Reinforced Concrete Design

Analysis and Design of One Way Concrete Slab

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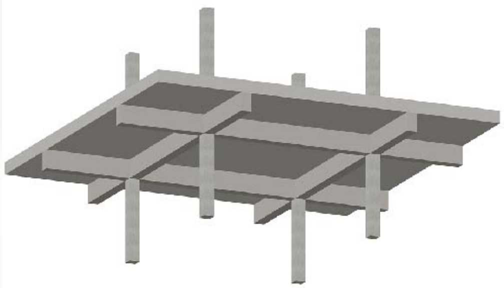
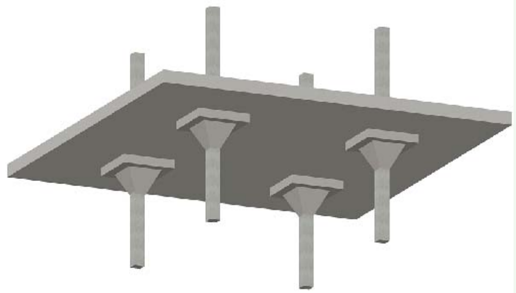
LOGO

The Type Of Slab

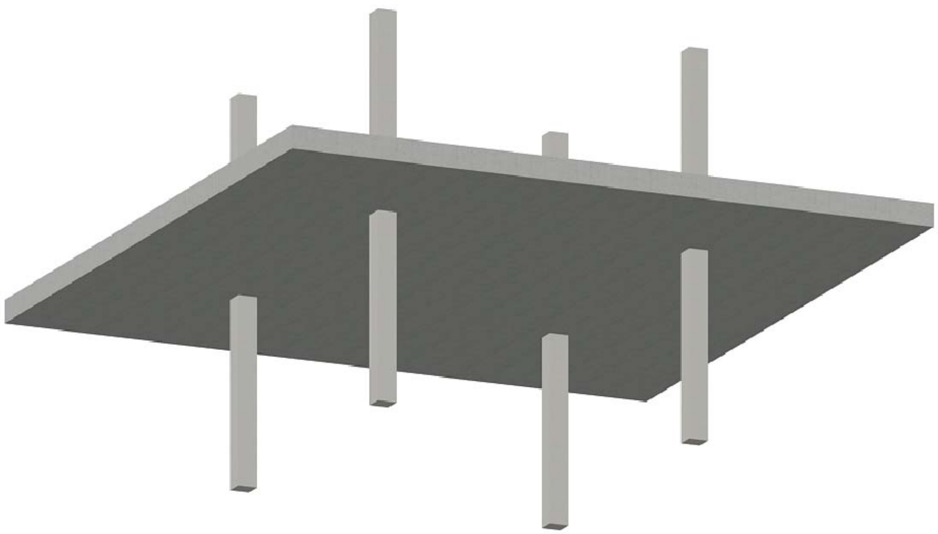
1-One-way slabs 2- Two-way floor systems   3-Flat slab   4- Waffle slab   5- Ribbed Slab



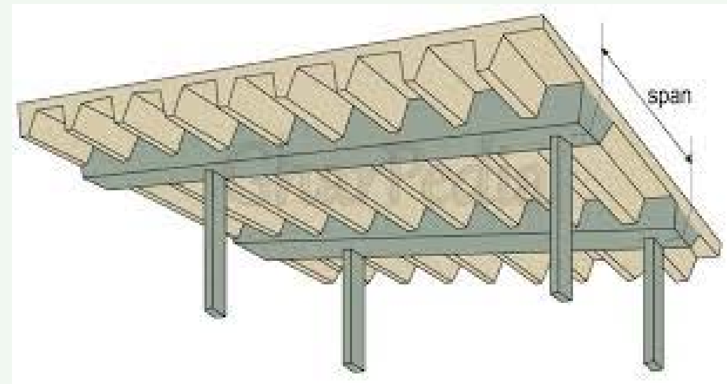
(a) Slab with beams

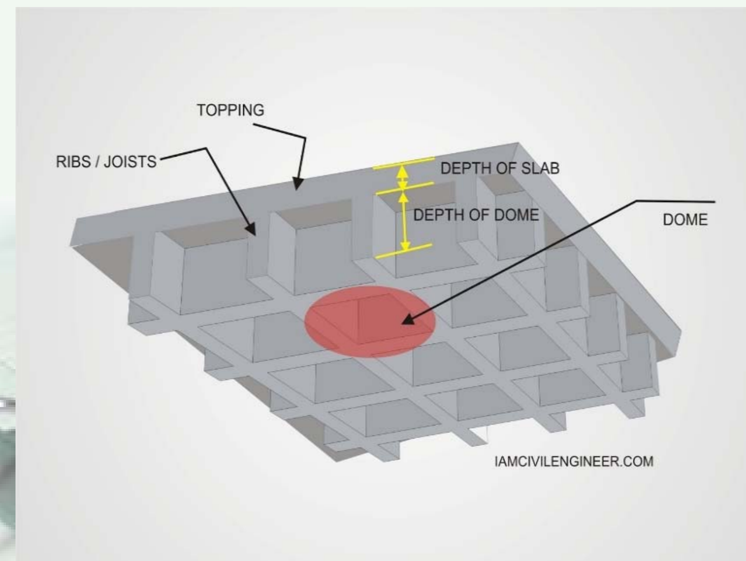


(b) Flat slab with capitals



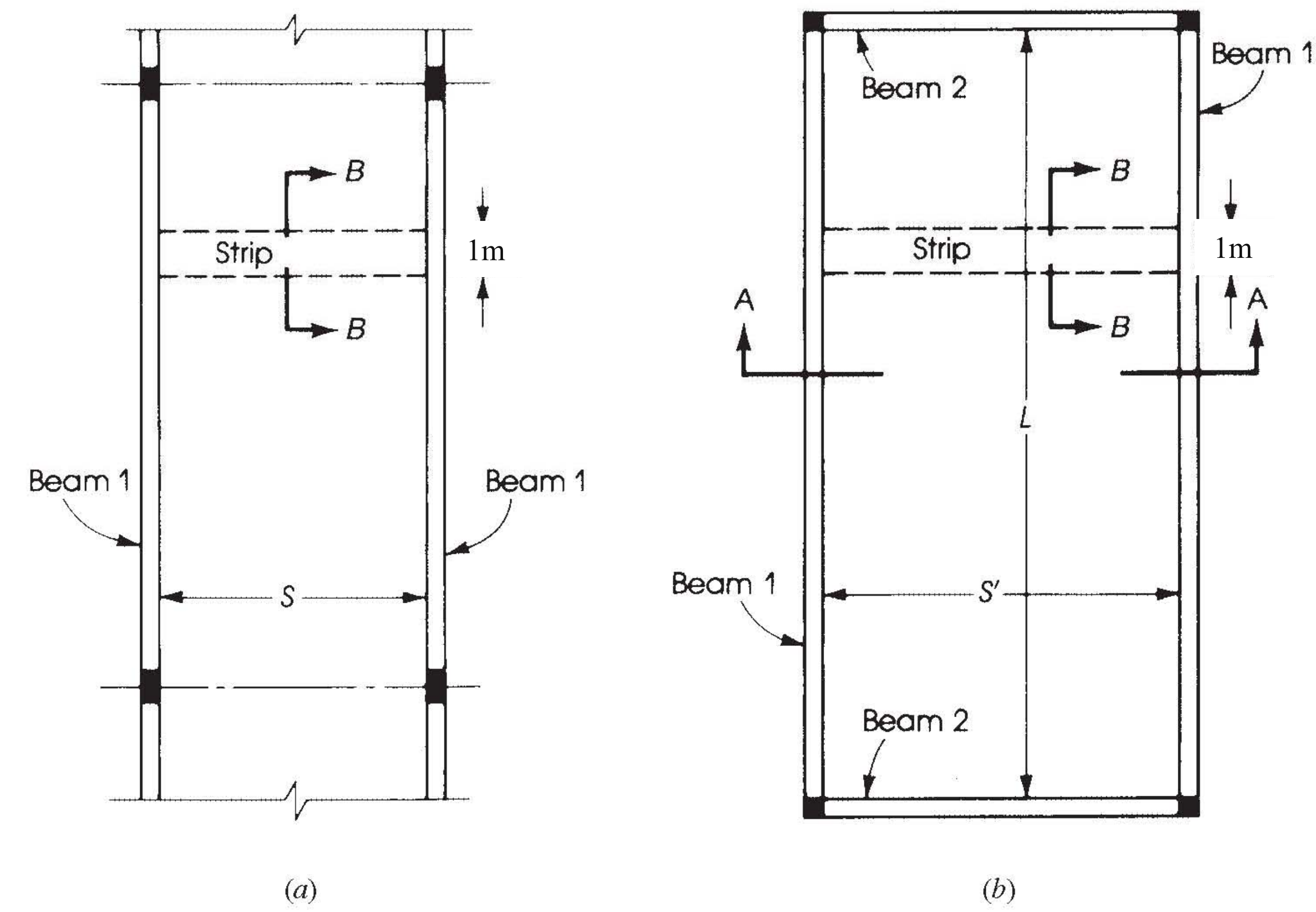
(c) Flat plate

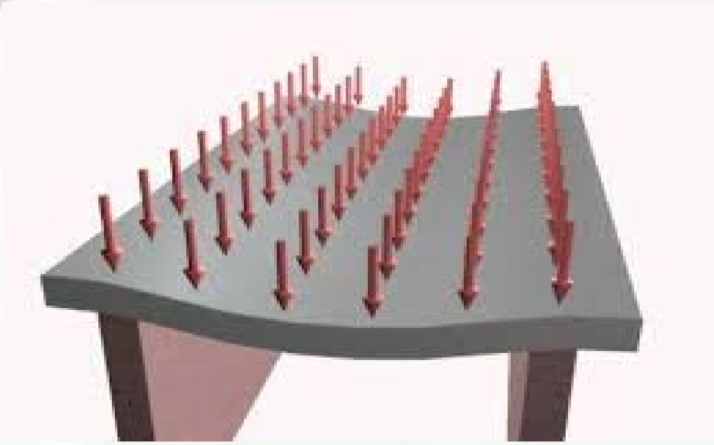


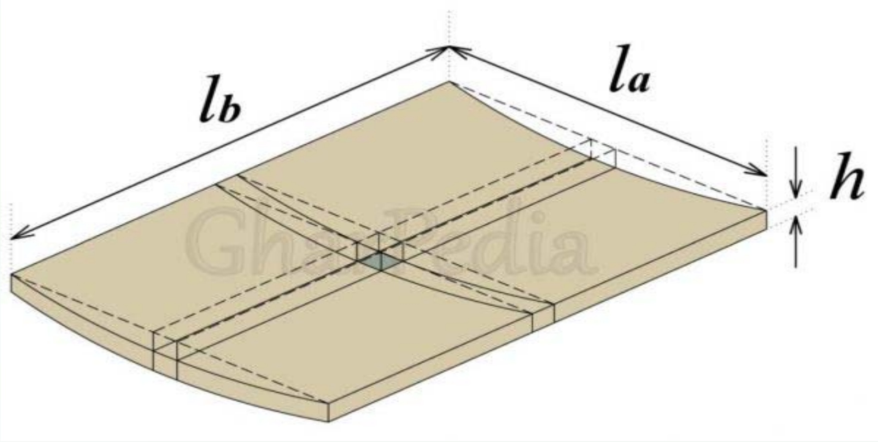


2. One Way Slab:

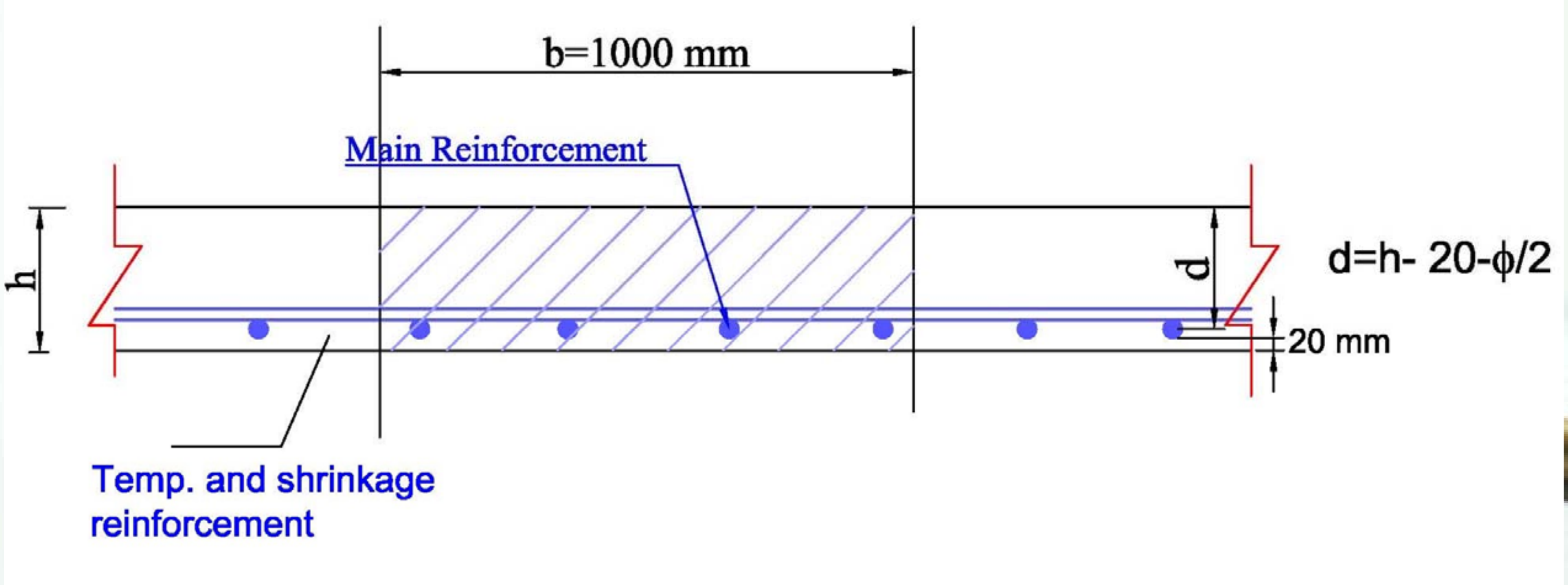
$$\frac{Longer Span}{Short Span} \geq 2$$







