Reinforced Concrete Design

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Chapter II

Flexural Analysis Reinforced Beam

Strength Design Approach

The analysis and design of a structural member may be regarded as the process of selecting the proper materials and determining the member dimensions such that the design strength is equal or greater than the required strength. The required strength is determined by multiplying the actual applied loads, the dead load, the assumed live load, and other loads, such as wind, seismic, earth pressure, fluid pressure, snow, and rain loads, by load factors. These loads develop external forces such as bending moments, shear, torsion, or axial forces, depending on how these loads are applied to the structure.

In proportioning reinforced concrete structural members, three main items can be investigated:

1 .The safety of the structure, which is maintained by providing adequate internal design strength.

2 .Deflection of the structural member under service loads. The maximum value of deflection must be limited and is usually specified as a factor of the span, to preserve the appearance of the structure.

3 .Control of cracking conditions under service loads. Visible cracks spoil the appearance of the structure and permit humidity to penetrate the concrete, causing corrosion of steel and consequently weakening the reinforced concrete member. The ACI Code implicitly limits crack widths to 0.016 in. (0.40 mm) for interior members and 0.013 in. (0.33 mm) for exterior members. Control of cracking is achieved by adopting and limiting the spacing of the tension bar.

It is worth mentioning that the strength design approach was first permitted in the United States in 1956 and in Britain in 1957. The latest ACI Code emphasizes the strength concept based on specified strain limits on steel and concrete that develop tension-controlled, compression controlled, or transition conditions.

ASSUMPTIONS

Reinforced concrete sections are heterogeneous (nonhomogeneous), because they are made of two different materials, concrete and steel. Therefore, proportioning structural members by strength design approach is based on the following assumptions:

1. Strain in concrete is the same as in reinforcing bars at the same level, provided that the bond between the steel and concrete is adequate.
2. Strain in concrete is linearly proportional to the distance from the neutral axis.
3. The modulus of elasticity of all grades of steel is taken as Es = (200,000MPa or N/mm^2). The stress in the elastic range is equal to the strain multiplied by Es.
4. Plane cross sections continue to be plane after bending.
5. Tensile strength of concrete is neglected because (a) concrete's tensile strength is about 10% of its compressive strength, (b) cracked concrete is assumed to be not effective, and (c) before cracking, the entire concrete section is effective in resisting the external moment.
6. The method of elastic analysis, assuming an ideal behavior at all levels of stress, is not valid. At high stresses, non-elastic behavior is assumed, which is in close agreement with the actual behavior of concrete and steel.
7. At failure the maximum strain at the extreme compression fibers is assumed equal to 0.003 by the ACI Code provision.
8. For design strength, the shape of the compressive concrete stress distribution may be assumed rectangular, parabolic, or trapezoidal. In this text, a rectangular shape will be assumed (ACI Code, Section 22.2).

TYPES OF FLEXURAL FAILURE AND STRAIN LIMITS

Three types of flexural failure of a structural member can be expected depending on the percentage of steel used as explained before.

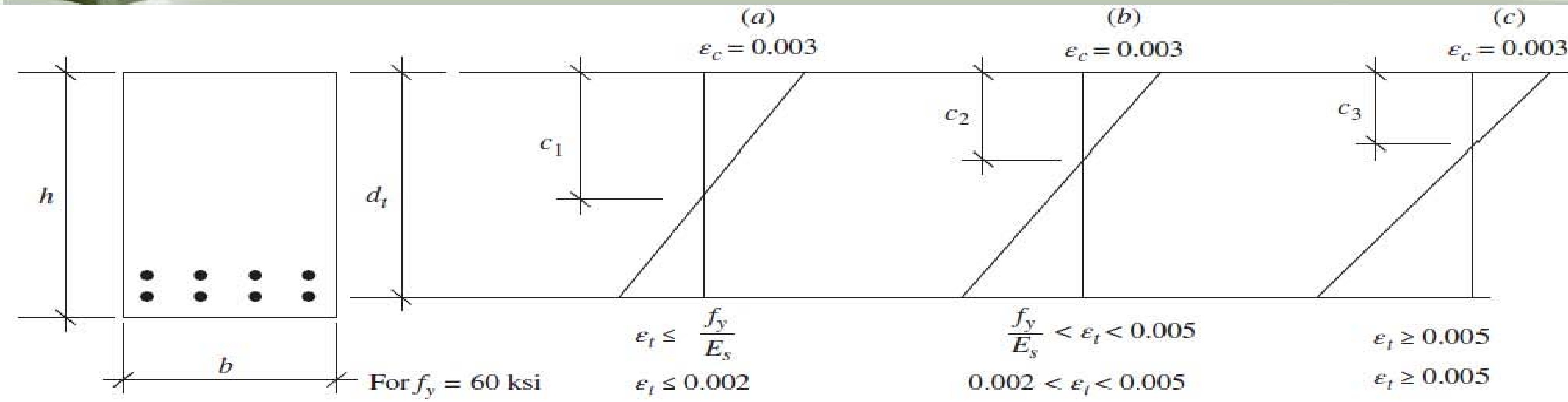
It can be assumed that concrete fails in compression when the concrete strain reaches 0.003.A range of 0.0025 to 0.004 has been obtained from tests and the ACI Code, Section 22.2.2.1, assumes a strain of 0.003.

In beams designed as tension-controlled sections, steel yields before the crushing of concrete. Cracks widen extensively, giving warning before the concrete crushes and the structure collapses. The ACI Code adopts this type of design. In beams designed as balanced or compression-controlled sections, the concrete fails suddenly, and the beam collapses immediately without warning. The ACI Code does not allow this type of design.

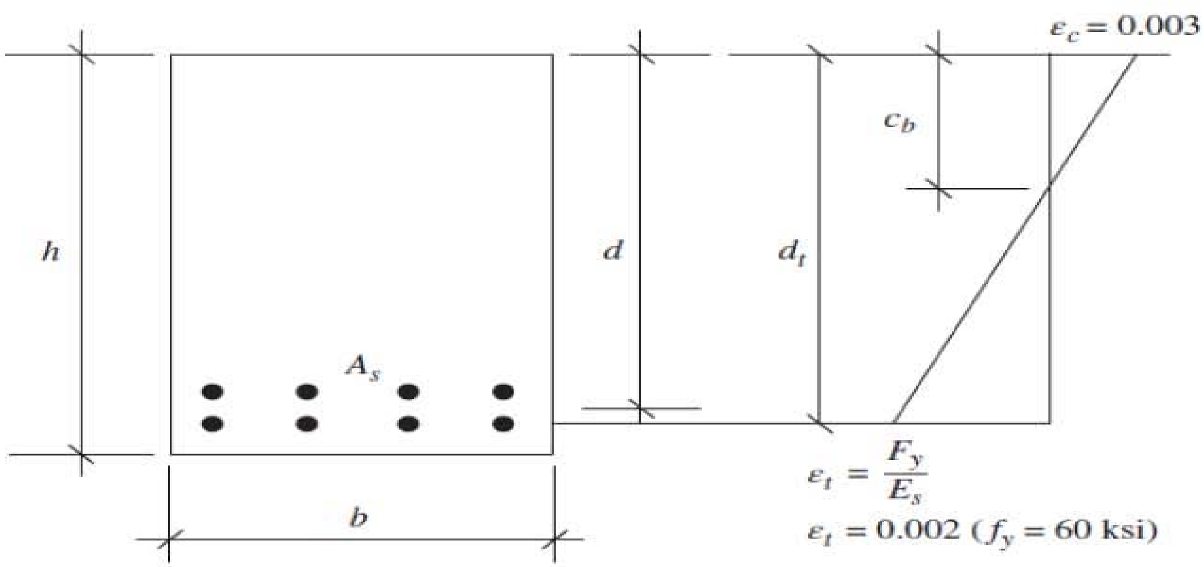
Strain Limits for Tension and Tension-Controlled Sections

The design provisions for both reinforced and pre-stressed concrete members are based on the concept of tension or compression-controlled sections, ACI Code, Section 21.2. Both are defined in terms of net tensile strain (NTS), (ε_t), in the extreme tension steel at nominal strength, exclusive of pre-stress strain. Moreover, two other conditions may develop: (1) the balanced strain condition and (2) the transition region condition. These four conditions are defined as follows:

1. Compression-controlled sections are those sections in which the net tensile strain, NTS, in the extreme tension steel at nominal strength is equal to or less than the compression-controlled strain limit at the time when concrete in compression reaches its assumed strain limit of 0.003,(εc = 0.003). For grade 60 steel, (f_y = 420 MPa), the compression-controlled strain limit may be taken as a net strain of 0.002, Fig. a. This case occurs mainly in columns subjected to axial forces and moments.
2. Tension-controlled sections are those sections in which the NTS, εt, is equal to or greater than 0.005 just as the concrete in the compression reaches its assumed strain limit of 0.003, Fig. c.
3. Sections in which the NTS in the extreme tension steel lies between the compression controlled strain limit (0.002 for f_y = 420 MPa) and the tension-controlled strain limit of 0.005 constitute the transition region, Fig. b.
4. The balanced strain condition develops in the section when the tension steel, with the first yield, reaches a strain corresponding to its yield strength, f_y or $\varepsilon s = f_y/E_s$, just as the maximum strain in concrete at the extreme compression fibers reaches 0.003, Fig. d.



Strain limit distribution, $c_1 > c_2 > c_3$: (*a*) compression-controlled section, (*b*) transition region, and (*c*) tension-controlled section.



d. Balanced strain section (occurs at first yield or at distance d_t).

In addition to the above four conditions, Section 9.3.3.1 of the ACI Code indicates that the net tensile strain, εt, at nominal strength, within the transition region, shall not be less than 0.004 for reinforced concrete flexural members without or with an axial load less than 0.10 f′c Ag, where Ag=gross area of the concrete section.

