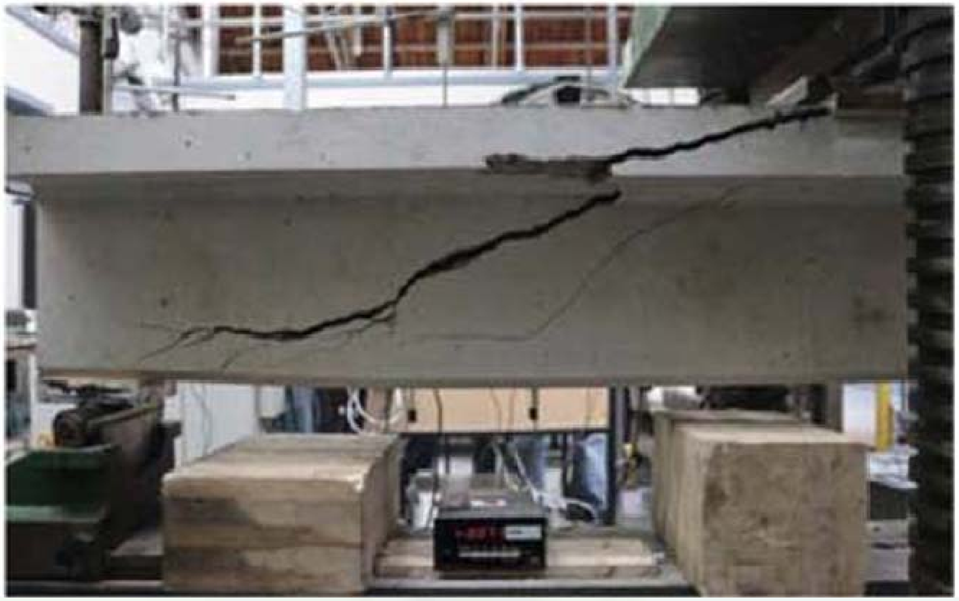
The presence of shear reinforcement in a concrete beam restrains the growth of inclined cracking. Moreover, ductility is increased, and a warning of failure is provided. If shear reinforcement is not provided, brittle failure will occur without warning. Accordingly, a minimum area of shear reinforcement is specified by the Code. The ACI Code, Section 9.6.3.3, requires all stirrups to have a minimum shear reinforcement area, Av, equal to:

$$A_{v,min} = greater\ of \begin{Bmatrix} 0.062\sqrt{f_c'}\,\dfrac{b_w\, s}{f_{yt}} \\ 0.35\,\dfrac{b_w\, s}{f_{yt}} \end{Bmatrix}$$

where b_w is the width of the web and S is the spacing of the stirrups. The minimum amount of shear reinforcement is required whenever V_u exceeds $\phi V_c / 2$, except in:

1. Slabs and footings.
2. Concrete floor joist construction.
3. Beams where the total depth (h) does not exceed 10 in.(250 mm), 2.5 times the flange thickness for T-shaped flanged sections, or one-half the web width, whichever is greatest.
4. The beam is integrated with slab, h not greater 24 in.(600 mm) and not greater than the larger of 2.5 times the thickness of the flange and 0.5 times the width of the web.

Shear Failure



6.3 Maximum Shear Carried by Web Reinforcement Vs

To prevent a shear–compression failure, where the concrete may crush due to high shear and compressive stresses in the critical region on top of a diagonal crack, the ACI Code, Section 22.5.1.2, requires that V_s shall not exceed

$(0.66\sqrt{f_c'})\, b_w d$. If V_s exceeds this value, the section should be increased.

6.4 Maximum Spacing of Stirrups

To ensure that a diagonal crack will always be intersected by at least one stirrup. Maximum spacing of legs of shear reinforcement along the length of the member and across the width of the member shall be in accordance with the ACI Code, Table 9.7.6.2.2.

Table 9.7.6.2.2—Maximum spacing of legs of shear reinforcement

Required V_s		Maximum s, mm			
		Nonprestressed beam		Prestressed beam	
		Along length	Across width	Along length	Across width
$\le 0.33\sqrt{f_c'}b_w d$	Lesser of:	$d/2$	d	$3h/4$	$3h/2$
		600			
$> 0.33\sqrt{f_c'}b_w d$	Lesser of:	$d/4$	$d/2$	$3h/8$	$3h/4$
		300			

This is based on the assumption that a diagonal crack develops at 45° and extends a horizontal distance of about d. In regions of high shear, where Vs exceeds $(0.33\sqrt{f_c'})b_w d$, the maximum spacing between stirrups must not exceed d/4. This limitation is necessary to ensure that the diagonal crack will be intersected by at least three stirrups. When V_s exceeds the maximum value of $(0.66\sqrt{f_c'})b_w d$, this limitation of maximum stirrup spacing does not apply, and the dimensions of the concrete cross section should be increased.

A second limitation for the maximum spacing of stirrups may also be obtained from the condition of minimum area of shear reinforcement. A minimum A_v is obtained when the spacing s is maximum.

A third limitation for maximum spacing is 600 mm. when $V_s \leq (0.33\sqrt{f_c'})\ b_w d$ and 300mm. when V_s is greater than $(0.33\sqrt{f_c'})b_w d$ but less than or equal to $(0.66\sqrt{f_c'})b_w d$. The least value of all maximum spacing must be adopted. The ACI Code maximum spacing requirement ensures closely spaced stirrups that hold the longitudinal tension steel in place within the beam, thereby increasing their dowel capacity, V_d (Fig. 5.5).

6.5 DESIGN OF VERTICAL STIRRUPS

Stirrups are needed when $Vu \geq \phi Vc$. Minimum stirrups are used when Vu is greater than 0.5 ϕVc but less than ϕV. This is achieved by using no.3 (10 mm) stirrups placed at maximum spacing. When Vu is greater than ϕV, stirrups must be provided. The spacing of stirrups may be less than the maximum spacing and can be calculated using

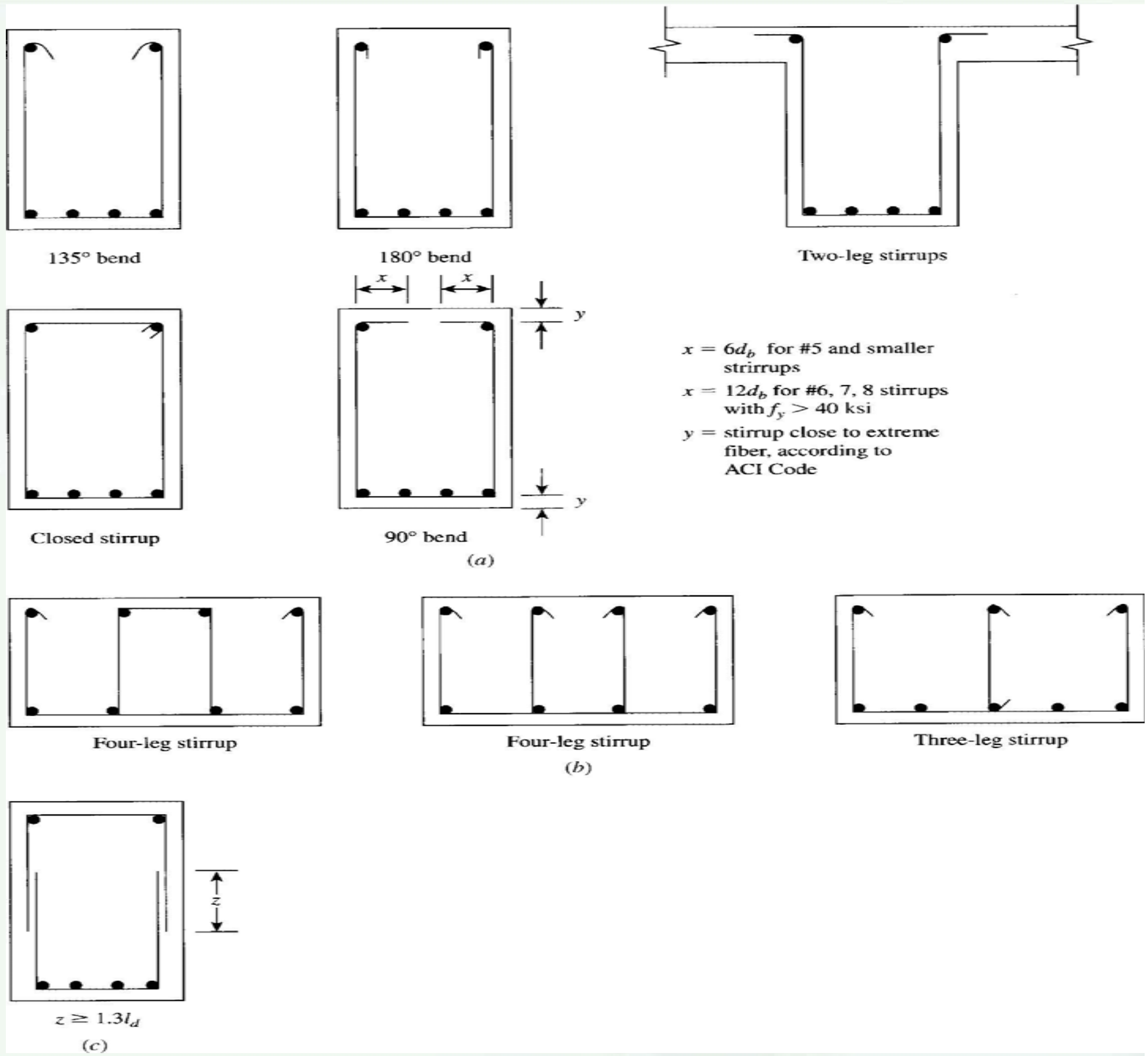
$$S = \frac{A_v\, f_{yt} d}{V_s}$$

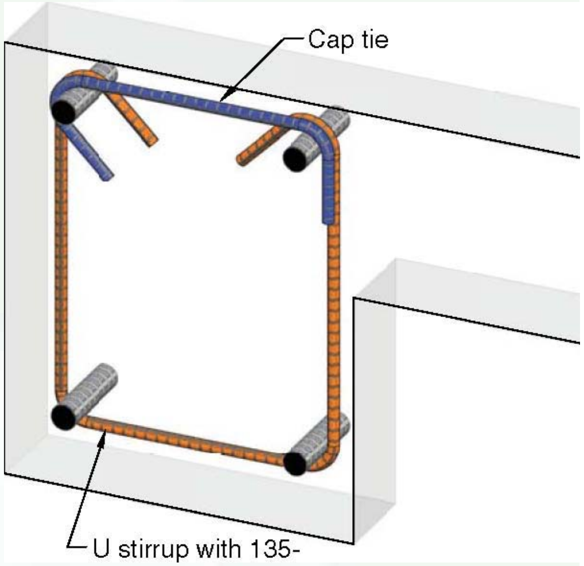
Figure Stirrup types:

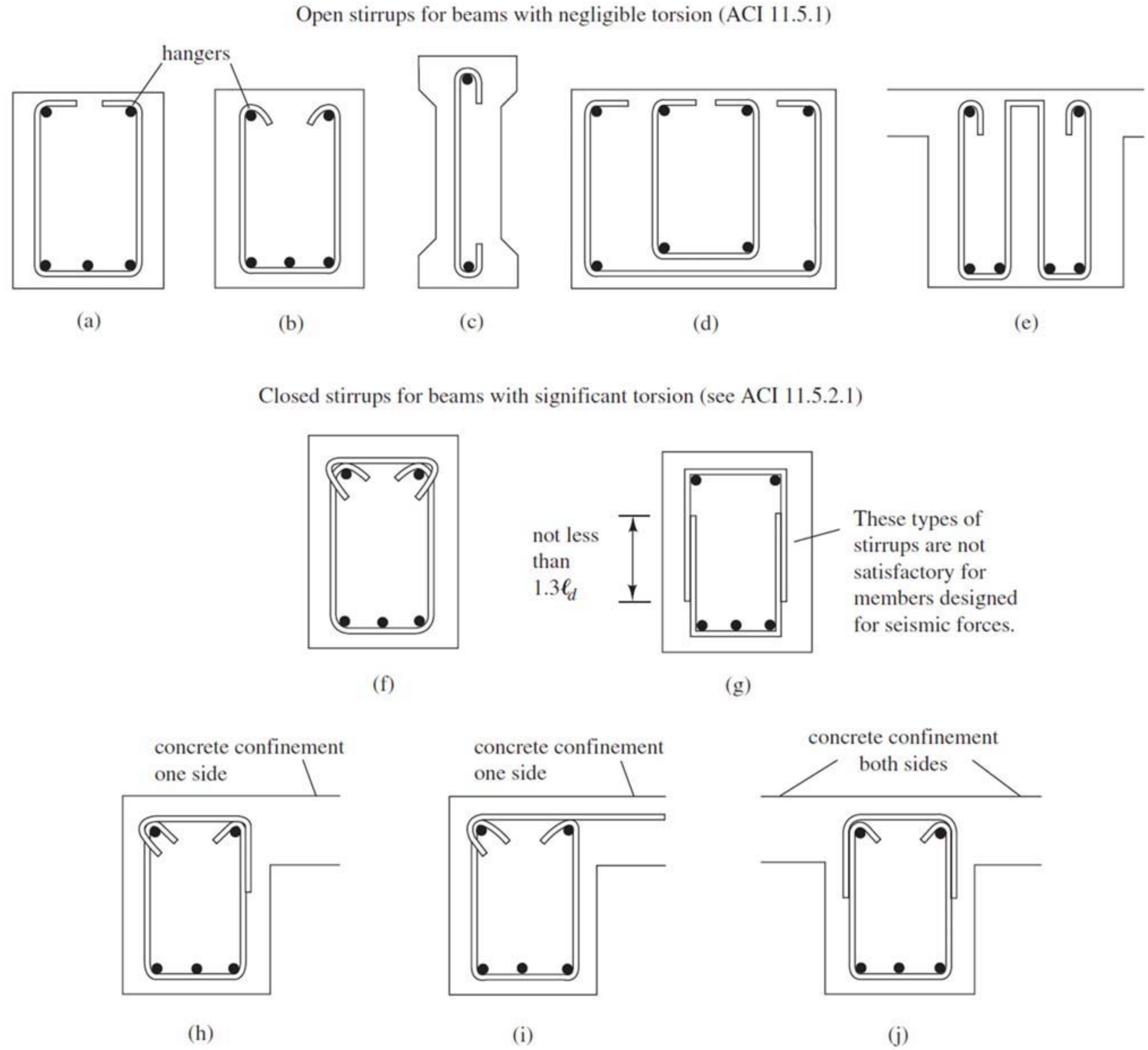
(a)U-stirrups enclosing longitudinal bars, anchorage lengths, and closed stirrups;