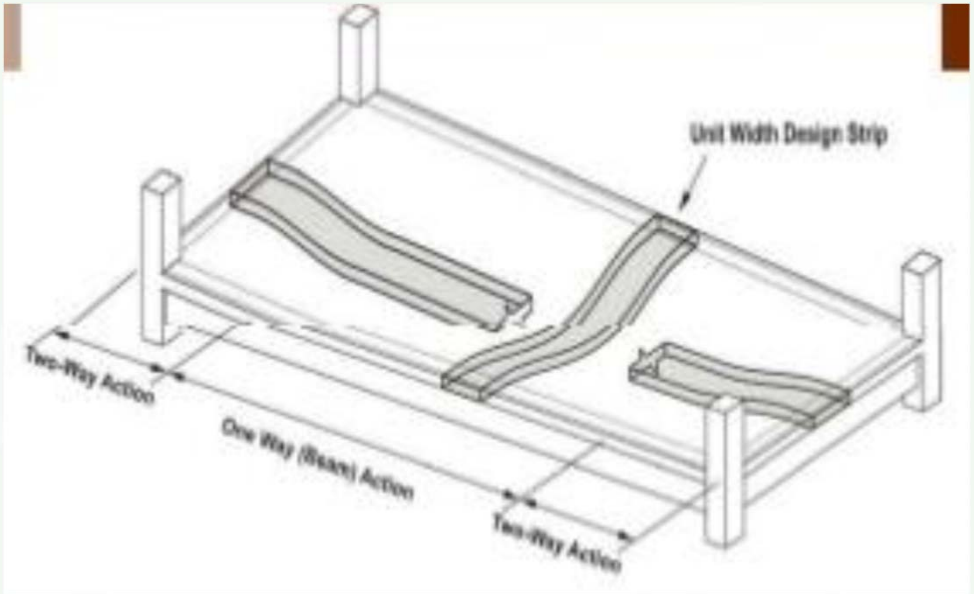
One way Slab Deflection



One Way Slab Section

2. Two Way Slab:

$$\frac{Longer Span}{Short Span} < 2$$



Two way Slab Deflection

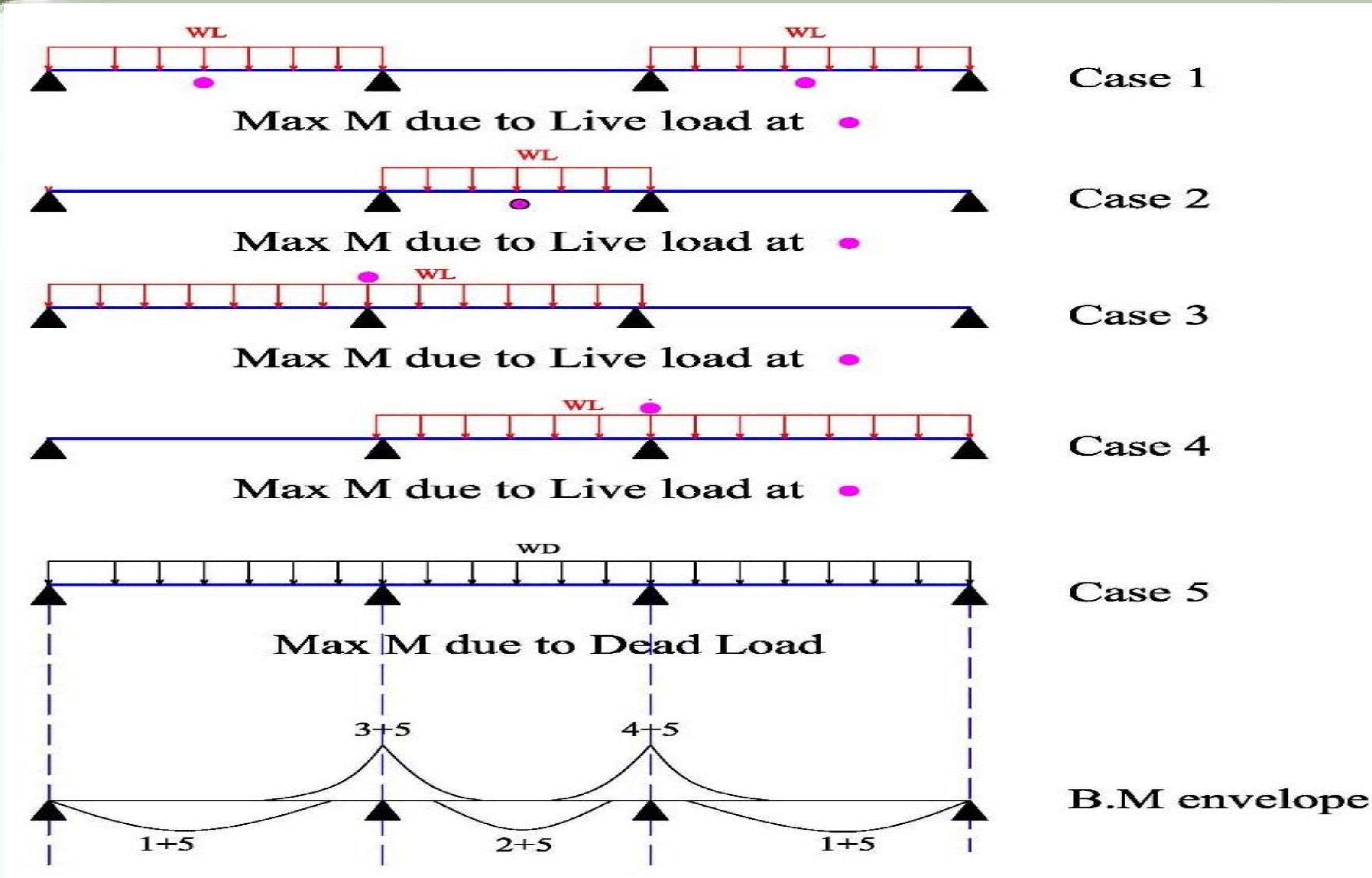
Design of One way slab

For simply supported slab the max. positive bending moment

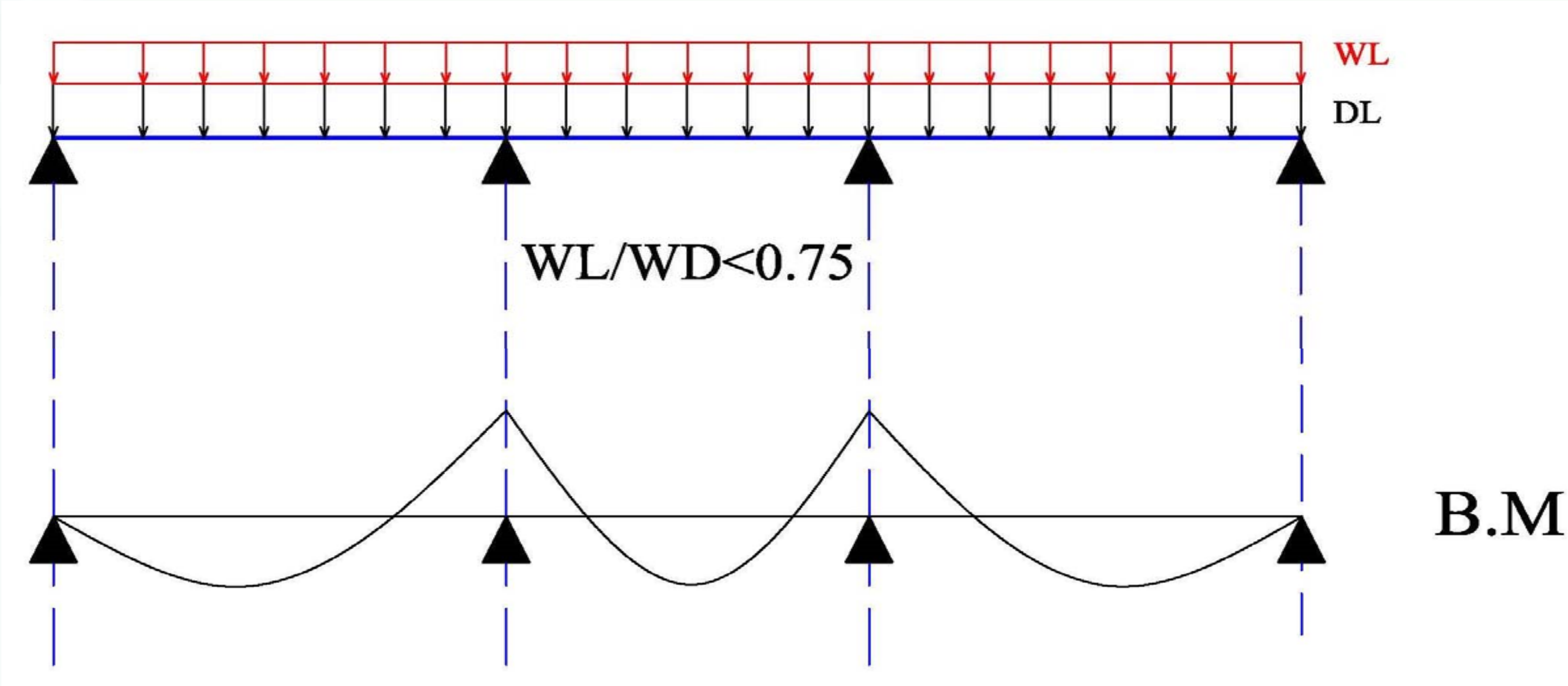
$+M = \frac{Wu \; ln^2}{8}$ where wu = KN/m^2 , ln = clear span in short direction

While for continuous span the Moment at mid and support should be calculated according to method of structures analysis. To find the Critical section the live should arrange to the spans in different position to get the envelope of bending moment Diagram as shown below:

Loading Cases



Bending moment envelopes for the critical section when the WL/WD> 0.75



Bending moment for the critical section when the WL/WD < 0.75

Thank You

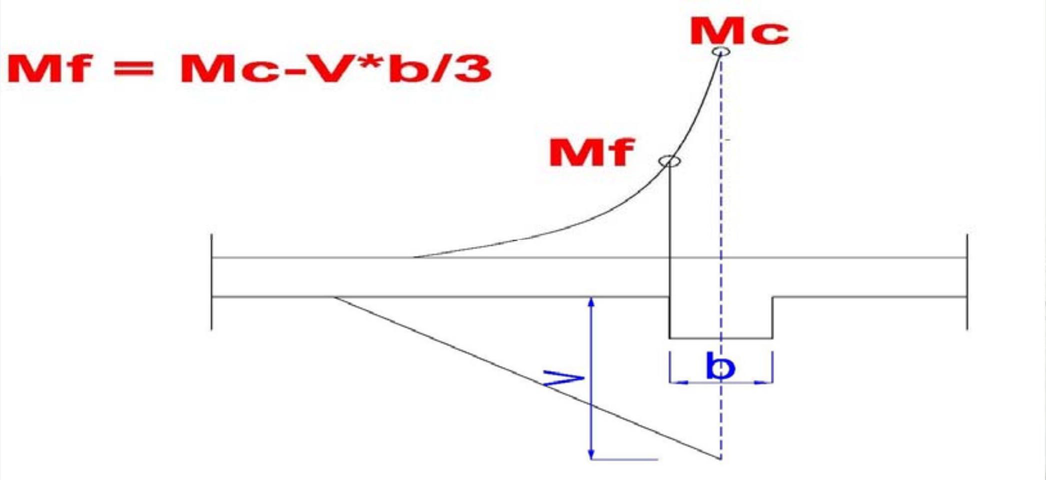
………….To be Continued

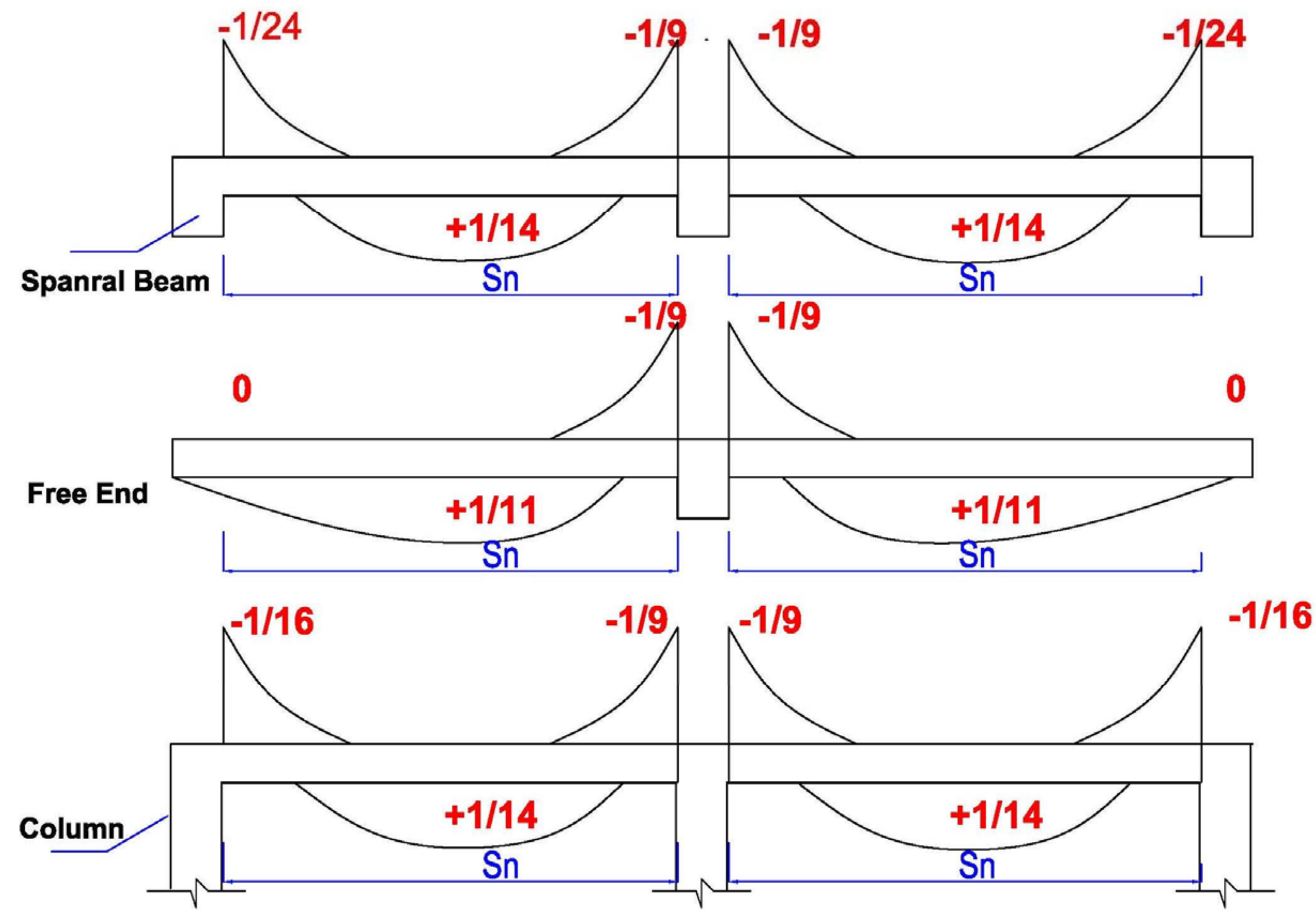
ACI Code Coefficient Methods (ACI Item 6.5)

1. Members are prismatic.
2. Load are uniform ally distributed.
3. Live Load $\leq 3 \times Dead\ Load$
4. There are at least two span.
5. The longer of the two adjacent spans does not exceed the shorter by more than 20 Percent (L ≤ 1.2 S)

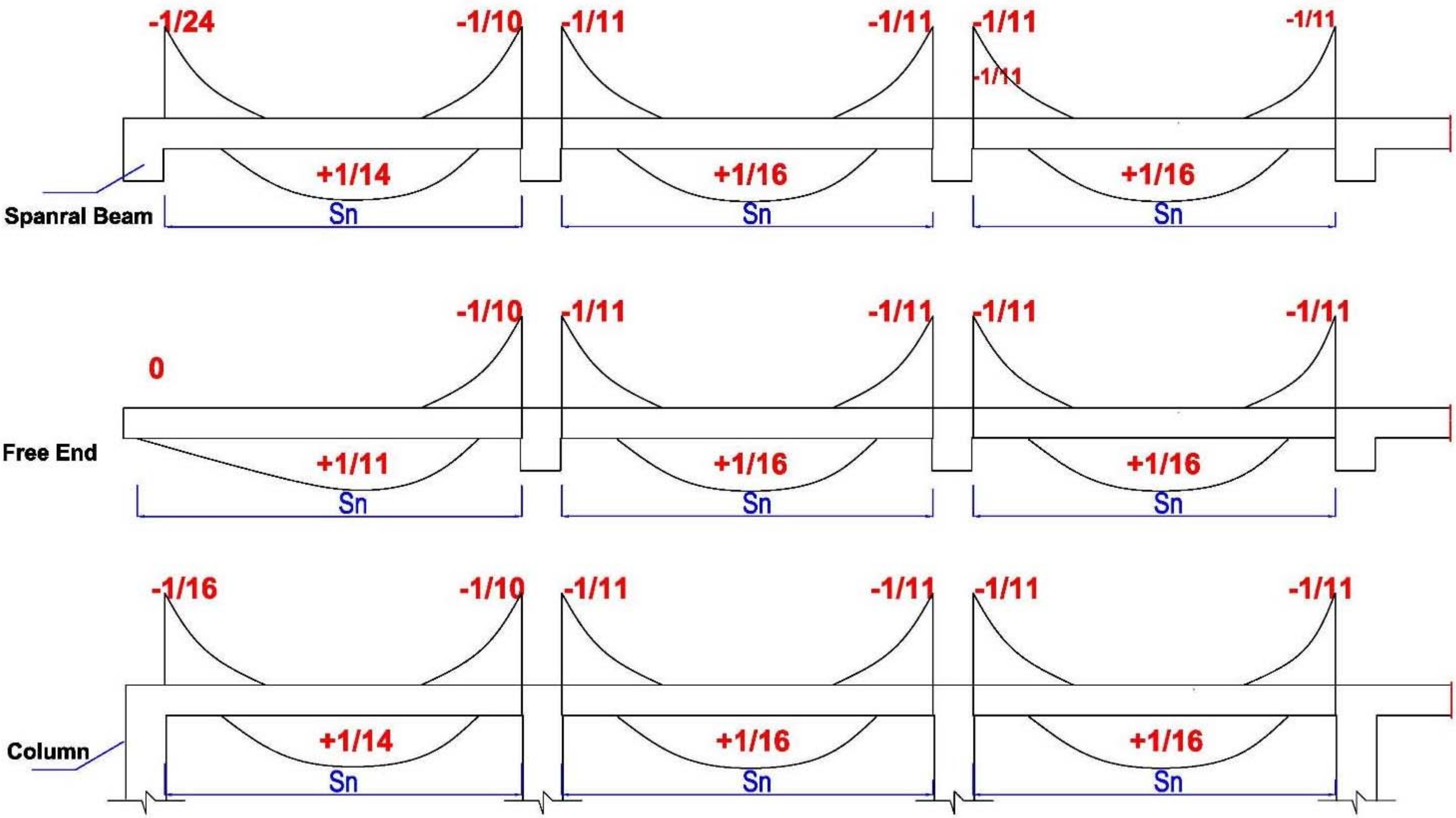
$Mu = (coefficient)\,(Wu\,Sn^2) = C_f\,Wu\,Sn^2$

S_n =clear span.




-1/24
-1/9
-1/9
-1/24
+1/14
+1/14
Spanral Beam
Sn
Sn
-1/9
-1/9
0
0
Free End
+1/11
+1/11
Sn
Sn
-1/16
-1/9
-1/9
-1/16
+1/14
+1/14
Column
Sn
Sn

Two Span


-1/24
-1/10
-1/11
-1/11
-1/11
-1/11
-1/11
+1/14
+1/16
+1/16
Spanral Beam
Sn
Sn
Sn
-1/10
-1/11
-1/11
-1/11
-1/11
0
Free End
+1/11
+1/16
+1/16
Sn
Sn
Sn
-1/16
-1/10
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Column
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Sn
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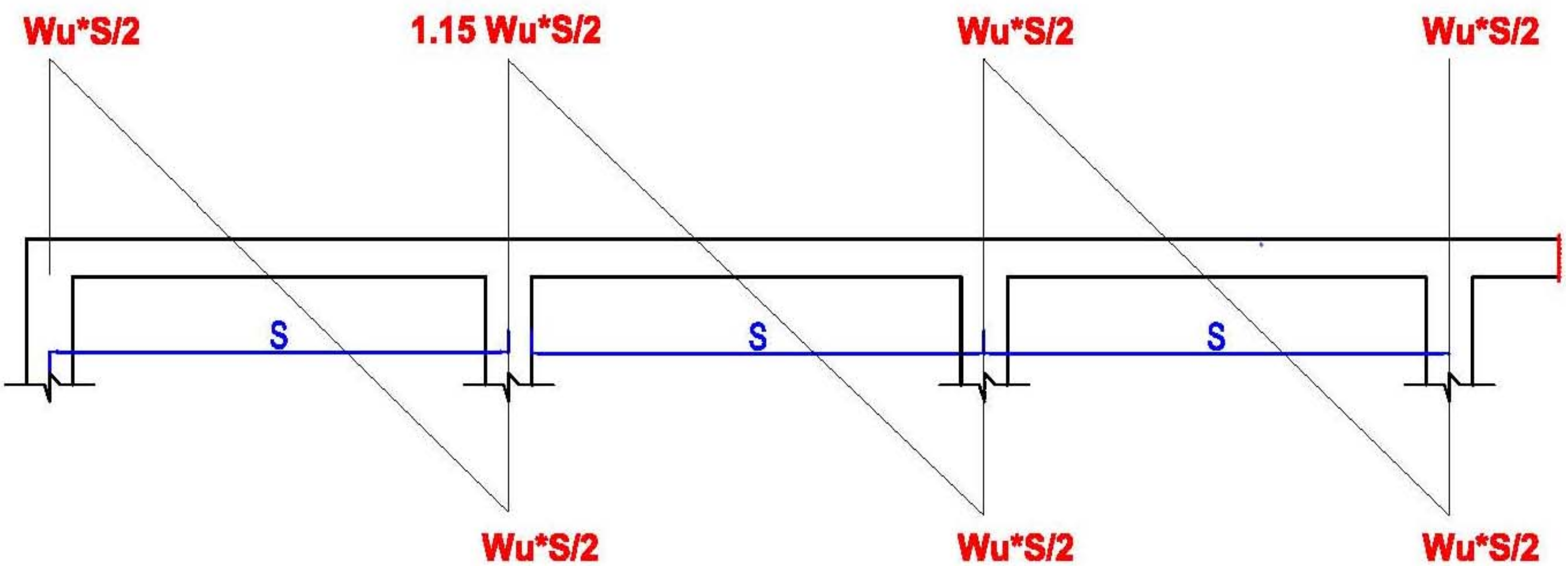
B.M Factors



Shear Force Factors

Design Limitations of ACI Code.

Design Limitations of ACI Code.

1- minimum depth of Slab (h) when Fy=420 Mpa for solid one way slab should follow the ACI item 7.3.1.1 for normal concrete only

Support condition	Minimum Thickness (h)
Simply Supported	L/20
One End Continuous	L/24
Both End Continuous	L/28
Cantilever	L/10

For f_y other than 420 MPa, these values shall be multiplied by $(0.4 + fy/700)$

2- Deflection is to be checked when the slab supports are attached to construction likely to be damaged by large deflections. Deflection limits are set by the ACI Code, Table 24.2.2.

3- It is preferable to choose slab depth to the nearest 10 mm.

4. Shear should be checked, although it does not usually control.

5-Concrete cover in slabs shall not be less than (20 mm) at surfaces not exposed to weather

or ground for bar dia. 36 mm and less, while concrete cover not less than 40 mm for bar greater than 36 mm in dia. ACI table 20.6.13.1

6-In structural one way slabs of uniform thickness, the minimum amount of reinforcement (As min. in the direction of the span shall not be less than that required for shrinkage and temperature

reinforcement (ACI Code, Sections 7.6.1 and 24.4.3).

7. The main reinforcement maximum spacing shall be the lesser of three times the slab thickness (3 * h) and 450 mm . (ACI Code, Section 7.7.2.3).

8- In addition to main reinforcement, steel bars at right angles to the main must be provided. this additional steel is called secondary, distribution, shrinkage, or temperature reinforcement.

Temperature and shrinkage reinforcement .

