Reinforced Concrete Design

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Chapter III
Flexural Design of Reinforced Concrete

Introduction

In the previous chapter, the analysis of different reinforced concrete sections was explained. Details of the section were given, and we had to determine the design moment of the section. In this chapter, the process is reversed: The external moment is given, and we must find safe, economic, and practical dimensions of the concrete section and the area of reinforcing steel that provides adequate internal moment strength.

Rectangular Sections With Tension Reinforcement Only

From the analysis of rectangular singly reinforced sections the following equations were derived for tension-controlled sections, where f′c and fy are in MPa:

$$\rho_b = \frac{0.85\, fc'}{fy} \left(\frac{600}{\mathbf{600 + fy}}\right) \left(\frac{\mathbf{dt}}{\mathbf{d}}\right)$$

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.008}\right) \rho_b \quad or \quad \rho_{max} = \frac{3}{8} \frac{\beta_1}{m} \left(\frac{d_t}{d}\right)$$

$$m = \frac{fy}{0.85 f'c}$$

$$\beta_1 = 0.85 - 0.05 \left(\frac{f'c - 28}{7}\right) \geq 0.56$$

$For\ fy = 420\ Mpa$ $\rho_{max} = 0.6375\ \rho_b$

$For\ fy = 280\ Mpa$ $\rho_{max} = 0.55\ \rho_b$

$For\ fy = 350\ Mpa$ $\rho_{max} = 0.594\ \rho_b$

$$\beta_1 = 0.85 - 0.05\frac{f'c - 28}{7} \geq 0.56$$

$$m = \frac{fy}{0.85f'c}$$

It should be clarified that the designer has a wide range of choice between a large concrete section and relatively small percentage of steel ρ , producing high ductility and a small section with a high percentage of steel with low ductility. A high value of the net tensile strain, εt, indicates a high ductility and a relatively low percentage of steel. The limit of the net tensile strain for tension-controlled sections is 0.005 ,with =0.9. The strain limit of 0.004 can be used with a reduction in ϕ . If the ductility index is represented by the ratio of the net tensile strain , εt, to the yield strain, εy=fy/Es, the relationship between εt , / b , , and εt/εy is shown in Table below for fy=420 MPa . Also, the ACI Code, Section 6.6.5.1, indicates that εt should be ≥ 0.0075 for the redistribution of moments in continuous flexural members producing a ductility index of 3.75. It can be seen that adopting $\varepsilon_t \geq 0.005$ is preferable to the use of a higher steel ratio, ρ / ρ b ,with ε_t =0.004 , because the increase in Mn is offset by a lower ϕ. The value of ε_t=0.004 represents the use of minimum steel percentage of 0.00333 for f′c=28 Mpa and fy=420 Mpa .This case should be avoided.

$For\ fy = 420\ Mpa$

ε_t	0.004	0.005	0.006	0.007	0.0075	0.008	0.009	0.010	0.040
ρ/ρ_b	0.714	0.625	0.555	0.500	0.476	0.454	0.417	0.385	0.117
$\varepsilon_t/\varepsilon_y$	2.0	2.5	3.0	3.5	3.75	4.0	4.5	5.0	20
ϕ	0.82	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9

The value of ε_t between $\varepsilon_t = 0.005$ and $\varepsilon_t = 0.004$ can be calculated from Eq.:

$$\phi = 0.65 + (\varepsilon t - 0.002)\left(\frac{250}{3}\right).$$

The design moment equations were derived in the previous chapter in the following forms:

$$\phi Mn = Mu = \phi R\, bd^2$$

$$R = \emptyset \rho f y\left(1 - \frac{1}{2}\rho\, m\right)$$

This equation have two unknown, this can be find by assumes $\rho \leq \frac{1}{2}\rho_{max}$ for and also assume value of b then we can find the value of h

For design purpose , two method can be adopted:

A- First Case

The knowns is Mu and the properties of used material and the unknowns is As , d , b

1- assume $\rho \leq \frac{1}{2}\rho_{max}$ and assume b

2- find value of R:

$$R = \emptyset \rho f y\left(1 - \frac{1}{2}\rho\, m\right) \quad \text{and} \quad m = \frac{fy}{0.85 f'c}$$

3- find the effective depth d from equation :

$$\phi Mn = Mn = \phi R\, bd^2$$

$$d = \sqrt{\frac{Mu}{\emptyset R\, b}}$$

4- Calculate As:

$As = \rho\, b\, d$

Then choose a suitable bar diameter numbers and calculate the total depth h considering the concrete cover (h should be choose around 10 mm)

B- Second Case

-The knowns $\boldsymbol{Mu}$ and the **dimension of section** according to the architectural requirement

- Unknown is the steel Area As

1- calculate R value

$$R = \frac{Mu}{\emptyset b d^2}$$

2- Calculate steel Ratio ρ from :

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right)$$

Then compare value of ρmin and ρmax with value of ρ

3- calculate the As

$$As = \rho\, b\, d$$

than find the no. of bars

If $\rho > \rho_{max}$,the section should be design as **Double reinforced section**

Spacing Of Reinforcement And Concrete Cover

Specifications

Figure below shows two reinforced concrete sections. The bars are placed such that the clear spacing shall be at least the greatest of (25mm), nominal bar diameter D, and (4/3) d_{agg}.(nominal maximum size of the aggregate) , (ACI Code, Section 25.2.1). Vertical clear spacing between bars in more than one layer shall not be less than (25mm), according to the ACI Code, Section 25.2.2. Also for reinforcement of more than two layers, the upper layer reinforcement shall be placed directly above the reinforcement of the lower layer. The width of the section depends on the number , n, and diameter of bars used. Stirrups are placed at intervals; their diameters and spacing depend on shear requirements, to be explained later. At this stage, stirrups of (10mm) diameter can be assumed to calculate the width of the section. There is no need to adjust the width ,b , if different diameters of stirrups are used. The specified concrete cover for cast-in-place and pre-cast concrete is given in the ACI Code, Section 20.6.1. Concrete cover for beams and girders is equal to (38mm), and that for slabs is equal to (20mm), when concrete is not exposed to weather or in contact with the ground.

Minimum Width of Concrete Sections

The general equation for the minimum width of a concrete section can be written in the form

$$b_{min} = n \times D + (n-1) \times s + 2 \times (stirrup\ diameter) + 2 \times (concrete\ cover)$$

Where:

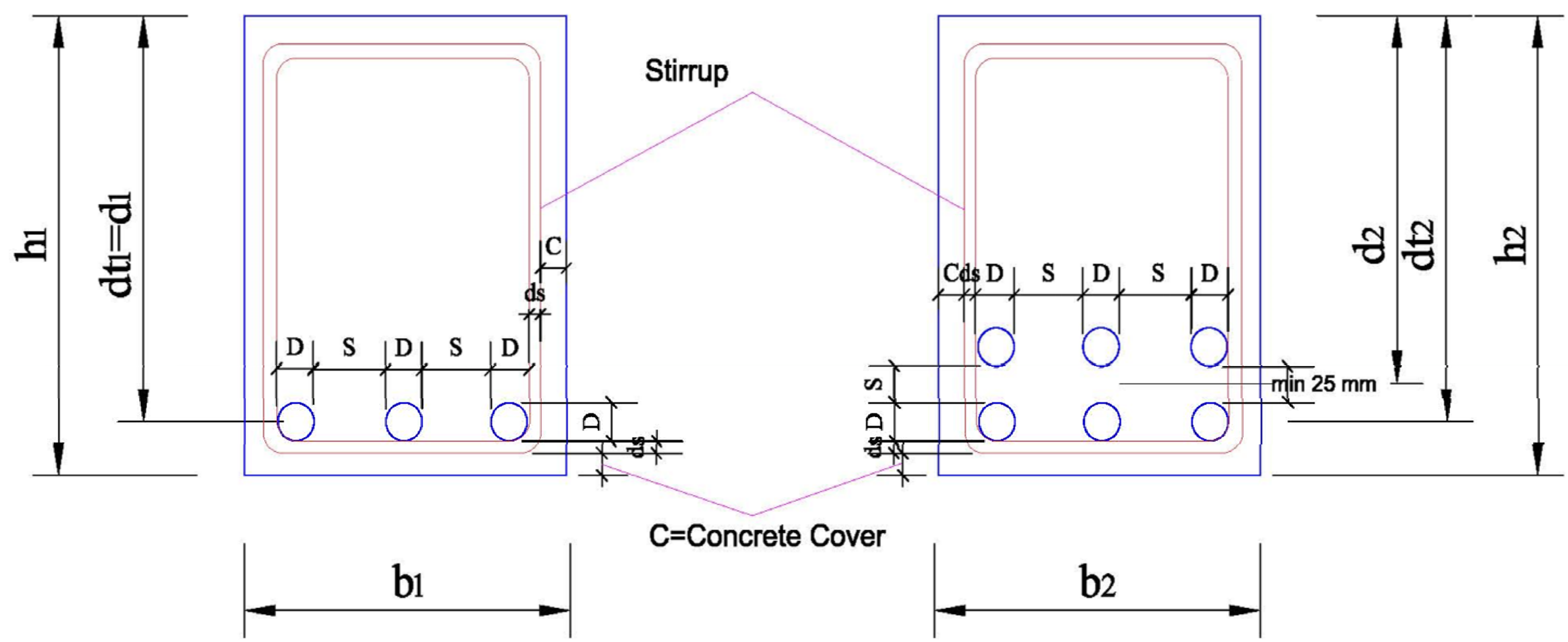
$n = number\ of\ bars$

$D = diameter\ of\ largest\ bar\ used$

$s = spacing\ between\ bars\ (equal\ to\ D\ or\ 25\ mm\ , whichever\ is\ larger)$

If the stirrup's diameter is taken equal to (10 mm) and concrete cover equals .(38mm),then

$$B_{min} = n \times D + (n-1) \times s + 96$$



This equation, if applied to the concrete section s in Fig, above becomes:

$b_1 = 3D + 2S + (96mm)$

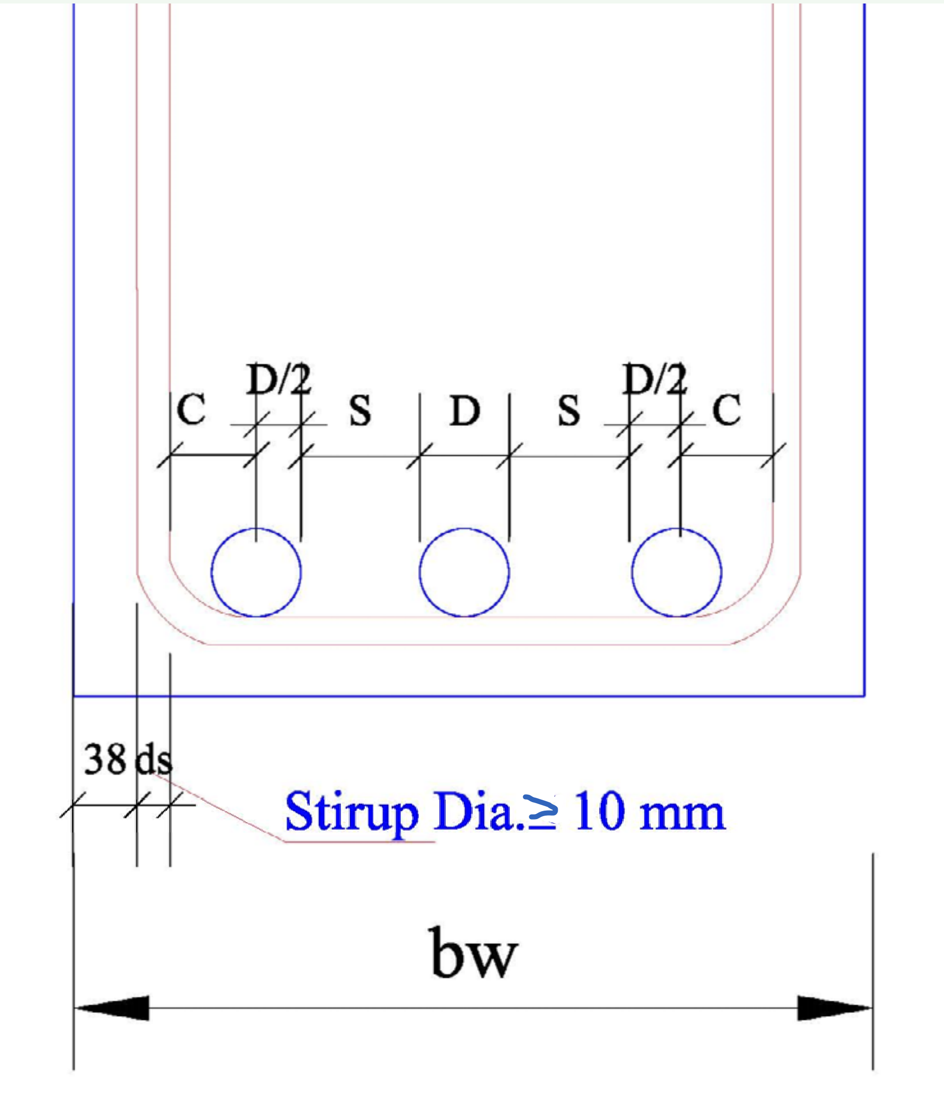
$b_2 = 3D + 2S + (96mm)$ *while for 4 bars* $b1 = 4D + 3S + (96mm)$

In fig. below , c = 20 mm when ds more than 10 mm

$$b_{min} = 2 \times 38 + 2\,ds + 2\,c + (n-1)(D+S)$$
$$b_{min} = 116 + 2ds + (n-1)\,(D+S)$$

If b is known then:

$$\text{Bar No.} = n = \frac{b - 116 - 2ds}{D+S} + 1$$



Minimum Over all Depth of Concrete Sections

The effective depth, d, is the distance between the extreme compressive fibers of the concrete section and the centroid of the tension reinforcement. The minimum total depth is equal to d plus the distance from the centroid of the tension reinforcement to the extreme tension concrete fibers, which depends on the number of layers of the steel bars .In application to the sections shown in Fig

$$h_1 = d_1 + \frac{D}{2} + ds. + 38\ mm \qquad One\ layer$$

$$h_2 = d_2 + \frac{25}{2} + D + ds + 38\ mm \qquad Two\ layer$$

When use bar diameter $\emptyset \leq 28\ mm$ then total depth calculated from :

$$h = d + 65\ mm \qquad one\ layer$$

Or

$$h = d + 90\ mm \qquad two\ layer$$

It should be mentioned that the minimum spacing between bars depends on the maximum size of the coarse aggregate used in concrete. The nominal maximum size of the coarse aggregate shall not be larger than one-Fifth of the narrowest dimension between sides of forms, or one-third of the depth of slabs, or three-fourths of the minimum clear spacing between individual reinforcing bars or bundles of bars (ACI Code, Section 26.4.2.1).

Example (1): Design a simply reinforced rectangular section to resist a factored moment of 490 KN.m using the maximum steel percentage ρ_{max} for tension-controlled sections to determine its dimension. Given: f′c=21 MPa fy=420 MPa.

Sol.

$for\ f'c = 21\ MPa\ \ then \quad \beta_1 = 0.85$

$$m = \frac{fy}{0.85\, f'c} = 23.53, \qquad \emptyset = 0.9$$

$$\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right) = \frac{0.85}{23.53}\left(\frac{600}{600 + 420}\right)(1) = 0.02125$$

$$\rho_{max} = \left(\frac{0.003 + fy/Es}{0.008}\right)\rho_b = \left(\frac{0.003 + 0.0021}{0.008}\right)\rho_b = 0.6375\, \rho_b$$

$\rho_{max} = 0.01355$

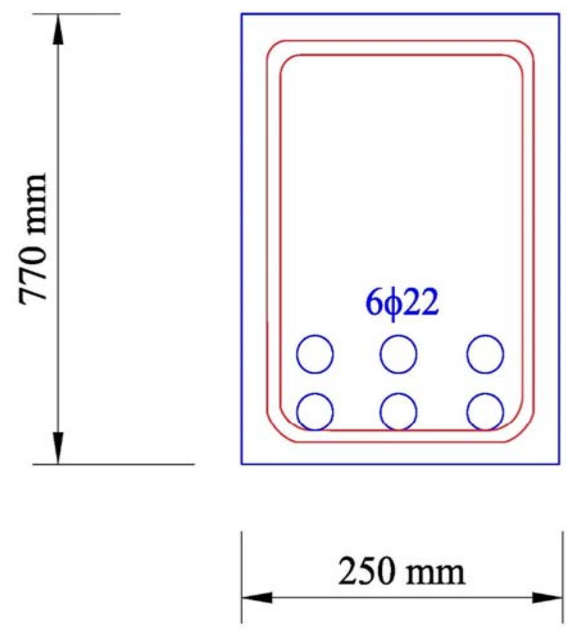
$$R = \rho fy\left(1 - \frac{1}{2}\rho\, m\right) = 0.01355 \times 420\left(1 - \frac{1}{2} \times 0.01355 \times 23.53\right) = 4.784\ MPa$$

$$Mn = \frac{Mu}{\emptyset} = Rbd^2$$

$$bd^2 = \frac{Mu}{\emptyset R} = \frac{490 \times 10^6}{0.9 \times 4.784} = 113805277\ mm^3$$

$Assume\ b\ and\ find\ d$

b mm	d mm	As mm^2
250	672.18	2298.86
300	613.61	2518.25
350	568.09	2720.00
400	531	2907.83



$If\ we\ choose\ b\ =\ 250\ mm, d\ =\ 672.18\ mm$

Ø 22 mm (380 mm^2).

$No.of\ Bars = n = \frac{2298.86}{380} = 6.04$ *Use 6 Bars*

$$Bar\ No.(n) = \frac{b\ - 116 - 2ds}{D + S} + 1$$

$ds = 10\ mm, D\ =\ 22\ mm, S = 25\ mm$

$$n = \frac{250\ - 116 - 2 \times 10}{22 + 25} + 1 = 3.42 = 3$$

$If\ use\ two\ layer\quad h =\ d + 90\ mm =\ 762.18\ mm\quad use\ h =\ 770\ mm\quad (increase\ the\ value\ for\ 10\ mm)$

Check the effective depth :

$$d = h - 38 - 10 - 22 - \frac{25}{2} = 770 - 38 - 10 - 22 - 12.5 = 687.5\ mm$$

$$b = 250\ mm$$

$$\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right)$$

$$d_t = 770 - 38 - 10 - \frac{22}{2} = 711\ \text{mm}$$

$$\rho_b = \frac{0.85}{23.53}\left(\frac{600}{600 + 420}\right)\left(\frac{711}{687.5}\right) = 0.02219$$

$$\rho_{max} = \left(\frac{0.003 + fy/Es}{0.008}\right)\rho_b = \left(\frac{0.003 + 0.0021}{0.008}\right) \times \rho_b = 0.6375\ \rho_b$$

$$\rho_{max} = 0.6375 \times 0.02219 = 0.014146$$

$$Mn = \frac{Mu}{\emptyset} = Rbd^2$$

$$\rho = \frac{As}{bd} = \frac{6 \times 380}{250 \times 687.5} = 0.01326 < \rho_{max}$$

$$R = \rho fy\left(1 - \frac{1}{2}\rho\, m\right) = 0.01326 \times 420\left(1 - \frac{1}{2}\, 0.01326 \times 23.53\right) = 4.7$$

$$Mn = 4.7 \times 250 \times 687.52 = 555.37\ KN.m$$

$$Mu = \emptyset Mn = 0.9 \times 555.37 = 499.83\ KN.m \qquad Mu = 490 KN.m \qquad OK$$

Example (2): Design a simply reinforced rectangular section with steel percentage $\rho = 0.5\,\rho_{max}$ of previous example

Sol:

$\rho = 0.5\,\rho_{max}$ then tension Controlled section $\phi=0.9$

$\rho = 0.5 \times (0.01368)$ (previous Example Exa. (1))

$\rho = 0.00684$

$$R = \emptyset \rho f y \left(1 - \frac{1}{2}\rho m\right)$$

$$= 0.9 \times (0.00684) \times (420)\big(1 - 0.5 \times (0.00684) \times (23.53)\big) = 2.642$$

$$\mathrm{d} = \sqrt{\frac{Mu}{\emptyset b\ R}} = \sqrt{\frac{490 \times 106}{0.9\ b \times 2.642}}$$

Assume *b* to find *d*:

b mm	d mm	As mm²
250	907.9	1552.5
300	828.8	1700.7
350	767.3	1836.9
400	717.8	1963.8