(b) Multi leg stirrups; and

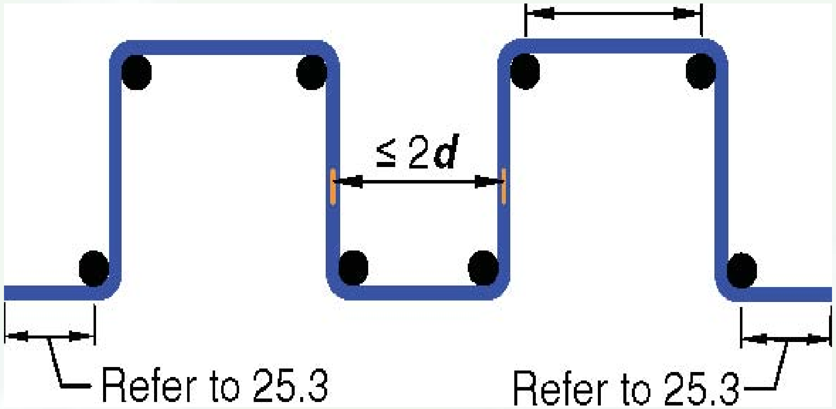
(c) Spliced stirrups.

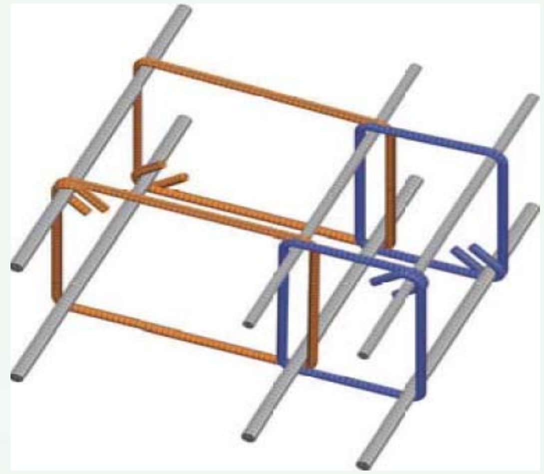


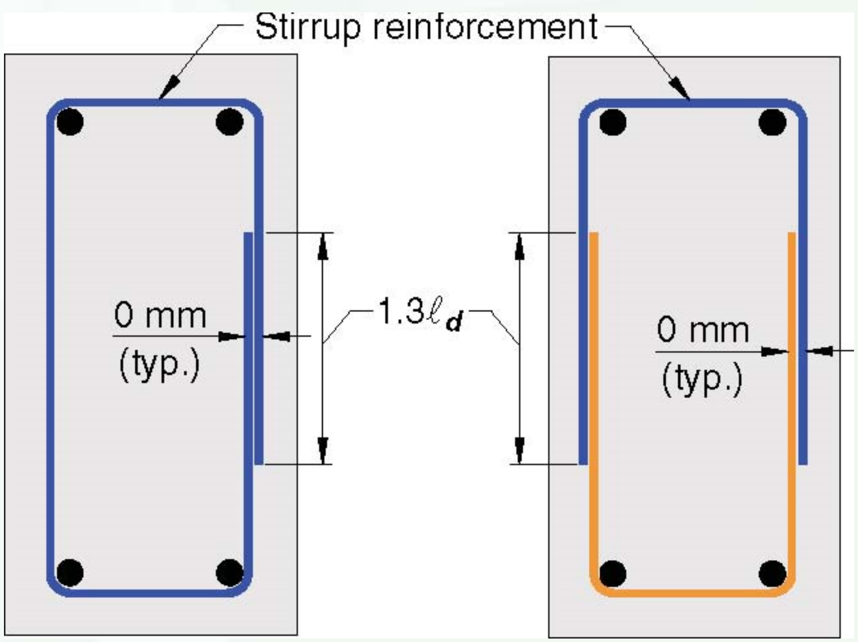




Types of stirrups.


≤ 2d
Refer to 25.3
Refer to 25.3




Stirrup reinforcement
0 mm
(typ.)
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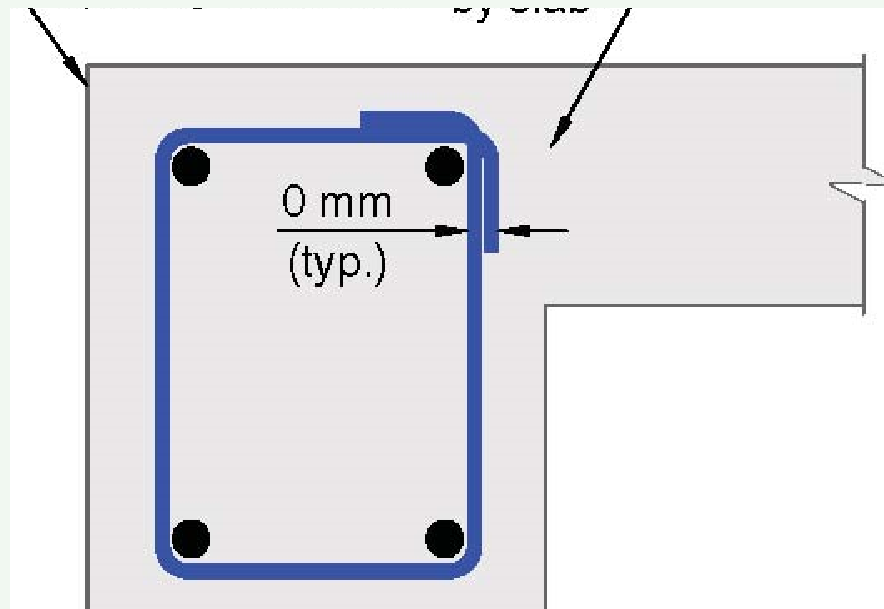
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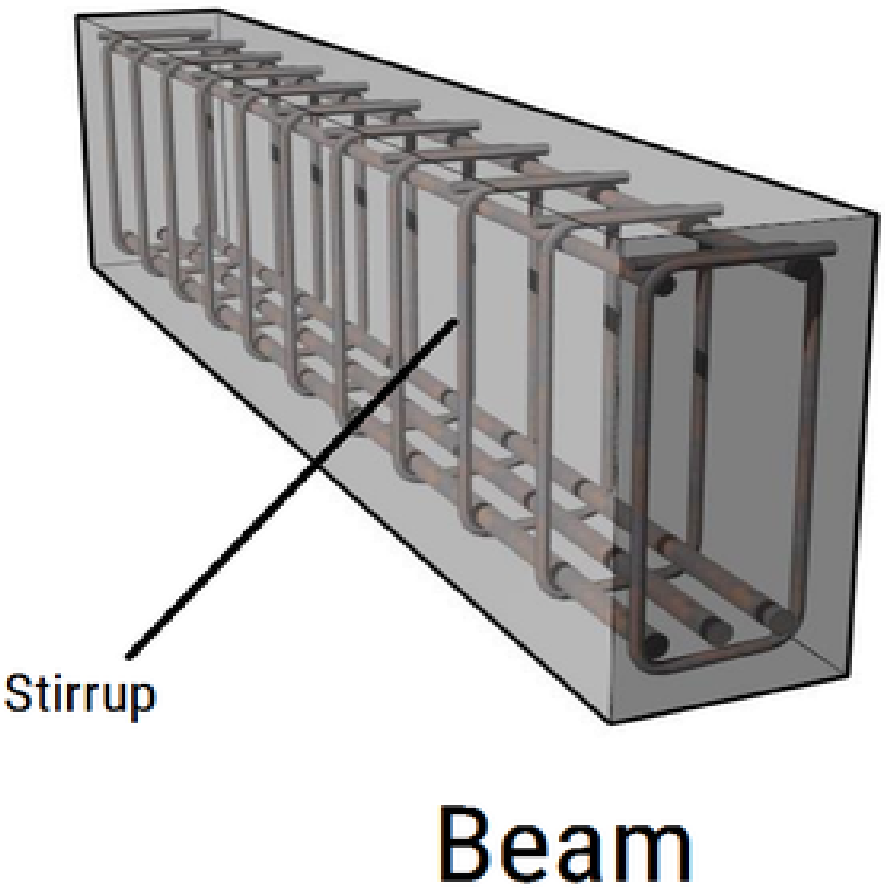
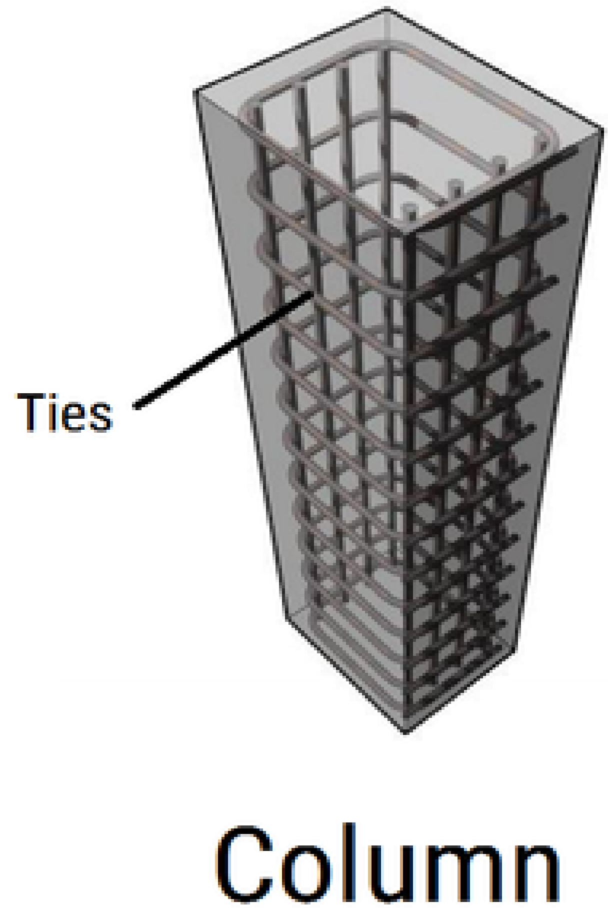
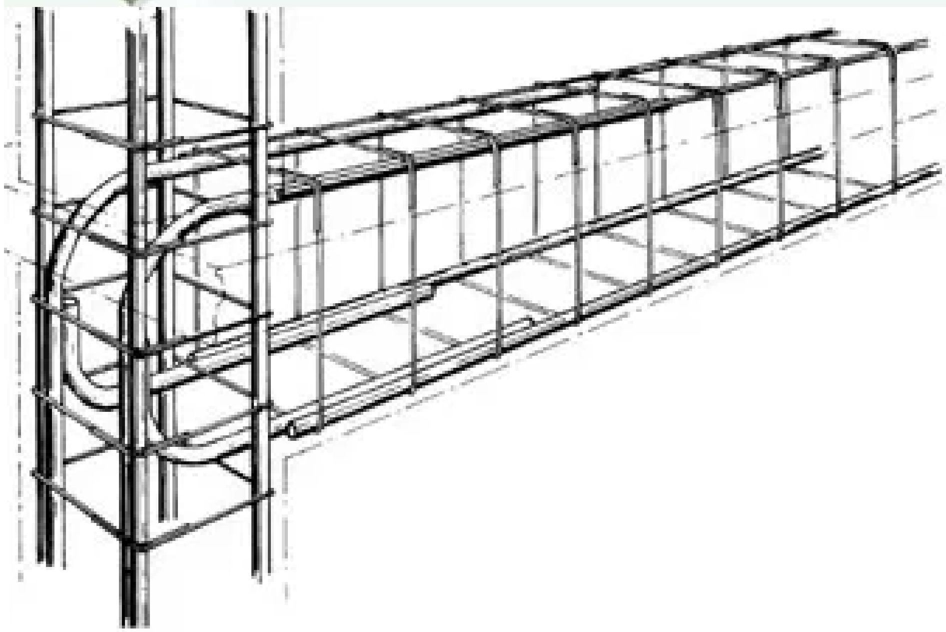
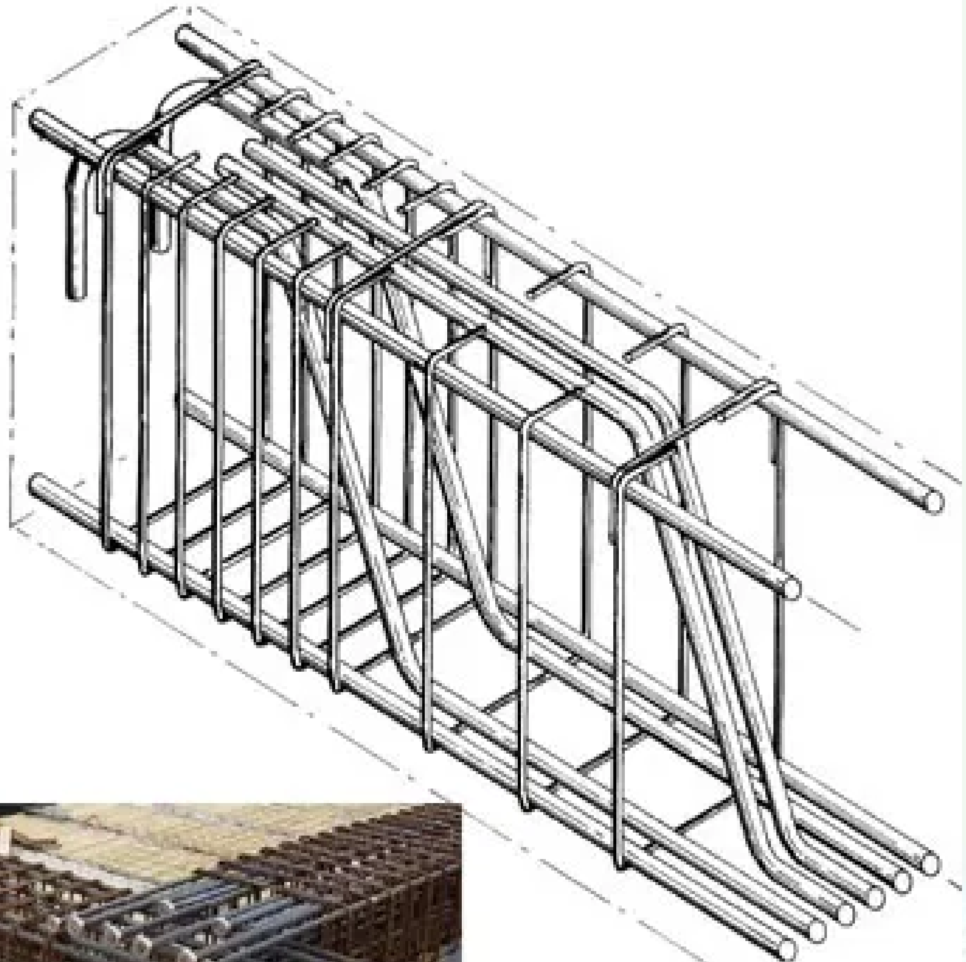
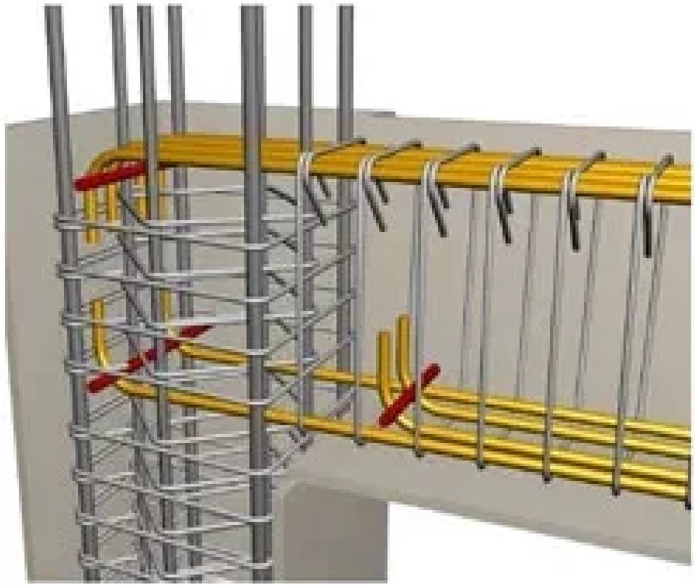