The minimum reinforcement should be equal or greater than: ACI 2019 24.4.3.2

$$\rho_{min} = 0.0018$$

Max spacing of reinforcement should be greater than

1. $5 * h$ $(h = slab\ thickness)$
2. $450\ mm$

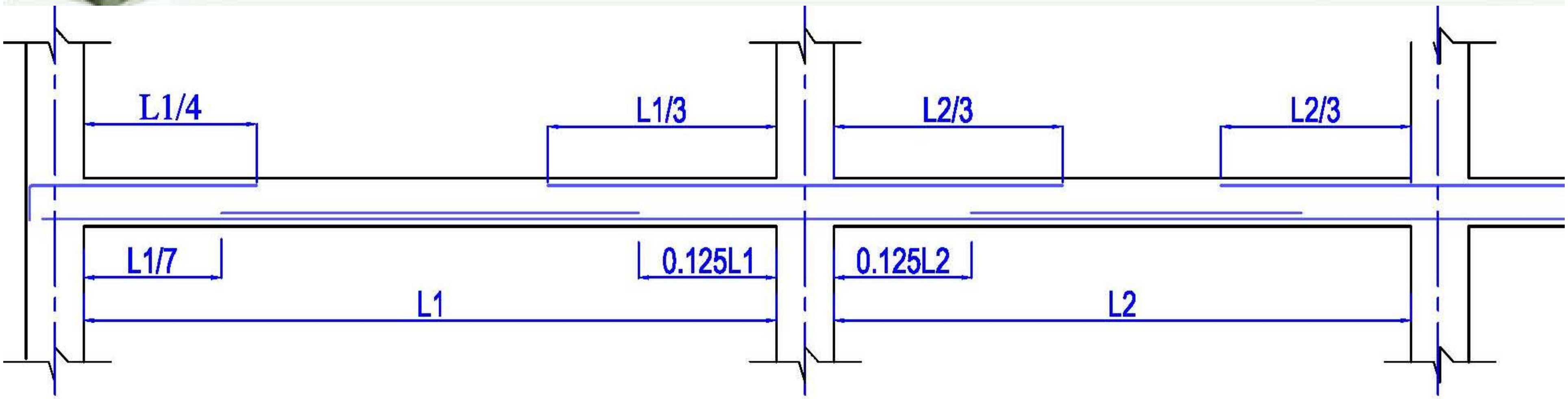
Choose the smaller of above value

The width of slab strip= 1000 mm

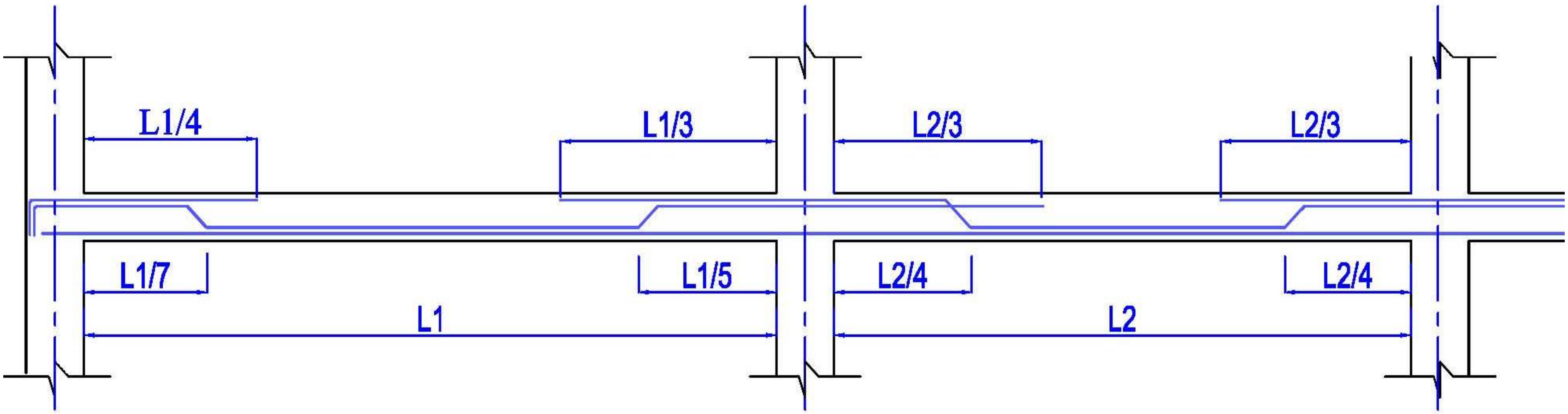
$As_{min} = \rho_{min} \times b \times h = 0.0018 \times b \times h$ minimum shrinkage and temperature steel

Reinforcement Details

In continuous one-way slabs, the steel area of the main reinforcement is calculated for all critical sections, at midspans, and at supports. There is two reinforcement system can be applied
1- Straight-bar.
2-Bent-bar, or trussed system.
straight and bent bars are placed alternately in the floor slab.
The location of bent points should be checked for flexural, shear, and development length requirements.

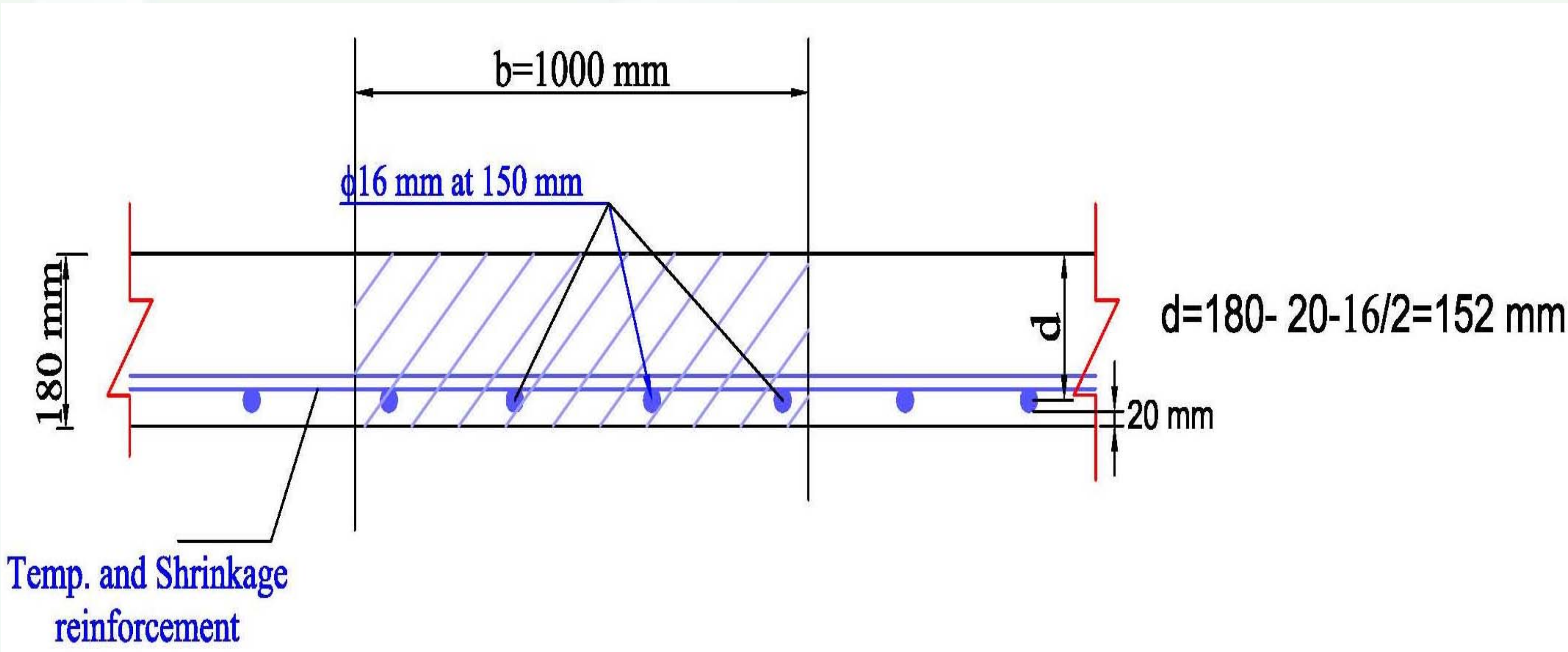


Straight Bar



Bent Bar

Example (1):calculate the design moment of one way solid slab that has a total depth of h=180 mm and is reinforced with 16 mm diameter bar spaced s =150 mm, used fc =21 MPa , fy = 420 MPa

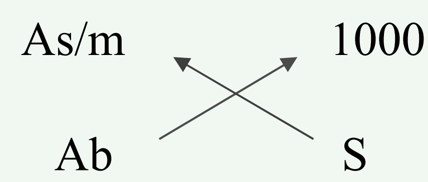


Sol.

$$d = h - \text{concrte cover } (20) - \frac{\emptyset}{2}$$

$$d = 180 - 20 - \frac{16}{2} = 152mm$$

$Taking\ width\ strip\ =\ 1000\ mm$



$$As/m = 1000 \times \frac{Ab}{S}$$

$A_b = 201\ mm^2$ (Ab for bar diameter = 16 mm)

$$As/m = 1000 \times \frac{Ab}{s} = 1000 \times 201/150 = 1340\ mm^2$$

Check $As_{min} = \rho_{min} \times b \times h = 0.0018 \times 1000 \times 180 = 270\ mm^2$

$$m = \frac{fy}{0.85f'c} = \frac{420}{0.85 \times 21} = 23.53$$

claculate $\rho_b = \frac{\boldsymbol{\beta}_1}{m}\left(\frac{600}{600+fy}\right)\left(\frac{dt}{d}\right) = \frac{0.85}{23.53}\left(\frac{600}{600+420}\right)(1) = 0.02127$

and calculate $\rho_{max} = \left(\frac{0.003+\frac{fy}{Es}}{0.008}\right)\rho_b = \left(\frac{0.003+\frac{420}{200000}}{0.008}\right) \times 0.02127 = 0.01355$

$As_{max} = \rho_{max} \times b \times d = 0.01355 \times 1000 \times 152 = 2059mm^2$

$As/m = 270\ mm^2 < As/m = 1340\ mm^2 < Asmax = 2059\ mm^2$

Tension Control $\emptyset = 0.9$

-Section Capacity ϕMu

$C = T$

$0.85 f'c\ a\ b = As\ fy$

$a = \dfrac{1340 \times 420}{0.85 \times 21 \times 1000} = 31.5\ mm$

$\phi Mu = 0.8\ As \times fy\ \left(d - \dfrac{a}{2}\right)$

$= 0.9 \times 1370 \times 420 \times \left(152 - \dfrac{31.5}{2}\right)$

$= 69\ KN.m/m$

Or

$Mu = \phi Rbd^2$

$R = \rho fy(1 - 0.5\rho m)$

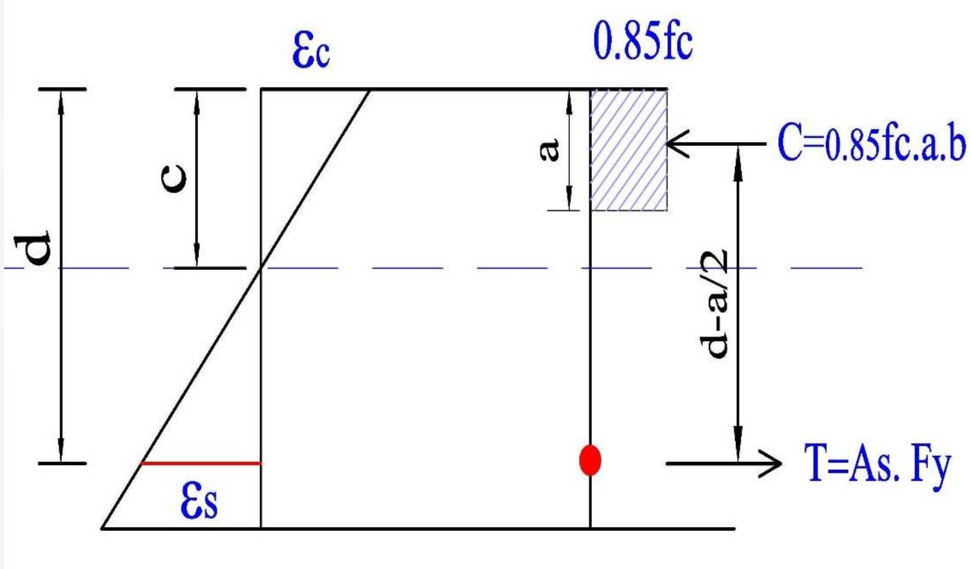
$\rho = \dfrac{As}{bd} = 0.00882$

$R = 0.00882 \times 420 \times (1 - 0.5 \times 0.00882 \times 23.53)$

$= 3.32$

$Mu = \phi Rbd^2 = 0.9 \times 3.32 \times 1000 \times 152^2$

$Mu = 69\ KN.m/m$



Example (2):Determine the allowable uniform live load of example (1) that can be applied on the simply supported slab with span 4.9 m and carries a uniform dead load (excluding self weight) of 4.8 KN/m² .

Sol.

$\phi M_n = 69\ KN.m/m$ (*example* (1))

$$Mu = \phi\, Mn = \frac{w_u \times L^2}{8}$$

$$69 = \frac{Wu \times 4.92}{8} \qquad \therefore W_u = 23\ KN/m^2$$

$$W_u = 1.2\ D.L + 1.6\ L.L$$

Self weight of slab $= h\,.b.1.\gamma c = 0.18 \times 1 \times 1 \times 24 = 4.32\ KN/m^2$

$$23 = 1.2 \times (4.32 + 4.8) + 1.6 \times W_L$$

$$W_L = 7.54\ KN/m^2$$

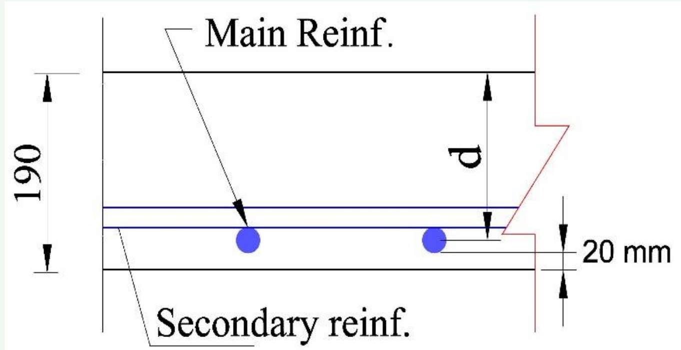
Example (3): Design a 3.65 m simply supported slab to carry a uniform dead load (excluding self weight) of 5.75 KN/m^2 and a uniform Live load of 4.8 KN/m^2 , normal concrete , fc =21 Mpa, fy= 420 Mpa.

Sol.

❖ *Minimum Slab thickness, fy = 420 Mpa and simply supported slab*

$$h = \frac{L}{20} = \frac{3650}{20} = 182.5\ mm \quad (ACI\ code\ Table\ 7.3.1.1)$$

$Use\ h = 190\ mm$



❖ *Applied load*

$$Wu = 1.2\ DL + 1.6\ WL$$

$$Self\ weight\ of\ slab = 0.19 \times 1 \times 1 \times 24 = 4.56\ KN/m^2$$

$$Wu = 1.2 \times (4.56 + 5.75) + 1.6 \times 4.8 = 20.05\ KN/m/m$$

For 1 m strip width

$$Mu = wu\frac{L^2}{8} = 20.05 \times \frac{3.652}{8} = 33.39 KN.m/m$$

$$d = h - 20 - \frac{\phi}{2} = 190 - 20 - \frac{12}{2} = 164\ mm \qquad (use\ \phi = 12\ mm\ Ab = 113\ m2)$$

$$m = \frac{fy}{0.85 f'c} = 23.53$$

$$Mu = \phi\ Rbd^2$$

$$R = \frac{Mu}{\phi\ bd^2} \qquad assume\ Tension\ control, use\ \phi = 0.9$$

$$R = \frac{33.39 \times 10^6}{0.9 \times 1000 \times 164^2} = 1.379$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right)$$

$$\rho = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2 \times 23.53 \times 1.379R}{420}}\right) = 0.00342$$

$$As/m \;=\; \rho . b . d = \; 0.00342 \times 1000 \times 164 = 561\, mm^2/m$$

$$As \text{ min.} = \; 0.0018 \times b \times h = \; 0.0018 \times 1000 \times 190 \; = 342\, mm^2/m$$

$$calculate \;\; \rho_b \; = \; \frac{\boldsymbol{\beta}_1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right) = \frac{0.85}{23.53}\left(\frac{600}{600 + 420}\right)(1) = 0.02127$$

$$\text{and calculate} \;\; \rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.008}\right)\rho_b = \left(\frac{0.003 + \frac{420}{200000}}{0.008}\right) \times 0.02127 = 0.01355$$

$$As_{\text{ max.}} = \; 0.01355 \times 1000 \times 164 = \; 2222\, mm^2/m \;\; > \; As/m \; = 561\, mm^2 \qquad (\, OK)$$

$$Ab \; = \; 113\, mm^2 \qquad S = \; 1000 \times \frac{113}{561} = 201\, mm$$

❖ *Check maximum spacing* $\qquad 3 \times hf \; = \; 3 \; \times 190 = \; 570\, mm \;\; or \;450\, mm$

$$use\; S = \; 190 \;\; mm\; c/c$$

Secondary steel (shrinkage and temperature reinforcement)

$\rho_{min} = 0.0018$

$As\ \min = 0.0018 \times b \times h = 00.0018 \times 190 \times 1000 = 342\ mm^2$

$If\ use\ \phi\ 10mm \quad Ab = 78\ mm^2$

$$S = \frac{1000 \times 78}{342} = 228\ mm^2/m < 5 \times hf = 5 \times 190 = 950\ mm < 450\ mm \qquad (OK)$$

$Use\ \phi 10\ mm\ at\ 220\ mm\ c/c$

$Check\ the\ shear\ requirement$

$$V_u = \frac{Wu \times L}{2} = 20.05 \times 3.65 = 36.59\ KN/m$$

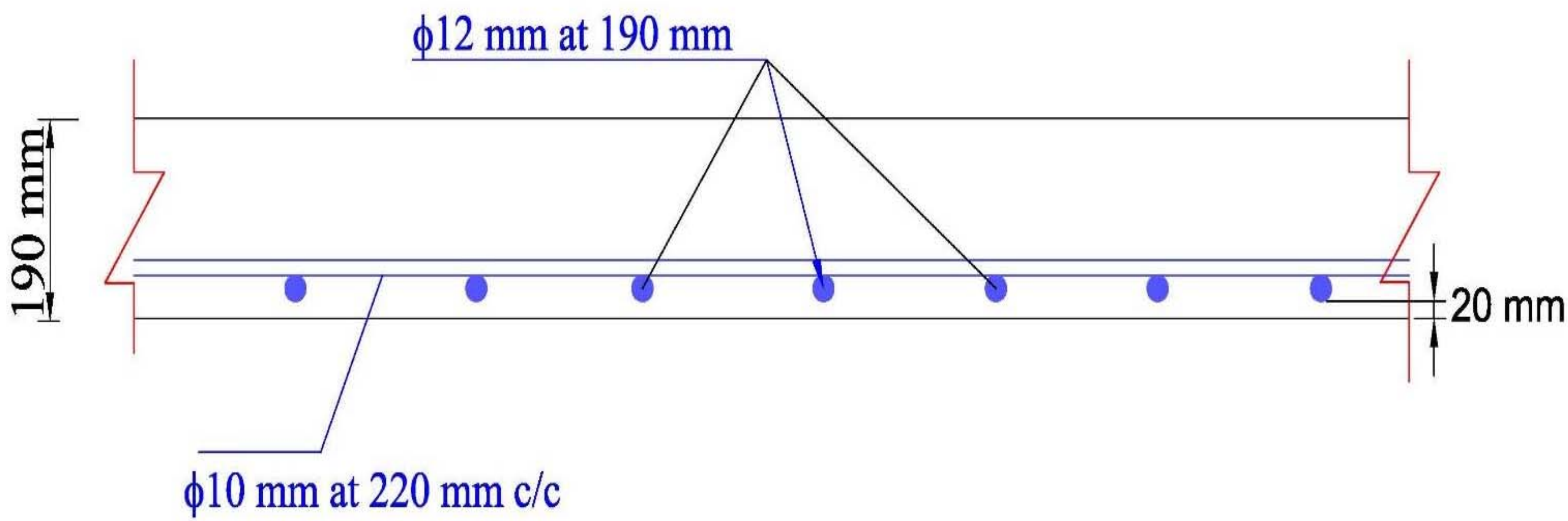
The critical section at d distance from the face of support