Note that d_t in Fig. above, is the distance from the extreme concrete compression fiber to the extreme tension steel, while the effective depth, d, equals the distance from the extreme concrete compression fiber to the centroid of the tension reinforcement. These cases are summarized in Table below:

Table 1 Strain Limits of Figure above

Section Condition	Concrete Strain	Steel Strain	Notes (f_y = 60 ksi) 420MPa
Compression controlled	0.003	$\varepsilon_t \leq f_y/E_s$	$\varepsilon_t \leq 0.002$
Tension controlled	0.003	$\varepsilon_t \geq 0.005$	$\varepsilon_t \geq 0.005$
Transition region	0.003	$f_y/E_s < \varepsilon_t < 0.005$	$0.002 < \varepsilon_t < 0.005$
Balanced strain	0.003	$\varepsilon_s = f_y/E_s$	$\varepsilon_s = 0.002$
Transition region (flexure)	0.003	$0.004 \leq \varepsilon_t < 0.005$	$0.004 \leq \varepsilon_t < 0.005$

LOAD FACTORS

For the design of structural members, the factored design load is obtained by multiplying the dead load by a load factor and the specified live load by another load factor. The magnitude of the load factor must be adequate to limit the probability of sudden failure and to permit an economical structural design. The choice of a proper load factor or, in general, a proper factor of safety depends mainly on the importance of the structure (whether a courthouse or a warehouse), the degree of warning needed prior to collapse, the importance of each structural member (whether a beam or column), the expectation of overload and the accuracy of calculations.

Based on historical studies of various structures, experience, and the principles of probability, the ACI Code adopts a load factor of 1.2 for dead loads and 1.6 for live loads. The dead-load factor load. Moreover, the choice of factors reflects the degree of the economical design as well as the degree of safety and serviceability of the structure. It is also based on the fact that the performance of the structure under actual loads must be satisfactorily within specific limits.

If the required strength is denoted by U (ACI Code, Section 5.3.1), and those due to wind and seismic forces are W and E, respectively, according to the ACI and ASCE 7-10 Codes (American society of civil Engineering) , the required strength, U, shall be the most critical of the following factors:

1. In the case of dead, live, and wind loads,

U= 1.4 D
U= 1.2 D+ 1.6L
U= 1.2 D+ 1.0L+1.0 W
U= 0.9 D+1.0 W
U= 1.2 D+ (1.0L+0.5 W)

2. In the Case of Dead Load , Live and seismic load (earthquake) forces , E

U= 1.2 D+ 1.0L+1.0E
U= 0.9 D+ 1.0E

3. For load combination due to roof live load , Lr , rain Load ,R, Snow load ,S, in additional to dead , live load , wind , and earthquake load:

U= 1.2 D + 1.6L+0.5 (Lr or S or R)
U= 1.2 D + 1.6 (Lr or S or R) + (1.0 L or 0.5 W)
U= 1.2 D + 1.0 W + 1.0 L + 0.5 (Lr or S or R)
U= 1.2 D + 1.E + 1.0 L+ 0.2 S

4. Where fluid load F is present, it shall be included as follows:

U= 1.4 (D + F)
U= 1.2 D + 1.2F + (L or 0.5 W)+ 1.6(Lr or S or R)
U= 1.2 D + 1.2F + 1.0 W + L + 0.5 (Lr or S or R)
U= 1.2 D + 1.2F + 1.0 E + L + 0.2 S
U= 0.9 (D+F) + 1.0 E

STRENGTH REDUCTION FACTOR ϕ

The nominal strength of a section, say Mn, for flexural members, calculated in accordance with the requirements of the ACI Code provisions must be multiplied by the strength reduction factor, ϕ, which is always less than 1. The strength reduction factor has several purposes:

1 .To allow for the probability of understrength sections due to variations in dimensions, material properties, and inaccuracies in the design equations.

2 .To reflect the importance of the member in the structure.

3 .To reflect the degree of ductility and required reliability under the applied loads

The ACI Code, Table 21.2.1, specifies the following values to be used

A higher ϕ factor is used for tension-controlled sections than for compression-controlled sections, because the latter sections have less ductility and they are more sensitive to variations in concrete strength. Also, spirally reinforced compression members have a ϕvalue of 0.75 compared to 0.65 for tied compression members; this variation reflects the greater ductility behavior of spirally reinforced concrete members under the applied loads. In the ACI Code provisions, the ϕ factor is based on the behavior of the cross section at nominal strength, (Pn, Mn), defined in terms of the NTS, εt, in the extreme tensile strains, as given below. For tension-controlled members,ϕ= 0.9. For compression-controlled members, ϕ= 0.75 (with spiral reinforcement) and ϕ= 0.65 for other members.

For tension-controlled sections	ϕ= 0.9
For Compression -controlled sections	
a- with Spiral Reinforcement	ϕ= 0.75
b- other Reinforced member	ϕ= 0.65
For Plain Concrete	ϕ= 0.60
For Shear and Torsion	ϕ= 0.75
For Bearing on Concrete	ϕ= 0.65
For Strut and Tie model	ϕ= 0.75

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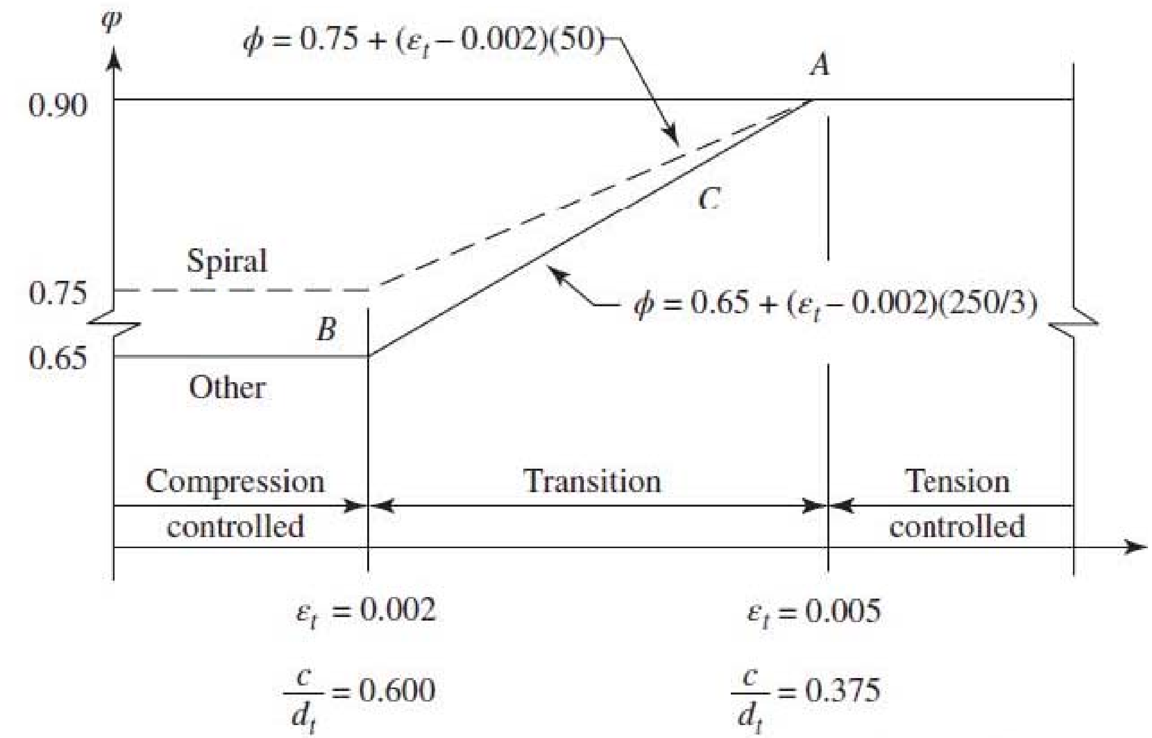
For the transition region, ϕmay be determined by linear interpolation between 0.65 (or 0.75) and 0.9. Figure 3.6a shows the variation of $\phi$for grade 60 steel (420 Mpa) . The linear equations are as follows:

$\emptyset = 0.75 + (\varepsilon_t - 0.002)(50)$	For Spiral Members
$\emptyset = 0.65 + (\varepsilon_t - 0.002)\left(\frac{250}{3}\right)$	For Other Members

Alternatively Ø may be determined in the transition region , as a function of (c/dt) for grade 60 (fy 420 Mpa) steel as follows:

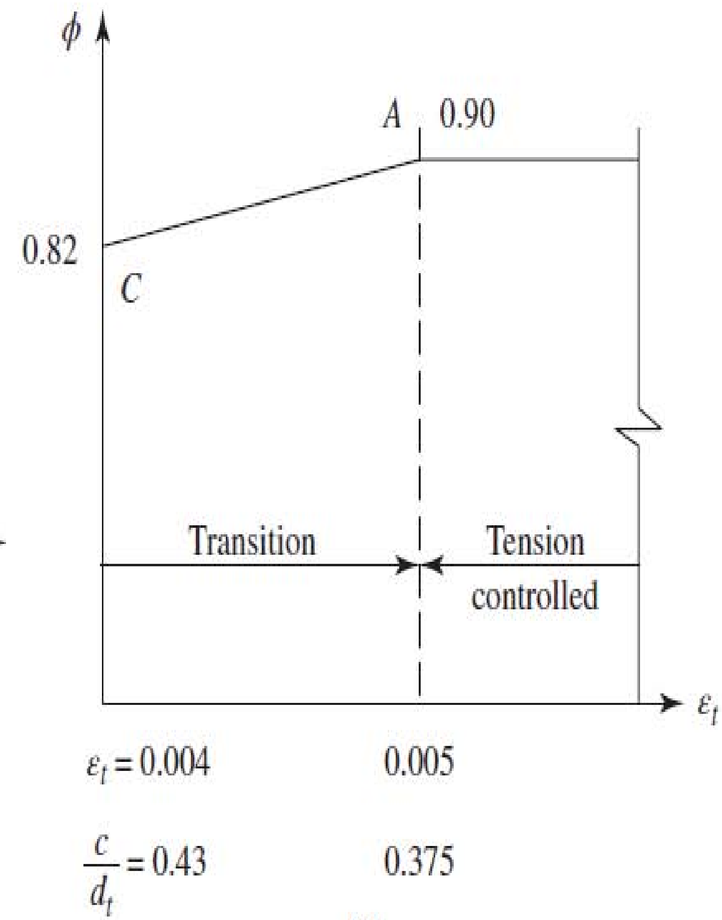
$\emptyset = 0.75 + 0.15\left(\frac{1}{c/dt} - \frac{5}{3}\right)$ …………….	For Spiral Members
$\emptyset = 0.65 + 0.15\left(\frac{1}{c/dt} - \frac{5}{3}\right)$ ………………	For Other Members