Stirrup
Beam


Ties
Column







7. DESIGN PROCEDURE ACCORDING ACI-2019

The design procedure for shear using vertical stirrups according to the ACI Code can be summarized as follows:

1. Calculate the factored shearing force, V_u, from the applied factored forces acting on the structural member. The critical design shear value is at a section located at a distance d from the face of the support.

Let $V_n = \dfrac{V_u}{\phi}$

2. Calculate V_c by:

$$for\ A_v \geq A_{v,min} \quad V_c = either\ of \begin{cases} 0.17\,\lambda\sqrt{f_c'}\,b_w\,d & Eq.a \\ 0.66\,\lambda\,(\rho_w)^{1/3}\sqrt{f_c'}\ b_w\,d & Eq.b \end{cases}$$

$$for\ A_v < A_{v,min} \quad V_c = 0.66\,\lambda_s\,\lambda\,(\rho_w)^{1/3}\sqrt{f_c'}\ b_w\,d \qquad Eq.c$$

And shall consider the following :

$$V_c \leq 0.42\,\lambda\sqrt{f_c'}\,b_w\,d.$$

$$\lambda s = \sqrt{\frac{2}{1 + 0.004\,d}} \leq 1$$

3. Calculate $0.083\,\lambda\sqrt{f_c'}\;b_w\,d = 0.5\,V_c\;\ldots\ldots\ldots, Eq.a$

4. $\boldsymbol{a}.\ If\ V_n < 0.5\,V_{c,Eq.a},\quad no\ shear\ reinforcement\ is\ needed.$

$\boldsymbol{b}.\ If\ 0.5\,V_{c,Eq.a} < V_n \le V_c\quad minimum\ shear\ reinforcement\ is\ required.$

Can use no.3 (dia.10 mm) U-stirrups spaced at maximum spacing, as explained in step 8.

$\boldsymbol{c}.\ If\ V_n > V_c,\ shear\ reinforcement\ must\ be\ provided\ according\ to\ steps\ 5\ through\ 8.$

5. $If\ V_n > V_c$, calculate the shear to be carried by shear reinforcement:

$$V_n = V_c + V_S \quad or \quad V_S = V_n - V_c$$

6. Calculate:

$V_{C1} = 0.33\sqrt{f_c'}\, b_w\, d$ and $V_{C2} = 0.66\sqrt{f_c'}\, b_w\, d = 2\, V_{C1}$ $then$:

$If\ \ V_S > V_{C2}$ increase the dimensions of the section.

$If\ \ V_S < V_{C2}$ proceed in the design

7. Calculate the stirrups spacing based on

$$S_1 = \frac{A_v\, f_{yt}\, d}{V_s}$$

8. Determine the maximum spacing allowed by the ACI Code. The maximum spacing is the least of S_1, S_2 and S_3: where

$S_2 = \frac{d}{2} \leq 600\ mm$, if $V_S \leq V_{C1}$ or $S_2 = \frac{d}{4} \leq 300\ mm$, if $V_S > V_{C1}$

$$S_3 = smaller\ of\ \begin{Bmatrix} \dfrac{A_v\, f_{yt}}{0.062\sqrt{f_c'}\, b_w} \\ \dfrac{A_v\, f_{yt}}{0.35\, b_w} \end{Bmatrix}$$

$then,$ $S_{max} = Min\ (S_1, S_2\ and\ S_3)\ (Practical\ value)$.

9. The ACI Code did not specify a minimum spacing. Under normal conditions, a practical minimum S may be assumed to be equal to 75 mm. for d ≤ 500 mm. and 100 mm. for deeper beams. If S is considered small, either increase the stirrup bar number or use multiple-leg stirrups.

10. For circular sections, the area used to compute V_c is the diameter times the effective depth d, where d=0.8 times the diameter, ACI Code, Section 22.5.2.2.

Where:

V_c	Shear resistance of the concrete
λ_s	Factor for considering the component height
λ	Factor for normal or lightweight concrete
ρ_w	Longitudinal reinforcement ratio of the tension reinforcement
f_c'	Concrete compressive strength
N_u	Design axial force
A_g	cross-sectional area
b_w	Width of the cross-section
d	Effective depth

Reinforced Concrete Design

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LOGO

Example (1): A simply supported beam has a rectangular section with b=300mm., d=540 mm, and h=600 mm. and is reinforced with 4 φ 25 mm bars. Check if the section is adequate for each of the following factored shear forces. If it is not adequate, design the necessary shear reinforcement in the form of U-stirrups. Use $f_c' = 28$ MPa and f_{yt} =420 MPa. Assume normal-weight concrete. When :

(a) Vu = 52 kN, (b) Vu = 104 kN, (c) Vu = 243 kN, (d) Vu = 337 kN, (e) Vu = 560 kN

Solution

$Calculate\ V_c$:

$$V_c = 0.17\,\lambda\sqrt{f_c'}\;\; b_w\, d = 0.17 \times 1 \times \sqrt{28} \times 300 \times 540 = 145728\ N \approx 146\ kN$$

$Calculate\ \ 0.5\ V_c$

$$0.5\ V_c = \frac{146}{2} = 73\ KN$$

$V_{C1} = 0.33\sqrt{f_c'}\, b_w\, d = 0.33 \times \sqrt{28} \times 300 \times 450 = 236\ kN$ and

$$V_{C2} = 0.66\sqrt{f_c'}\, b_w\, d = 2\ V_{C1} = 472\ kN$$

(a) $V_u = \ 52\ kN$

$$V_n = \frac{V_u}{\emptyset} = \frac{52}{0.75} = 69.33\ kN$$

$\therefore\ V_n\ (69.3\ kN) < \ 0.5\ V_c\ (73\ kN)$

$\therefore\ no\ shear\ reinforcement\ is\ needed.$

(b) $V_u = 104\ kN$

$$V_n = \frac{V_u}{\emptyset} = \frac{104}{0.75} = 139\ kN$$

$\therefore\ 0.5\,V_c\ (73\ kN) < V_n\ (139\ kN) < V_c\ (146\ kN)$

$\therefore$ *minimum shear reinforcement is required.*

$$\therefore S_2 = \frac{d}{2} \leq 600\ mm$$

$$\therefore S_2 = \frac{540}{2} = 270\ mm \leq 600\ mm \quad \therefore S_2 = 270\ mm$$

Use ϕ 10 mm therefore $A_v = 2\ leg = 2 \times (10^2 \times \frac{\pi}{4}) = 157\ mm^2$

$$S_3 = smaller\ of \begin{Bmatrix} \dfrac{A_v\, f_{yt}}{0.062\sqrt{f_c'}\ b_w} \\ \dfrac{A_v\, f_{yt}}{0.35\ b_w} \end{Bmatrix} = min \begin{Bmatrix} \dfrac{157 \times 420}{0.062\sqrt{28} \times 300} \\ \dfrac{157 \times 420}{0.35 \times 300} \end{Bmatrix}$$

$$S_3 = min \begin{Bmatrix} 670\ mm \\ 628\ mm \end{Bmatrix} \quad \therefore S_3 = 628\ mm$$

$\therefore S_{max} = \min(S_2\ and\ S_3) = 270\ mm$

$\therefore$ Use ϕ 10 mm @ 260 mm c/c , U-stirrups

$V_S = \boldsymbol{Vn} - \boldsymbol{Vc}$	$\boldsymbol{Vc} = 0.17\,\lambda\sqrt{f_c'}\;\; b_w\, d$
	$S_1 = \frac{A_v\, f_{yt} d}{V_s}$ (calculated)
$V_S < V_{C\mathbf{1}}$	$\therefore S_2 = \frac{d}{2}, \qquad or\ S_2 = 600\ mm$
$V_{C\mathbf{1}} < V_S < V_{C\mathbf{2}}$	$S_2 = \frac{d}{4}, \qquad or\ S_2 = 300\ mm$
$V_S > V_{C\mathbf{2}}$	Change Section Dimension
	$S_3 = smaller\ of \quad \left\{ \begin{matrix} \frac{A_v\, f_{yt}}{0.062\sqrt{f_c'}\, b_w} \\ \frac{A_v\, f_{yt}}{0.35\, b_w} \end{matrix} \right\}$ (calculated)

(c) $V_u = 243\ kN$

$$V_n = \frac{V_u}{\emptyset} = \frac{243}{0.75} = 324\ kN$$

$\therefore\ V_c\ (146\ kN) < V_n\ (324\ kN)$

$\therefore\ shear\ reinforcement\ must\ be\ provided\ and\ calculate\ V_s$

$V_S = V_n - V_c$

$V_S = 324 - 146 = 178\ kN$

$V_S\ (178\ kN) < Vc_1(236kN) < V_{C2}(472\ kN) \qquad \therefore\ the\ dimensions\ of\ the\ sec.is\ OK$

Calculate the stirrups spacing, Use ϕ 10 mm therefore $A_v = 157\ mm^2$

$$S_1 = \frac{A_v\ f_{yt} d}{V_s} = \frac{157 \times\ 420 \times 540}{178 \times 10^3} = 200\ mm$$