$V_{ud} = Vu - w_u \times d$

$= 35.59 - 20.05 \times 0.164 = 33.3\ KN/m$

$$\phi\ Vc = \phi \times (0.17\sqrt{f'c}\ b.d) = 0.75 \times 0.17 \times \sqrt{21} \times 1000 \times 164 = 95.82\ KN/m$$

$Vud < \phi\ Vc \qquad (OK)$


ϕ12 mm at 190 mm
190 mm
20 mm
ϕ10 mm at 220 mm c/c

Thank You

Example
For Design Floor system One Way Slab

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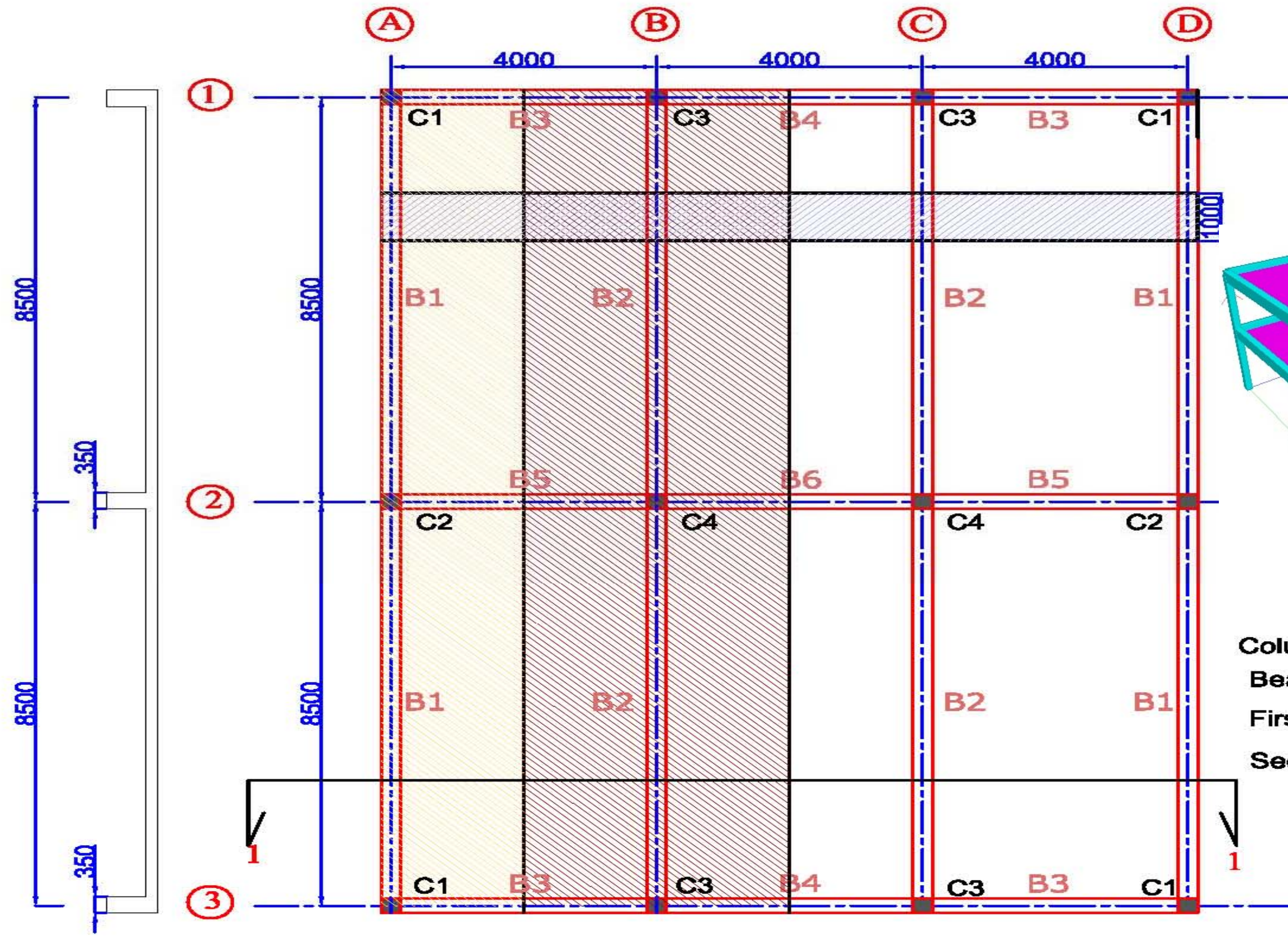
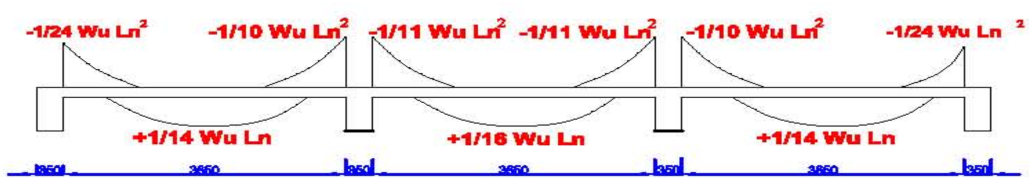


Column Dim. =350 x350 mm

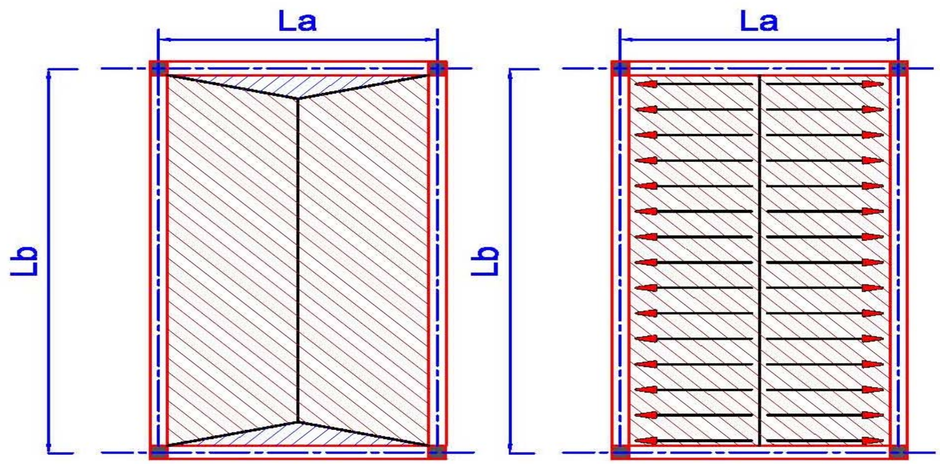
Beam width. =350 mm

First Floor height. =4 m

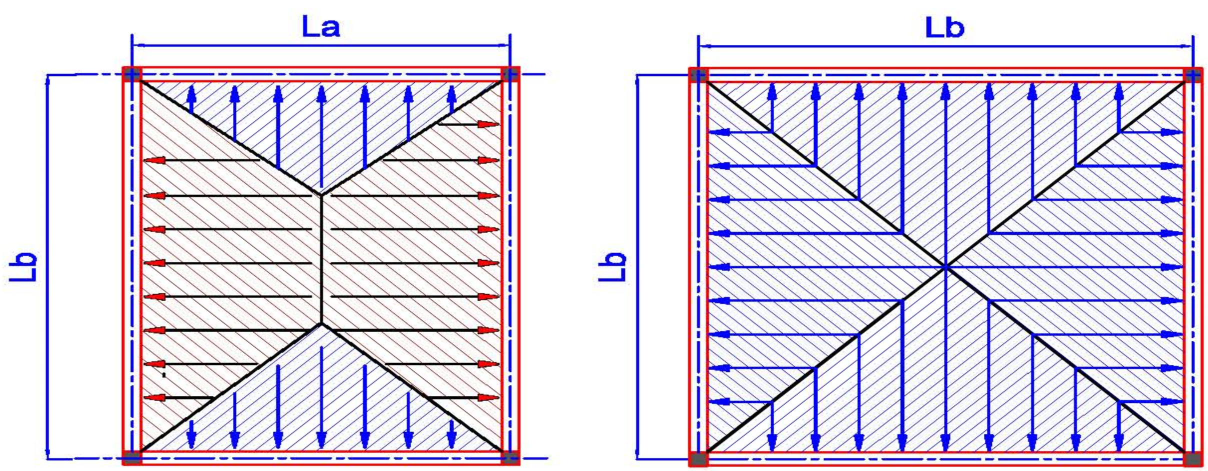
Second Floor height. =3.2 m



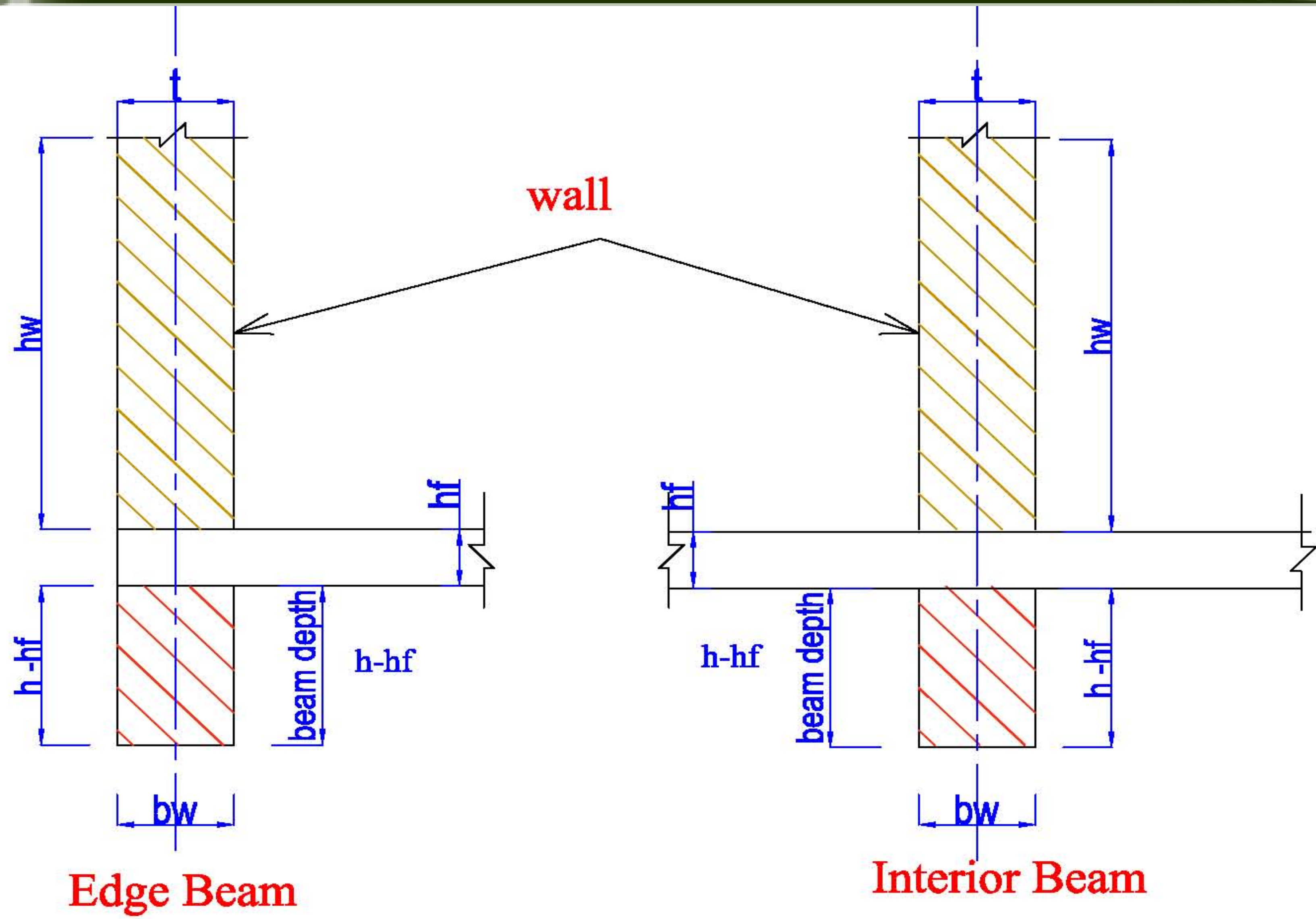
Section 1-1

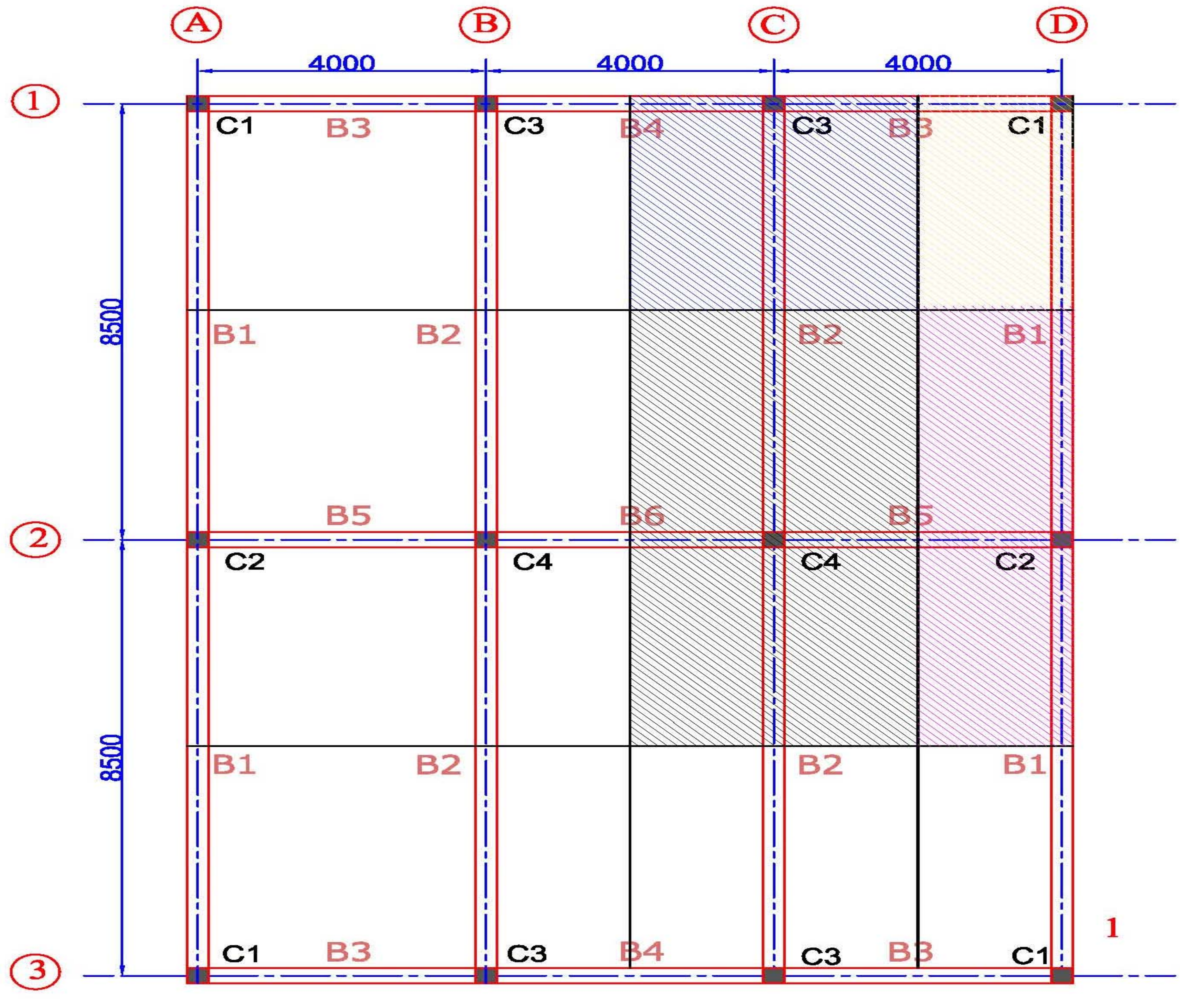


One way slab
Lb/La >= 2



Two way slab
Lb/La < 2


t
t
wall
hw
hw
hf
hf
h -hf
beam depth
h-hf
h-hf
beam depth
h -hf
bw
bw
Edge Beam
Interior Beam


A
B
C
D
4000
4000
4000
1
2
3
8500
8500
C1
B3
C3
B4
C3
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C1
B1
B2
B2
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B5
B6
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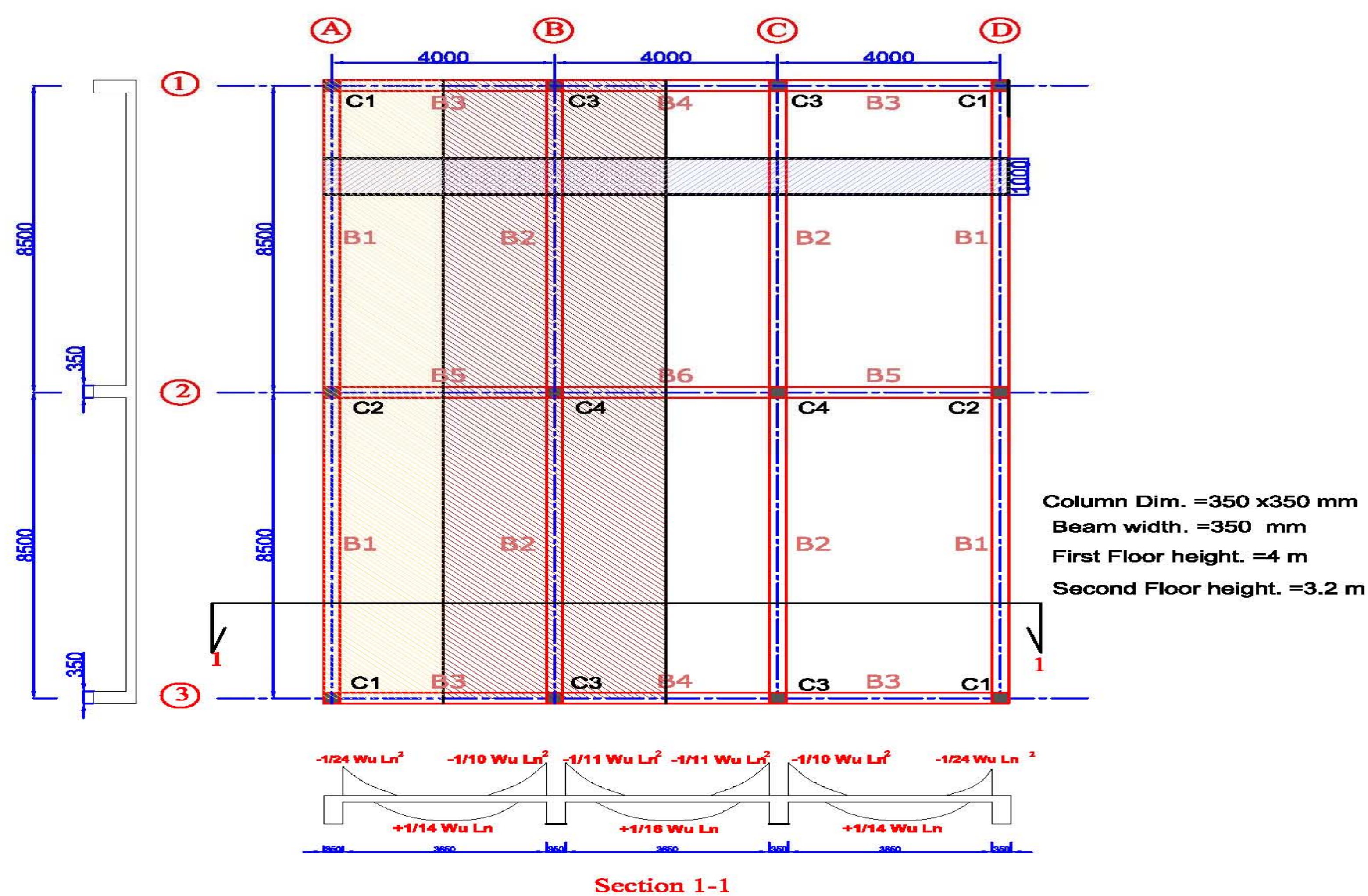
C2
C4
C4
C2
B1
B2
B2
B1
C1
B3
C3
B4
C3
B3
C1

Example (4): Design the one way slab system shown in fig below , subjected to the superimposed dead load 2 KN/m^2 and live load 3 KN / m^2 assume normal concrete , f'c =21 Mpa, and fy= 280 Mpa.