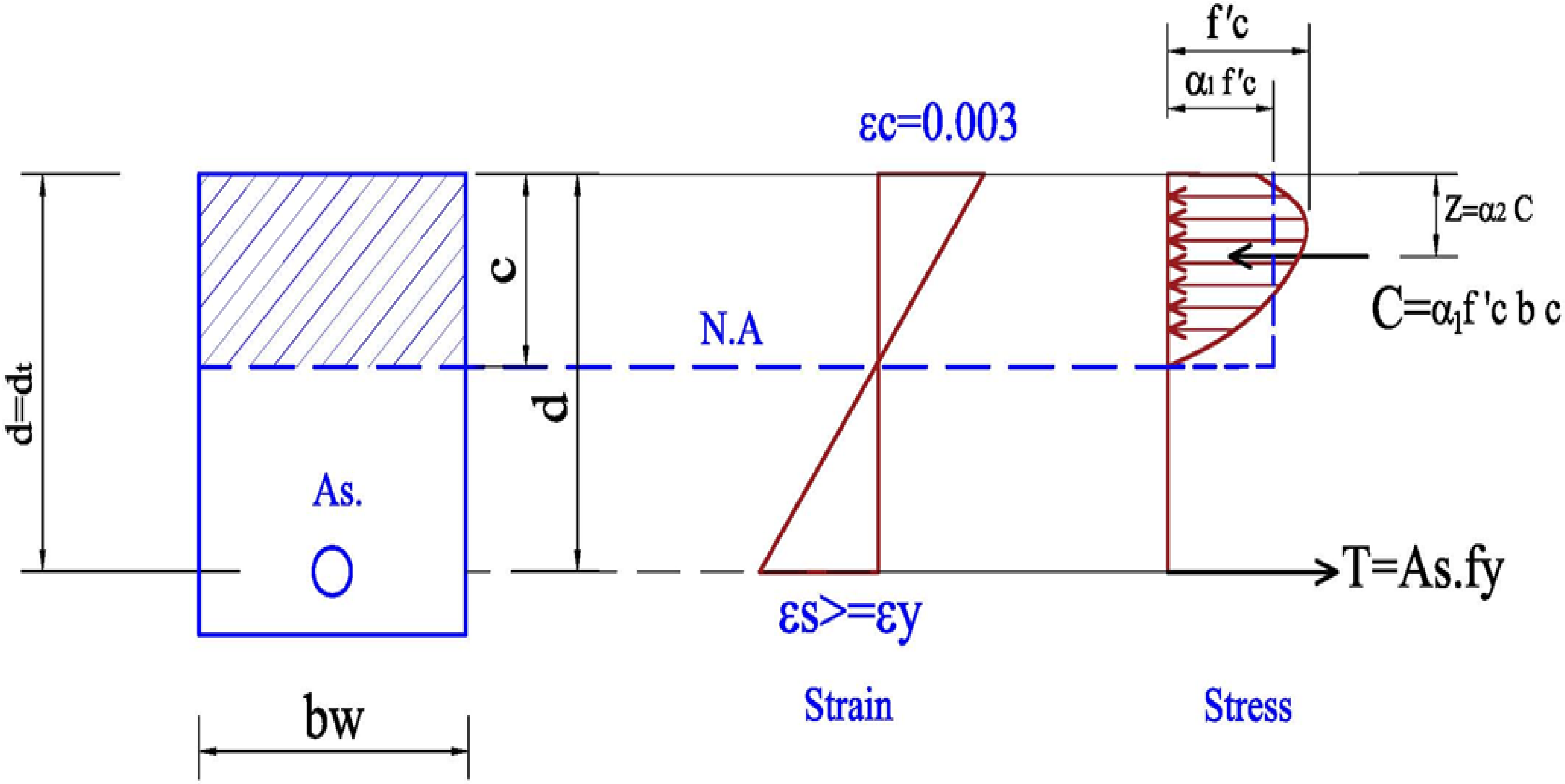
$$\text{Spiral } \phi = 0.75 + 0.15\left[\frac{1}{c/d_t} - \frac{5}{3}\right]$$

$$\text{Other } \phi = 0.65 + 0.25\left[\frac{1}{c/d_t} - \frac{5}{3}\right]$$

(e)



(f)


f'c
α1 f'c
εc=0.003
Z=α2 C
C=α1f 'c b c
c
d
d=dt
N.A
As.
T=As.fy
εs>=εy
bw
Strain
Stress

EQUIVALENT COMPRESSIVE STRESS DISTRIBUTION

The distribution of compressive concrete stresses at failure may be assumed to be a rectangle, trapezoid, parabola, or any other shape that is in good agreement with test results.

When a beam is about to fail, the steel will yield first if the section is under reinforced, and in this case the steel is equal to the yield stress. If the section is over reinforced, concrete crushes first and the strain is assumed to be equal to 0.003, which agrees with many tests of beams and columns. A compressive force, C, develops in the compression zone and a tension force, T, develops in the tension zone at the level of the steel bars. The position of force T is known because its line of application coincides with the center of gravity of the steel bars. The position of compressive force C is not known unless the compressive volume is known and its center of gravity is located. If that is done, the moment arm, which is the vertical distance between C and T, will consequently be known.
In Fig. above, if concrete fails, $\varepsilon_c = 0.003$, and if steel yields, as in the case of a balanced section, $f_s = f_y$.
The compression force C is represented by the volume of the stress block, which has the non-uniform shape of stress over the rectangular hatched area of b*c. This volume may be considered equal to $C = b\, c\, (\alpha_1 f'c)$, where $\alpha_1 f'c$ is an assumed average stress of the non-uniform stress block.
The position of compression force C is at a distance z from the top fibers, which can be considered as a fraction of the distance c (the distance from the top fibers to the neutral axis), and z can be assumed to be equal to $\alpha_2 C$, where $\alpha_2 < 1$. The values of $\alpha 1$ and $\alpha 2$ have been estimated from many tests, and their values are as follows:

α_1 = 0.72 for f'c ≤ (28MPa); it decreases linearly by 0.04 for every (7MPa) greater than (28 MPa)

$$\alpha_1 = 0.72 - 0.04 \times (f'c - 28)/7$$

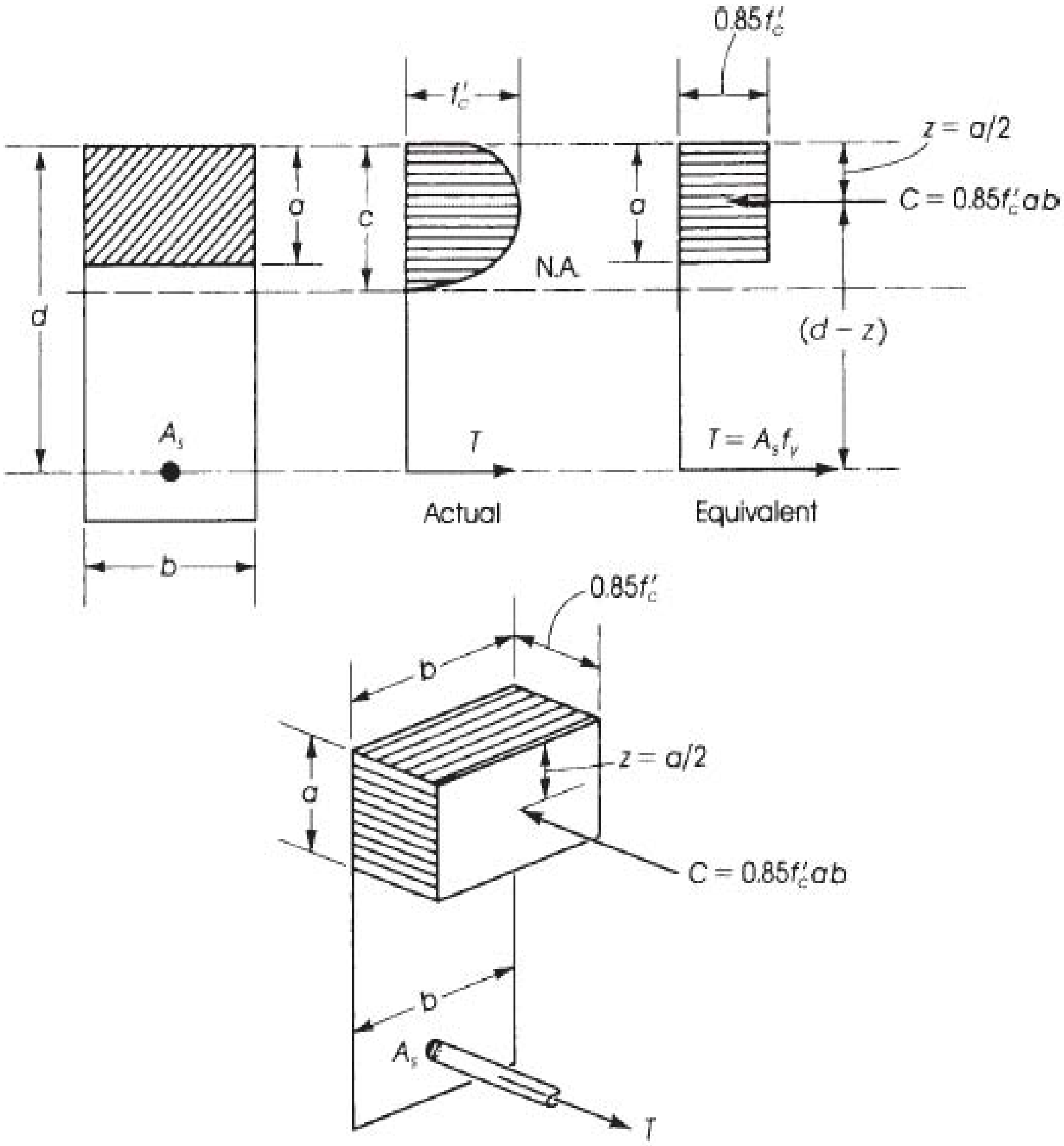
α_2 = 0.425 for f'c < (28MPa); it decreases linearly by 0.025 for every (7MPa) greater than (28MPa)