$For\ V_S\ (178\ kN) <\ V_{C1}(236\ kN)$

$$\therefore S_{\ 2} = \frac{d}{2} \leq 600\ mm$$

$$\therefore S_{\ 2}\ \frac{540}{2} = 270\ mm \leq 600\ mm \quad \therefore S_{\ 2} = 270\ mm \quad and$$

$$S_3 = smaller\ of \begin{Bmatrix} \dfrac{A_v\ f_{yt}}{0.062\sqrt{f_c'}\ b_w} \\ \dfrac{A_v\ f_{yt}}{0.35\ b_w} \end{Bmatrix} = min \begin{Bmatrix} \dfrac{157 \times\ 420}{0.062\sqrt{28}\ \times 300} \\ \dfrac{157 \times\ 420}{0.35\ \times 300} \end{Bmatrix}$$

$$S_3 = min \begin{Bmatrix} 670\ mm \\ 628\ mm \end{Bmatrix} \qquad \therefore S_3 = 628\ mm$$

$$S_{max} = \min(S_1\ , S_2\ and\ S_3) = 200\ mm$$

∴ Use ϕ 10 mm @ 200 mm c/c , U-stirrups

(d) $V_u = 337\ kN$

$$V_n = \frac{V_u}{\emptyset} = \frac{337}{0.75} = 449\ kN$$

$\therefore\ V_c\ (146\ kN) < V_n\ (449\ kN)$

$\therefore\ shear\ reinforcement\ must\ be\ provided\ and\ calculate\ V_s$

$V_S = V_n - V_c$

$V_S = 449 - 146 = 303\ kN$

$V_S\ (303\ kN) < V_{C2}(472\ kN) \qquad \therefore\ the\ dimensions\ of\ the\ sec.is\ OK$

Calculate the stirrups spacing, Use ϕ 10 mm therefore $A_v = 157\ mm^2$

$$S_1 = \frac{A_v\ f_{yt} d}{V_s} = \frac{157 \times\ 420 \times 540}{303 \times 10^3} = 117\ mm$$

$For\ V_S\ (303\ \ kN) >\ V_{C1}(236\ kN)$

$$\therefore S_{\ 2} = \frac{d}{4} \leq 300\ mm$$

$$\therefore S_{\ 2} = \frac{540}{4} = 135\ mm \leq 300\ mm \quad \therefore S_{\ 2} = \ 135\ mm$$

$$and \qquad S_3 = smaller\ of \begin{Bmatrix} \dfrac{A_v\ f_{yt}}{0.062\sqrt{f_c'}\ b_w} \\ \dfrac{A_v\ f_{yt}}{0.35\ b_w} \end{Bmatrix} = min \begin{Bmatrix} \dfrac{157 \times\ 420}{0.062\sqrt{28}\ \times 300} \\ \dfrac{157 \times\ 420}{0.35\ \times 300} \end{Bmatrix}$$

$$S_3 = \ min \begin{Bmatrix} 670\ mm \\ 628\ mm \end{Bmatrix} \qquad \therefore S_3 = \ 628\ mm$$

$S_{max} = \min(S_1\ , S_2\ and\ S_3) = 117\ mm$

∴ Use ϕ 10 mm @ 110 mm c/c , U-stirrups

(e) $V_u = \ 560\ kN$

$$V_n \ = \frac{V_u}{\emptyset} = \frac{560}{0.75} = \ 747\ kN$$

$\therefore \ \ V_c\ (146\ kN) < V_n\ (747\ kN)$

$\therefore\ \ shear\ reinforcement\ must\ be\ provided\ and\ calculate\ V_s$

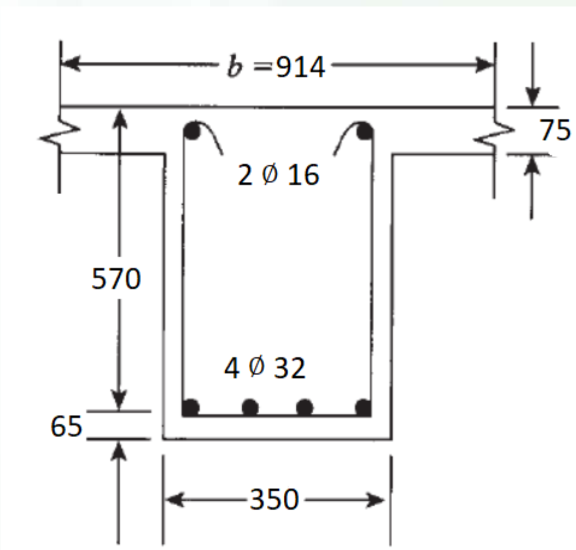
$V_S \ = V_n - V_c$

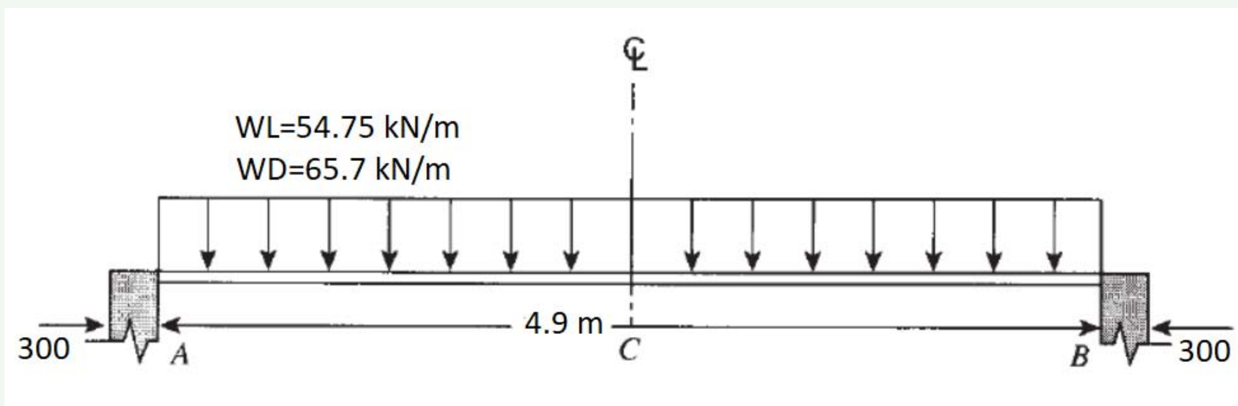
$V_S \ = 747 - 146 = 601\ kN$

$V_S\ (601\ kN) > \ V_{C2}(472\ kN) \qquad \therefore Not\ OK\ and\ change\ the\ dimensions\ of\ the\ section.$

Example (2)

A 5.2 m, span simply supported beam has a clear span of 4.9 m and carries uniformly distributed dead and live loads of 65.7 kN/m and 54.75 kN/m, respectively. The dimensions of the beam section and steel reinforcement are shown in Fig. below. Check the section for shear and design the necessary shear reinforcement. Given $f_c' = 21\ MPa$ normal-weight concrete and $f_{yt} = 420\ MPa$.





Solution

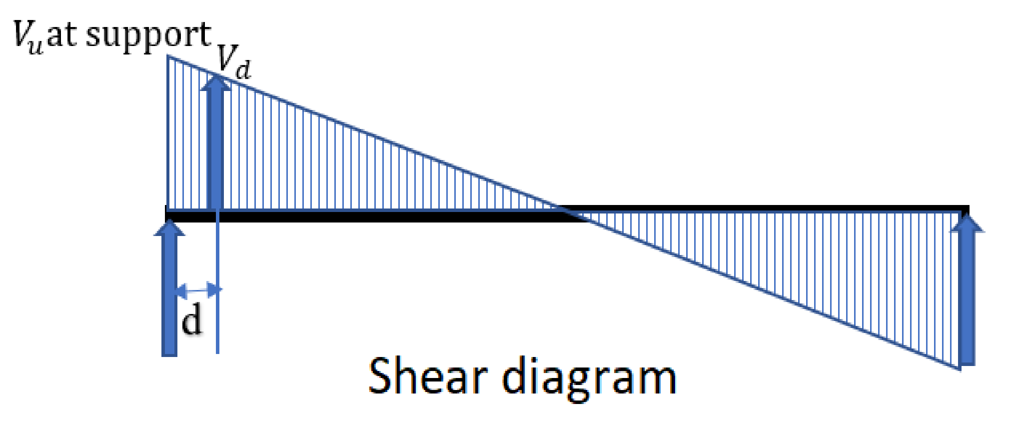
Calculate W_u

$W_u = 1.2\,WD + 1.6\,WL$

$W_u = 1.2\,(65.7) + 1.6\,(54.75) = 166.44\ kN/m$

Calculate V_u (at face of support)

$$V_{u,f} = \frac{W_u\ l}{2} = \frac{166.44\ \times 4.9}{2} = 407.8\ kN$$



Design V_u (at distance d from the face of the support) = $V_{u,d} = V_{u,f} - W_u\ d$

$V_{u,d} = V_{u,f} - W_u\ d = 407.8 - 166.44 \times 0.57 = 313\ kN$

$\therefore V_u = 313\ kN$

$$V_n = \frac{V_u}{\emptyset} = \frac{313}{0.75} = 417\ kN$$

Calculate V_c

$V_c = 0.17\,\lambda\sqrt{f_c'}\ b_w\ d = 0.17 \times 1 \times \sqrt{21} \times 350 \times 570$

$V_c = 155\ kN$

Calculate $0.5\ V_c$

$$0.5\ V_c = 0.5Vc == \frac{155}{2} = 77.5\ kN$$

$V_{C1} = 0.33\sqrt{f_c'}\, b_w\, d = 0.33 \times \sqrt{21} \times 350 \times 570 = 302\ kN$

and $V_{C2} = 0.66\sqrt{f_c'}\, b_w\, d = 2\, V_{C1} = 604\ kN$

$\therefore\ V_c\ (155\ kN) < V_n\ (417\ kN)$

$\therefore\ shear\ reinforcement\ must\ be\ provided\ and\ calculate\ V_S$

$V_S\ = V_n - V_c$

$V_S\ = 417 - 155 = 262\ kN$

$V_S\ (262\ kN) <\ V_{C2}(604\ kN)$ $\quad\therefore the\ dimensions\ of\ the\ sec. is\ OK$

Calculate the stirrups spacing, Use ϕ 10 mm, therefore $A_v = 157\ mm^2$

$$S_1 = \frac{A_v\, f_{yt} d}{V_s} = \frac{157 \times\ 420 \times 570}{262 \times 10^3} = 143\ mm$$

$For\ V_S\ (262\ \ kN) <\ V_{C1}(302\ kN)$

$$\therefore S_{\ 2} = \frac{d}{2} \leq 600\ mm$$

$$\therefore S_{\ 2} = \frac{570}{2} = 285\ mm \leq 600\ mm \quad \therefore S_{\ 2} =\ 285\ mm$$

$$S_3 = smaller\ of \begin{Bmatrix} \dfrac{A_v\, f_{yt}}{0.062\sqrt{f_c'}\ b_w} \\ \dfrac{A_v\, f_{yt}}{0.35\ b_w} \end{Bmatrix} = min \begin{Bmatrix} \dfrac{157 \times\ 420}{0.062\sqrt{21}\ \times 350} \\ \dfrac{157 \times\ 420}{0.35\ \times 350} \end{Bmatrix}$$

$$S_3 =\ min \begin{Bmatrix} 663\ mm \\ 538\ mm \end{Bmatrix} \quad \therefore S_3 =\ 538\ mm$$

$$S_{max} = \min(S_1\ , S_2\ and\ S_3) = 143\ mm \approx 130\ mm$$

∴ Use ϕ 10 mm @ 130 mm c/c

From shear diagram, the shear force on beam not constant and decrease to zero in center of beam, therefore using the spacing (S= 130 mm) for all beam is not economic, because this value (S= 130 mm) determined according to maximum shear force at distance d from support. So, for such cases when shear force not constant, the beam can divide to 3 or 2 zones according to the following.

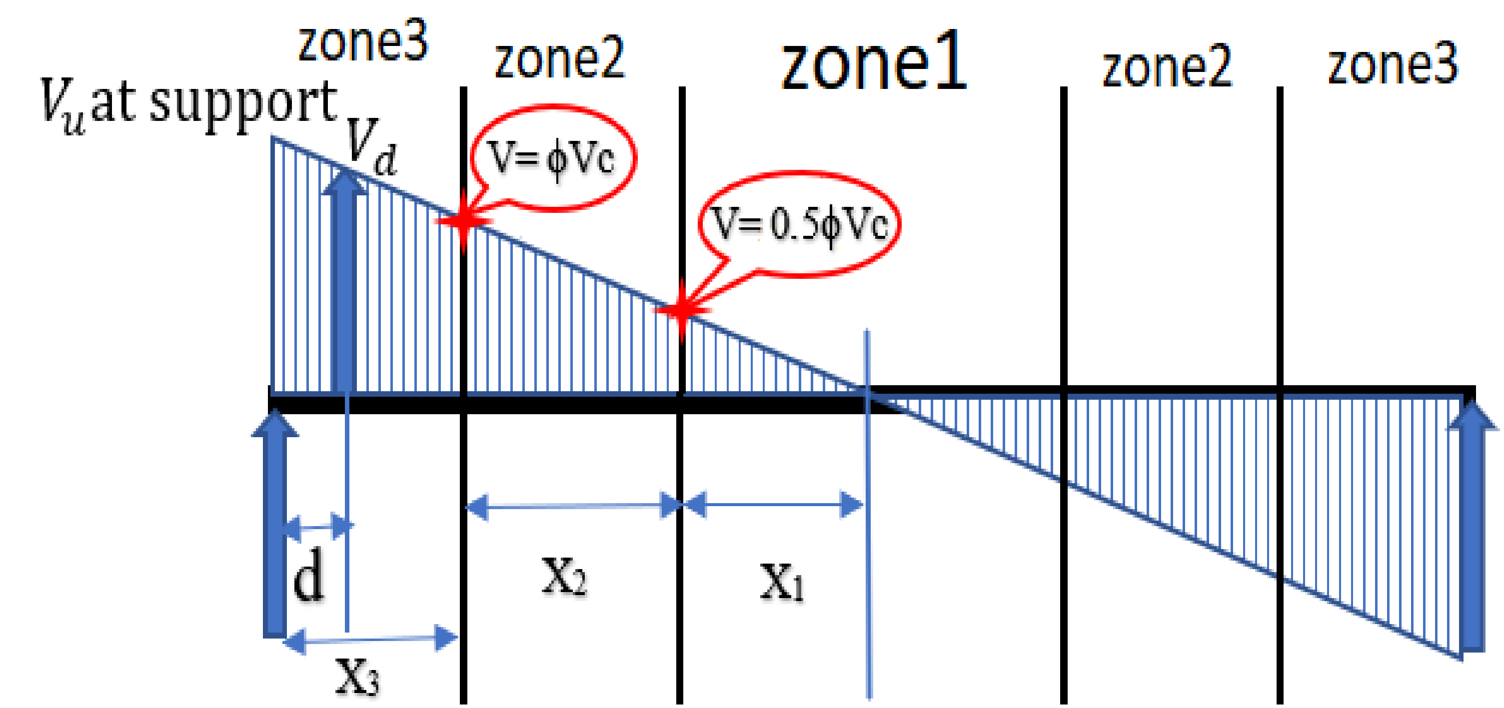
Zone 1: $V_n\ <\ 0.5\, V_{c,Eq.a},$ $no\ shear\ reinforcement\ is\ needed.$