Sol.

❖ Minimum Slab thickness, (assume one end continuous) and $Fy = 280\ Mpa$

$h = L/24 = 4000/24 = 166.6\ mm$ (*ACI code Table 7.3.1.1*)

$Use\ h = 170\ mm$

$use\ \phi = 12\ mm\ Ab = 113\ m2$

$$d = h - 20 - \frac{\phi}{2} = 170 - 20 - \frac{12}{2} = 144\ mm$$

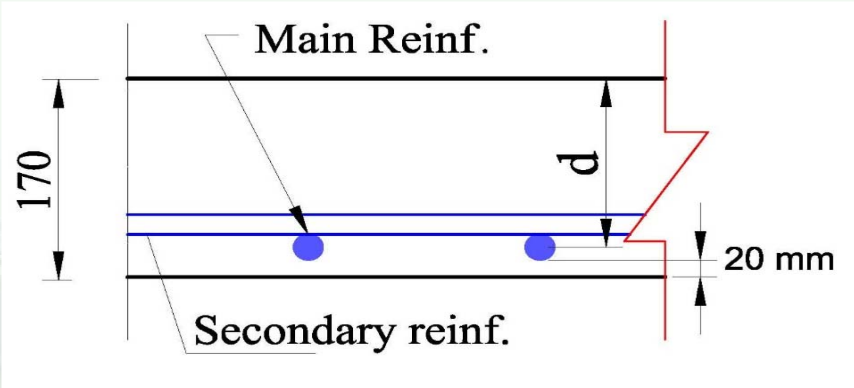
❖ ***Applied load***

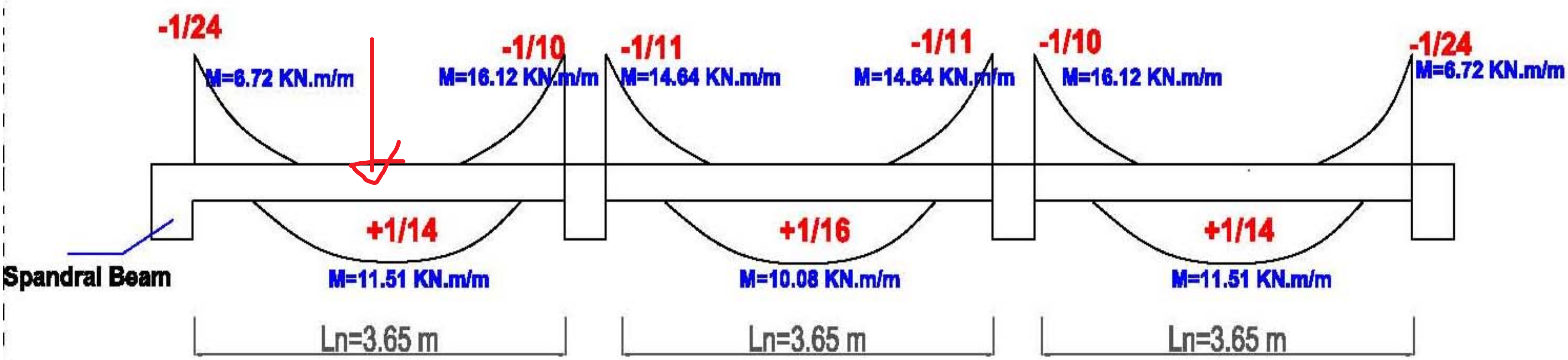
$Wu = 1.2\ DL + 1.6\ WL$

Self weight of slab $= 0.17 \times 1 \times 1 \times 24 = 4.08\ KN/m^2$

$Wu = 1.2 \times (4.08 + 2) + 1.6 \times 3 = 12.1\ KN/m/m$

$$m = \frac{fy}{0.85 f'c} = \frac{280}{0.85 \times 25} = 15.69$$





❖ *Moment Calculation*

−External span

$1 - \text{External Negative Moment} = -M = \left(-\frac{1}{24}\right) Wu\, ln^2 = \left(-\frac{1}{24}\right) \times 12.1 \times 3.65^2 = 6.72\ KN.m/m$

$2 - \text{Positive Moment} = +M = \left(-\frac{1}{14}\right) Wu\, ln^2 = \left(\frac{1}{14}\right) \times 12.1 \times 3.65^2 = 11.51\ KN.m/m$

$3 - \text{Internal Negative Moment} = -M = \left(-\frac{1}{10}\right) Wu\, ln^2 = \left(-\frac{1}{10}\right) \times 12.1 \times 3.65^2 = 16.12\ KN.m/m$

−Interior span

$1 - \text{Internal Negative Moment} = -M = \left(-\frac{1}{11}\right) Wu\, ln^2 = \left(-\frac{1}{11}\right) \times 12.1 \times 3.65^2 = 14.65\ KN.m/m$

$2 - \text{Positive Moment} = +M = \left(-\frac{1}{16}\right) Wu\, ln^2 = \left(\frac{1}{16}\right) \times 12.1 \times 3.65^2 = 10.08\ KN.m/m$

$3 - \text{Internal Negative Moment} = -M = \left(-\frac{1}{11}\right) Wu\, ln^2 = \left(-\frac{1}{11}\right) \times 12.1 \times 3.65^2 = 14.65\ KN.m/m$

No.	Details	External Span			Internal Span	
		-M Exterior supp.	+ M Mid Span	- M Interior supp.	-M Exterior supp.	+M Exterior supp.
1	Mu * 10^6 (N.mm)	6.72	11.51	16.12	14.65	10.08
2	b (mm)	1000	1000	1000	1000	1000
3	d (mm)	144	144	144	144	144
4	$R=Mu/(\phi bd^2)$	0.36	0.617	0.863	0.785	0.54
5	$\rho=1/m(1-(1-\sqrt{1-2mR/fy})$	0.001299	0.002243	0.00316		0.001959
6	As =ρ.b.d (mm^2)	187	323	456	413	282
7	As min = ρ.b.h (${mm}^2$) ρmin = 0.0018	306	306	306	306	306
8	As Provided (choosed)	306	306	456	413	306
9	S= 1000*Ab/As (mm)	369	369	247	273	369
10	Smax= 3*h=510 0r 450 mm	450	450	450	450	450
11	S (choosed)	369	369	247	273	369
12	Used Spacing S (use ϕ12 mm)	360	360	240	270	360

Due to $fy = 280\ MPa \quad (from\ ACI\ code\ 24.4.3.2)$

$\rho_{min} = 0.0018$

As shrinkage and Temperature $= \rho_{min} \times b \times h = 0.018 \times 1000 \times 170 = 306$ mm²/m

Use ϕ 10 mm ($Ab = 78\ mm^2$)

$$S = \frac{1000 \times 78}{288} = 229\ mm$$

Check Maximum spacing for shrinkage and temperature steel

$S_{max} = 5 \times h = 5 \times 170 = 850\ mm \quad or \quad 450 < s_{max}$

$Use\ \phi\ 10\ mm\ \ 220mm\ c/c$

$S_{\ max} = 5 \times h = 5 \times 170 = 850\ mm\ or\ 450 < s\ max$

Check for Shear:

$$Vu = 1.15\frac{W_u L_n}{2}$$

$$Vu = 1.15 \times 12.1 \times \frac{3.65}{2} = 25.39\ KN/m$$

$$Vu,d = Vu - wu \times d = 25.39 - 12.1 \times 0.144 = 23.65 KN/m$$

$$\phi\,Vc = \phi \times (0.17\sqrt{f'c}\,b.d) = 0.75 \times 0.17 \times \sqrt{21} \times 1000 \times 144 = 84.12\ KN/m$$

$$Vud < \phi\,Vc \qquad (\,OK\ section\ safe\ for\ shear.)$$

Reinforcement Details

1- Bent Bar

A-Additional steel at exterior support (-M)

If we assume that 50% of positive steel will bent then:

As provided will be ϕ12 at 660 mm c/c

$$As/m = \frac{1000 \times 113}{660} = 171\ mm^2/m$$

As (required)/m =340 mm^2/m

then As additional = $As_{req.} - As_{provided} = 340 - 171 = 169\ mm^2/m$

$$S = \frac{1000 \times 113}{169} = 668mm \quad use = \quad 12@660\ mm\ c/c$$

B-Additional steel at Interior support (-M)

If we assume that 50% of positive steel will bent then:

As provided will be ϕ12 at 660 mm c/c from left side :

$$As/m = \frac{1000 \times 113}{660} = 171\, mm^2/m$$

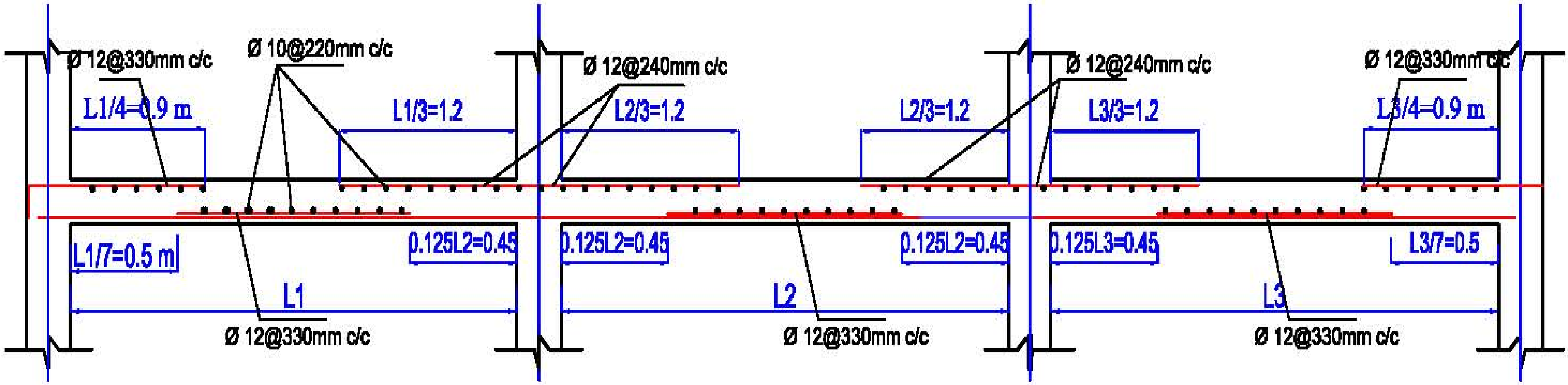
So the Total As provided from both side = 2×171 = 342 mm^2/m

As (required)/m =456 mm^2/m

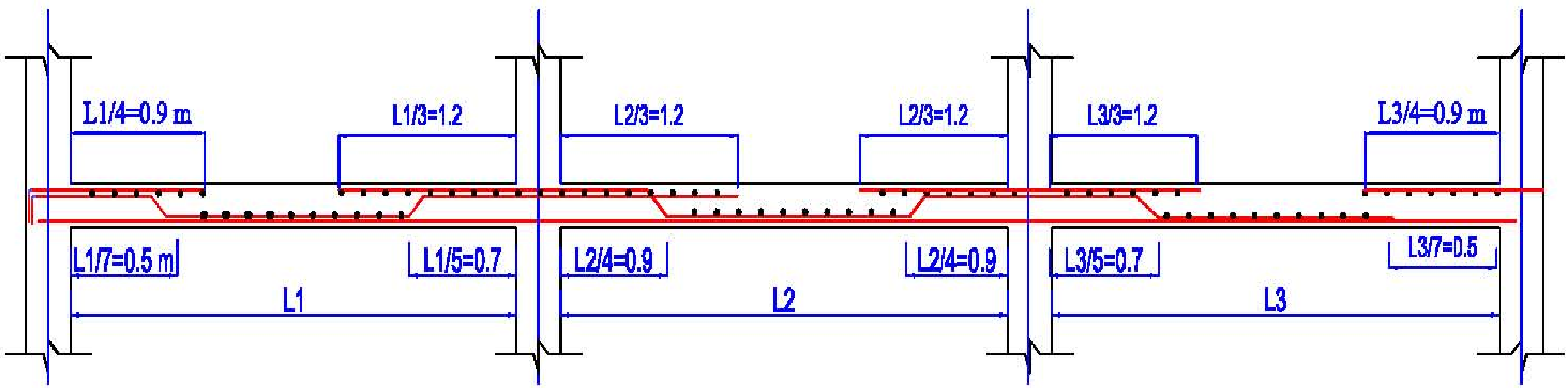
then As additional = As req. – As provided = 456-342=114 mm^2/m