$$\alpha_2 = 0.425 - 0.025 \times (f'c - 28)/7$$

The decrease in the value of $\alpha 1$ and $\alpha 2$ is related to the fact that high-strength concretes show more brittleness than low-strength concretes.

To derive a simple rational approach for calculations of the internal forces of a section, the ACI Code adopted an equivalent rectangular concrete stress distribution, which was first proposed by C.S. Whitney and checked by Mattock and others. A concrete stress of 0.85 f'c is assumed to be uniformly distributed over an equivalent compression zone bounded by the edges of the cross section and a line parallel to the neutral axis at a distance ($a=\beta_1 c$) from the fiber of maximum compressive strain, where c is the distance between the top of the compressive section and the neutral axis. The fraction β_1 is 0.85 for concrete strengths f'c ≤ (28MPa) and is reduced linearly at a rate of 0.05 for each (7MPa) of stress greater than (28MPa) with a minimum value of 0.65.

$$\boxed{\beta_1 = 0.85 - 0.05 \times \left(\frac{f'c - 28}{7}\right) \geq 0.65}$$



0.85f'c
f'c
z = a/2
C = 0.85f'c ab
a
c
a
d
N.A.
(d − z)
As
T
T = As fy
Actual
Equivalent
b
0.85f'c
b
z = a/2
a
C = 0.85f'c ab
b
As
T

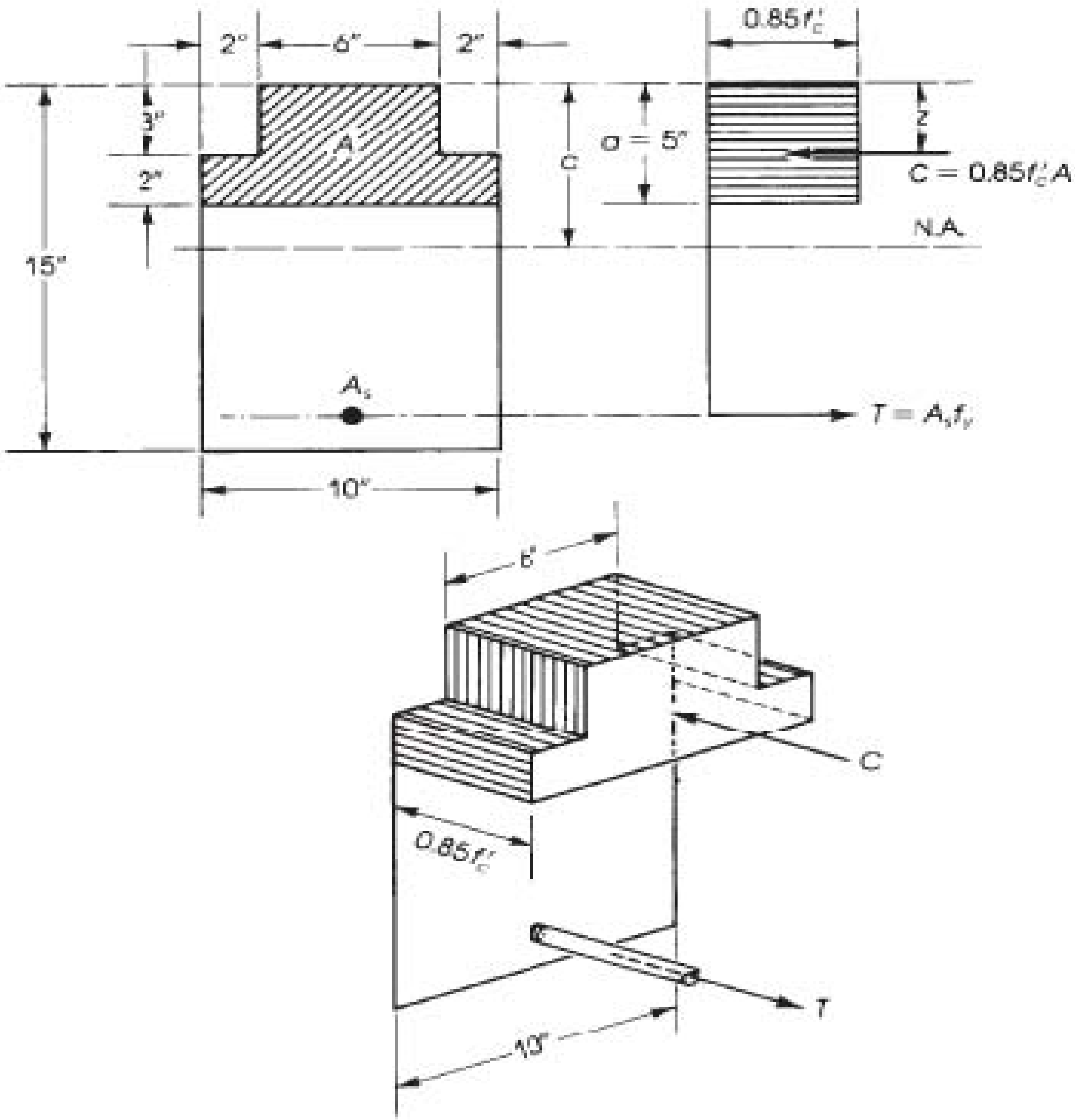


Figure 3.10 Forces in a nonrectangular section.

SINGLY REINFORCED RECTANGULAR SECTION IN BENDING

The balanced condition is achieved when steel yields at the same time as the concrete fails, and that failure usually happens suddenly. This implies that the yield strain in the steel is reached (εy =fy/Es) and that the concrete has reached its maximum strain of 0.003.

The percentage of reinforcement used to produce a balanced condition is called the balanced steel ratio, ρb.

This value is equal to the area of steel, As, divided by the effective cross section bd.

$$\rho_b = \frac{As_{balanced}}{bd}$$

Where:

b = width of compression face of member

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement

Two basic equations for the analysis and design of structural members are the two equations of equilibrium that are valid for any load and any section:

1 .The compression force should be equal to the tension force; otherwise, a section will have linear displacement plus rotation:

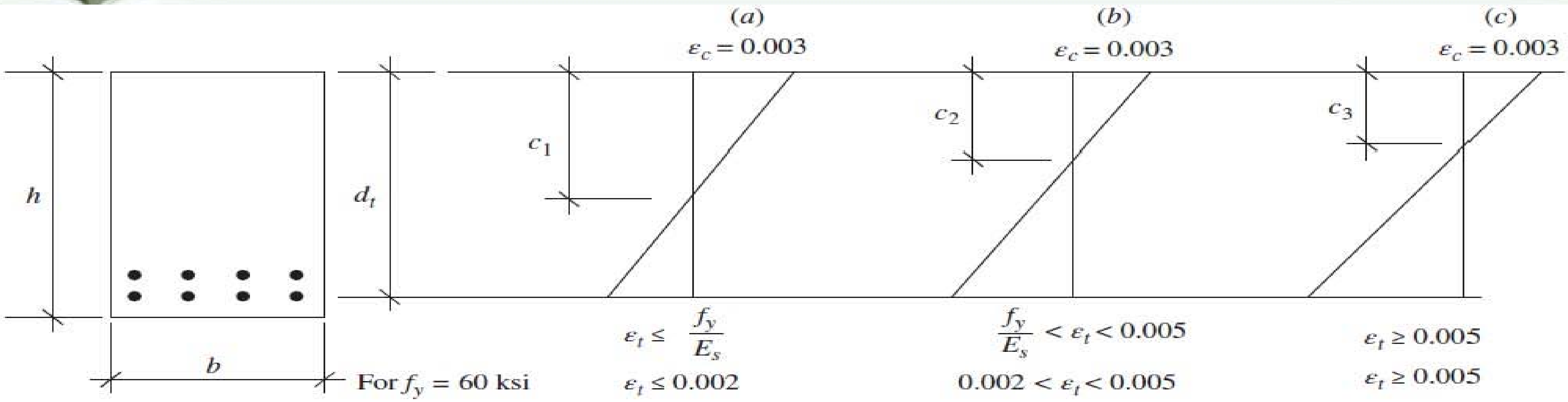
$$C = T$$

2 .The internal nominal bending moment, Mn, is equal to either the compressive force, C, multiplied by its arm or the tension force, T, multiplied by the same arm:

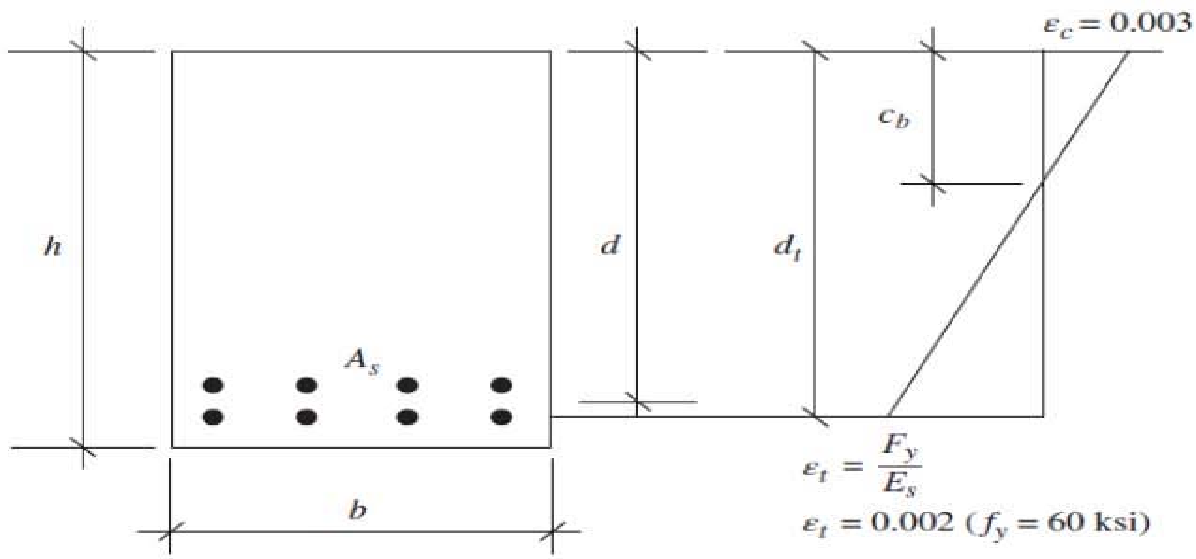
$$M_n = C(d - z) = T(d - z)$$

(Mu = ϕMn after applying a reduction factor ϕ)

The use of these equations can be explained by considering the case of a rectangular section with tension reinforcement. The section may be balanced, under reinforced, or over reinforced, depending on the percentage of steel reinforcement used.



Strain limit distribution, $c_1 > c_2 > c_3$: (*a*) compression-controlled section, (*b*) transition region, and (*c*) tension-controlled section.



d. Balanced strain section (occurs at first yield or at distance d_t).

Balanced Section

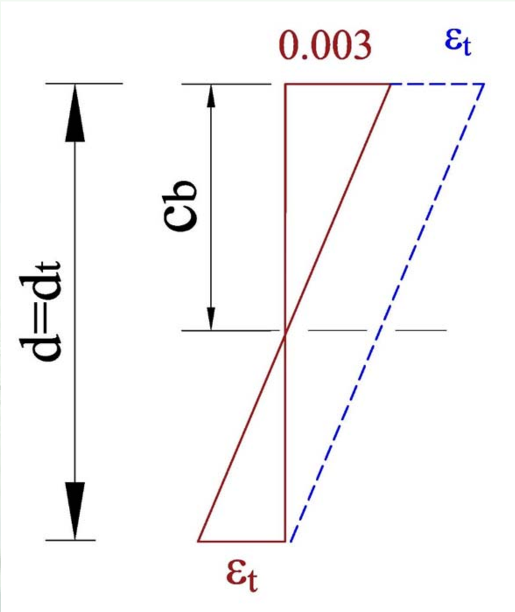
Let us consider the case of a balanced section, which implies that at maximum load the strain in concrete equals 0.003 and that of steel equals the first yield stress at distance dt divided by the modulus of elasticity of steel, fy/Es. This case is explained by the following steps.

Step 1. From the strain diagram

$$\frac{\varepsilon_t}{d_t - C_b} = \frac{0.003}{C_b} \qquad or \qquad \frac{C_b}{d_t - C_b} = \frac{0.003}{\frac{f_y}{E_s}}$$

From triangular relationships (where C_b is c for a balanced section) and by adding the numerator to the denominator,

$$\frac{C_b}{d_t} = \frac{0.003}{0.003 + \frac{f_y}{E_s}}$$

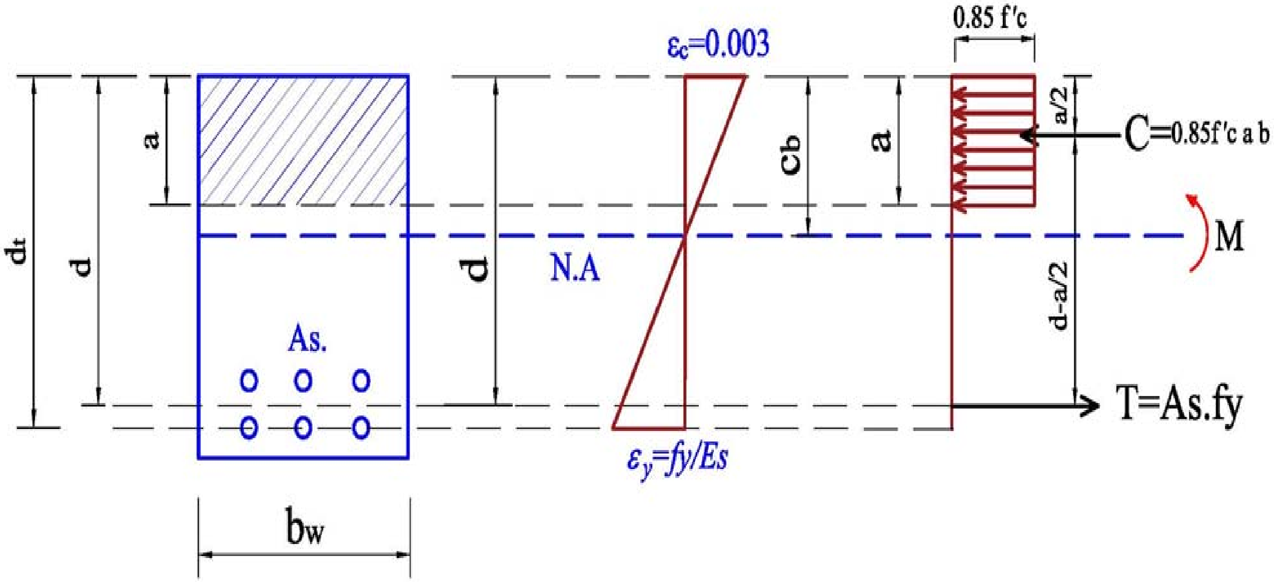


If $Es = 200000\ Mpa$
Then :

$$C_b = \left(\frac{600}{600+fy}\right) dt \ldots\ldots (1)$$

$C = T$ $\rightarrow$ $0.85\ f_c'\ b\ a = As\ fy$

$$a = \frac{As\ fy}{0.85 f_c'\ b} \quad \ldots\ldots (2)$$



β_1 C= a effective depth
While β_1= 0.85 when fc' ≤ 28 MPa

$$\rho_b = \frac{A_s\ balanced}{bd} = \frac{As_{\ b}}{bd}$$

$or \quad As_b = \rho_b\, b\ d \ldots\ldots (3)$
Substitute in eq. (2)

$$0.85 f_c'\ b\ a = fy\ \rho_b \times b \times d$$

$$\rho_b = \frac{0.85 f_c'\ \ a}{fy \times\ d} = \frac{0.85 f_c'\ \ (\beta_1 c)}{fy \times\ d}$$

Cb from equation (1) then

$$\rho_b = \frac{0.85\ f_c' \beta_1}{fy}\left(\frac{600}{\mathbf{600 + fy}}\right)\left(\frac{\mathbf{dt}}{\mathbf{d}}\right) \ldots\ldots (4)$$

While the nominal Moment $Mn = C(d - z) = T(d - z)$ $\left(where \quad z = \frac{a}{2}\right)$

$$\therefore a = \frac{As\, fy}{0.85\, f_c'\, b}$$

$$M_n = C\left(d - \frac{a}{2}\right) = T\left(d - \frac{a}{2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Or:

$$M_n = A_s\, fy\left(d - \frac{a}{2}\right)$$

To get the usable design moment ϕMn, the previously calculated Mn must be reduced by the capacity reduction factor:

$$\emptyset Mn = \emptyset\, A_s fy\left(d - \frac{A_s fy}{1.7\, f_c' b}\right)$$

while $\rho_b = \frac{As}{bd}$ or $As = \rho_b bd$ then :

$$\emptyset Mn = \emptyset fy\, \rho\, bd\left(d - \frac{\rho\, bd\, fy}{1.7\, f_c' b}\right)$$

$$\boxed{\emptyset Mn = \emptyset fy\, \rho\, bd^2\left(1 - \frac{\rho fy}{1.7\, f_c'}\right) \ldots\ldots (6)}$$

Or $\emptyset Mn = R_u\, bd^2$

$$R_u = \emptyset \rho fy\left(1 - \frac{\rho fy}{1.7\, f_c'}\right), \qquad a = \frac{As\, fy}{0.85\, f_c'\, b} \text{ lead to } \frac{a}{d} = \frac{\rho fy}{0.85\, f_c'} \ldots\ldots\ldots\ldots (7)$$

For more simlified, *let* $m = \dfrac{fy}{0.85\, fc'}$ then :

$$R_u = \emptyset \rho f y \left(1 - \frac{1}{2}\, \rho\, m \right) \ldots \ldots ..(8)$$

Then :

$$\rho_b = \frac{0.85\, f_c' \beta_1}{fy} \left(\frac{600}{600 + fy} \right) \left(\frac{dt}{d} \right)$$

$$\boxed{\rho_\mathrm{b} = \frac{\beta_1}{m} \left(\frac{600}{600+fy} \right) \left(\frac{dt}{d} \right) \ldots \ldots .(9)}$$