Use $b = 300$, $then\ d = 828.2\ mm$, $As = 1700.7\ mm^2$

Use $\emptyset\ 25\ mm$

$Ab = 490\ mm^2$

$$No.of\ bars = n = \frac{1700.7}{490} = 3.47\ mm \quad use\ 4\ bar$$

To find the bw , how many bars can be contains :

$$n = \frac{b - 116 - 2 \times ds}{D + s} + 1 = \frac{300 - 116 - 2 \times 10}{25 + 25} + 1 = 4.28 \quad use\ 4\ bar$$

Use One layer

Find total depth of Beam h

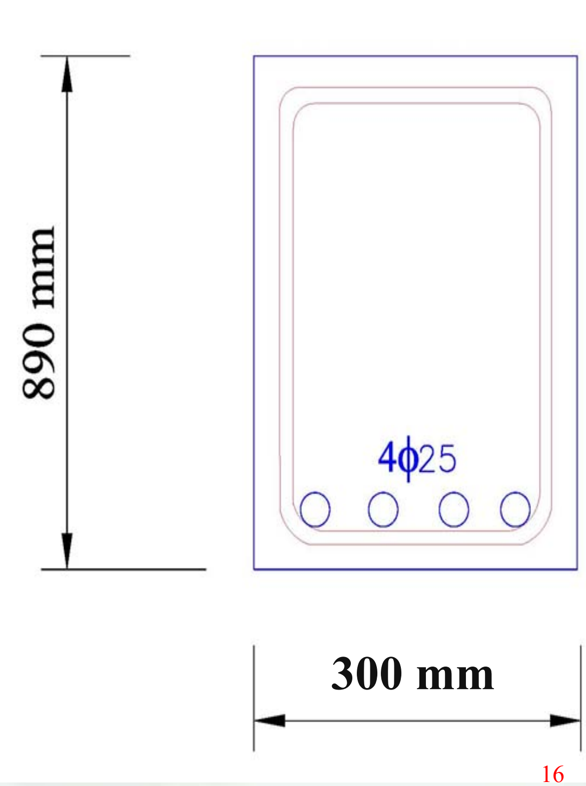
$$h = d + 38 + ds + \frac{D}{2}$$

$$= 828.8 + 38 + 10 + \frac{25}{2} = 889.3\ mm$$

Use h= 890 mm

Note that in this example (2) , the value of h use less than calculated nearest 10 mm , cause the provided steel area is larger than required area and this allow to use h less than calculated .

While in example (1) the selected h was greater than the calculated cause the provided steel area was less than required in very small a mount



$$d = 890 - 38 - 10 - \frac{25}{2} = 829.5\ mm$$

$$\rho = \frac{As}{b\ d} = \frac{4 \times 490}{300 \times 829.5} = 0.007876$$

$$R = \emptyset \rho f y \left(1 - \frac{1}{2}\ \rho\ m \right) = 0.07876\ (420)\big(1 - 0.5\ (0.007876)(23.53)\big) = 3.0\ MPa$$

$$Mn = 3\ \times 300\ \times (829.5)^2 = 619.26\ KN.m$$

$$\emptyset Mn = Mu = 0.9 \times 619.26 = 557.33\ KN.m > Mu = 490\ KN.m$$

Example (3): Find the necessary reinforcement for a given section that has a width of 250 mm and a total depth of 500mm ,if it is subjected to an external factored moment of 222 KN. m. Given: f'c= 28 mPa and fy= 420 mPa .

Solution

Assume one layer of steel

$$d = h - 65\,mm = 500 - 65 = 435\,mm$$

$$R = \frac{Mn}{bd^2} = \frac{Mu}{\emptyset bd^2} = \frac{222 \times 10^{6}}{0.9 \times 250 \times 435^2} = 5.214$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right)$$

$$m = \frac{fy}{0.85\,f'c} = \frac{420}{0.85 \times 28} = 17.65$$

$$\rho = \frac{1}{17.65}\left(1 - \sqrt{1 - \frac{2 \times 17.65 \times 5.214}{420}}\right) = 0.01419$$

$$\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right) = \frac{0.85}{17.65}\left(\frac{600}{600 + 420}\right)(1) = 0.028328$$

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.008}\right)\rho_b = \frac{0.0051}{0.008} \times \rho_b = 0.6375\,\rho_b = 0.6375 \times 0.028328 = 0.018059 > \rho = 0.01419$$

Tension Controlled section $\emptyset = 0.9$

$As = \rho \times b \times d = 0.01419 \times 250 \times 435 = 1543.1\ mm^2$

Use Ø 20 mm (A_b= 314 mm^2)

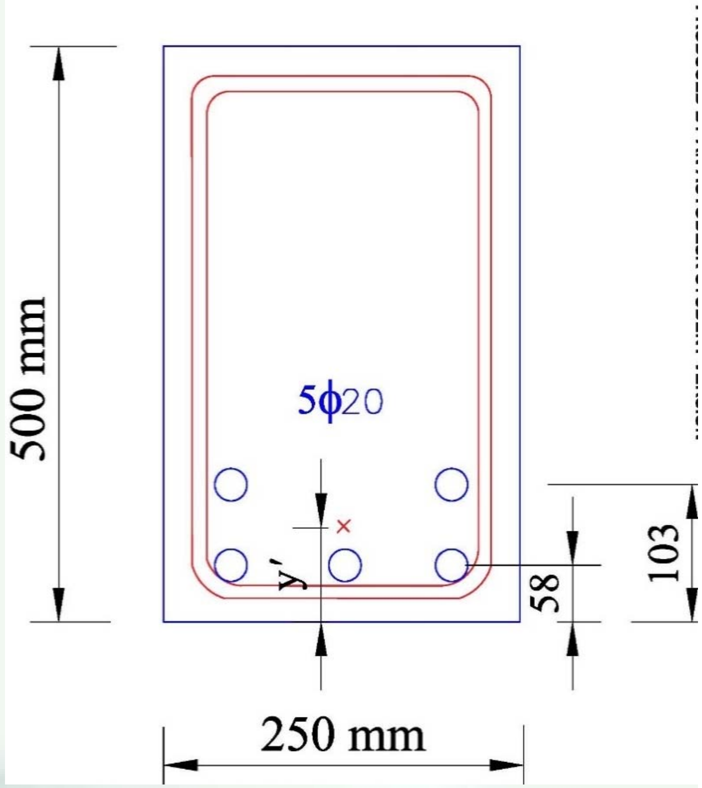
$No.of\ bars = \frac{1543.1}{314} = 4.91$ $\quad$ $Use\ 5Ø\ 20\ mm$

Check spacing between bars

$n = \frac{b - 116 - 2 \times ds}{D + s} + 1 = \frac{250 - 116 - 2 \times 10}{20 + 25} + 1 = 3.53 \quad use\ 3\ bars$

Need two layers

Or increased the steel bar area



$$y' = \frac{2 \times 314 \times 103 + 3 \times 314 \times 58}{5 \times 314} = 76\ mm$$

$d = h - 76 = 500 - 76 = 424\ mm$

$\rho = \frac{5 \times 314}{bd} = \frac{5 \times 314}{250 \times 424} = 0.01481$

$R = \rho fy\left(1 - \frac{1}{2}\rho m\right) = 0.01481 \times 420\left(1 - \frac{1}{2}(0.01481 \times 17.65)\right) = 5.407$

$ØMn = Mu = ØRbd^2 = 0.9 \times 5.407 \times 250 \times 4242 = 218.72\ KN.m \quad < Mu\ applied$

Note: we can start solution by assuming two layer and d=h-90 mm

Thank You……….

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Rectangular Sections With Compression Reinforcement

A singly reinforced section has its moment strength when ρ_{max} of steel is used. If the applied factored moment is greater than the internal moment strength, as in the case of a limited cross section, a doubly reinforced section may be used, adding steel bars in both the compression and the tension zones. Compression steel will provide compressive force in addition to the compressive force in the concrete area.

The procedure for designing a rectangular section with compression steel when M_u, $f'c$, fy , b, d, and d'

are given can be summarized as follows:

When Mu > Ø Mn_{max}

1- calculate $\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600+fy}\right)\left(\frac{dt}{d}\right)$

and calculate $\rho_{max} = \left(\frac{0.003+\frac{fy}{Es}}{0.008}\right)\rho_b$ or calculate $A_{s1} = \rho_1 bd$ (maximum steel area as singly reinforced).

where $\rho_1 = 0.75\,\rho_{max}$ to ρ_{max} , and As_1 and its preferable to use $\rho_1 = 0.75\,\rho_{max}$ using to produces moment equal to Mn_1

$$R = \rho_1 fy\left(1 - \frac{1}{2}\rho\, m\right) \quad and \quad Mn_1 = Rbd^2\ or\ Mu_1 = \emptyset Rbd^2$$

2. Calculate $M_{u2} = M_u - M_{u1}, or\ Mn_2 = Mn - Mn_1$, the moment to be resisted by compression steel.
3. Calculate the As_2 in tension zone where ;

$$As = As_1 + As_2 \quad and \quad As_2 = \frac{Mn_2}{fy(d-d')}$$

4. Calculate the compression stress at the compression steel and check the condition :

$$\rho_1 = \rho - \rho' \geq \left(\frac{\beta_1 d'}{m\,d}\right)\left(\frac{600}{600-fy}\right)$$