$$S = \frac{1000 \times 113}{114} = 991mm$$

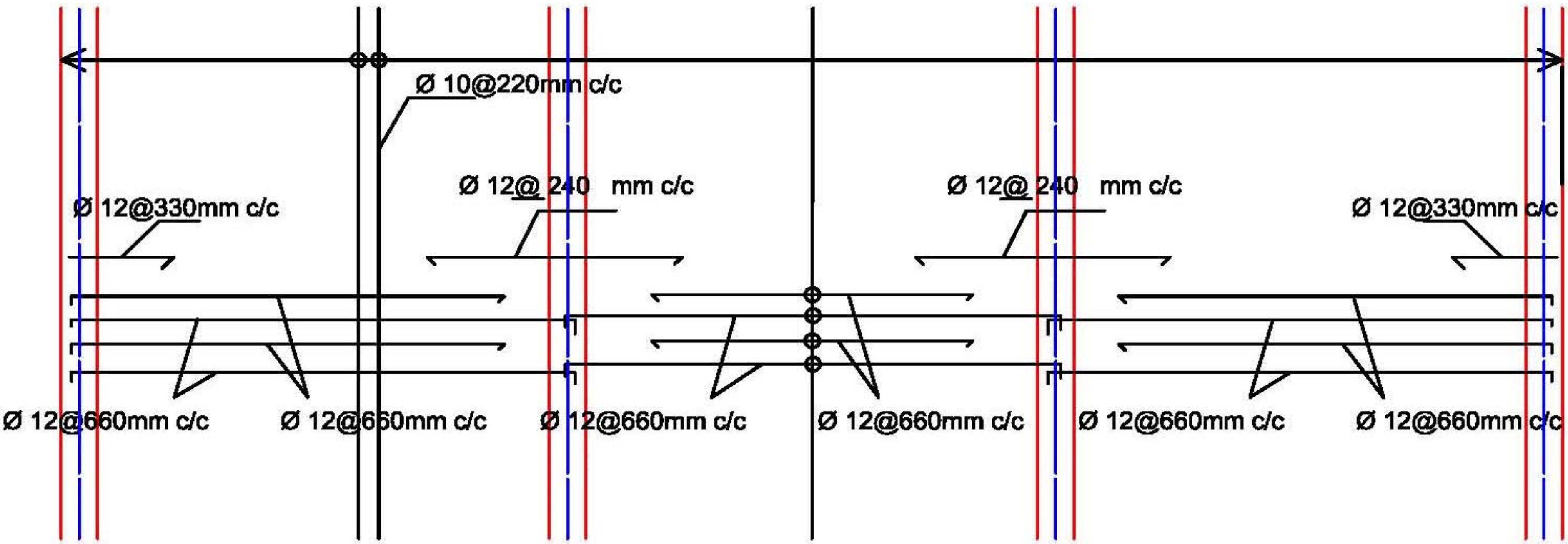
use = ϕ12@990 mm c/c

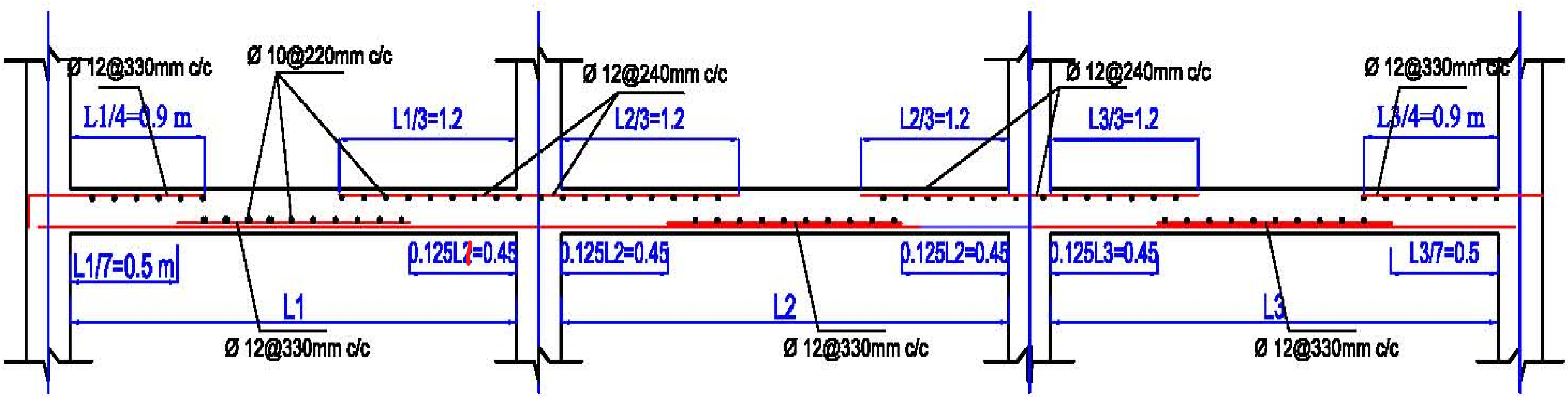


Straight Bar

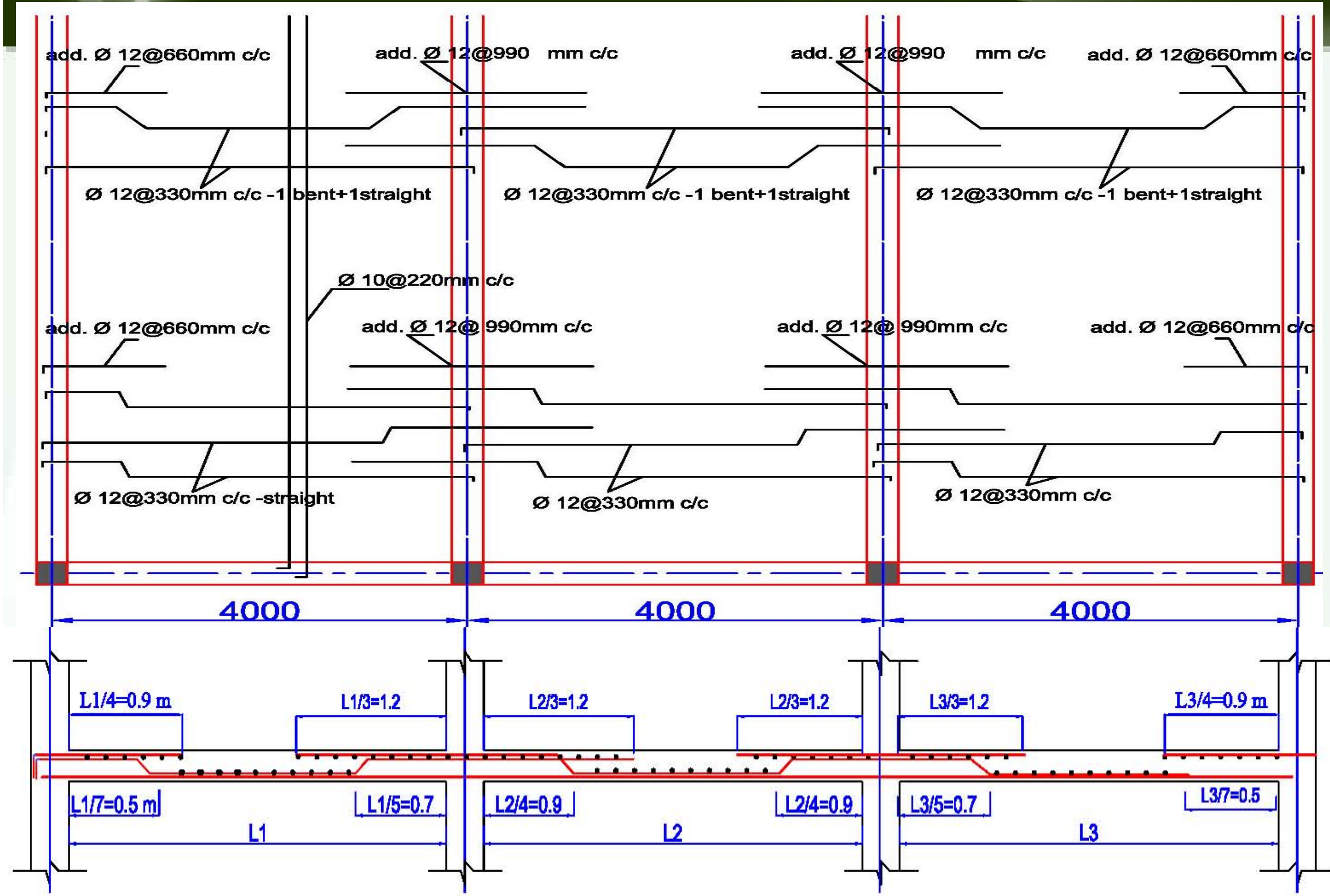


Bent Bar

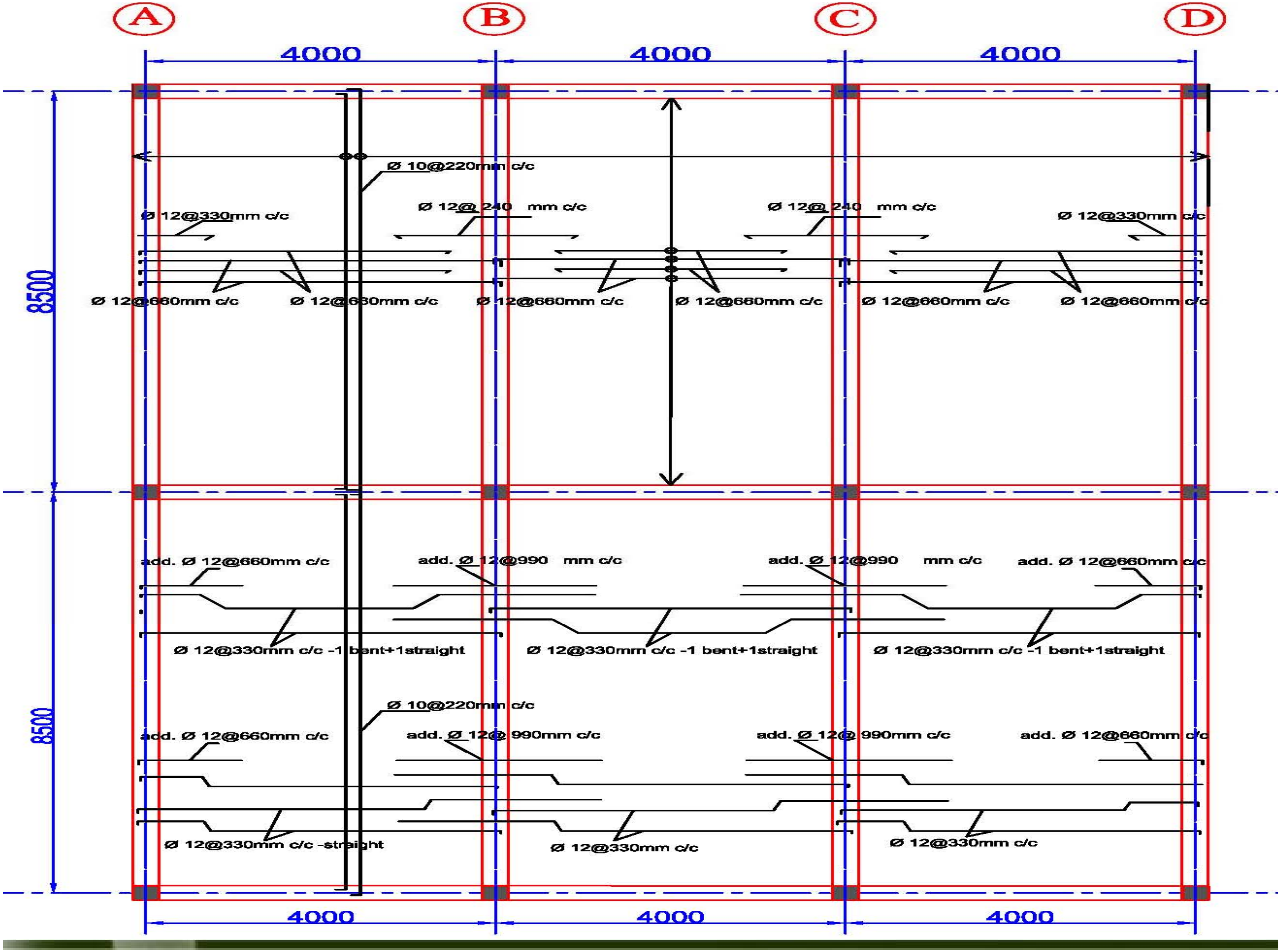




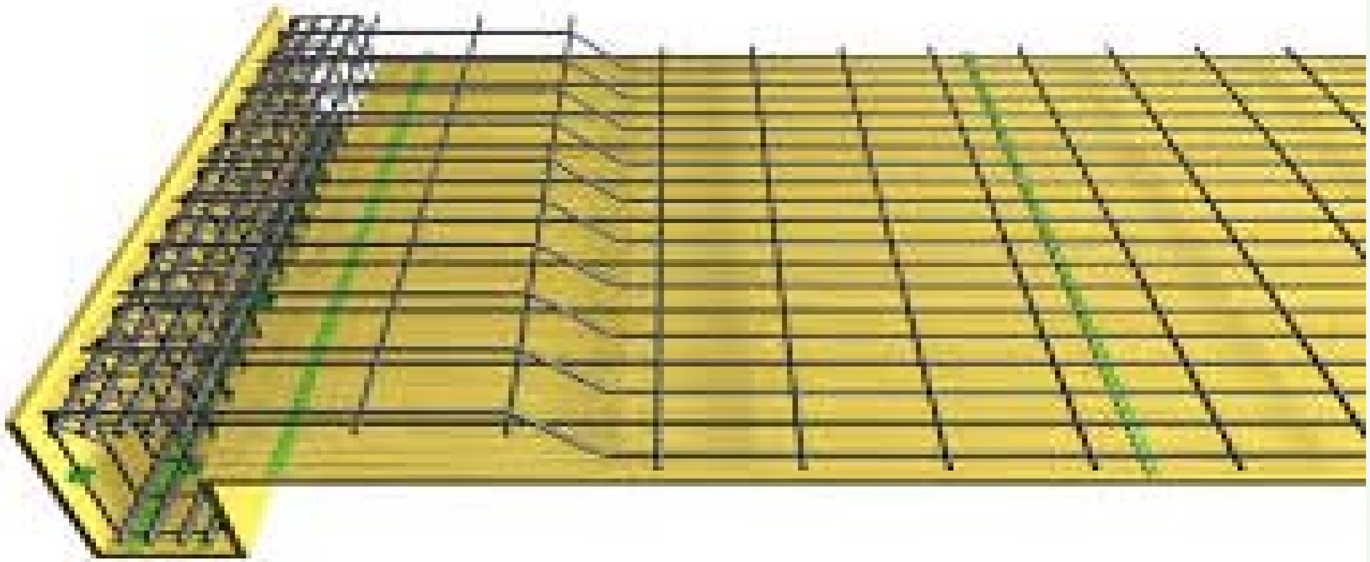
Straight Bar

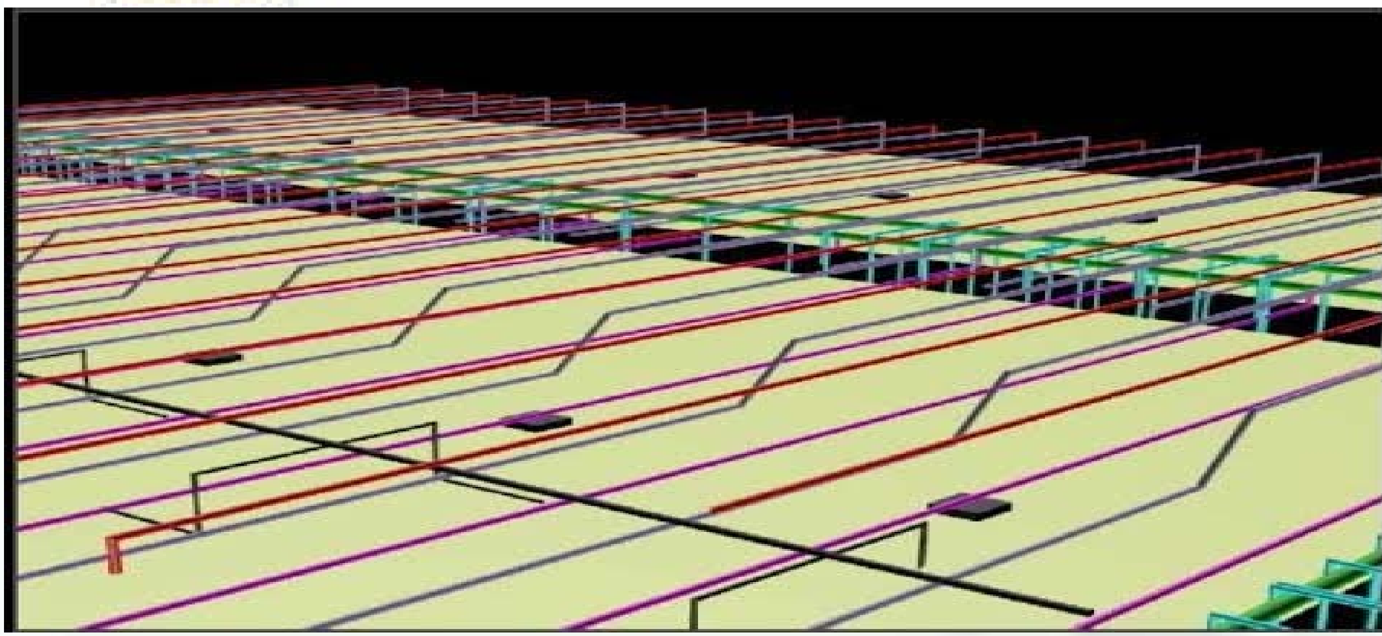

add. Ø 12@660mm c/c
add. Ø 12@990 mm c/c
add. Ø 12@990 mm c/c
add. Ø 12@660mm c/c
Ø 12@330mm c/c -1 bent+1straight
Ø 12@330mm c/c -1 bent+1straight
Ø 12@330mm c/c -1 bent+1straight
Ø 10@220mm c/c
add. Ø 12@660mm c/c
add. Ø 12@ 990mm c/c
add. Ø 12@ 990mm c/c
add. Ø 12@660mm c/c
Ø 12@330mm c/c -straight
Ø 12@330mm c/c
Ø 12@330mm c/c
4000
4000
4000
L1/4=0.9 m
L1/3=1.2
L2/3=1.2
L2/3=1.2
L3/3=1.2
L3/4=0.9 m
L1/7=0.5 m
L1/5=0.7
L2/4=0.9
L2/4=0.9
L3/5=0.7
L3/7=0.5
L1
L2
L3

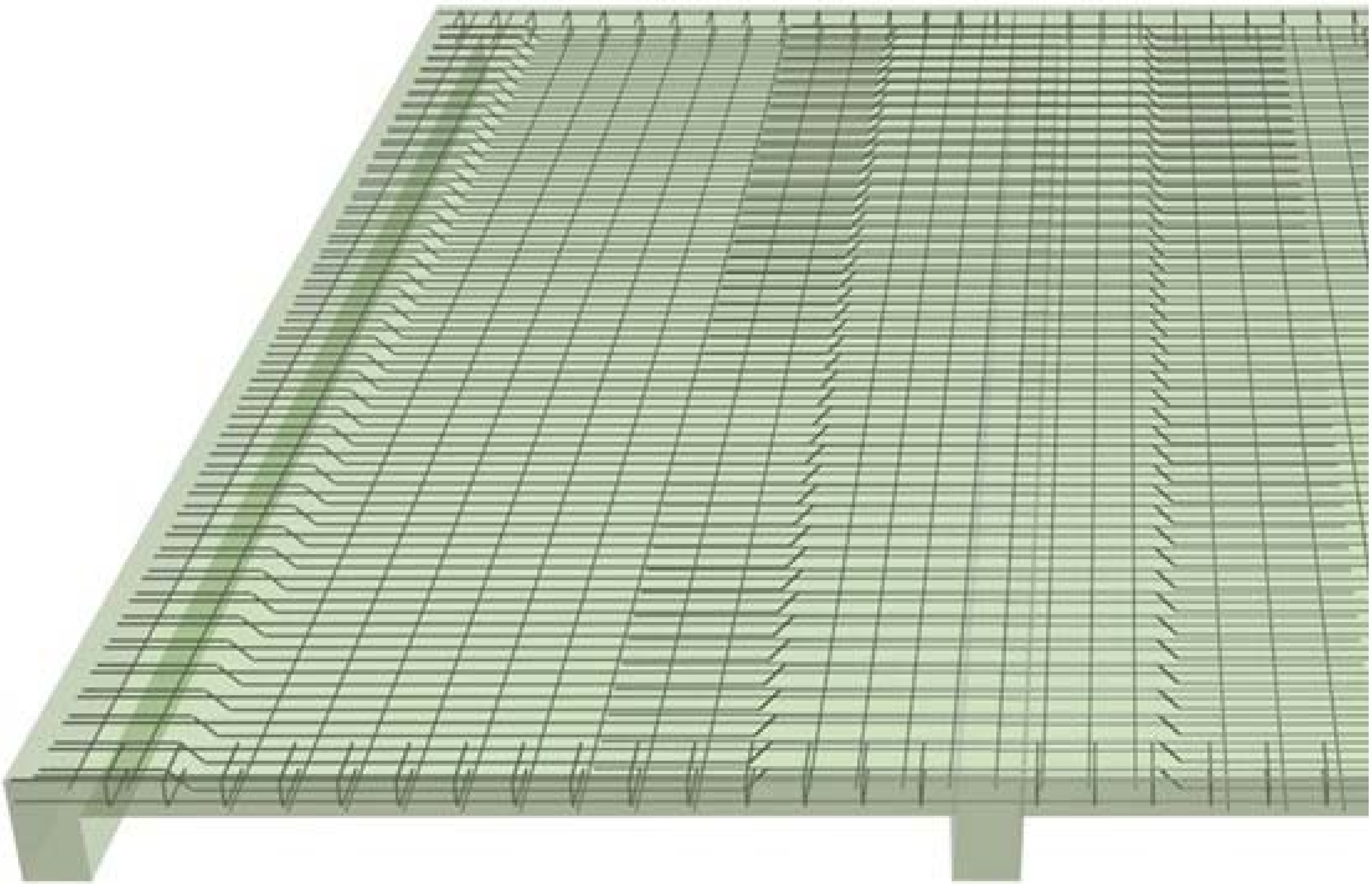
Bent Bar



A
B
C
D
4000
4000
4000
8500
8500
Ø 10@220mm c/c
Ø 12@330mm c/c
Ø 12@ 240 mm c/c
Ø 12@ 240 mm c/c
Ø 12@330mm c/c
Ø 12@660mm c/c
Ø 12@660mm c/c
Ø 12@660mm c/c
Ø 12@660mm c/c
Ø 12@660mm c/c
Ø 12@660mm c/c
add. Ø 12@660mm c/c
add. Ø 12@990 mm c/c
add. Ø 12@990 mm c/c
add. Ø 12@660mm c/c
Ø 12@330mm c/c -1 bent+1straight
Ø 12@330mm c/c -1 bent+1straight
Ø 12@330mm c/c -1 bent+1straight
Ø 10@220mm c/c
add. Ø 12@660mm c/c
add. Ø 12@ 990mm c/c
add. Ø 12@ 990mm c/c
add. Ø 12@660mm c/c
Ø 12@330mm c/c -straight
Ø 12@330mm c/c
Ø 12@330mm c/c
4000
4000
4000



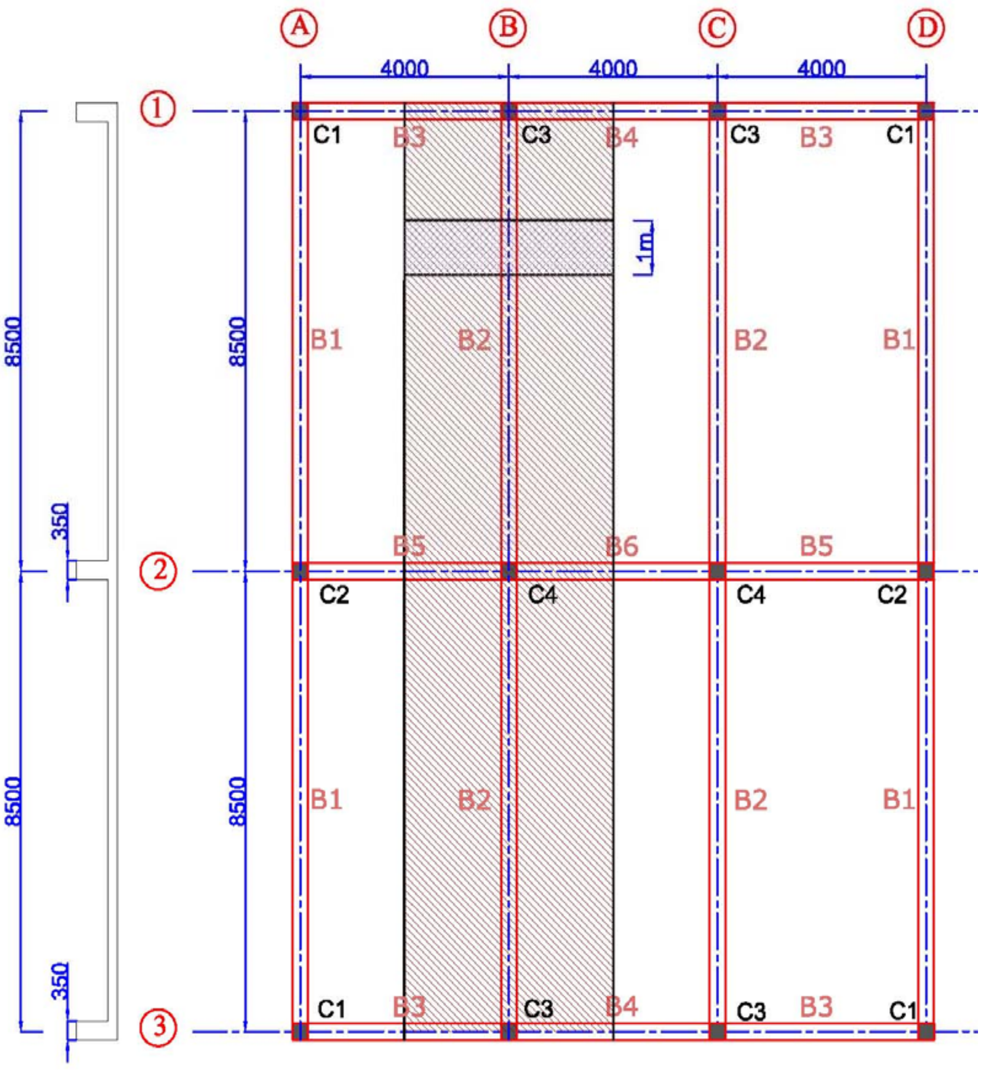




Thank You

. . . .