*Zone*2: $0.5\, V_{c,Eq.a}\ <\ V_n\ \leq V_c$ $minimum\ shear\ reinforcement\ is\ required.$

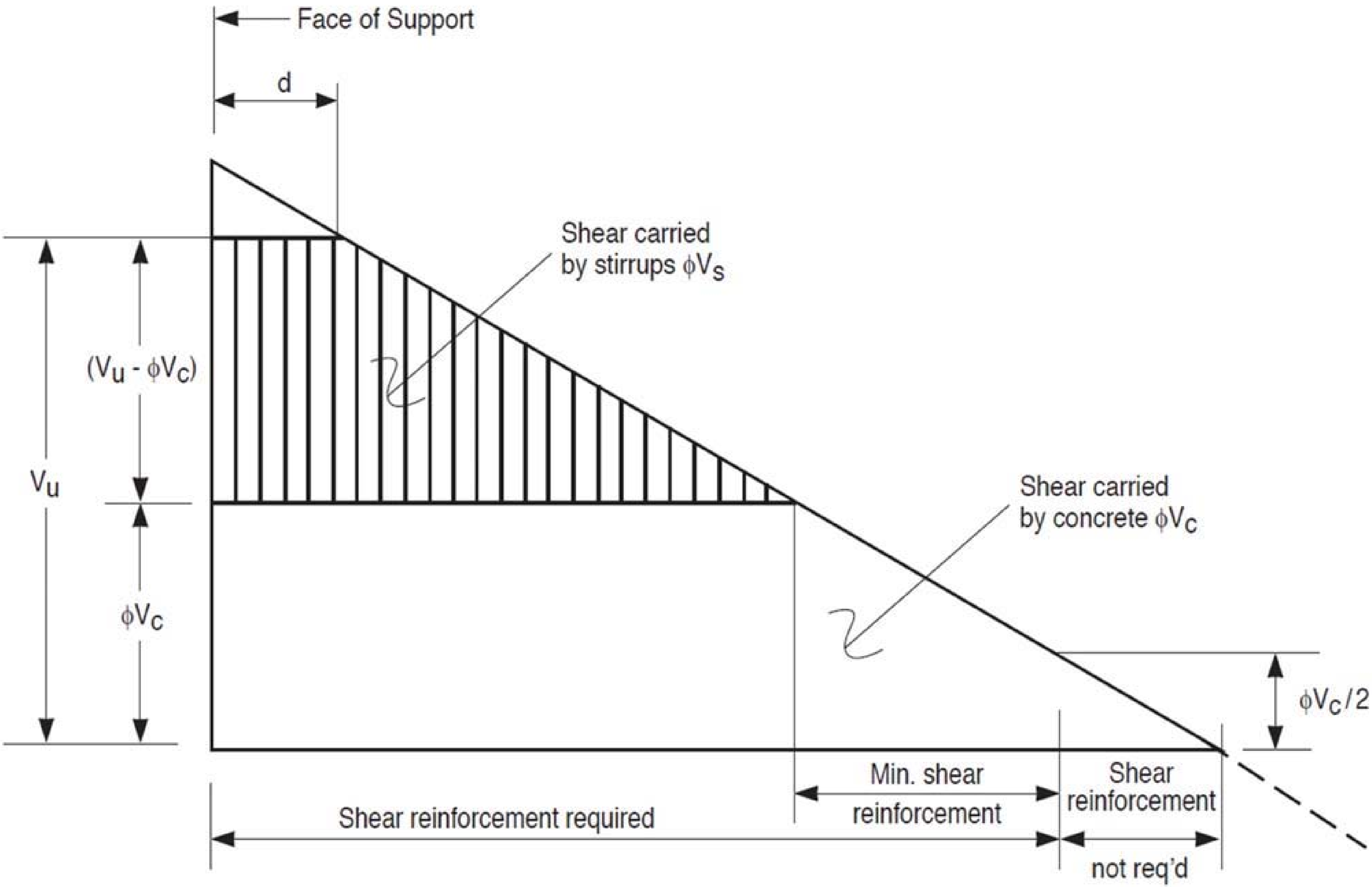
*Zone*3: $V_n\ > V_c$ $shear\ reinforcement\ is\ required.$

Zone 1 and zone 2 can be consider as one zone with minimum shear reinforcement.

It is easy to locate these zones as shown below, for zone1, by determine the location of V = 0.5ϕVc (x_1) and for zone2, by determine the location of V= (x_2).



zone3
zone2
zone1
zone2
zone3
V_u at support
V_d
V= ϕVc
V= 0.5ϕVc
d
X3
X2
X1



Face of Support
d
Shear carried
by stirrups ϕV_s
$(V_u - \phi V_c)$
V_u
Shear carried
by concrete ϕV_c
ϕV_c
$\phi V_c/2$
Min. shear
reinforcement
Shear
reinforcement
not req'd
Shear reinforcement required

For zone1, $V = 0.5\phi Vc = \phi \times 77.5 = 58.13\ kN$, from similarity of triangles

$$\frac{V_{u,f}}{l/2} = \frac{0.5\phi V_c}{x_1}$$

$$x_1 = \frac{0.5\phi V_c\, l}{2\, V_{n,f}} = \frac{0.75 \times 77.5 \times 4.9}{2 \times 407.8} = 0.35\ mm$$

For this distance of x_1 from center, no shear reinforcement is needed.

For zone2, $V = \phi Vc = 0.75 \times 155 = 116.25\ kN$, from similarity of triangles

$$\frac{V_{u,f}}{l/2} = \frac{\phi V_c}{x_1 + x_2}$$

$$x_1 + x_2 = \frac{\phi V_c\, l}{2\, V_{u,f}} = \frac{0.75 \times 155 \times 4.9}{2 \times 407.8} = 0.7\ mm$$

$$x_2 = 0.7 - 0.35 = 0.35\ mm$$

$$For\ this\ distance\ of\ x_2, minimum\ shear\ reinforcement\ is\ required$$

$$S_{max} = \min\left(S_2\ and\ S_3\right) = 285\ mm \approx 275\ mm$$

∴ Use ϕ 10 mm @ 275 mm c/c

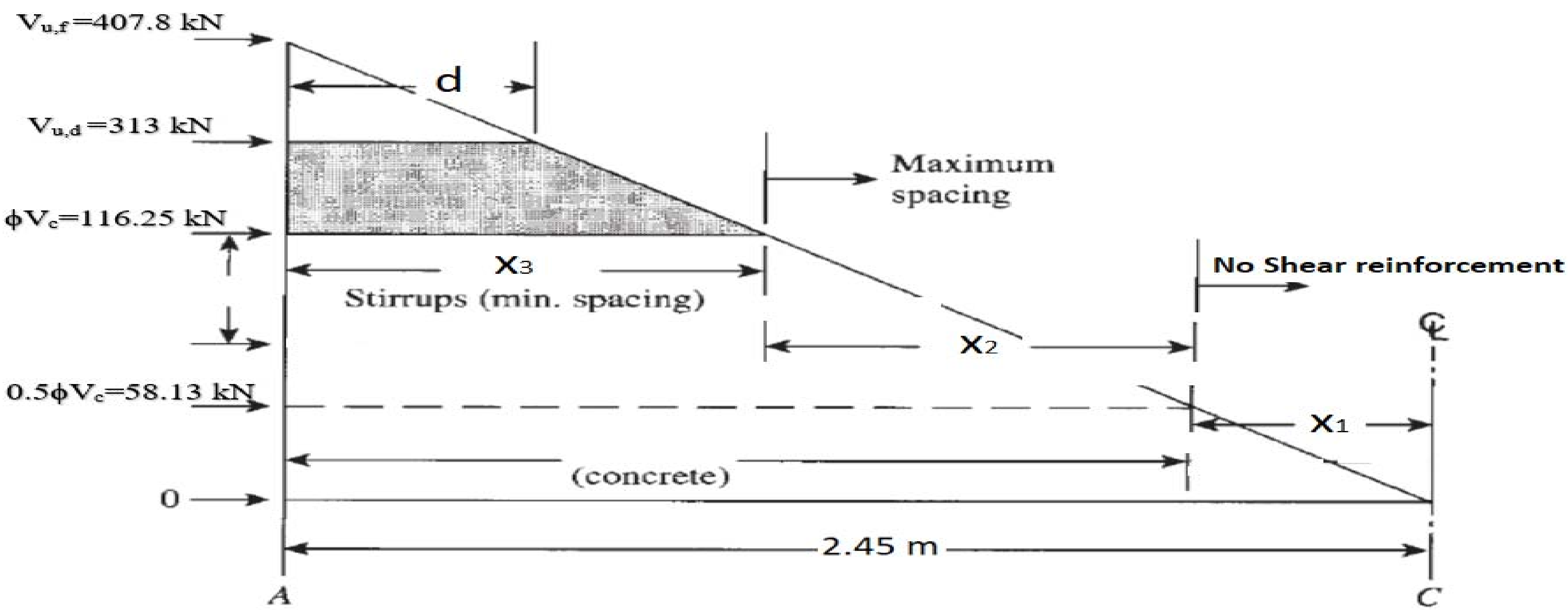
Actually, we can use min. shear reinforcement for $x_1 + x_2$.

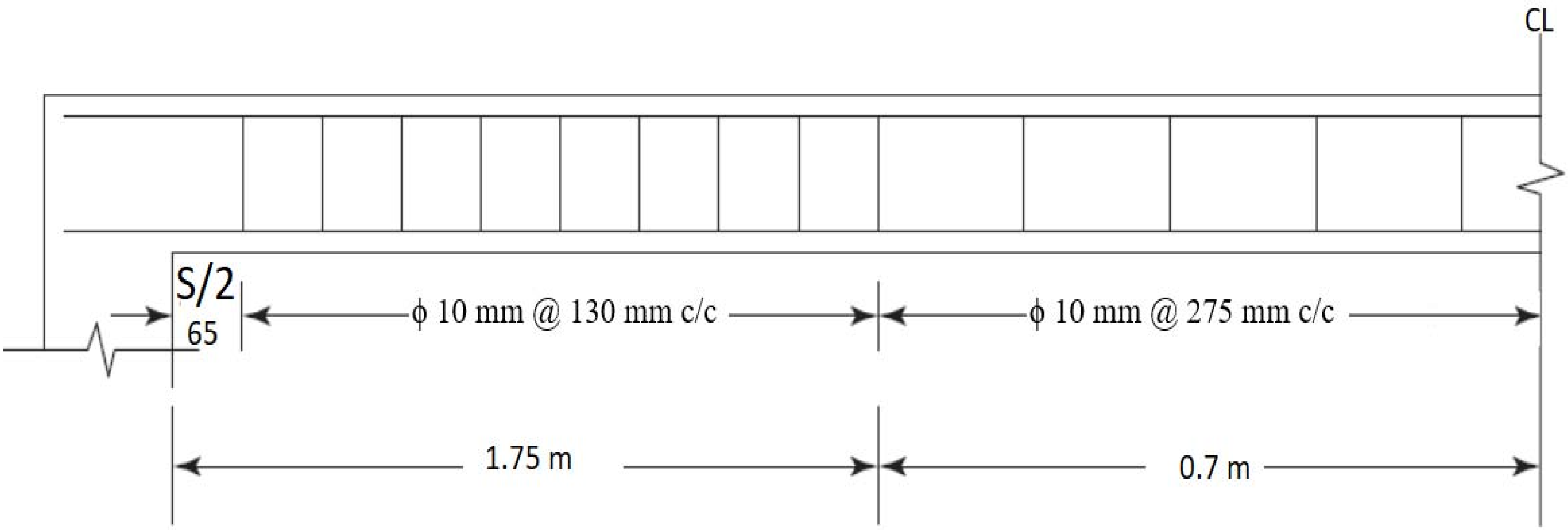
For zone 3,

$$x_3 = \frac{l}{2} - (x_1 + x_2) = \frac{4.9}{2} - 0.7 = 1.75\ mm$$

$$S_{max} = \min(S_1, S_2\ and\ S_3) = 143\ mm \approx 130\ mm$$

∴ Use ϕ 10 mm @ 130 mm c/c





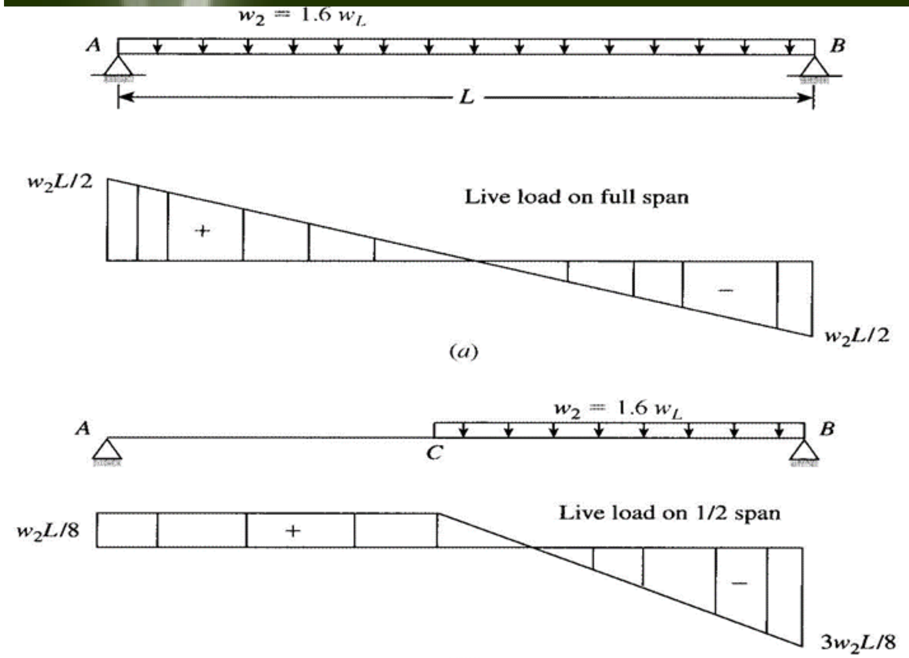
Distribution of stirrups.