It the condition is checked then: $fs' = fy$
And If not then $fs' < fy$ and $fs'\,calculated\ from\ formula$:

$$fs' = 600\left(1 - \frac{\beta_1 d'}{\rho_1\, m\, d}\right) \le fy$$

$$fs' = 600\left(\frac{C - d'}{C}\right) = 600\left(1 - \frac{d'}{C}\right)$$

$$a = \rho_1\, m\, d \quad and \quad C = \frac{a}{\beta_1}$$

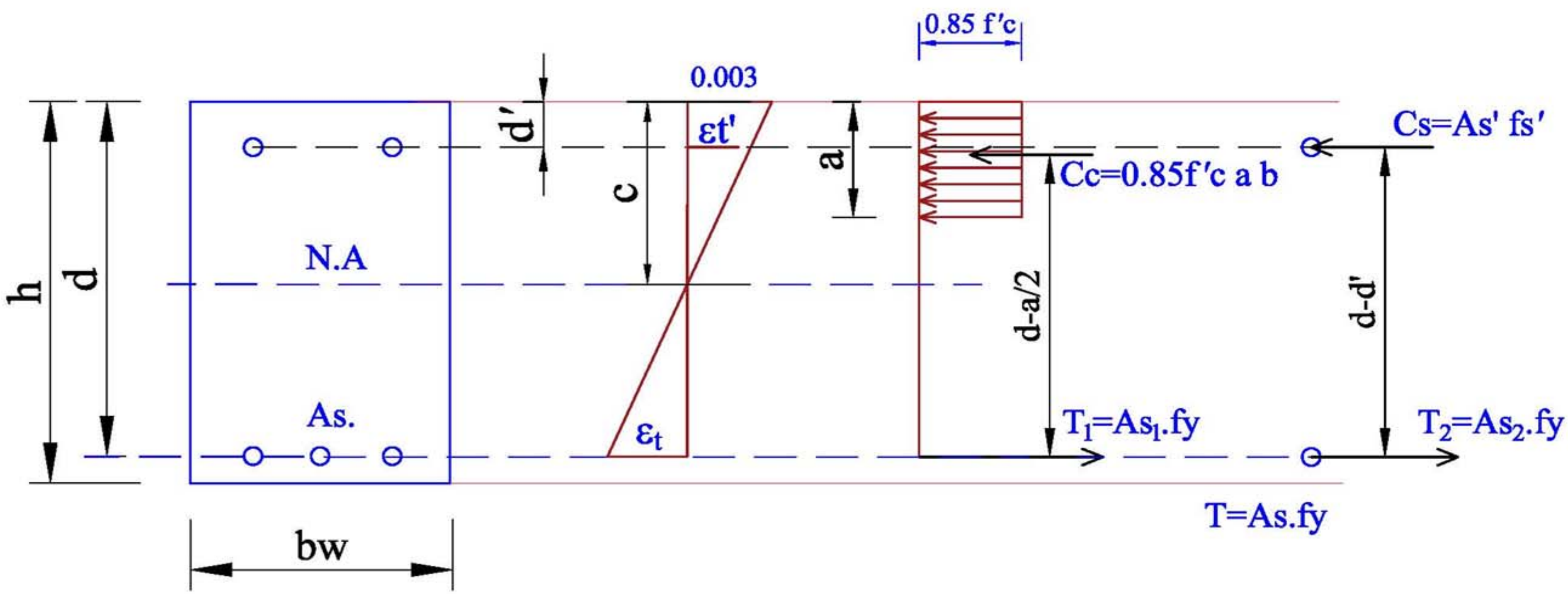
$$\therefore\ C = \frac{\rho_1\, md}{\beta_1}$$

$$\therefore fs' = 600\left(1 - \frac{\beta_1 d'}{\rho_1\, m\, d}\right)$$

In case of $fs' = fy$ use $As' = As_2$

and $fs' < fy$ use $As' = As_2 \times \left(\frac{fy}{fs'}\right)$

5. Choose the Tension steel bar diameter and compression steel bar whether can arrange in single layer



Example (4): A beam section is limited to a width $b = 250$mm. and a total depth $h = 550$ mm and has to resist a factored moment of 307 KN.m. Calculate the required reinforcement. Given: $f'_c = 21$ mPa and $f_y = 350$ mPa. $d' = 65$ mm.

Solution

Determine the design moment strength that is allowed for the section as singly reinforced based on tension-controlled conditions;

Assume (Two layer of steel) (assume ϕ 28 mm)

Then $d = h - 90 = 550 - 90 = 460mm$

$$dt = 460 + \frac{25}{2} + \frac{28}{2} = 486.5mm$$

1− calculate $\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right)$

$$m = \left(\frac{fy}{0.85\, f'c}\right) = \frac{350}{0.85 \times 21} = 19.61 \qquad \text{and} \qquad \beta_1 = 0.85$$

$$\rho_b = \frac{0.85}{19.61}\left(\frac{600}{600 + 350}\right)\left(\frac{486.5}{460}\right) = 0.02896$$

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.008}\right)\rho_b = \left(\frac{0.003 + \frac{350}{200000}}{0.008}\right) \times 0.02896 = 0.01719 \quad or \quad \rho_{max} = \frac{3}{8} \times \frac{\beta_1}{m}\left(\frac{dt}{d}\right) = 0.01719$$

$$Mu = \emptyset R b d^2$$

$$R = \frac{307 \times 10^6}{0.9 \times 250 \times 460^2} = 6.448$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right) = \frac{1}{19.61}\left(1 - \sqrt{1 - \frac{2 \times 19.61 \times 6.448}{350}}\right) = 0.02413 > \rho_{max} = 0.01719$$

Then Design the section as Double Reinforced Section (D.D.R.S)

2− Assume $\rho_1 = 0.75\, \rho_{max}$

$\rho_1 = 0.75 \times 0.01719 = 0.01289$

$As_1 = \rho_1\, b\, d = 0.01289 \times 250 \times 460 = 1482.6\, mm^2$

$R = \rho_1 fy \left(1 - \frac{1}{2}\, \rho\, m\right) = 0.01289 \times 350 \left(1 - \frac{1}{2} \times 0.01289 \times 19.61\right) = 3.94$

$Mn_1 = Rbd^2 = 3.94 \times 250 \times 460^2 = 208.43\, KN.m$

3 − $Mn_2 = Mn - Mn_1 = \left(\frac{307}{0.9}\right) - 208.43 = 132.68\, kN.m$

4- Calculate the Total Tension Steel

$$As_2 = \frac{Mn_2}{fy\,(d - d')} = \frac{132.68 \times 10^6}{350\,(460 - 65)} = 959.7\, mm^2$$

$As = As_1 + As_2 = 1482.6 + 959.7 = 2442.3\ \text{mm}^2$

5- Check the stress in compression steel

$$\rho_1 = \rho - \rho' \ngeq \left(\frac{\beta_1 d'}{m\, d}\right)\left(\frac{600}{600 - fy}\right) = \left(\frac{0.85 \times 65}{19.61 \times 460}\right)\left(\frac{600}{600 - 350}\right) = 0.014699$$

$f's < fy = 350\text{MPa}$ NOT O.K

$$f's = 600\left(1 - \frac{\beta_1\, d'}{\rho_1\, m\, d}\right) = 600\left(1 - \frac{0.85 \times 65}{0.01289 \times 19.61 \times 460}\right) = 314.9\ MPa$$

$$As' = As_2 \times \left(\frac{fy}{fs'}\right) = 959.7 \times \left(\frac{350}{314.9}\right) = 1066.7\ mm^2$$

$For\ \emptyset 25\ mm\ (\ Ab = 490\ mm^2)$

$Use\ 5\ \phi\ 25\ mm = (\ 5 \times 490 = 2450\ mm^2) > As\ required = 2442.3\ mm^2$

$For\ As'\ Use\ 3\ \phi\ 22\ mm = (\ 3 \times 380 = 1140\ mm^2) > As'$ required = 1066.7 mm²

6- Check no of Bars in one layer

use stirrup diameter ds = 10 mm

$$n = \frac{b - 116 - 2 \times ds}{D + s} + 1 = \frac{250 - 116 - 2 \times 10}{25 + 25} + 1 = 3.28 = 3$$

$$d' = 38 + 10 + \frac{22}{2} = 59\ mm$$

$$dt = 550 - 38 - 10 - \frac{25}{2} = 489.5\ mm$$

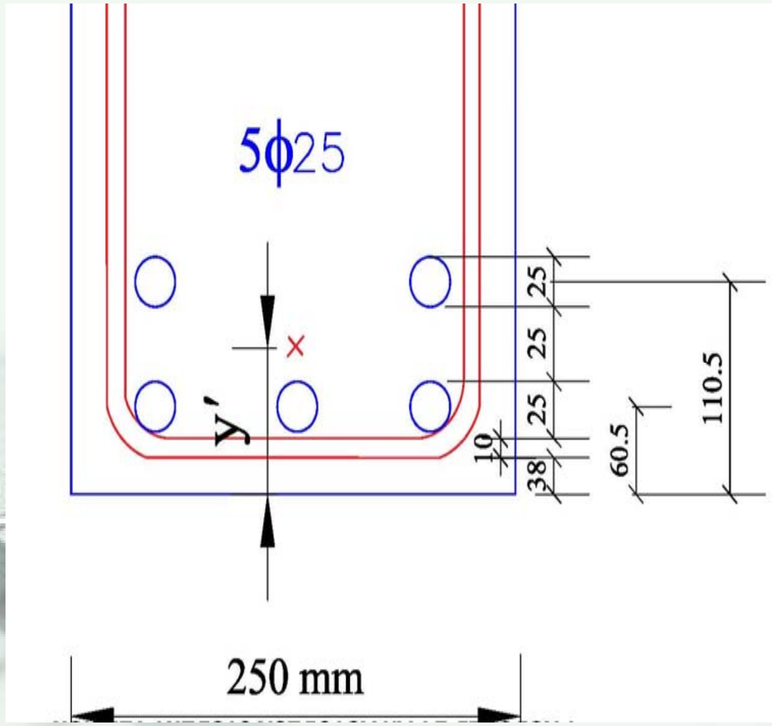
$$d = h - y'$$

$$y' = \frac{3 \times 490 \times 60.5 + 2 \times 490 \times 110.5}{5 \times 490} = 80.5\ \text{mm}$$

$$d = 550 - 80.5 = 469.5\ mm$$

$$\rho_b = \frac{0.85}{19.61}\left(\frac{600}{600 + 350}\right)\left(\frac{489.5}{469.5}\right) = 0.02854$$

$$\rho_{max} = \left(\frac{0.003 + \frac{350}{200000}}{0.008}\right) \times 0.02854 = 0.01695$$



$$\rho = \frac{As}{bd} = \frac{2450}{250 \times 469.5} = 0.020873$$

$$\rho' = \frac{As'}{bd} = \frac{1140}{250 \times 469.5} = 0.00971$$

Check again the stress in compression steel

$$\rho_1 = 0.01116 = \rho - \rho' \not\geq \left(\frac{\beta_1 d'}{m\ d}\right)\left(\frac{600}{600 - fy}\right) = \left(\frac{0.85\ \times 59}{19.61 \times 469.5}\right)\left(\frac{600}{600 - 350}\right) = 0.01307$$

Then : $f's < fy = 350\text{MPa}$

Check the failure at the tension steel :

$\rho - \rho' < \rho_{max}$

$\rho - \rho' = 0.020873 - 0.00971 = 0.01116 < \rho_{max} = 0.01695$ O.K

To find the $f's$ use the direct method where: (also can use other method to find a and c)