Example (5): From previous Ex. (4) Design the interior beams shown in fig below , where the slabs is subjected to the superimposed dead load 2 KN/m^2 and live load 3 KN / m^2 , normal concrete , f'c =21 mPa, and fy= 280 mPa.



Sol.

1- Interior Beam

-Self weight of drop part of beam =?

Assume Wu (self weight= 5 KN/m) (check Later)

$Wu = 1.2WD + 1.6\,WL$

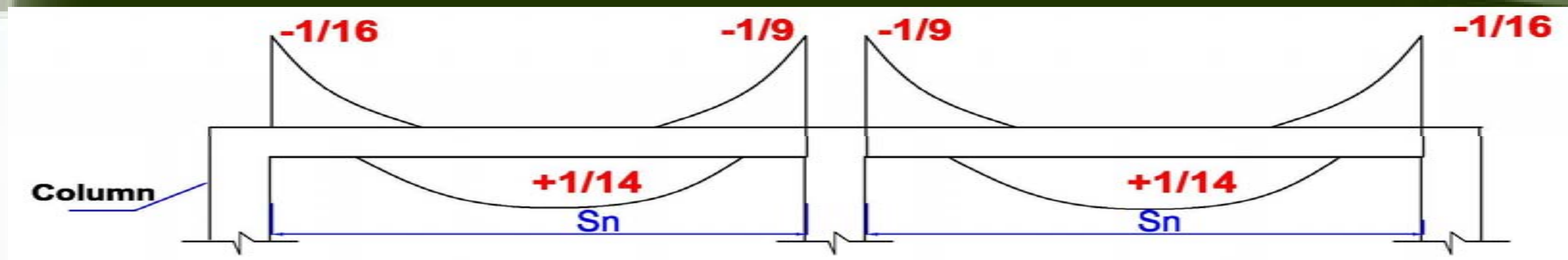
$Wu\ slab = 12.1KN/m^2$

$Wu\ (Load\ on\ beam\ from\ slab/m)$

$= 12.1 \times 4 = 48.4KN/m$

$Wu\ (\ on\ beam\ )\ =\ 1.2\ \times\ 5 + 48.4 = 54.4\ KN/m$

-Calculate Moments (Using ACI code coefficient)



$M = Cf\ Wu\ Sn^2$ $\qquad$ $Sn = 8.5 - 0.35 = 8.15m$

$Negative\ M\ at\ exterior\ support = \left(-\frac{1}{16}\right) \times 54.4 \times 8.152 = 225.8\ KN.m$

$Positive\ M\ at\ mid\ span = \left(\frac{1}{14}\right) \times 54.4 \times 8.152 = 258.1\ KN.m$

$Negative\ M\ at\ interior\ support = \left(-\frac{1}{9}\right) \times 54.4 \times 8.152 = 401.49\ KN.m$

<u>*Flexural Design*</u>

$m = \frac{fy}{0.85f'c} = \frac{280}{0.85 \times 21} = 15.69$

$\rho b =$ $\quad$ (*or you can assume* $\rho = 0.5\ \rho max$)

$\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right)$ $\qquad$ $\left(\frac{dt}{d}\right) = 1$

$= \frac{0.85}{15.69} \times \left(\frac{600}{600 + 280}\right) = 0.03693$

$used = 0.5\ \rho\ b = 0.01846$

$R = \rho\ fy(1 - 0.5\ \rho\ m) = 0.01846 \times 280 \times (1 - 0.5 \times 0.01846 \times 15.69) = 4.42$

$Mu = \phi\ Rbd^2$

$$d^2 = \frac{Mu}{\phi\, R\, b} = 401.49 \times 10^6/(0.9 \times 4.42 \times 350)$$

$d = 537\ mm$

$h = 537 + 90 = 627\ mm \quad (two\ layer\ of\ steel)$

$Use\ b \times h = 350 \times 630\ mm$

$Check\ the\ self\ weigh\ of\ beam$

$W_D = 1.2\, Wo\ (self\ Wt.)$

$W_D(self\ wt.) = 1.2 \times 0.35 \times (0.63 - 0.17) = 4.64\ KN/m$

$So\ the\ correct\ Wu = 48.4 + 4.64 = 53.04 KN/m$

Check The ACI code requirement for Minimum Depth of Beam (deflection Control) *ACI Table 9.3.1.1*

Simply supported = L/16

One end Continuous = L/18.5

Both end Continuous =L/21

Cantilever =L/8

If fy not equal 420 MPa then h min. shall be multiplied by factor =(0.4+fy/700) for normal concrete

-And if we use lightweight concrete ((14.4 to 18.4 KN/m^3*) the above value of h shall be multiplied with greater of :*

1- 1.65-0.0003γc *(*γc*= concrete unit weight)*

2- 1.09

In this example the span is one end continuous

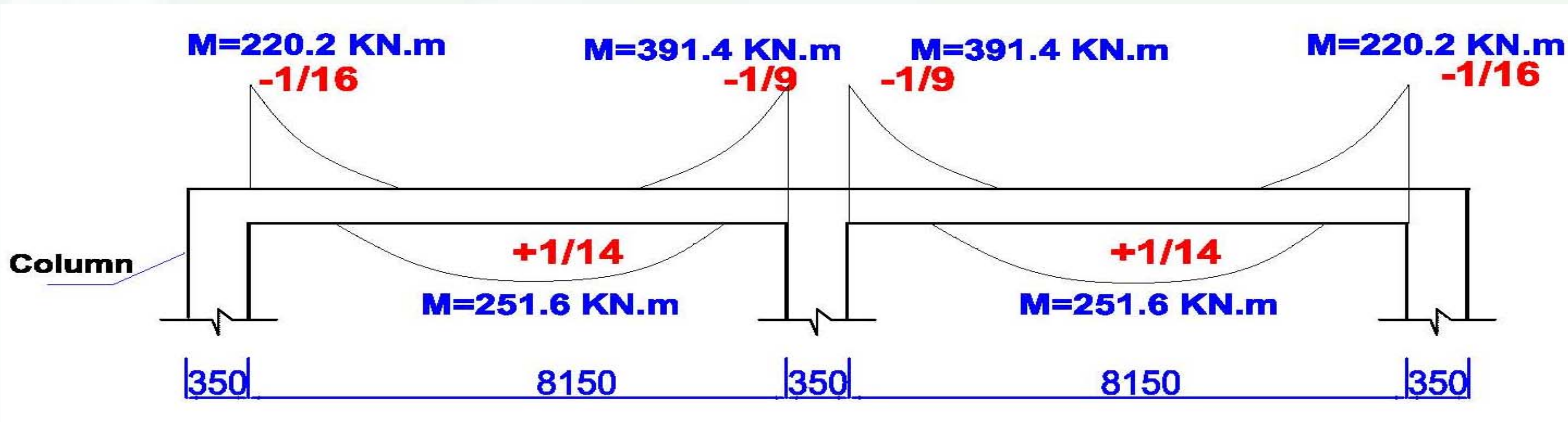
$$h_{min} = \frac{L}{18.5} = \frac{8500}{18.5} = 460\ mm$$

$but\ Fy\ not\ equal\ 420\ Mpa$

$So\ \ f = (0.4 + 280/700) = 0.8$

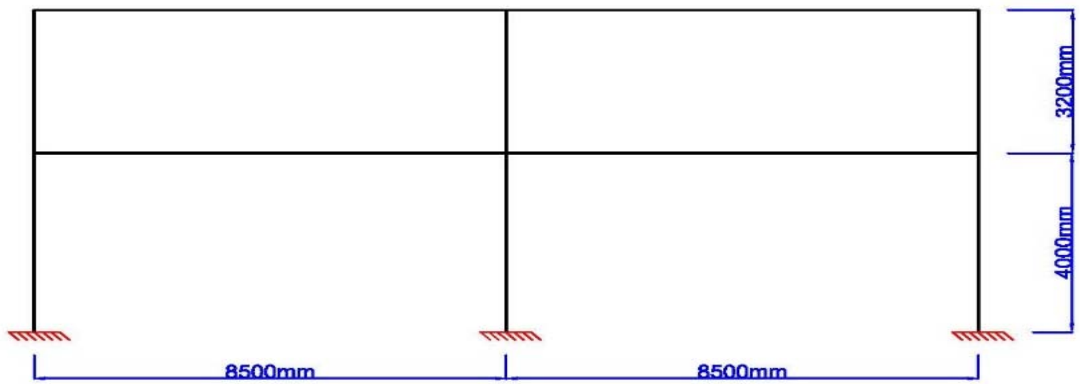
$Corrected\ h_{\min} = 0.8 \times 460 = 368\ mm < \ h\ used\ = 630\ mm.$

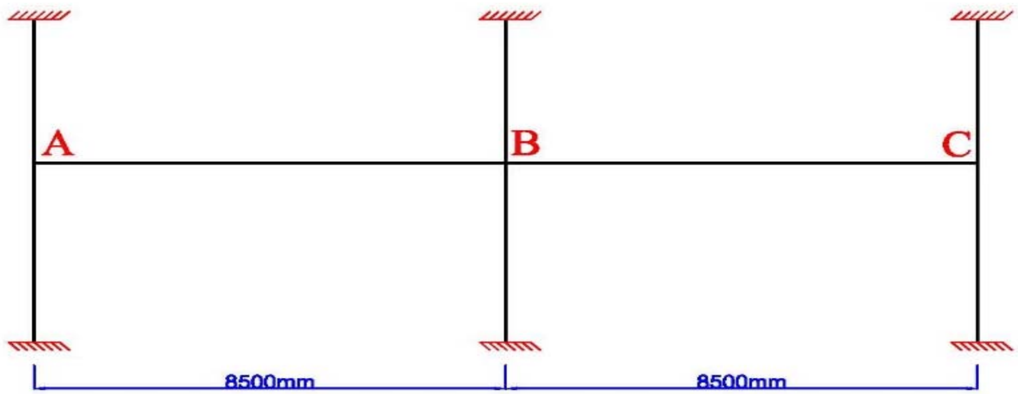
Note: The moment should be corrected according to the modified Wu

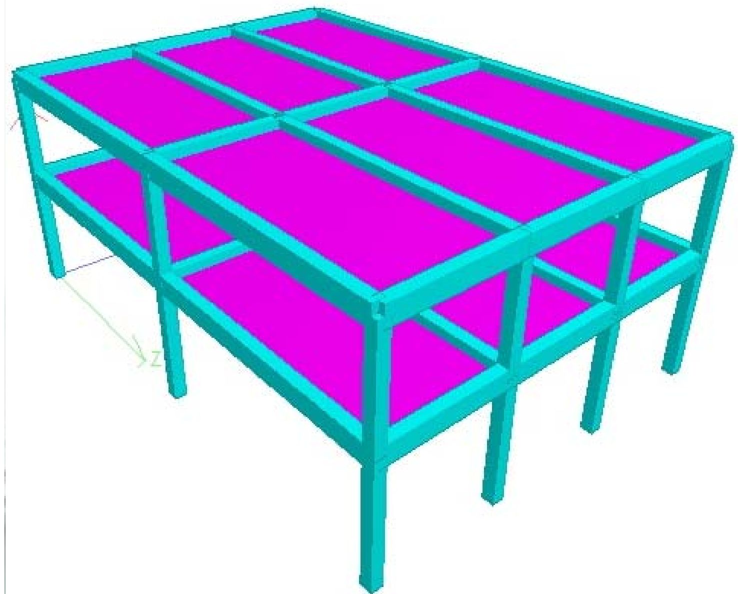


Beams and Column Moment Calculation

- *By using substitute frame method* (*Moment distribution method)*







Calculate The stiffness of members at each node

$$K = \frac{EI}{L}$$

When use the same concrete properties for whole structure E will be same and constant for all members then :

$$K = \frac{I}{L}$$

K = stiffness of member (mm^3)

I = moment of Inertia (mm^4)

L = length of member (mm)

$$For\ beams\ I_b = \frac{bh^3}{12} = \frac{350 \times 630^3}{12} = 72.93 \times 10^6\ mm^4$$

$$K = \frac{I}{L} = \frac{72.93 \times 10^6}{8500} = 858 \times 10^3$$

$$Stiffness\ of\ Upper\ Column = 350 \times \frac{350^3}{12} = 12.505 \times 10^6\ mm^4$$

$$Kc = \frac{Ic}{hc} = \frac{12.505 \times 10^6}{3200} = 390.8 \times 10^3\ mm^3$$

$$Lower\ Column\ Kc = \frac{12.505 \times 10^6}{4000} = 312.6 \times 10^3\ mm^3$$

Distributed factor (DF) or **Relative stiffness** $= \left(\frac{\frac{Ib}{Lb}}{\frac{Ic}{ht} + \frac{Ic}{hb} + 2 * \frac{Ib}{Lb}} \right)$

$$\text{Relative stiffness for beams at B} = \frac{858 \times 10^3}{390.5 \times 10^3 + 312.6 \times 10^3 + 2 \times 858 \times 10^3} = 0.355$$

$$\text{Relative stiffness for beams at A\&C} = \frac{858 \times 10^3}{390.5 \times 10^3 + 312.6 \times 10^3 + 858 \times 10^3} = 0.55$$

$$\text{Fixed end Moment } = \frac{WuL^2}{12} = 53.04 \times \frac{8.52}{12} = 319.35\ KN/m$$

$$Where\ WL/WD < \frac{3 \times 4}{(4.08 + 2) \times 4 + 3.864} = 0.43 < 0.75$$

No need to Use the loading case for Envelope

0.55	0.355	0.355	0.55
+319.35 -175.64	-319.35 0	+319.35 0	-319.34 +175.64
0	-87.82	+87.82	0
+143.71	-407.17	+407.17	-143.71
203.94		*203.94*	

$$M\ positive = \frac{wuL^2}{8} - \left(\frac{M1 + M2}{2}\right) = \frac{53.04 \times 8.52}{8} - \left(\frac{143.71 + 407.17}{2}\right) = 203.94\ KN.m$$

$$Rc = Ra = \frac{wuL}{2} - \frac{M2 - M1}{L}$$

$$= \frac{53.04 \times 8.5}{2} - \frac{407.17 - 143.71}{8.5} = {\color{red}194.42\ KN}$$

$$Rb1 = Rb2 = \frac{WuL}{2} + \frac{M2 - M1}{L}$$

$$= 225.42 + 30.99 = {\color{red}256.43\ KN}$$

$$Vu\ at\ face\ of\ support\ A\ = Ra - \frac{wu \times x}{2} = 194.42 - 53.04 \times 0.175 = {\color{red}185.14KN} \qquad (x = \frac{350}{2} = 0.175)$$

$$Vu\ at\ face\ of\ support\ B = Rb - \frac{wu \times x}{2} = 256.43 - 53.04 \times 0.175 = {\color{red}247.15\ KN}$$

$$Moment\ at\ face\ of\ support\ A = 194.42 \times 0.175 - 143.71 - \frac{53.04 \times 0.1752}{2} = {\color{red}110.5\ KN.m}$$

$$Moment\ at\ face\ of\ support\ B = 256.43 \times 0.175 - 407.17 - \frac{53.04 \times 0.1752}{2} = {\color{red}363.11\ KN.m}$$

$To\ calculate\ the\ positive\ moment:$

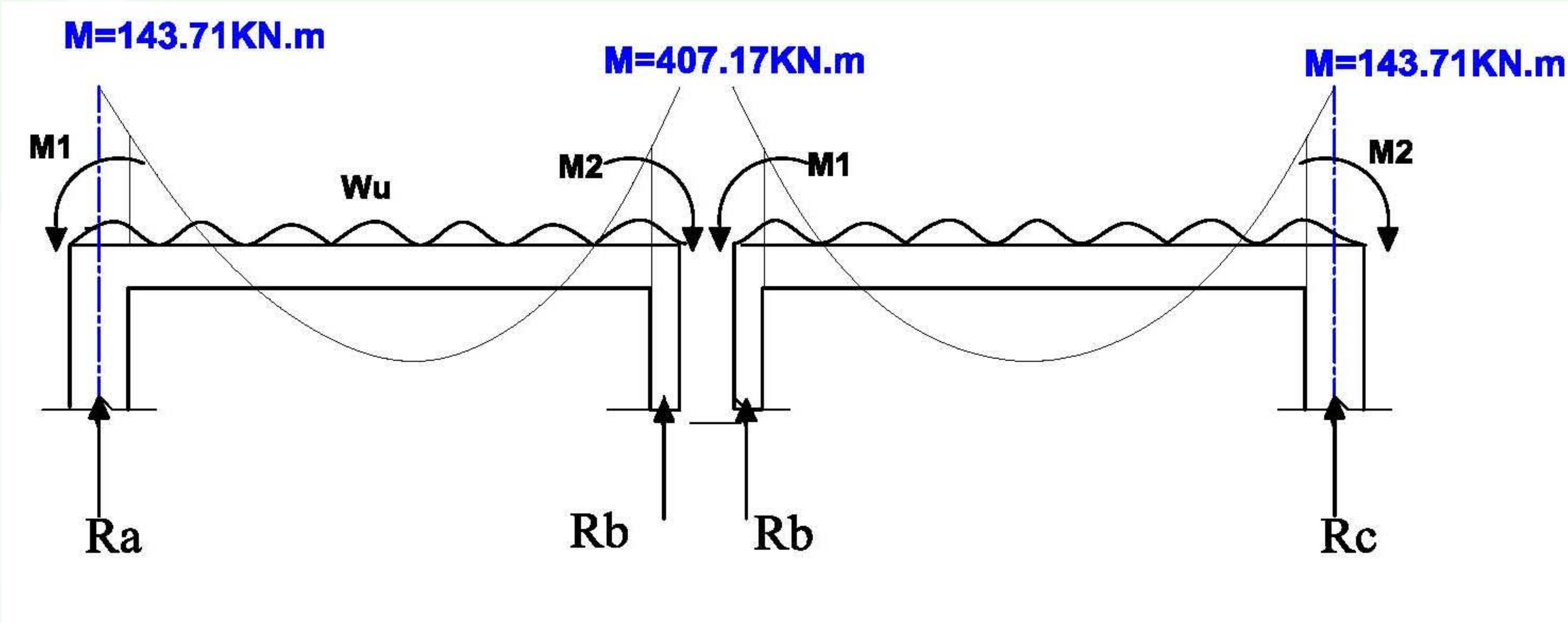
$Shear\ force\ = 0\ at\ X$

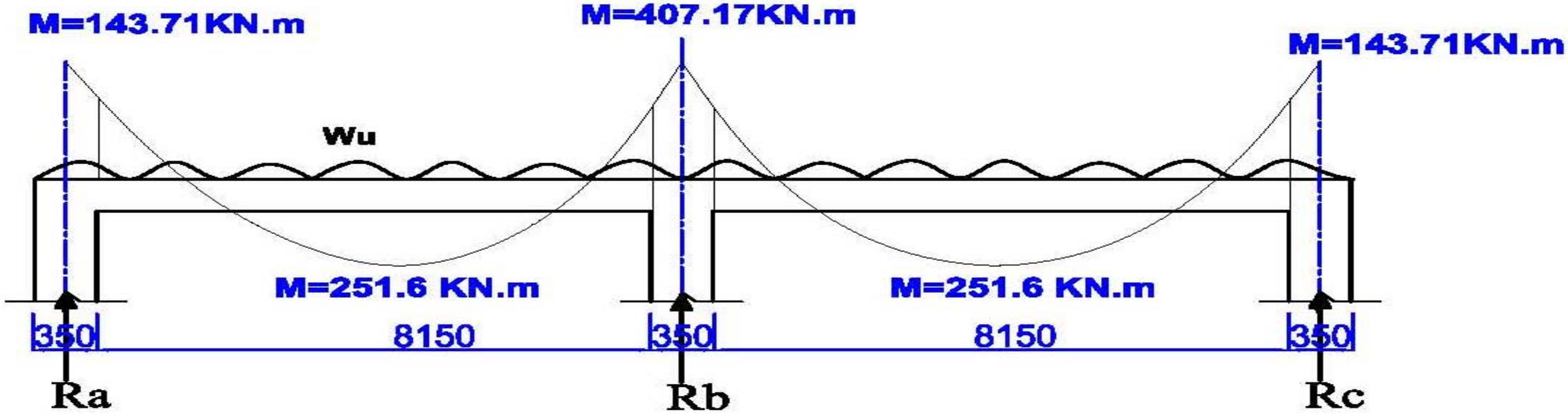
$$X = \frac{194.42 \times 8.5}{194.42 + 256.43} = {\color{red}3.66\ m}$$

$$Mu = 0 = Ra \times x - 143.71 - \frac{wu \times x^2}{2} = 194.42x - 143.71 - 53.04x^2/2$$