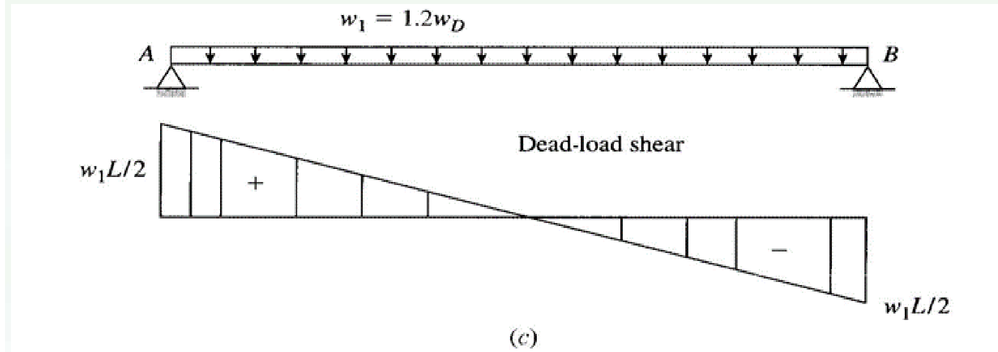
SHEAR FORCE DUE TO LIVE LOADS

In example 2, it was assumed that the dead and live loads are uniformly distributed along the full span, producing zero shear at midspan. Actually, the dead load does exist along the full span, but the live load may be applied to the full span or part of the span, as needed to develop the maximum shear at midspan or at any specific section. Figure 5.15a shows a simply supported beam with a uniform load acting on the full span. The shear force varies linearly along the beam, with maximum shear acting at support A.

In the case of live load, $W_2 = 1.6W\ L$, the maximum shear force acts at support A when W_2 is applied on the full span, Fig. 5.14a. The maximum shear at midspan develops if the live load is placed on half the beam, BC (Fig. 5.14b), producing Vu at midspan equal to $W_2L/8$. Consequently, the design shear force is produced by adding the maximum shear force due to the live load (placed at different lengths of the span) to the dead-load shear force (Fig. 5.14c) to give the shear distribution shown in Fig. 5.14d. It is common practice to consider the maximum shear at support A to be $WuL/2 = (1.2WD + 1.6WL)L/2$, whereas Vu at midspan is $W_2L/8 = (1.6\ W\ L)L/8$ with a straight-line variation along AC and CB, as shown in Fig. 5.14d. The design for shear in this case will follow the same procedure explained in Example 2. If the approach is applied to the beam in Example 2, then

Vu (at A) = 407.8 kN and Vu (at midspan) = $(1.6 \times 54.75)(4.9/8) = 53.66$ kN.


$w_2 = 1.6\ w_L$
A
B
L
$w_2L/2$
Live load on full span
+
−
$w_2L/2$
(a)
$w_2 = 1.6\ w_L$
A
B
C
$w_2L/8$
Live load on 1/2 span
+
−
$3w_2L/8$


$w_1 = 1.2w_D$
A
B
$w_1L/2$
Dead-load shear
+
−
$w_1L/2$
(c)

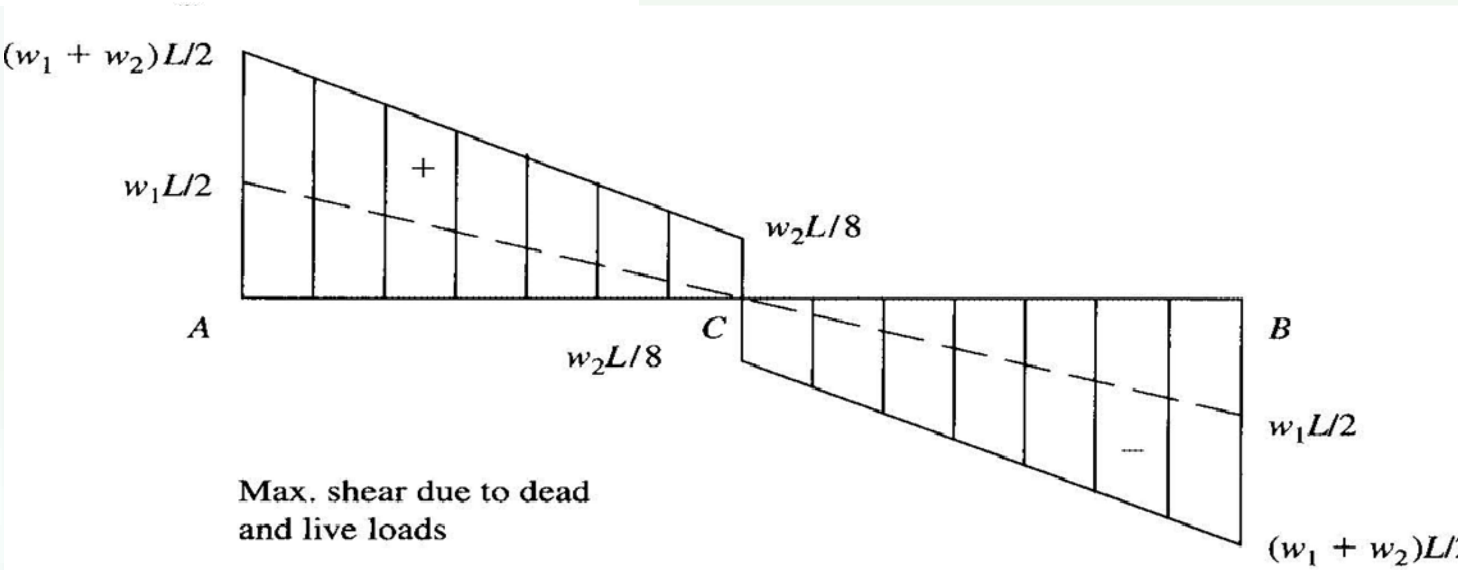
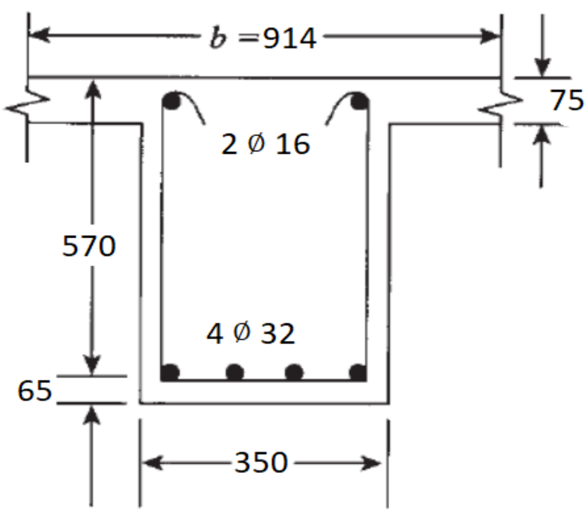
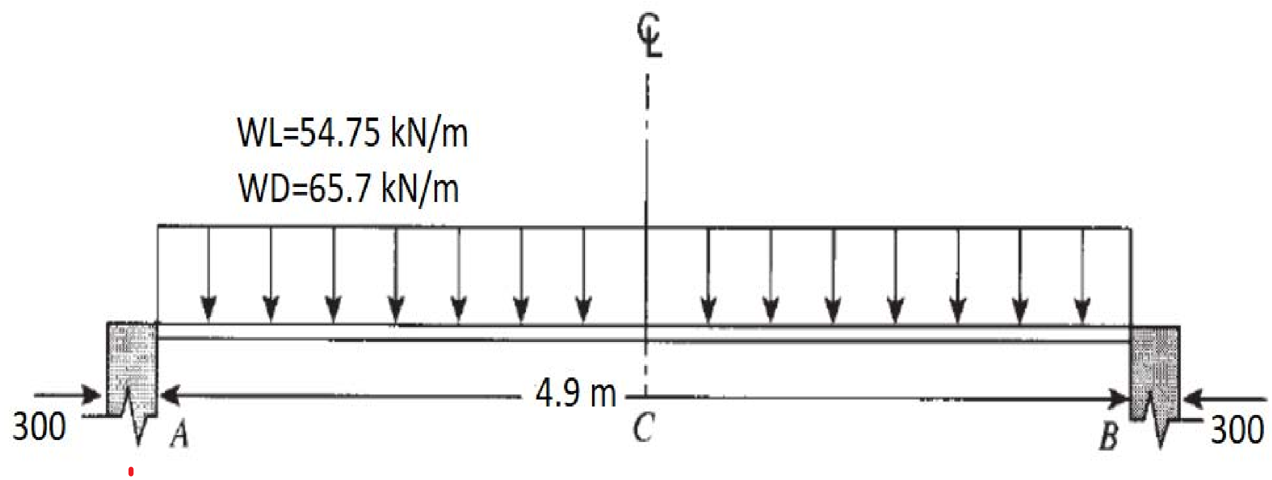

$(w_1 + w_2)L/2$
$w_1L/2$
+
$w_2L/8$
A
C
B
$w_2L/8$
$w_1L/2$
−
$(w_1 + w_2)L/2$
Max. shear due to dead
and live loads

Fig. 5.14

Example 3

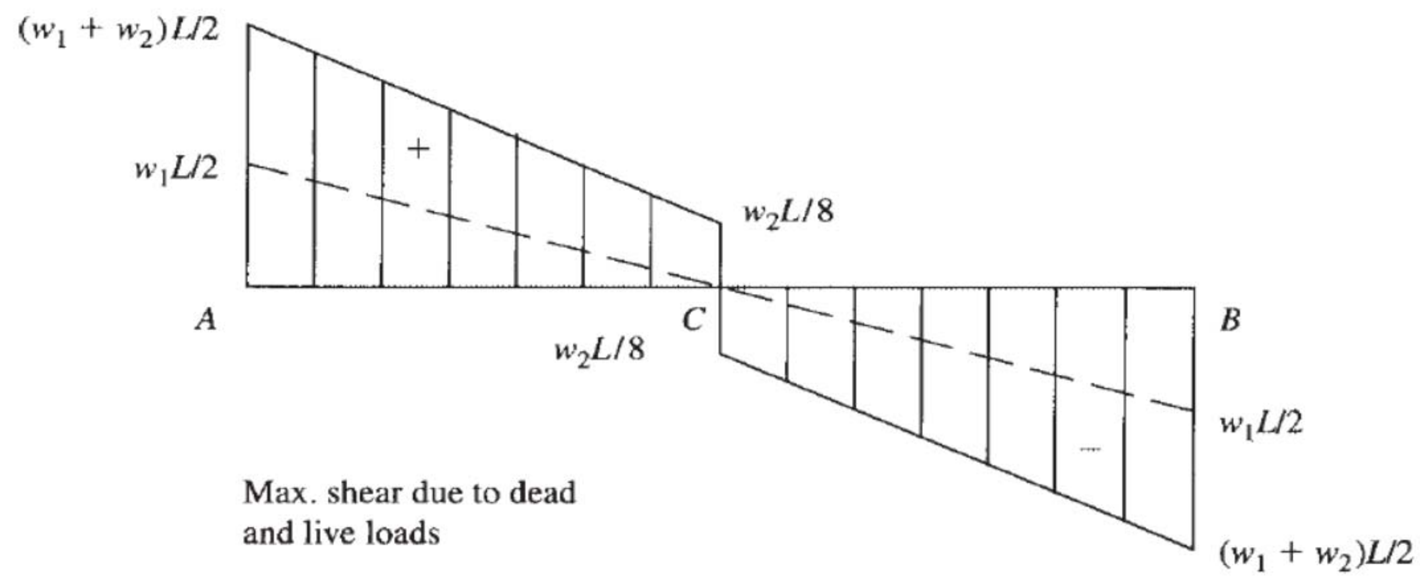
A 5.2 m, span simply supported beam has a clear span of 4.9 m and carries uniformly distributed dead and live loads of 65.7 kN/m and 54.75 kN/m, respectively. The dimensions of the beam section and steel reinforcement are shown in Fig. below. Check the section for shear and design the necessary shear reinforcement by taking the effect of placing of live load to produce maximum shear at mid-span. Given *fc'=21 MPa* normal-weight concrete and *fyt=420 MPa*.