$A\ a^2 - Ba - C = 0$

$A = 1,$

$$B = m\ d\left(\rho - \frac{600}{fy}\rho'\right)$$

$$C = \frac{600}{fy}\beta_1\, m\ d\ d'\rho'$$

$$a = \frac{1}{2}\left[B + \sqrt{B^2 + 4\ A\ C}\right] \qquad C = \frac{a}{\beta}$$

$$B = 19.61 \times 469.5 \left(0.020873 - \frac{600}{350} \times 0.01116\right) = 16.03$$

$$C = \frac{600}{350} \times 0.85 \times 19.61 \times 469.5 \times 59 \times 0.01116 = 8833.4$$

$$a = \frac{1}{2}\left[16.03 + \sqrt{16.03^2 + 4 \times 1 \times 8833.4}\right] = 102.3mm$$

$$C = \frac{a}{\beta} = \frac{102.3}{0.85} = 120.35\,mm$$

$$fs' = \left(\frac{c-d'}{c}\right)\left(\frac{120.35-59}{120.35}\right) = 305.87\,MPa$$

$$\emptyset Mn = \emptyset\left[(Asfy - As'fs')\left(d - \frac{a}{2}\right) + As'fs'(d - d')\right]$$

$$= 0.9\left[(2450 \times 350 - 1140 \times 305.87) \times \left(469.5 - \frac{102.3}{2}\right) + 1140 \times 305.87 \times (469.5 - 59)\right]$$

$$= 320.4\,KN.m > Applied\ Mu = 307\,KN.m \qquad O.K$$

Another method to calculate *a and c*

$$f's = 600\left(1 - \frac{\beta 1\, d'}{\rho_1\, m\, d}\right) = 600\left(1 - \frac{0.85 \times 59}{0.011163 \times 19.61 \times 469.5}\right) = 307.2\ MPa$$

$$fs' = 600\left(\frac{c - d'}{c}\right)$$

$$307.2C = 600\,C - 35400$$

$$C = 120.9\ mm$$

$$a = 102.7\ mm \quad ok$$

Example (5): A beam section is limited to a width b = 300mm. and a total depth h = 500 mm and is subjected to a factored moment of 405 kN.m. Determine the necessary reinforcement. Given: f'_c = 28 MPa and f_y = 420 mPa, d'=65 mm.

Solution

1- Design the section considering single reinforced section (assume two layer of steel)

$$d = h - 90 = 500 - 90 = 410\,mm$$

$$R = \frac{Mn}{b \times d^2} = \frac{405 \times 10^6}{0.9 \times 300 \times 4102} = 8.923$$

$$m = \frac{fy}{0.85\,f'c} = \frac{420}{0.85 \times 28} = 17.65$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right)$$

$$\rho_b = \frac{0.85}{17.65}\left(\frac{600}{600 + 420}\right) = 0.028329$$

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.008}\right)\rho_b$$

$$= \left(\frac{0.003 + 0.0021}{0.008}\right)\rho_b = 0.6375 \times 0.028329 = 0.01806$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right) = \frac{1}{17.65}\left(1 - \sqrt{1 - \frac{2 \times 17.65 \times 8.923}{420}}\right) = 0.028329 > \rho_{max} = 0.01806$$

The Beam section should be design as D.D.R S

assume ρ_1 vary from 0.75 $\rho_{\max}$ to $\rho_{\max}$

$$Use\ \rho_1 = 0.9\,\rho_{\max} = 0.9 \times 0.01806 = 0.016254$$

$$As_1 = \rho_1 \times b \times d = 0.016254 \times 300 \times 410 = 1999.24\ mm^2$$

$$\rho_1 = 0.016254 = \rho - \rho' \ngeq \left(\frac{\beta_1 d'}{m\ d}\right)\left(\frac{600}{600 - fy}\right) = \left(\frac{0.85 \times 65}{17.65 \times 410}\right)\left(\frac{600}{600 - 420}\right) = 0.0254$$

So the $fs' < fy$

Check the stress in steel at compression zone from Formula:

$$f's = 600\left(1 - \frac{\beta d'}{\rho_1\, m\ d}\right) = 600\left(1 - \frac{0.85 \times 65}{0.016254 \times 17.65 \times 410}\right) = 318.16\ MPa$$

$$As' = As_2\left(\frac{fy}{fs'}\right)$$

$$As_2 = \frac{Mn_2}{fy(d - d')}$$

$$Mn_2 = Mn - Mn_1$$

$$Mn_1 = Rbd^2$$

$$R = \rho_1 fy\left(1 - \frac{1}{2}\rho_1 m\right)$$

$$R = 0.016254 \times 420\left(1 - \frac{1}{2} \times 0.016254 \times 17.65\right) = 5.848$$

$$Mn_1 == 5.848 \times 300 \times 410^2 = 294.9\ kN.m$$

$$Mn_2 = \frac{Mu}{\emptyset} - Mn_1 = \frac{405}{0.9} - 294.9 = 155.1\ KN.m$$

$$As_2 = \frac{Mn_2}{fy(d-d')} = \frac{155.18106}{420(410-65)} = 1070.4\ mm^2$$

$$As' = As_2 \times \left(\frac{fy}{fs'}\right) = 1070.4 \times \left(\frac{420}{381.16}\right) = 1179.5\ mm^2$$

$$As = As_1 + As_2 = 1999.24 + 1070.4 = 3070mm^2$$

As , Use 5 Ø 28 mm = 3075 mm^2

As' , Use 2 Ø 28 mm = 1230 mm^2

$$n = \frac{b - 116 - 2 \times ds}{D+S} + 1 = \frac{300 - 116 - 2 \times 10}{28 + 25} + 1 = 3.09$$

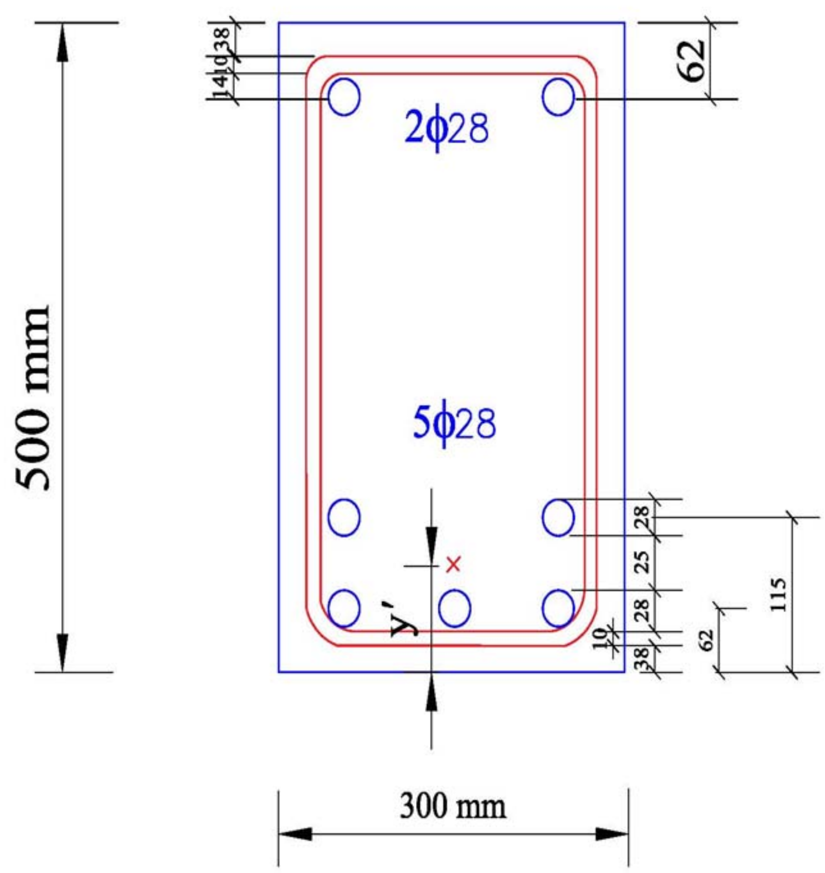
$$y' = \frac{2 \times (615) \times 115 + 3 \times (615) \times 62}{5 \times 615} = 83.2\ mm$$

$$d' = 38 + 10 + \frac{28}{2} = 62\ mm$$

$$dt = 500 - 38 - 10 - \frac{28}{2} = 438mm$$

$$d = 500 - y' = 416.8\ mm$$

$$\rho_b = \frac{438}{416.8} \times 0.028329 = 0.02977$$



$\rho_{max} = 0.6375\, \rho_b = 0.6375 \times 0.0297 = 0.01898$

$$\rho = \frac{3075}{300 \times 416.8} = 0.02459$$

$$\rho' = \frac{As'}{bd} = \frac{1230}{300 \times 416.8} = 0.009837$$

Check the stress in compression steel :

$$\rho_1 = 0.01475 = \rho - \rho' \ngeq \left(\frac{\beta_1 d'}{m\, d}\right)\left(\frac{600}{600 - fy}\right) = \left(\frac{0.85 \times 62}{17.65 \times 416.8}\right)\left(\frac{600}{600 - 420}\right) = 0.023879$$

So the $fs' < fy = 420\ MPa$

To find the value of fs' there is two method , Direct Method and Indirect Method

1- Direct Method

$A\, a^2 - Ba - C = 0$

$A = 1,$

$$B = m\, d\left(\rho - \frac{600}{fy}\rho'\right)$$

$$C = \frac{600}{fy}\beta_1\, m\, d\, d'\rho'$$

$$a = \frac{1}{2}\left[B + \sqrt{B^2 + 4\, A\, C}\,\right],$$

$$C = \frac{a}{\beta}$$

Find the constant ;

$$B = 17.65 \times 411.5 \left(0.02459 - \frac{600}{420} * 0.009837\right) = 76.53$$

$$C = \frac{600}{420} \times 0.85 \times 17.65 \times 411.5 \times 62 \times 0.009837 = 5378.85$$

$$a = \frac{1}{2}\left[76.53 + \sqrt{76.53^2 + 4 \times 1 \times 5378.85}\right] = 120.989 mm$$

$$C = \frac{a}{\beta} = \frac{120.989}{0.85} = 142.34\ mm$$

$$f's = 600\left(\frac{c - d'}{c}\right) = 600\left(\frac{142.34 - 62}{142.34}\right) = 338.7\ mPa$$