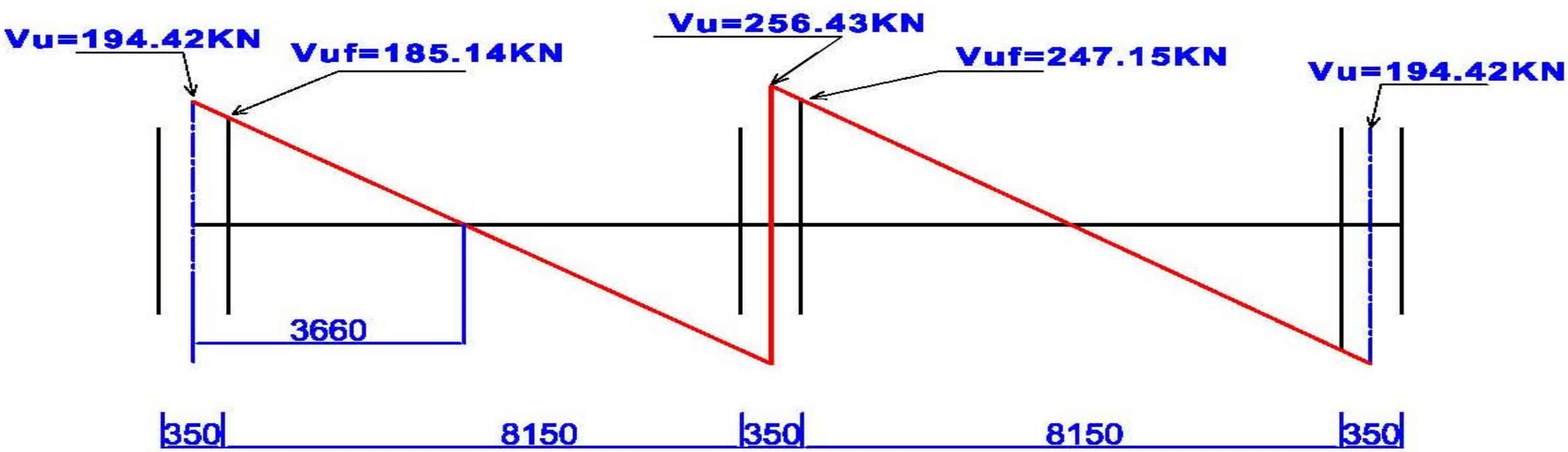
${\color{red}x = 0.83\ m}$

$$Find\ the\ \max positive\ moment\ = Ra \times x - 143.71 - 53.04 \times \frac{3.662}{2} = {\color{red}212.62\ KN.m}$$


M=143.71KN.m
M=407.17KN.m
M=143.71KN.m
M1
Wu
M2
M1
M2
Ra
Rb
Rb
Rc

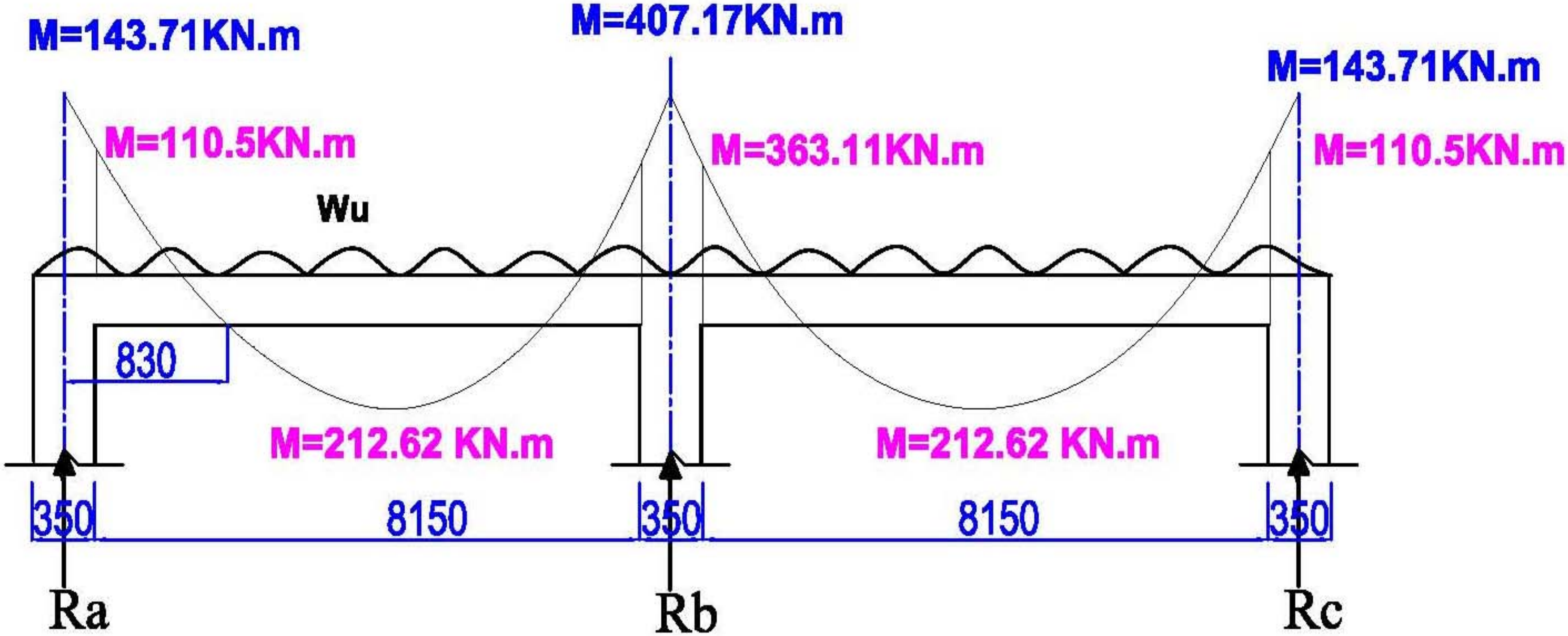


B.M.D



Dimension in mm

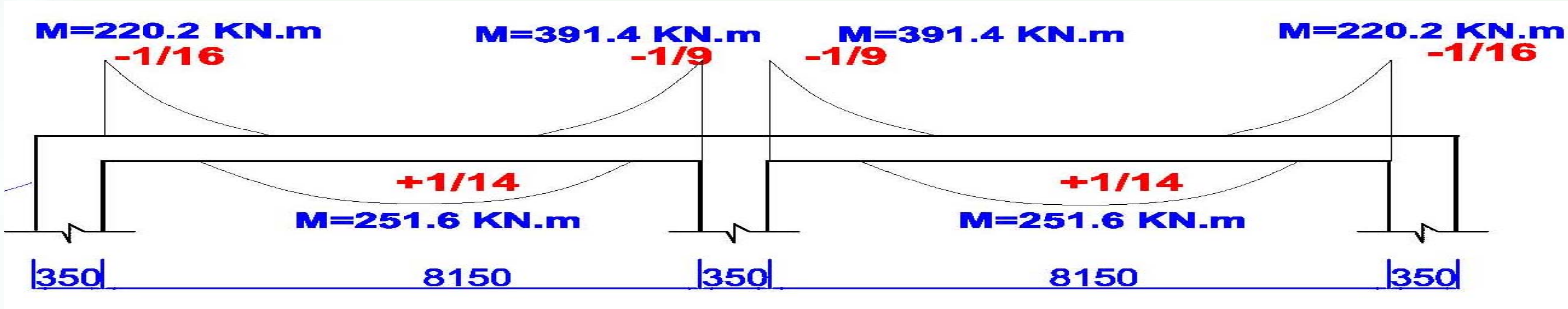
S.F.D



B.M.D

Dimension in mm

Design of interior Beam (B2) *(using ACI Coefficient Methods)*



-Negative Moment

$1-\ Exterior\ support \quad (-M = 220.2KN.m)$

$R = Mu/(\phi\ bd^2)$ $\longrightarrow$ $R = \dfrac{220.2\times10^6}{0.9\times350\times565^2} = 2.19$

$m = 15.69$

$\rho = \frac{1}{m}\times(1-\sqrt{1-\frac{2mR}{fy}})$

$$= \frac{1}{15.69}\times\left(1-\sqrt{1-2\times2.19\times\frac{15.69}{280}}\right) = 0.008371 > \rho\min. = \frac{1.4}{fy} = 0.005$$

$As = \rho bd = 0.008371\times350\times565 = 1655\ mm^2$

$Use\ \ 4\ \phi\ 25\ mm\ = 1964\ mm^2$

$(\ or\ you\ can\ use\ 6\ \phi\ 20 = 1884\ mm^2\ in\ two\ layer\ then\ we\ have\ to\ corrected\ the\ calculation)$

Check spacing

$$n = \frac{b-116-2ds}{D+S}+1$$

$$= \frac{350-116-20}{25+25}+1 = 4.3\ Bar \qquad (OK)$$

$2-$ *Interior support* $(-M = 391.4 KN.m)$

$$R = \frac{Mu}{\phi\ bd^2}$$

Assume two layer of steel bar

$$d = 630 - 90 = 540\ mm$$

$$R = \frac{391.4 \times 10^6}{0.9 \times 350 \times 540^2} = 4.26$$

$$m = 15.69$$

$$\rho = \frac{1}{m} \times \left(1 - \sqrt{1 - \frac{2mR}{fy}}\right)$$

$$= \frac{1}{15.69} \times \left(1 - \sqrt{1 - 2 \times 4.26 \times \frac{15.69}{280}}\right) = 0.01766 > \rho\min. = \frac{1.4}{fy} = 0.005$$

$$As = \rho bd = 0.01766 \times 350 \times 540 = 3337\ mm^2$$

$Use\ 8\,\phi\,25\ mm = 3928\ mm^2 \quad (two\ layer)$

$(or\ you\ can\ use\ 4\,\phi\,25 + 4\,\phi\,22 = 3484 mm^2\ in\ two\ layer)$

Check spacing

$$n = \frac{b - 116 - 2ds}{D + S} + 1$$

$$= \frac{350 - 116 - 20}{25 + 25} + 1 = 5.25\ Bar \qquad (OK)$$

Check for maximum steel ratio

calculate $\boldsymbol{\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600+fy}\right)\left(\frac{dt}{d}\right)}$

and calculate $\rho_{max} = \left(\frac{0.003+\frac{fy}{Es}}{0.008}\right)\rho_b$

$d = 540\ mm\ , dt = 565\ mm$

$\rho b = 0.85/15.69 \times (\ 600/(600 + 280)) \times (565/540) = 0.03865$

$$\rho_{max} = \frac{\left(0.003 + \frac{280}{200000}\right)}{0.008} \times \rho b = 0.55 \times \rho b = 0.02126$$

$\rho_{max} = 0.02126 > \rho = 0.01766 > \rho min = 0.005 \qquad (OK)$

$Positive\ \ Moment\ (\ M = 251.6KN.m)$

$We\ have\ T\ section\ \ ---- to\ find\ the\ be$

$1 - \ be = 16t\ +\ bw = 16 \times 170 + 350 = 3070mm$

$2 - \ be\ = \frac{L}{4} = \frac{8500}{4} = 2125\ mm$

$3 - \ be = S = \ 4000\ mm$

$Choose\ be\ \ = 2125\ mm$

$Assume\ \ block\ stress\ depth\ \ = a\ \ = h = 170\ mm$

$$R = \frac{Mu}{\phi\ \ bd^2}$$

$Assume\ two\ layer\ of\ steel\ bar$

$d = 630 - 90 = 540\ mm$

$R = 251.6 \times 10^6/(\ 0.9 \times 2125 \times 540^2) = \ 0.45$

$m = 15.69$

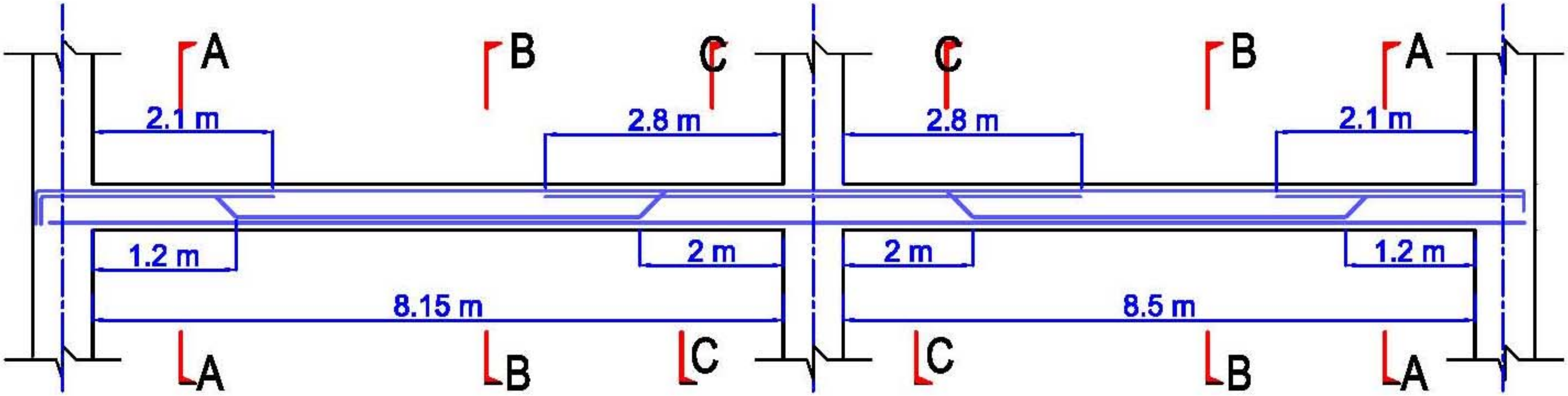
$$\rho = \frac{1}{m} \times \left(1 - \sqrt{1 - \frac{2mR}{fy}}\right)$$

$$= \frac{1}{15.69} \times \left(1 - \sqrt{1 - 2 \times 0.45 \times \frac{15.69}{280}}\right) = \ 0.001632\ <\ \rho_{min}. = \frac{1.4}{fy} = 0.005$$

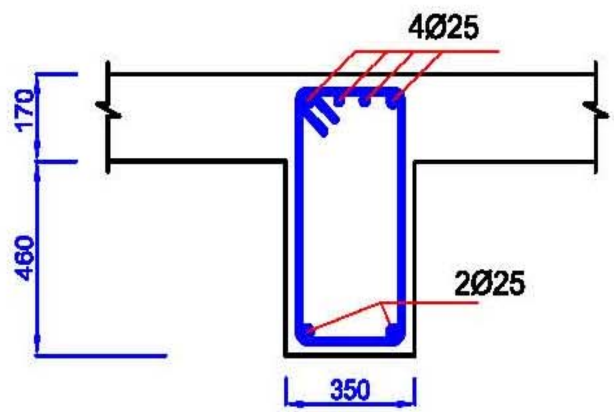
$As = \ \rho_{min}\, bd = \ 0.005 \times 350 \times 540 = 945\ mm^2$

$a = \ \rho.m.d = 0.005 \times 15.69 \times 540 = 42.8\ mm < \ 170\ mm$ $(\ design\ as\ rectangular\ section\ )$

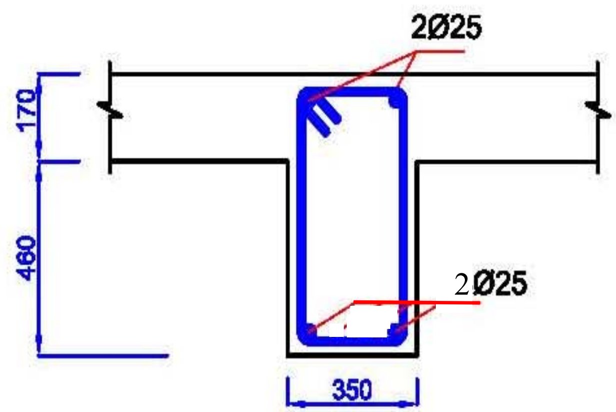
$Use\ \ \ 2\phi\ 25\ mm\ = \ 982mm^2$ $(one\ layer)$



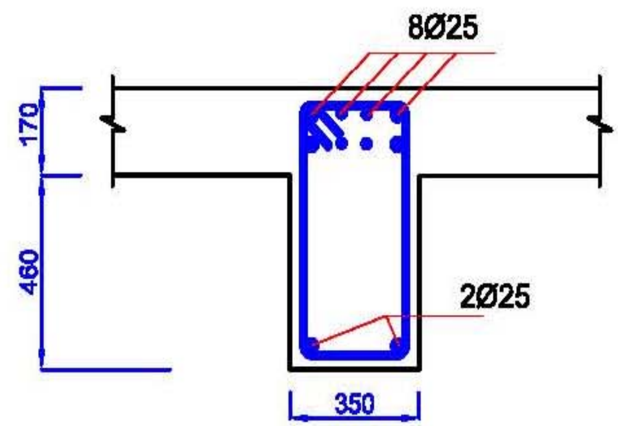
Bent Beams



Beam B2
Section A-A



Beam B2
Section B-B



Beam B3
Section C-C

Thank You

……………To be Continued