Solution

As shown above in figure the maximum shear force will be



(d)

$W_1 = 1.2\,WD = 1.2(65.7) = 78.84\ kN/m$
$W_2 = 1.6\,WL = 1.6\,(54.75) = 87.6\ kN/m$

Calculate V_u (at face of support)

$$V_{u,f} = \frac{(W_1 + W_2)\,l}{2} = \frac{(78.84 + 87.6)\ \times 4.9}{2} = 407.8\ kN$$

$$V_{u,m}\ (at\ midspan)\ = \frac{W_2\,l}{8}\ (87.6)\ (4.9\ /\ 8)\ =\ 53.66\ kN$$

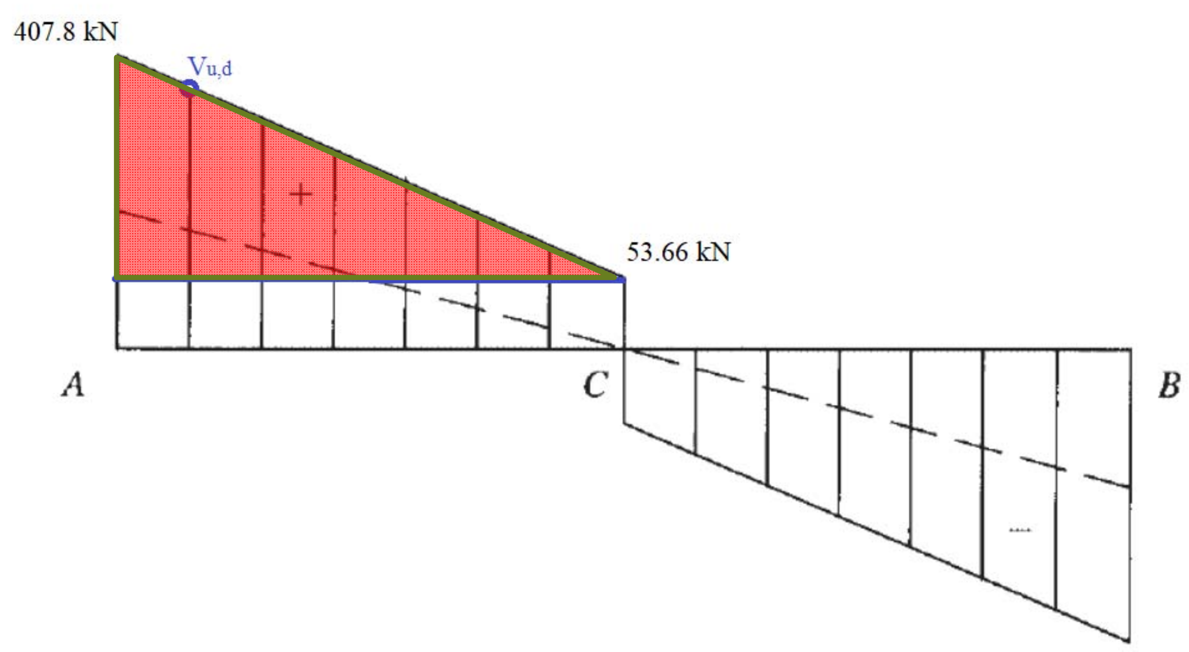
Calculate $V_{u,d}$ (at distance d from the face of the support) from similarity of triangles

$$\frac{V_{u,d} - 53.66}{\frac{l}{2} - d} = \frac{V_{u,f} - 53.66}{l/2}$$

$$\frac{V_{u,d} - 53.66}{\frac{4.9}{2} - 0.57} = \frac{407.8 - 53.66}{4.9/2}$$

$$V_{u,d}\ =\ 325.4\ kN$$

$$\therefore V_{n,d} = \frac{V_{u,d}}{\emptyset} = \frac{325.4}{0.75}\ =\ 434\ kN$$



$Calculate\ V_c$

$V_c = 0.17\,\lambda \sqrt{f_c'}\ b_w\, d = 0.17 \times 1 \times \sqrt{21} \times 350 \times 570$

$V_c = 155\ kN$

$Calculate\ \ 0.5\, V_c$

$0.5\, V_c = \frac{155}{2} = 77.5\ kN$

$V_{C1} = 0.33 \sqrt{f_c'}\, b_w\, d = 0.33 \times \sqrt{21} \times 350 \times 570 = 302\ kN$

and $\quad V_{C2} = 0.66 \sqrt{f_c'}\, b_w\, d = 2\, V_{C1} = 604\ kN$

$\therefore\ \ V_c\ (155\ kN) < V_n\ (434\ kN)\ \ \therefore\ \ shear\ reinforcement\ must\ be\ provided\ and\ calculate\ V_S$

$V_S\ = V_{n,d} - V_c$

$V_S\ = 434 - 155 = 279\ kN$

$V_S\ (279\ kN) <\ V_{C2}(604\ kN)$

$\therefore the\ dimensions\ of\ the\ sec. is\ OK$

Calculate the stirrups spacing, Use ϕ 10 mm, therefore $A_v = 157\ mm^2$

$$S_1 = \frac{A_v\, f_{yt} d}{V_s} = \frac{157 \times\ 420 \times 570}{279 \times 10^3} = 134\ mm$$

$For\ V_S\ (279\ \ kN) <\ V_{C1}(302\ kN)$

$$\therefore S\ _2 = \frac{d}{2} \leq 600\ mm$$

$\therefore S\ _2 = \frac{570}{2} = 285\ mm \leq 600\ mm \quad \therefore S\ _2 = 285\ mm$ and :

$$S_3 = smaller\ of \begin{Bmatrix} \dfrac{A_v\ f_{yt}}{0.062\sqrt{f_c'}\ b_w} \\ \dfrac{A_v\ f_{yt}}{0.35\ b_w} \end{Bmatrix} = min \begin{Bmatrix} \dfrac{157 \times\ 420}{0.062\sqrt{21}\ \ \times 350} \\ \dfrac{157 \times\ 420}{0.35\ \ \times 350} \end{Bmatrix}$$

$$S_3 =\ min \begin{Bmatrix} 663\ mm \\ 538\ mm \end{Bmatrix} \qquad \therefore S_3 =\ 538\ mm$$

$$S_{max} = \min(S_1\ , S_2\ and\ S_3) = 134\ mm$$

∴ Use ϕ 10 mm @ 130 mm c/c

From shear diagram, the shear force on beam not constant and decrease to 53.66 kN in center of beam, therefore using the spacing (S= 130 mm) for all beam is not economic, because this value (S= 130 mm) determined according to maximum shear force at distance d from support. So, for such cases when shear force not constant, the beam can divide to 3 or 2 zones according to the following.

$Zone\ 1:\ V_n < 0.5\, V_{c,Eq.a},\qquad no\ shear\ reinforcement\ is\ needed.$

$Zone2:\ 0.5\, V_{c,Eq.a} < V_n \le V_c\qquad minimum\ shear\ reinforcement\ is\ required.$

$Zone3:\ V_n > V_c\qquad shear\ reinforcement\ is\ required.$

Zone 1 and zone 2 can be consider as one zone with minimum shear reinforcement.
It is easy to locate these zones as shown below, by determine the location of V= ϕVc (distance x1)