2- The In direct Method

$find\ a\ when\ fy = 420\ MPa$

$$a = \frac{As\ fy - As'fs'}{0.85 f'c\ b} = \frac{3075 \times 420 - 1230 \times 420}{0.85 \times 28 \times 300} = 108.23\ mm \qquad (1st\ attempt)$$

$$C = \frac{108.23}{0.85} = 127.34\ mm$$

$$f's = 600\left(\frac{c - d'}{c}\right) = 600\left(\frac{127.34 - 62}{127.34}\right) = 307.9\ MPa \quad < 420\ Mpa$$

$$a = \frac{As\ fy - As'fs'}{0.85 f'c\ b} = \frac{3070 \times 420 - 1230 \times 307.9}{0.85 \times 28 \times 300} = 127.55 \text{ mm} \qquad (2nd\ attempt)$$

and C= 150 mm

$$fs' = 600\left(\frac{150 - 62}{150}\right) = 352 MPa$$

$$a = \frac{As\,fy - As'fs'}{0.85f'c\,b} = \frac{3070 \times 420 - 1230 \times 352}{0.85 \times 28 \times 300} = 119.95\text{ mm} \qquad and\ C = 141.12\ mm \qquad (3rd\ attempt)$$

$$fs' = 600\left(\frac{141.12 - 62}{141.12}\right) = 336.4MPa$$

$$a = \frac{As\,fy - As'fs'}{0.85f'c\,b} = \frac{3070 \times 420 - 1230 \times 336.4}{0.85 \times 28 \times 300} = 122.6\text{ mm} \qquad and\ C = 144.28\ mm \qquad (4th\ attempt)$$

$$fs' = 600\left(\frac{144.28 - 62}{144.28}\right) = 342.16.\,MPa$$

$$a = \frac{As\,fy - As'fs'}{0.85f'c\,b} = \frac{3070 \times 420 - 1230 \times 342.16}{0.85 \times 28 \times 300} = 121.6\text{ mm} \qquad and\ C = 143.1\ mm \qquad (5th\ attempt)$$

$$fs' = 600\left(\frac{143.1 - 62}{143.1}\right) = 340.1MPa$$

$$\emptyset Mn = \emptyset\left[(Asfy - As'fs')\left(d - \frac{a}{2}\right) + As'fs'(\,d - d')\right]$$

$$= 0.9\left[(3075 \times 420 - 1230 \times 338.7) \times \left(416.8 - \frac{121.4}{2}\right) + 1230 \times 338.7 \times (416.8 - 62)\right]$$

$$= 413.4\text{ KN.m} > Applied\ Moment\ \ Mu = 405\ KN.m \quad OK$$

Reinforced Concrete Design

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LOGO

FLEXURAL DESIGN OF T- BEAM CONCRETE SECTION

Introduction

T-Beams RC floors normally consist of slabs and beams that are cast monolithically. The two act together to resist loads and because of this interaction, the effective section of the beam is a T or L section. T-section for interior beams L-section for exterior beams.

Normally, the thickness of slab varies between 100 mm and 200 mm and the web width its from 200 mm to 400 mm and its often known. Effective depth and As reinforcement quantity will be calculated. When effective stress block depth less than hf of slab thickness that's lead to design the Beam as a rectangular section while with a greater than hf , the section will be true T- section

Two Known Case for Design Procedures :

1- d is known and As should be calculated

A-Check the section is behave like rectangular section or T section . Assume a = hf and calculate the moment produce by the two flanges :

$$Mn_f(\,flange) = \emptyset\, 0.85\, f'c\; b.h_f\;(\,d - \frac{h_f}{2})$$

B- if the applied moment $Mu\; >\; Mn_f$ then :

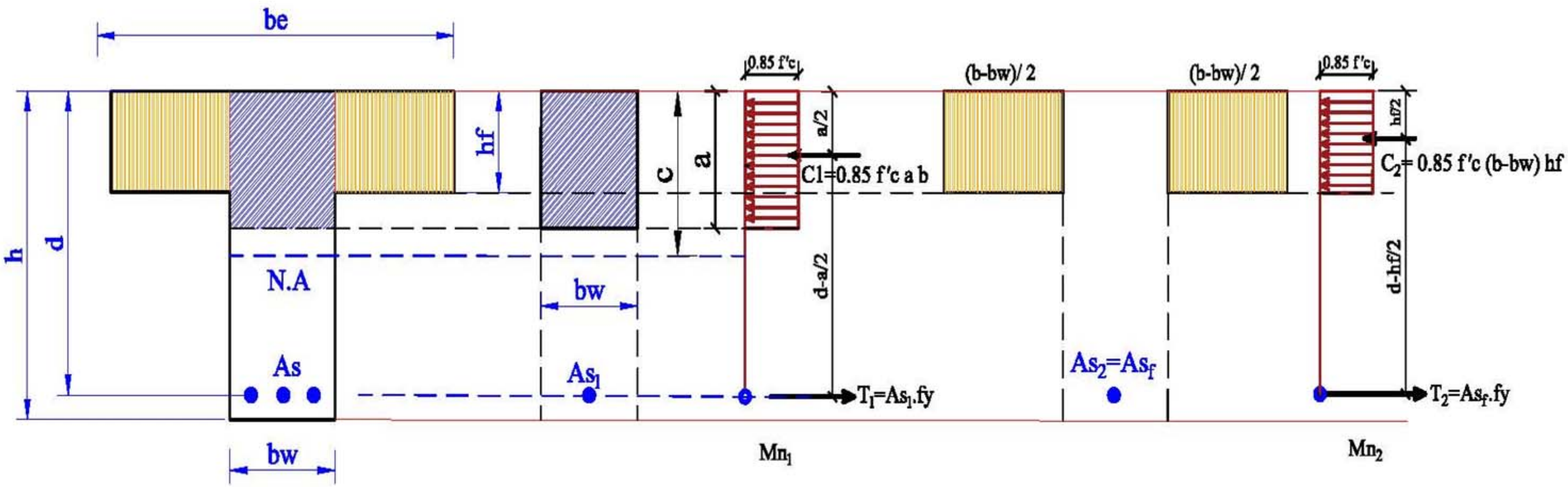
$a\; >\; h_f$ section should be design as T- Section

and if the applied moment $Mu\; <\; Mn_f$ then :

$a\; <\; hf$ and the section should be design as rectangular section ($b\; d$)

$$R = \frac{Mu}{\emptyset bd^{\,2}}$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right)$$


be
hf
h
d
N.A
As
bw
bw
As1
c
a
0.85 f'c
a/2
C1=0.85 f'c a b
d-a/2
T1=As1.fy
Mn1
(b-bw)/ 2
(b-bw)/ 2
As2=Asf
0.85 f'c
hf/2
C2= 0.85 f'c (b-bw) hf
d-hf/2
T2=Asf.fy
Mn2

$As = \rho\, bd > As_{\,min}$

In T- section case calculate :

$Asf = \dfrac{(b - bw)h_f}{m}$ $\quad$ $same \ \ (Asf.fy = 0.85\, f'c.(b - bw)h_f)$

$Mu_2 = \emptyset\, Asf\, fy \left(d - \dfrac{h_f}{2}\right)$

$Mu = Mu_1 + Mu_2$

$R = \dfrac{Mu_1}{\emptyset bd^2},$ $\quad$ $\rho = \dfrac{1}{m}\left(1 - \sqrt{1 - \dfrac{2mR}{fy}}\right)$

$As_1 = \rho\, bd$ and Total $As = As_1 + As_2$

2- When As and d is Unknown:

A- Assume $a = hf$ then we can calculated the steel area at tension zone with equal the compression force for flange

$As_{ft} = \dfrac{b\, hf}{m}$ $\quad$ $or \quad (As_{ft}.fy = 0.85\, f'c\, b\, hf)$

B- calculate d depending on calculated Asf and the Applied Mu

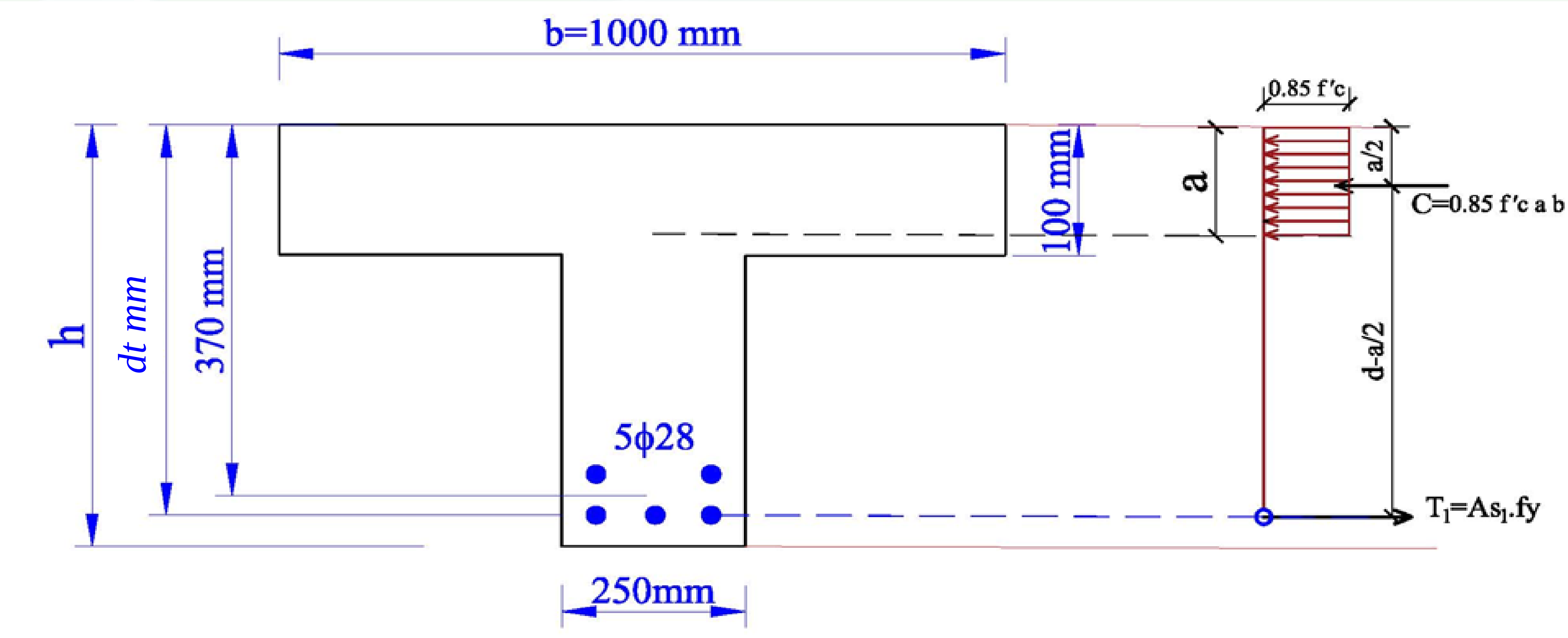
$Mu = \emptyset\, Asft\, fy \left(d - \dfrac{h_f}{2}\right)$

$$or \qquad d = \frac{Mu}{\emptyset\, Asft\, fy} + \frac{h_f}{2}$$

If d is a suitable then :

$h = d + 90$ (for two layer) and
$h = d + 65$ (for one layer)

Example (6): The T-beam section Shown below has a width bw = 250 mm, a flange width be =1000 mm , a flange thickness = 100 mm and effective depth d = 370 mm. Determine the necessary reinforcement if the applied factored moment Mu= 380KN.m . Given: f'_c = 21 MPa and f_y = 420 Mpa.