For one steel layer $\left(\frac{d}{dt} \right) = 1$

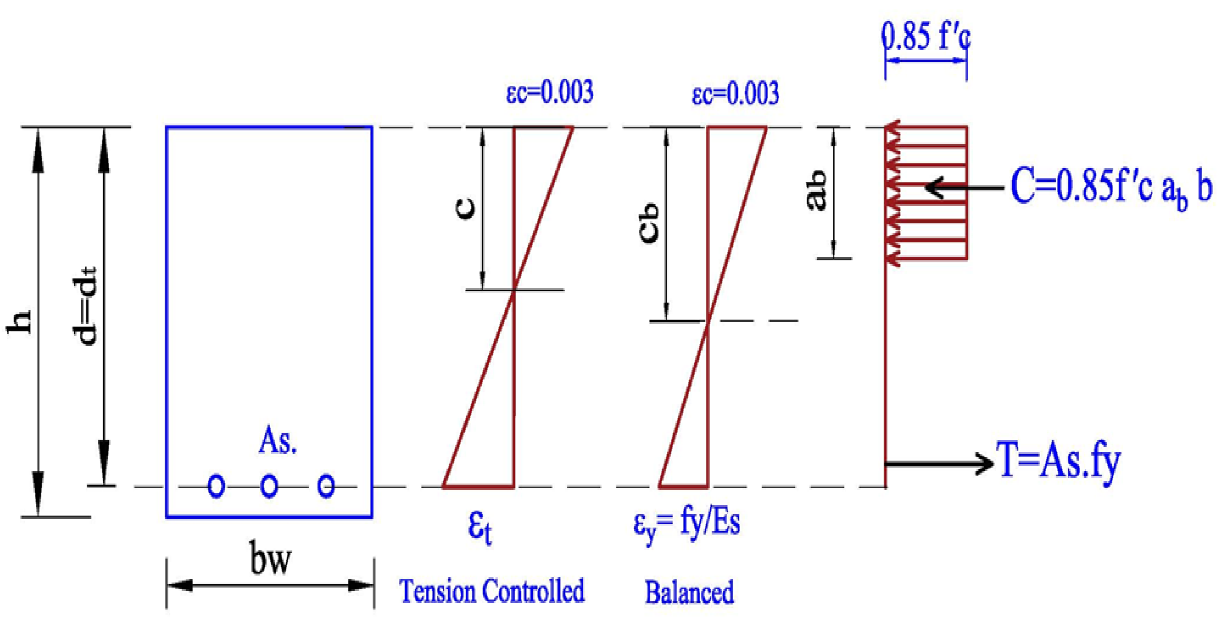
Upper Limit of Steel Percentage

The upper limit ort he maximum steel percentage ρmax, that can be used in a singly reinforced concrete section in bending is based on then net tensile strain in the tension steel, the balanced steel ratio, and the grade of steel used. The relationship between the steel percentage ρ in the section and the net tensile strain ε_t ,is as follows:

$$\varepsilon_t = \left(\frac{0.003 + \frac{fy}{E_s}}{\frac{\rho}{\rho b}}\right) - 0.003$$

$$For\ fy\ = 420\ MPa\ and\ \frac{fy}{E_s} = 0.002\ then$$

$$\varepsilon t = \left(\frac{0.005}{\frac{\rho}{\rho_b}}\right) - 0.003$$



These expressions are obtained by referring to Figure shown. For a balanced section,

$$C_b = \frac{a_b}{\beta_1} = \frac{As_b.fy}{0.85\ f_c'\ b\ \beta_1} = \frac{\rho_b\ fy\ d}{0.85\ f_c'\ \beta_1}$$

Similarly for any steel Ratio ρ:

$$c = \frac{\rho\ fy\ d}{0.85\ f_c'\ \beta 1} \quad and \quad \frac{c}{c_b} = \frac{\rho}{\rho_b}$$

Divide both sides by d to get:

$$\frac{c}{d} = \frac{\rho}{\rho_b} \times \frac{c_b}{d} \ldots\ldots\ldots\ldots..(10)$$

From the triangles of the strain diagrams,

$$\frac{c}{d} = \frac{0.003}{0.003 + \varepsilon_t}$$

$$\varepsilon_t = \left(\frac{0.003}{\frac{c}{d}}\right) - 0.003 \ldots\ldots\ldots.(11)$$

Similarly:

$$\frac{c_b}{d} = \frac{0.003}{0.003 + fy/Es} \ldots\ldots\ldots\ldots..(12)$$

Substitute in eq. (10)

$$\frac{c}{d} = \left(\frac{\rho}{\rho_b}\right)\left(\frac{c_b}{d}\right) = \left(\frac{\rho}{\rho_b}\right)\left(\frac{0.003}{0.003 + \frac{fy}{E_s}}\right) \ldots\ldots\ldots\ldots..(13)$$

Substitute in eq. (11)

$$\varepsilon_t = \frac{0.003}{c/d} - 0.003 = \left(\frac{0.003 + \frac{fy}{Es}}{\frac{\rho}{\rho_b}}\right) - 0.003$$

$$\frac{\rho}{\rho_b} = \frac{0.003 + \frac{fy}{Es}}{0.003 + \varepsilon_t} \;\dots\dots..(14)$$

$$\rho = \left(\frac{0.003 + \frac{fy}{Es}}{0.003 + \varepsilon_t}\right)\rho_b$$

$For\ fy = 420\ , Es = \ 200\ GPa, \qquad fy/E_s = \ 0.002$

$$\frac{\rho}{\rho_b} = \frac{0.0051}{0.003 + \varepsilon_t} \dots\dots$$

$The\ limit\ for\ tension\ to\ control\ is\ \varepsilon t \geq \ 0.005\ according\ to\ ACI. For\ \ \varepsilon_t = 0.005, becomes:$

$$\frac{\rho}{\rho_b} = \frac{0.0051}{0.008} = \frac{5.1}{8} = 0.6375$$

$\rho \leq 0.6375\ \rho_b$ Tension Control

For design purpose ε_t=0.005 and :

$\rho \leq \rho_{max}\ and\ \boldsymbol{\phi}\ = 0.9$

$$\boxed{\rho_{max} = \left(\frac{0.003 + \frac{fy}{E_s}}{0.008}\right)\rho_b \;\dots..(15)}$$

$\boldsymbol{Subistitute}\ \rho_b(\ \boldsymbol{Eq.9})\ \boldsymbol{gives}$:

$$\boxed{\rho_{max} = \frac{3}{8}\frac{\beta_1}{m}\left(\frac{d_t}{d}\right) \dots\dots\dots..(16)}$$

When $\rho > \rho_{max}$ section will be in transition state
then $\boldsymbol{\phi}$ will be between 0.65 and 0.9

Previously :

$$\boldsymbol{\phi} Mn = Ru\, bd^{2} \quad or \quad Mn = R_n\, bd^{2}$$

$$R_u = \emptyset \rho f y \left(1 - \frac{\rho f y}{1.7\, f_c'} \right)$$

$$m = \frac{fy}{0.85\, f_c'}$$

$$Then\ :\ R_u = \emptyset\, \rho f y \left(1 - \frac{1}{2} \rho\, m \right)$$

For one steel layer $(\frac{d}{d_t})$ =1, fy = 420 MPa, f_c'=28 MPa , ….. And m= 17.65

$$\boxed{\rho_b = \frac{\beta_1}{m} \left(\frac{600}{600+fy} \right) \left(\frac{dt}{d} \right)}$$

$$\rho_b = \frac{0.85}{17.65} \left(\frac{600}{600+420} \right) (1) = 0.0283$$

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{E_s}}{0.008} \right) \rho_b \ldots\ldots\ldots\ldots\ldots\ldots..(15)$$

$$= \left(\frac{0.003 + \frac{420}{200000}}{0.008} \right) \rho_b = 0.6375 \rho_b = 0.6375 \times 0.0283 = 0.01806$$

$$Ru_{\ max} = \emptyset \rho_{max} fy \left(1 - \frac{1}{2}\rho_{max}\, m\right)$$

$$Ru_{\ max} = 0.9\ (0.01806) \times 420\ \left(1 - \frac{1}{2}\ (0.01806 \times 17.65)\right)$$

$$Ru_{\ max} = 5.74$$

That's mean when $\rho > \rho_{max}$ ε_t< 0.005
and ACI cod 9.3.3.1 limited that should be not less than 0.004 in transition region

To keep enough ductility for beam when $\varepsilon_t = 0.004$

$$\frac{\rho}{\rho_b} = \frac{0.003 + \frac{fy}{Es}}{0.003 + \varepsilon_t}$$

$$\frac{\rho}{\rho_b} = \frac{0.003 + 0.0021}{0.003 + 0.004}$$

Then $\rho_{max\,\varepsilon_t} = 0.729\rho_b$

And $\emptyset$ calculated from

$\emptyset\ t = 0.65 + (\varepsilon_t - 0.002)\left(\frac{250}{30}\right)$ and when ε_t=0.004

$0.817 < \emptyset \leq 0.9$ and

$0.004 < \varepsilon_t \leq 0.005$

ρ_b= 0.0283 , fy = 420 MPa, f_c'=28 MPa , ….. and m= 17.65

$\rho_{max_, t}$= 0.729 *ρ_b= 0.0206

$$Ru_{\ max_, t} = \emptyset\rho_{max_, t} fy\left(1 - \frac{1}{2}\rho_{\ max_, t} m\right)$$

$$Rn_{\ max_, t} = 0.0206 \times 420\,(1 - 0.5 \times 0.0206 \times 17.65) = 7.08$$

$$\boxed{Ru_{\ max_, t} = 0.812 \times 7.08 = 5.75}$$

This value is very close from $Ru_{\ max}$, so increase the steel over the max ratio at the transition region does not increased effectively section capacity so its preferable to add steel at compression zone instead of over the $\rho_{max_, t}$