For zones 1 and 2, V=ϕVc = 0.75* 155 =116.25 kN, from similarity of triangles

$$\frac{V_{u,f} - 53.66}{l/2} = \frac{\phi V_c - 53.66}{x_1}$$

$$\frac{407.8 - 53.66}{4.9/2} = \frac{116.25 - 53.66}{x_1}$$

$$x_1 = 0.43\ mm$$

$For\ this\ distance\ of\ x_1, minimum\ shear\ reinforcement\ is\ required$

$S_{max} = \min(S_2\ and\ S_3) = 285\ mm \approx 280\ mm$

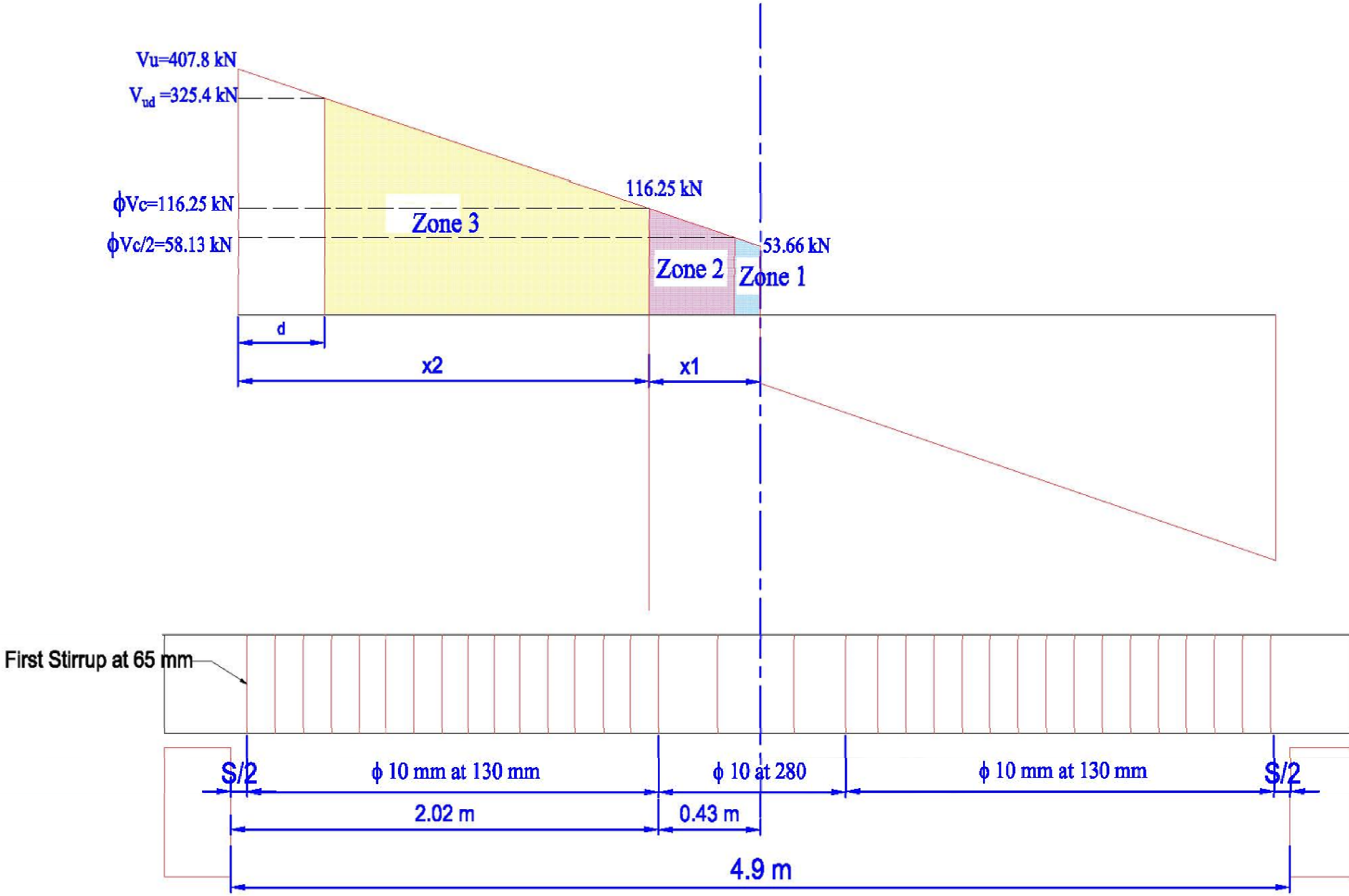
∴ Use ϕ 10 mm @ 280 mm c/c

For zone3,

$$x_2 = \frac{l}{2} - x_1 = \frac{4.9}{2} - 0.43 = 2.02\ m$$

$S_{max} = \min(S_1\ , S_2\ and\ S_3) = 134\ mm \approx 130\ mm$

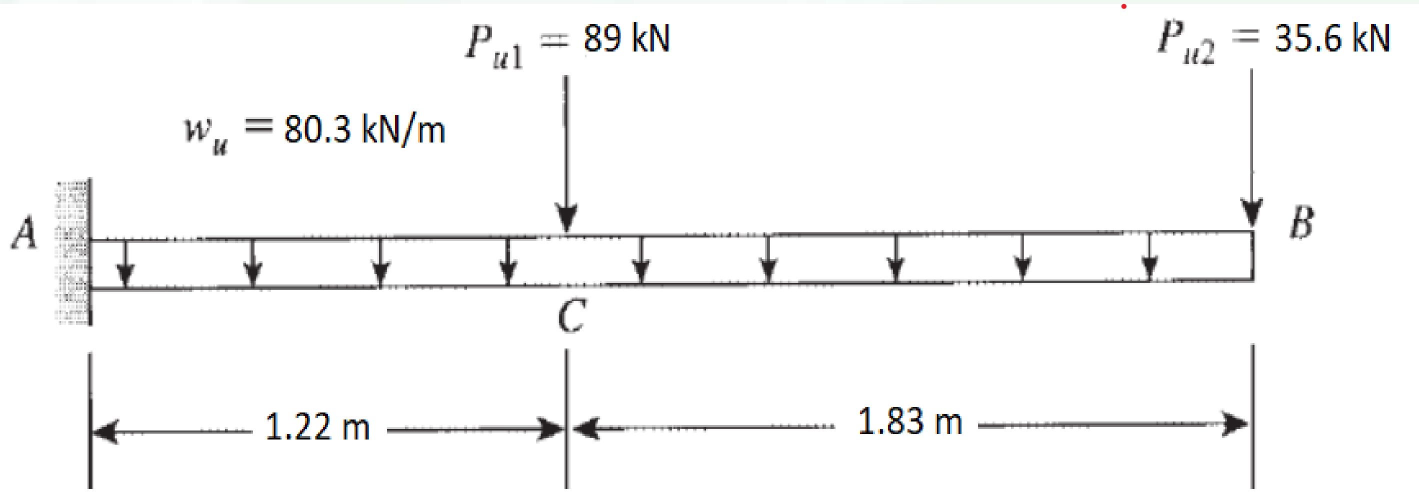
∴ Use ϕ 10 mm @ 130 mm c/c

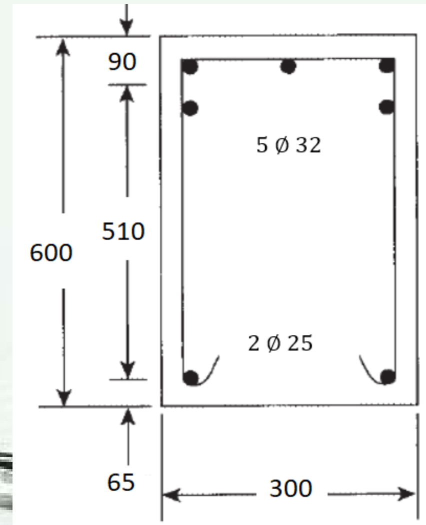


Vu=407.8 kN
Vud =325.4 kN
ϕVc=116.25 kN
ϕVc/2=58.13 kN
116.25 kN
53.66 kN
Zone 3
Zone 2
Zone 1
d
x2
x1
First Stirrup at 65 mm
S/2
ϕ 10 mm at 130 mm
ϕ 10 at 280
ϕ 10 mm at 130 mm
S/2
2.02 m
0.43 m
4.9 m

Example 4

A 3.05 m -span cantilever beam has a rectangular section and carries uniform and concentrated factored loads (self-weight is included), as shown in Fig. below. Using $f_c' = 28\,MPa$, normal-weight concrete and $f_{yt} = 420\,MPa$, design the shear reinforcement required for the entire length of the beam according to the ACI Code.





Solution

Calculate the shear force along the beam due to external loads:

$V_{u,f}(at\ support) = 80.3(3.05) + 89 + 35.6$

$= 369.52\ kN$

$V_{u,d}(at\ d\ distance) = 369.52 - 80.3 \times 0.51$

$= 328.56\ kN$

$V_{u,1.22L}(at\ 1.22\ left) = 369.52 - 80.3 \times 1.22$

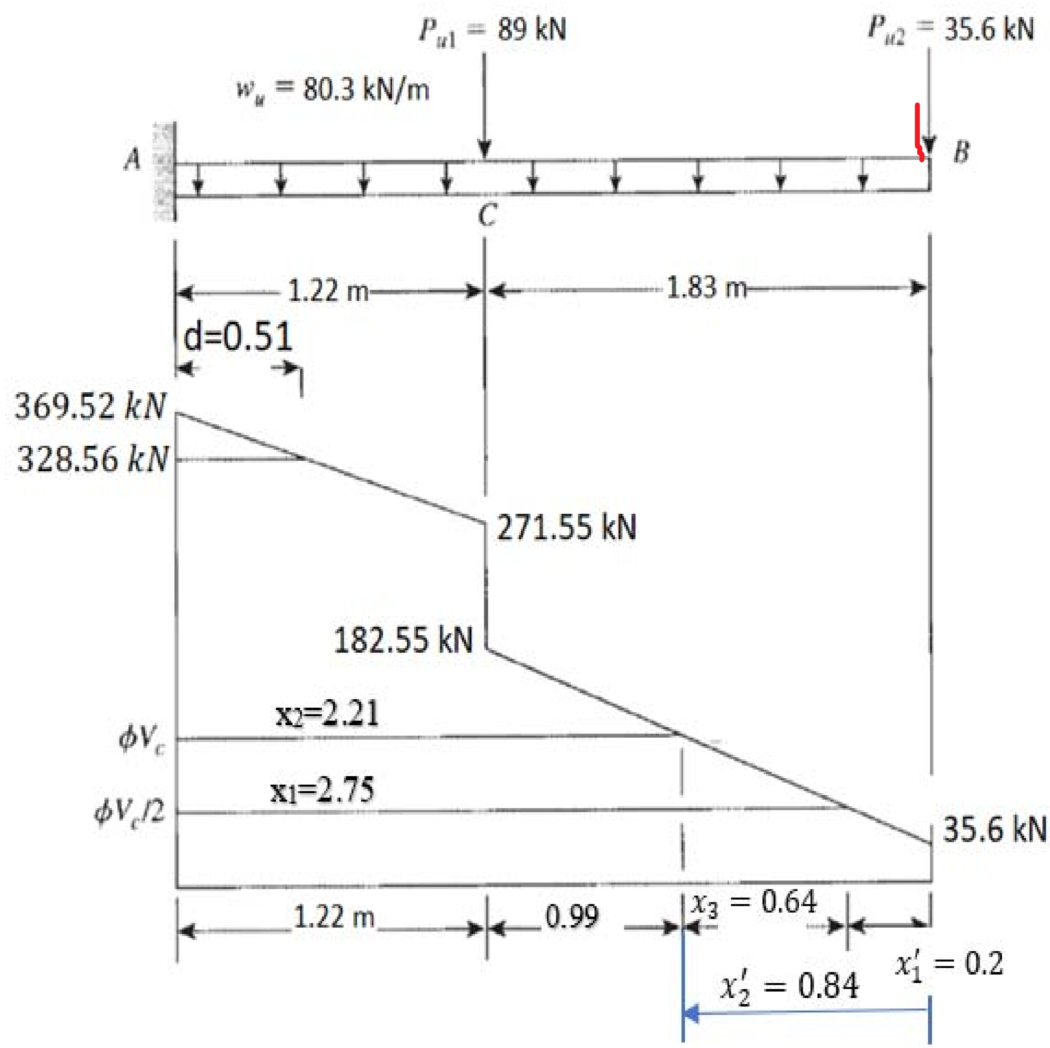
$= 271.55\ kN$

$V_{u,1.22R}(at\ 1.22\ right) = 271.55 - 89$

$= 182.55\ kN$

$V_{u,end}(at\ free\ end) = 35.6\ kN$

The shear diagram is shown below



$Calculate\ V_c$

$V_c = 0.17\,\lambda\sqrt{f_c'}\ b_w\,d = 0.17 \times 1 \times \sqrt{28} \times 300 \times 510$

$V_c = 137.6\ kN$

$Calculate\ \ 0.5\,V_c$

$0.5\,V_c = \frac{137.6}{2} = 68.8\ kN$

$V_{C1} = 0.33\sqrt{f_c'}\,b_w\,d = 0.33 \times \sqrt{28} \times 300 \times 510 = 267.2\ kN$

and $\quad V_{C2} = 0.66\sqrt{f_c'}\,b_w\,d = 2\,V_{C1} = 534.4\ kN$

$$V_n = \frac{V_{u,d}}{\emptyset} = \frac{328.56}{0.75} = 438.1\ kN$$

$\therefore\ V_c\ (137.6\ kN) < V_n\ (438.1\ kN)$

$\therefore\ shear\ reinforcement\ must\ be\ provided\ and\ calculate\ V_S$

$V_S = V_n - V_c$

$V_S = 438.1 - 137.6 = 300.5\ kN$

$V_S\ (300.5\ kN) < V_{C2}(534.4\ kN) \qquad \therefore\ the\ dimensions\ of\ the\ sec.\ is\ OK$

Calculate the stirrups spacing, Use ϕ 10 mm, therefore $A_v = 157\ mm^2$

$$S_1 = \frac{A_v\,f_{yt}d}{V_s} = \frac{157 \times\ 420 \times 510}{300.5 \times 10^3} = 112\ mm$$

$For\ V_S\ (300.5\ \ kN) > V_{C1}(267.2\ kN)$

$$\therefore S_{\,2} = \frac{d}{4} \leq 300\ mm$$

$$\therefore S_{\,2} = \frac{510}{4} = 127\ mm \leq 300\ mm \quad \therefore S_{\,2} = \ 127\ mm$$, and

$$S_3 = smaller\ of \left\{ \begin{matrix} \dfrac{A_v\ f_{yt}}{0.062\sqrt{f_c'}\ b_w} \\ \dfrac{A_v\ f_{yt}}{0.35\ b_w} \end{matrix} \right\} = \ min \left\{ \begin{matrix} \dfrac{157 \times\ 420}{0.062\sqrt{28}\ \times 300} \\ \dfrac{157 \times\ 420}{0.35\ \times 300} \end{matrix} \right\}$$

$$S_3 = \ min \left\{ \begin{matrix} 670\ mm \\ 628\ mm \end{matrix} \right\} \quad \therefore S_3 = \ 628\ mm$$

$S_{max} = \min(S_1\ , S_2\ and\ S_3) = 112\ mm \approx 110\ mm$

∴ Use ϕ 10 mm @ 110 mm c/c

From shear diagram, the shear force on beam not constant and decrease to 35.6 kN at free end of beam, therefore using the spacing (S= 110 mm) for all beam is not economic, because this value (S= 110 mm) determined according to maximum shear force at distance d from support. So, for such cases when shear force not constant, the beam can divide to 3 or 2 zones according to the following.

$Zone\ 1\!: V_n < 0.5\, V_{c,Eq.a},$ $\quad no\ shear\ reinforcement\ is\ needed.$

$Zone2\!: 0.5\, V_{c,Eq.a} < V_n \le V_c$ $\quad minimum\ shear\ reinforcement\ is\ required.$

$Zone3\!: V_n > V_c$ $\quad shear\ reinforcement\ is\ required.$

Zone 1 and zone 2 can be consider as one zone with minimum shear reinforcement. (d/2, 600 mm)

It is easy to locate these zones as shown below, for zone1, by determine the location of V = 0.5ϕVc (x_1) and for zone2, by determine the location of V= ϕVc (x_2).

For zone1,

V=0.5ϕVc = 0.5 *0.75* 137.6 =51.6 kN,

$$35.6 + 80.3\, x_1' = 0.5\phi V_c$$

$$x_1' = \frac{51.6 - 35.6}{80.3} = 0.2\, m \quad from\ free\ end$$

$$\therefore x_1 = 3.05 - 0.2 = 2.75\, m \quad from\ support$$

For this distance of x_1' from free end, no shear reinforcement is needed.

For zone2, V=ϕVc = 0.75*137.6 =103.2 kN, from similarity of triangles

$$35.6 + 80.3\, x_2' = \phi V_c$$

$$x_2' = \frac{103.2 - 35.6}{80.3} = \underline{0.84\, m} \quad from\ free\ end$$

$$\therefore x_2 = 3.05 - 0.84 = 2.21\, m \quad from\ support$$

$For\ the\ distance\ \ x_3 = x_2' - x_1' = 0.64\ m\ ,$

$minimum\ shear\ reinforcement\ is\ required$

$S_3\ or\ S_2\ ,\ \ S_2 = d/2, 600\ mm$

$$S_2\ = \frac{510}{2} = 255\ mm$$

∴ Use ϕ 10 mm @ 250 mm c/c

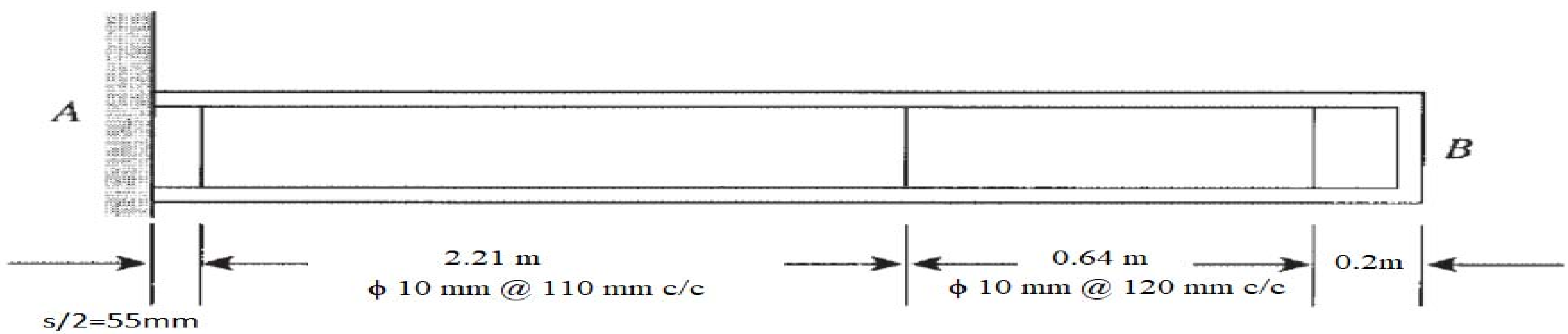
Actually, we can use min. shear reinforcement for all the distance x_2'.

For zone 3,

For the distance $x_2 = 2.21\ m$

$S_{max} = \min(S_1\ , S_2\ and\ S_3) = 112mm \approx 110\ mm$

∴ Use ϕ 10 mm @ 110 mm c/c



Distribution of stirrups.