1- Check the neutral axis depth

$$assume: \quad a = hf = 100\,mm$$

$$\emptyset Mn = \emptyset\,(0.85\,f'c)be\,hf\left(d - \frac{hf}{2}\right) = 0.9 \times 0.85 \times 21 \times 1000 \times 100\left(370 - \frac{100}{2}\right) = 514.08KN.m \ > 380KN.m$$

$$\therefore the\ section\ design\ as\ a\ Rectangular\ section\ with\ b = be = 1000\,mm$$

$$R = \frac{Mu}{\emptyset bd^2} = \frac{380 \times 10^6}{0.9 \times 1000 \times 370^2} = 3.084$$

$$m = \frac{fy}{0.85\,f'c} = \frac{420}{0.85 \times 21} = 23.53$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right) = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2 \times 23.53 \times 3.084}{420}}\right) = 0.008118$$

$$As = \rho\,bd = 0.008118 \times 1000 \times 370 = 3003.75\,mm^2$$

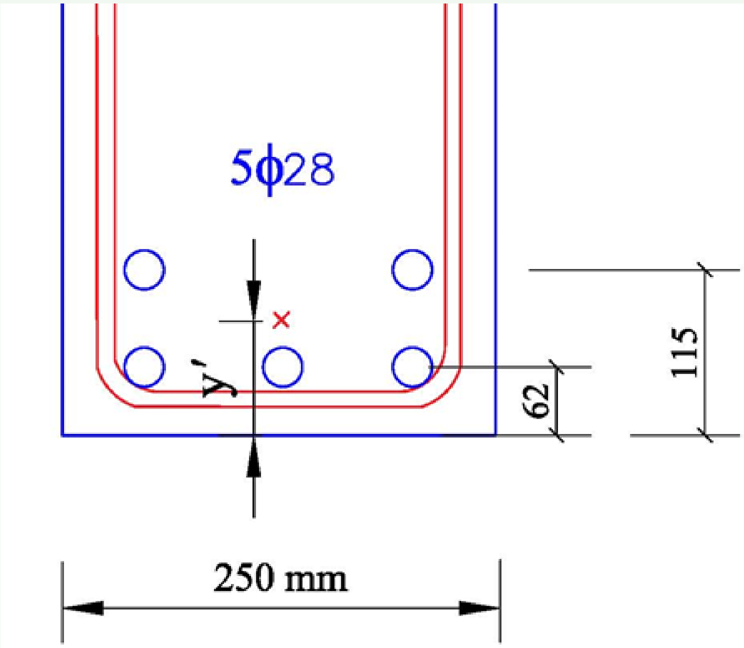
$$a = \rho\,m\,d = 0.008118 \times 23.53 \times 370 = 70.68\,mm < hf = 100\,mm$$

$$Total\ As = 5 \times 615 = 3075\,mm^2$$

$$\rho_w = \frac{3075}{250 \times 370} = 0.0332 > \rho_{min} = \frac{1.4}{420} = 0.0033$$

$$Max\ As = \frac{(b - bw)h_f}{m} + \rho_{\max}\ bw\ d$$

$$\rho_{\mathrm{b}} = \frac{\beta_1}{m}\left(\frac{600}{600 + 420}\right)\left(\frac{dt}{d}\right) = \frac{0.85}{17.65}\left(\frac{600}{600 + 420}\right)\left(\frac{dt}{d}\right)$$



$$y' = \frac{2 \times (615) \times 115 + 3 \times (615) \times 62}{5\ \times 615} = 83.2\ mm$$

$$h = 370 + y' = 370 + 83.5 = 453.5\ mm$$

$$dt = 453.5 - 38 - 10 - \frac{28}{2} = 391.5\ mm$$

$$\rho_{\mathrm{b}} = \frac{0.85}{23.53}\left(\frac{600}{600 + 420}\right)\left(\frac{391.5}{370}\right) = 0.02248$$

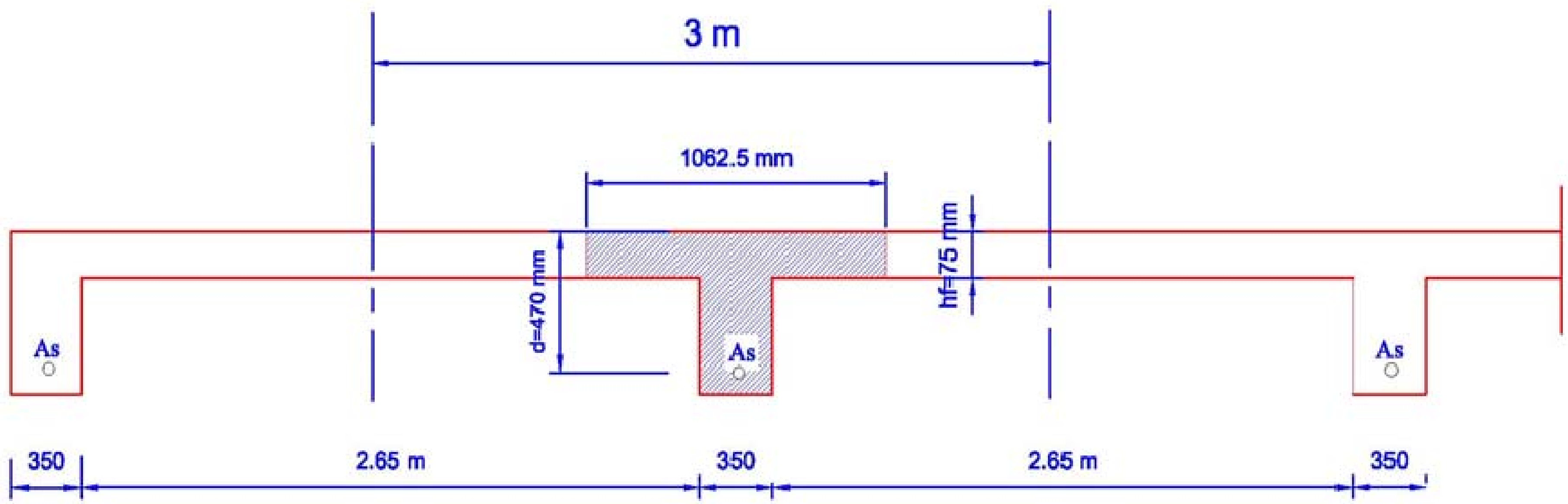
$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.008}\right)\rho_b = 06375 \times 0.02248 = 0.01433$$

$$Max\ As = \frac{(1000 - 250) \times 100}{23.53} + 0.01433 \times 250 \times 370 = 4513\ mm^2 > As = 3075\ mm^2$$

$$c = \frac{a}{\beta_1} = \frac{70.68}{0.85} = 83.15\ mm$$

$$\epsilon_t = \left(\frac{dt - c}{c}\right) \times 0.003 = \left(\frac{391.5 - 83.15}{83.15}\right) \times 0.003 = 0.0113 > 0.005\quad OK \qquad \emptyset = 0.9\ \ T.C$$

Example (7): The Floor system shown below consist of 75 mm slab thickness supported by 4.25 m span beam spaced 3 m on center. The beam have a web width bw = 350 mm and an effective depth d= 470 mm. Calculate the necessary reinforcement for a typical section interior beam if the factored applied moment Mu= 575 KN.m . $Given$: $f'_c = 21\, MPa$ and $f_y = 420\, mPa$, .



Solution :

find the effective be:

$1 - be = 16hf + bw = 16 \times 75 + 350 = 1550\ mm$

$2 - be = \frac{L}{4} = \frac{4250}{4} = 1062.5\ mm$

$3 - be\ = \ b\ (\ center\ to\ center\ adjacent\ panels) = \ 3000\ mm$

∴ be = 1062.5 mm

1-Design section as Rectangular Section :

$$R = \frac{Mu}{\emptyset bd^2} = \frac{575 \times 106}{0.9 \times 1062.5 \times 470^2} = 2.722$$

$$m = \frac{fy}{0.85\ f'c} = \frac{420}{0.85 \times 21} = 23.53$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right) = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2 \times 23.53 \times 2.722}{420}}\right) = 0.007069$$

$$a = \rho\ m\ d = 0.007069 \times 23.53 \times 470 = 78.18mm > hf\ = 75\ mm$$

∴ Design as T- Section

$$calculate \quad Asf = \frac{(b-bw)h_f}{m} = \frac{(1062.5-350)\times 75}{23.53} = 2271\,mm^2$$

$$Mu_2 = \emptyset Asf\, fy\left(d-\frac{hf}{2}\right) = 0.9\times 2271\times 420\times\left(470-\frac{75}{2}\right) = 371.27\,KN.m$$

$$\therefore Mu_1 = Mu - Mu_2 = 575 - 371.27 = 203.73\,KN.m$$

$$R = \frac{Mu}{\emptyset bd^2} = \frac{203.73\times 10^6}{0.9\times 350\times 470^2} = 2.928$$

$$\rho = \frac{1}{m}\left(1-\sqrt{1-\frac{2mR}{fy}}\right) = \frac{1}{23.53}\left(1-\sqrt{1-\frac{2\times 23.53\times 2.928}{420}}\right) = 0.007662$$

$$As_1 = \rho\, bd = 0.007662\times 350\times 470 = 1260.35\,mm^2$$

Total As= As_1+As_2=1260.35+2271=3531.35 mm^2

Use 6 Ø 28 mm = 3690 mm^2

$$a = \rho\, m\, d = 0.007662\ \times 23.53\times 470 = 84.73\,mm$$

$$c = \frac{a}{\beta 1} = \frac{84.73}{0.85} = 99.68\,mm$$

$$\epsilon t = \left(\frac{dt-c}{c}\right)\times 0.003 = \left(\frac{496.5-99.68}{99.68}\right)\times 0.003 = 0.01194 > 0.005 \quad OK$$