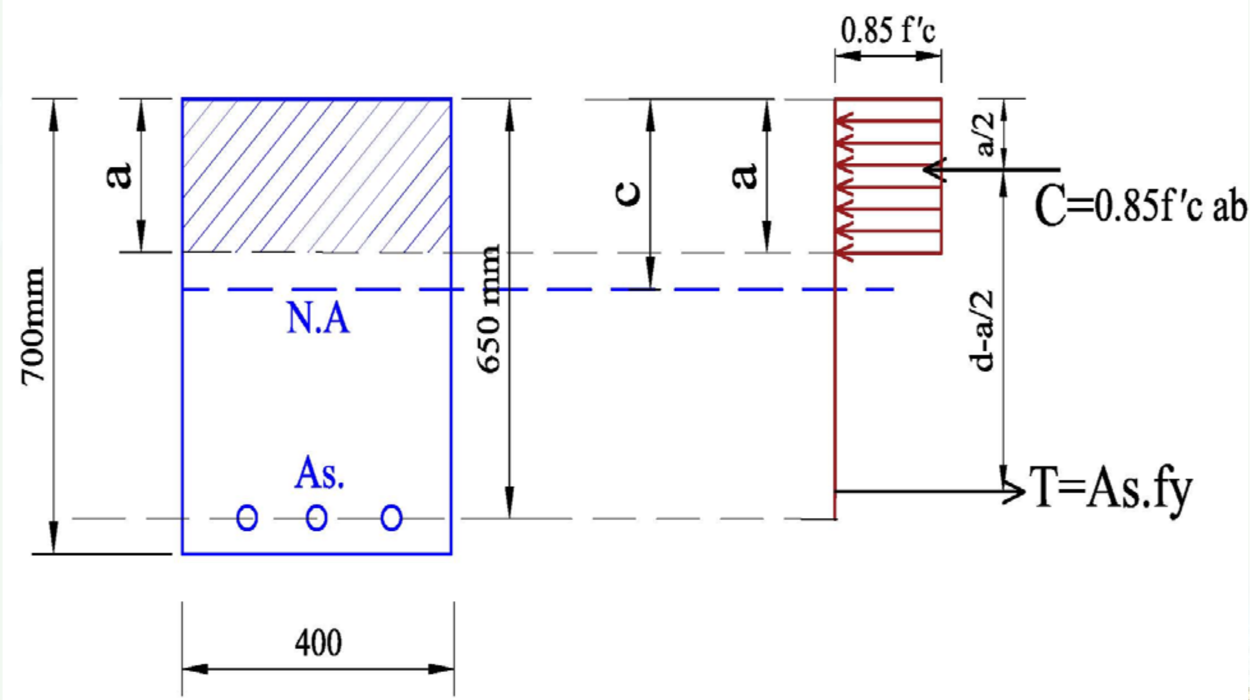
Example (1) : For the section shown below , calculate :

a- The balanced steel ratio.

b- The maximum reinforcement area allowed by ACI Code for a tension – controlled section and transition region.

c- The position of the Neutral axis and the depth of the equivalent compressive stress block for the tension – controlled section in b.

Given : $f_c' = 28$ MPa, $fy = 420$ MPa,



Solution

$$\rho_b = \frac{\boldsymbol{\beta}_1}{m}\left(\frac{600}{600+fy}\right)\left(\frac{dt}{d}\right)$$

$\boldsymbol{\beta}_1 = 0.85$ for $f_c' \leq 28$ Mpa

$$d_t = d \longrightarrow \frac{dt}{d} = 1$$

$$m = \frac{fy}{0.85\, f_c'} = \frac{420}{0.85 \times 28} = 17.65$$

$$\rho_b = \frac{0.85}{17.65}\left(\frac{600}{600+420}\right)(1) = 0.0283$$

$$A_{sb} = \rho_b \times b \times d = 0.0283 \times 650 \times 400 = 7358\, mm^2$$

b) $\varepsilon_t = 0.005$ for tension control

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{E_s}}{0.008}\right)\rho_b = \left(\frac{0.003 + \frac{420}{200000}}{0.008}\right) \times 0.0283 = 0.01804$$

$$As_{\max} = \rho_{\max} \times b \times d = 0.018043 \times 650 \times 400 = 4690.4\ mm^2$$

For Transition region , $\varepsilon_t = 0.004$

$$\rho_{max\,t} = \left(\frac{0.003 + \frac{fy}{E_s}}{0.007}\right)\rho_b = \left(\frac{0.003 + \frac{420}{200000}}{0.007}\right) \times 0.0283 = 0.0219$$

$$As_{max,t} = 0.0219 \times 400 \times 650 = 5694\ mm^2$$

$$\emptyset t = 0.65 + (\varepsilon_t - 0.002)\left(\frac{250}{3}\right) = 0.65 + (0.004 - 0.002)\left(\frac{250}{3}\right) = 0.817$$

c) Block stress depth *(Tension controlled)*

$$C = T$$

$$0.85\, f_c' \times a_{max} \times b = A_{s\,max}\, fy \qquad ....$$

$$a_{max} = \frac{As.fy}{0.85\, f_c'\, b}\,\frac{d}{d} = \rho_{max}\, m.d$$

$$a_{max} = 0.01804 \times 17.65 \times 650 = 206.96mm$$

$$or \qquad cmax = \frac{a_{max}}{\beta 1} = \frac{206.96}{0.85} = 243.48\ mm$$

Block stress depth at Transition zone

$$a = \frac{As.fy}{0.85\, f_c'\, b}\, \frac{d}{d} = \rho_{max,t}\, m.d$$

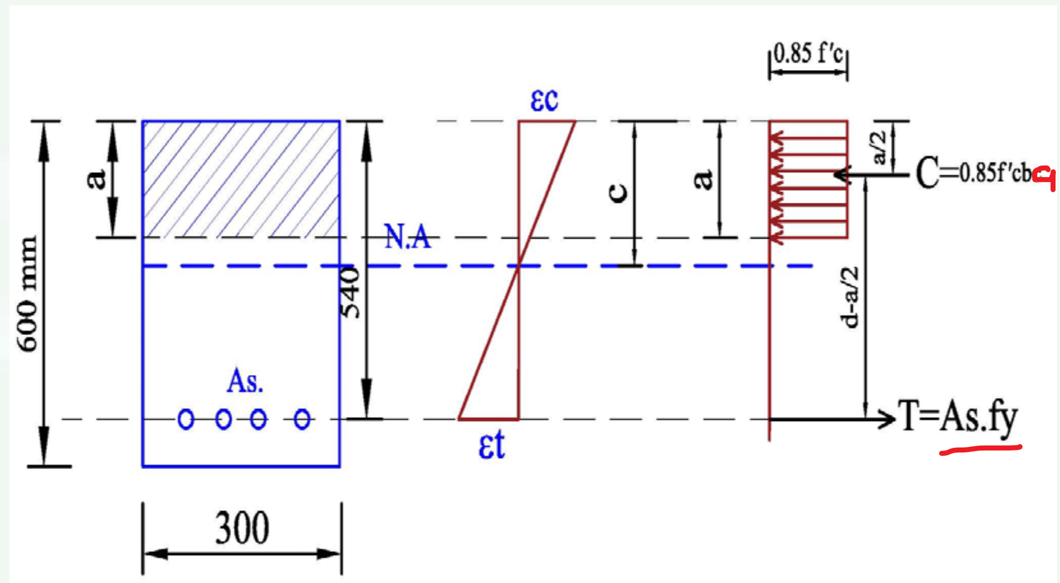
$$a_{max_{,}t} = 0.0219 \times 17.65 \times 650 = 251.25mm$$

Or :

$$c = \frac{a_{max_{,}t}}{\beta_1} = \frac{251.25}{0.85} = 295.6\ mm$$

Example (2) : Determine the design moment strength and the position of the neutral axis of the rectangular section shown below , if the reinforcement used is 4Ø 25 mm , Given : f'c = 28 Mpa, fy = 420 Mpa,

Solution:



$$As = 4Ø\,25\,mm = 4 \times 25^2 \times \frac{\pi}{4} = 1960\,mm^2$$

$$\rho = \frac{As}{bd} = \frac{1960}{300 \times 540} = 0.012098$$

$\rho < \rho_{\max} = (0.018040\ from\ Exa.1)\quad OK$

Tension Control

$\therefore$ Ø= 0.9

$C = T$

$0.85\, f_c' \times a \times b = As\, fy$

$0.85 \times 28\, a \times 300 = 1960 \times 420$

a= 115.29mm Or $a = \rho\, m.d = 0.012098 \times 17.65 \times 540 = 115.29$ mm

$$C = \frac{a}{\beta_1} = \frac{115.29}{0.85} = 135.64mm$$

($\boldsymbol{\beta}_1$= 0.85 for f_c' <= 28 Mpa)

$dt = d\ \ (one\ layer)$

$$\varepsilon_t = \frac{d - C}{C} \times \varepsilon_C = \frac{540 - 135.64}{135.64} \times 0.003 = 0.00894 > 0.005 \quad OK$$

Tension failure so ϕ=0.9

$$\emptyset Mn = Mu = \emptyset\, T\left(d - \frac{a}{2}\right) = \emptyset As\, fy\left(d - \frac{a}{2}\right)$$

$$= 0.9 \times 1960 \times 420 \times \left(540 - \frac{115.29}{2}\right) = 357.37 \times 10^6 N.mm = 357.37\ KN.m$$

Example (3) : Repeat Example (2) Using As = $4\emptyset\ 32\ mm$ (H.W)