$$dt = 470 + \frac{25}{2} + \frac{28}{2} = 496.5\text{ mm}$$

$$\therefore \emptyset = 0.9 \qquad T.C$$

$$\rho_{min} = \frac{1.4}{fy} = 0.0033 < \rho \quad OK$$

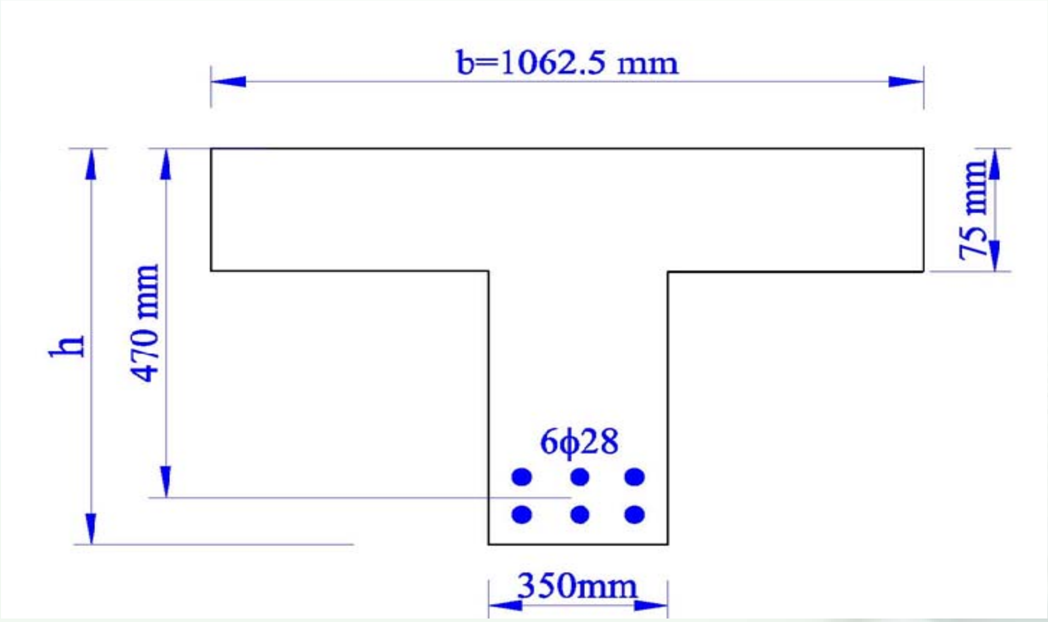
$$Max\ As = \frac{(b - bw)h_f}{m} + \rho_{\max}\ bw\ d$$

$$\rho_b = \frac{0.85}{23.53}\left(\frac{600}{600 + 420}\right)\left(\frac{496.5}{470}\right) = 0.02245$$

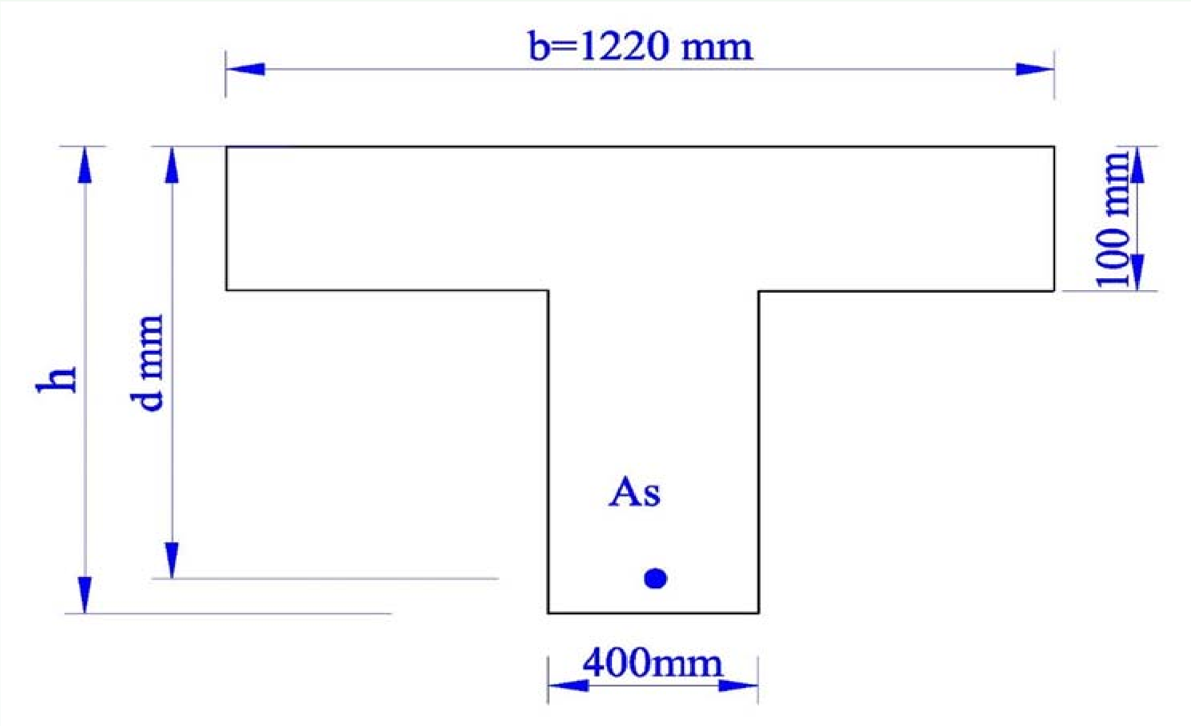
$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.008}\right)\rho_b$$

$$= \left(\frac{0.003 + 0.0021}{0.008}\right)\rho_b = 0.6375 \times 0.02245 = 0.01431$$

$$Max\ As = \frac{(1032.5 - 350) \times 75}{23.53} + 0.01431 \times 350 \times 470 = 4625\ mm^2 > 3690\ mm^2 \quad OK$$



Example (7): In slab beam, The flange width was determine to be =1220 mm , the web width was bw=400 mm , and the slab thickness was hf= 100 mm . Design T- section to resist an external factored moment Mu= 1100 kN.m . Given: $f'_c = 21\ MPa$ and $f_y = 420\ mPa,$.



Solution

d is unknown

So choose $a = h_f = 100\ mm$

$T = C$

$As_{ft}\, fy\ = 0.85\, f'c\ b\ hf$

$$As_{ft} = \frac{0.85\, f'c\ b\ hf}{fy} = \frac{0.85\ \times 21 \times 1220 \times 100}{420} = 5185\ mm^2$$

now calculate d from:

$$Mu = \emptyset Mn = \emptyset Asft\ fy\left(d - \frac{h_f}{2}\right) = 0.9 \times 5185 \times 420 \times \left(d - \frac{100}{2}\right)$$

$d = 661.24\ mm$

1- If we choose $d > 661.24\ mm$ (say 800 mm), in this case $a < hf$ and the section will be design as Rectangular section

$$R = \frac{Mu}{\emptyset bd^2} = \frac{1100 \times 10^6}{0.9 \times 11220 \times 800^2} = 1.565$$

$$m = \frac{fy}{0.85\, f'c} = \frac{420}{0.85 \times 21} = 23.53$$

$$\rho = \frac{1}{m}\left(1-\sqrt{1-\frac{2mR}{fy}}\right) = \frac{1}{23.53}\left(1-\sqrt{1-\frac{2\times 23.53\times 1.565}{420}}\right) = 0.003906$$

$$As = \rho bd = 0.003906\times 1220\times 800 = 3812\ mm^2$$

Use $8\ \emptyset\ 25\ mm\ (8\times 490 = 3920\ mm^2)$

$$\rho w = \frac{3920}{400\times 800} = 0.01225 > \rho_{min} = \frac{1.4}{420} = 0.0033$$

$$\rho_{max} = 0.6375\ \rho b$$

$$dt = d + \frac{25}{2} + \frac{25}{2} = 825\ mm$$

$$\rho_{max} = 0.6375\times\frac{0.85}{23.53}\left(\frac{600}{600+420}\right)\left(\frac{825}{800}\right) = 0.01397$$

$$Max\ As = As_f + \rho_{\max}\ bw\ d$$

$$Max\ As = \frac{(b-bw)h_f}{m} + \rho_{\max}\ bw\ d = \frac{(1220-400)\times 100}{23.53} + 0.01397\times 400\times 800 = 7955\ mm^2 > 3920\ mm^2\quad OK$$

2- If we choose $d < 661.24\ mm,$ (say 800 mm) in this case $a > hf$ and the section will be design as T $-$ section

$$Calculate\quad As_f = \frac{(b-bw)h_f}{m} = \frac{(1220-400)\times 100}{23.53} = 3484.9\ mm^2$$

$$Mu_2 = \emptyset Asf\ fy\left(d-\frac{h_f}{2}\right) = 0.9\times 3484.9\times 420\times\left(600-\frac{100}{2}\right) = 724.51\ KN.m$$

$$\therefore Mu_1 = Mu - Mu_2 = 1100 - 724.51 = 375.49\ KN.m$$

$$R = \frac{Mu}{\emptyset bd^2} = \frac{375.49 \times 106}{0.9 \times 400 \times 600^2} = 2.8973$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right) = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2 \times 23.53 \times 2.8973}{420}}\right) = 0.007573$$

$$As_1 = \rho\ bd = 0.007573 \times 400 \times 600 = 1817.5\ mm^2$$

Total As= As_1+As_2=1817.5+3484.9=5302.4 mm^2

Use 8 Ø 30 mm = 5648 mm^2

$$n = \frac{b - 116 - 2 \times ds}{D + s} + 1 = n = \frac{400 - 116 - 2 \times 10}{30 + 25} + 1 = 5.8\ \cong\ 5$$

$$dt = d + \frac{30}{2} + \frac{25}{2} = 627.5\ mm$$

$$a = \rho\ m\ d = 0.007573 \times 23.53 \times 600 = 106.9\ mm$$

$$c = \frac{a}{\beta_1} = \frac{106.9}{0.85} = 125.8\ mm$$

$$\epsilon_t = \left(\frac{dt - c}{c}\right) \times 0.003 = \left(\frac{627.5 - 125.8}{125.8}\right) \times 0.003 = 0.01196 > 0.005\quad (OK)$$

$\therefore \emptyset = 0.9 \qquad T.C$

Thank You……….