Lower limit or Minimum Percentage of Steel

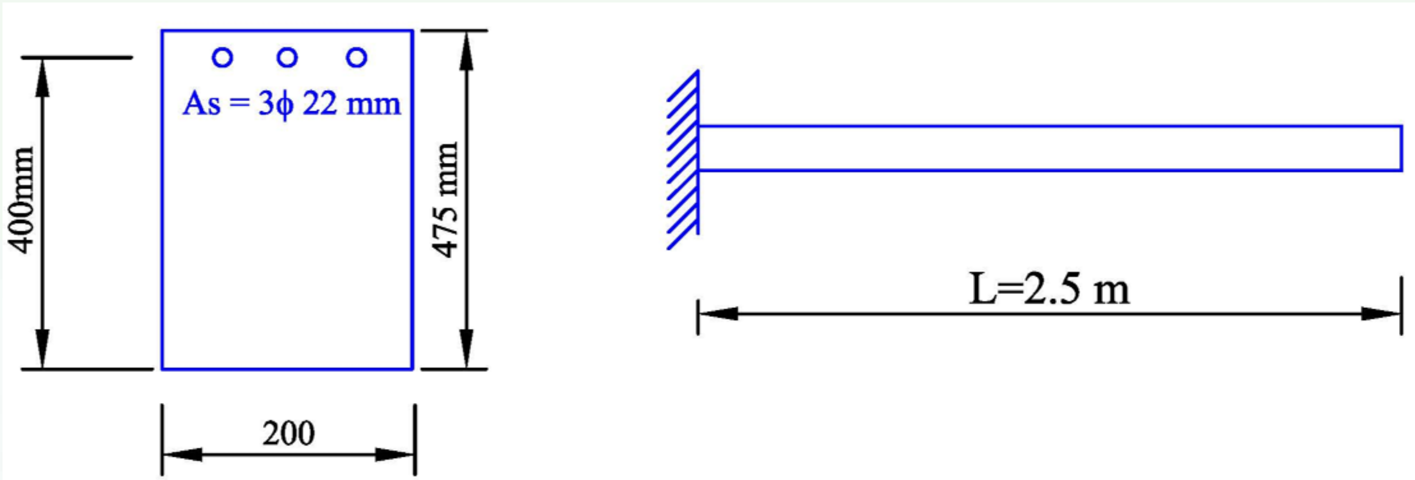
If the factored moment applied on a beam is very small and the dimensions of the section are specified (as is sometimes required architecturally) and are larger than needed to resist the factored moment , the calculation may show that very small or no steel reinforcement is required. In this case, the maximum tensile stress due to bending moment may be equal to or less than the modulus of rupture of concrete *fr*. If no reinforcement is provided, sudden failure will be expected when the first crack occurs, thus giving now warning. The ACI Code, Section 9.6.1, specifies a minimum steel area, As_{min} ,

$$As_{min} = \left(\frac{0.25\sqrt{f_c'}}{fy}\right)\mathrm{bw.d} \geq \left(\frac{1.4}{fy}\right) bw.d \ldots\ldots\ldots\ldots\ldots when\ f_c' = 31\ Mpa$$

$$\rho_{min} = \left\{\left(\frac{1.4}{fy}\right) \ldots\ldots\ldots\ldots\ldots\ldots.. For\quad f_c' < 31\ Mpa\right.$$

$$\rho_{min} = \left(\frac{0.25\sqrt{f_c'}}{fy}\right) \ldots\ldots\ldots\ldots. when\ f_c' \geq 31\ Mpa$$

Example (3) : A 2.5 m Span cantilever beam has a rectangular section and reinforced as shown below , The beam carries a dead load , including its self weight of 22 KN/m and a live load of 13 KN/m , using f_c'=28 MPa, fy = 420 MPa. Check if the beam is safe to carry above load.



Solution:

1- External Load

$$Wu = 1.2\,D.L + 1.6\,L.L = 1.2 \times 22 + 1.6 \times 13 = 47.2\,KN/m$$

$$Mu = \frac{Wu\,l^2}{2} = \frac{47.2 \times 2.5^2}{2} = 147.5\,KN.m$$

2- Check ε_t As ϕ22= 380 mm^2

$$a = \frac{As.fy}{0.85\,f_c'\,b} = \frac{3 \times 380 \times 420}{0.850 \times 28 \times 200} = 100.6\,mm$$

$$c = \frac{a}{0.85} = 118.35\,mm$$

$d_t = d = 400$ mm, $\phi = 0.9$

$$\varepsilon_t = \left(\frac{dt - c}{c}\right)\varepsilon_c$$

$$\varepsilon_t = \left(\frac{400 - 118.35}{118.35}\right) \times 0.003 = 0.00714 > 0.005\ (\varepsilon_t)$$

Or check

$$\rho = \frac{As}{bd} = \frac{3 \times 380}{200 \times 400} = 0.01425 < \rho_{max} = 0.01804$$

3- calculate :

$$\emptyset Mn = \emptyset As\ fy\left(d - \frac{a}{2}\right)$$

$$\emptyset Mn = 0.9 \times 3 \times 380 \times 420 \times \left(400 - \frac{100.6}{2}\right) = 150.69 KN.m$$

Other Solution

$$\rho = 0.01425 < \rho_{max} = 0.01804$$

$$m = \frac{fy}{0.85\ f_c'} = \frac{420}{0.85 \times 28} = 17.65$$

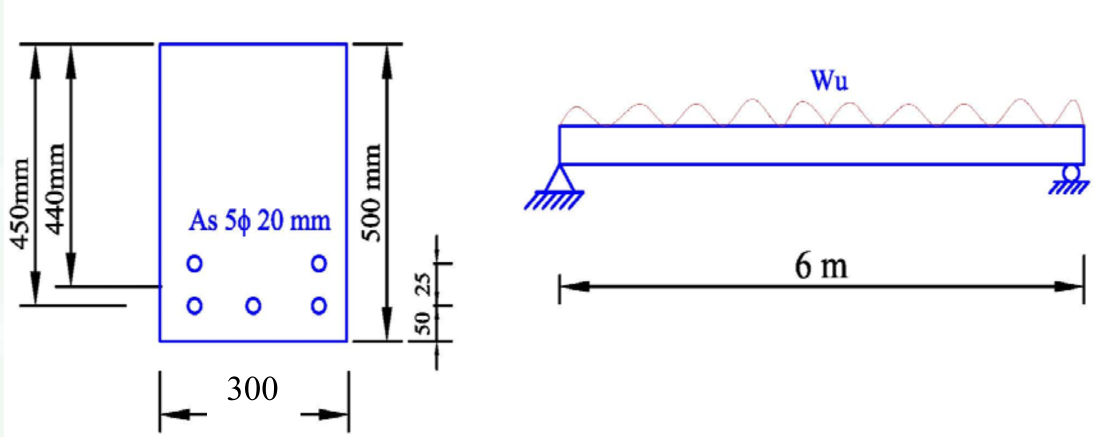
$$R = \rho fy\left(1 - \frac{1}{2}\rho\ m\right)$$

$$= 0.01425 \times 420\ (1 - 0.5 \times 0.01425 \times 17.65) = 5.23\ N/mm^2$$

$$\emptyset Mn = \emptyset \text{R b } d^2$$

$$= 0.9 \times 5.23 \times 200 \times 400^2 = 150.69\ KN.m$$

Example (4) : A simply supported beam have a span of 6 m . If the cross section is shown below , f_c'=21 MPa, fy = 420 MPa determine the allowable uninform load live load on the beam assuming the dead load is due to self weight of the beam , given b= 300 mm, h= 500 and reinforced with 5∅ 20 mm (1570 $mm2$).



Solution

Find the centroid of steel area

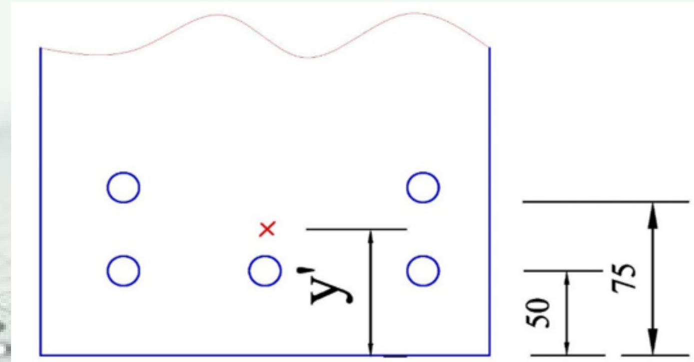
$$y' = \frac{3 \times 50(\,Asb) + 2 \times 75(Asb)}{5 \times (\,Asb)} = 60mm$$

$$d_t = h - 50 = 500 - 50 = 450mm$$

$$d = h - y' = 500 - 60 = 440 \text{ mm}$$

$$\rho_b = \frac{\beta 1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right)$$

$$m = \frac{fy}{0.85\, f_c'} = \frac{420}{0.85 \times 21} = 23.53 \;\; , \beta_1 = 0.85, (\, f_c' < 28\; MPa)$$



$$\rho_b = \frac{0.85}{17.65}\left(\frac{600}{600 + 420}\right)\left(\frac{450}{440}\right) = 0.02173$$

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.003 + \varepsilon t}\right)\rho_b = 0.6375\, \rho_b = 0.01385$$

$$\rho = \left(\frac{As}{bd}\right) = \left(\frac{5 \times 314}{300 \times 440}\right) = 0.01189 < \rho_{max} \qquad ok \quad (\emptyset = 0.9)$$

$$\rho_{min} = \left(\frac{1.4}{fy}\right) = \left(\frac{1.4}{420}\right) = 0.003 < \rho = 0.01189 \quad OK$$

$$\rho_{min} < \rho < \rho_{max}$$

$$\emptyset Mn = \emptyset \mathrm{R\,b}\,d^2$$

$$R = \rho fy\left(1 - \frac{1}{2}\rho\, m\right)$$

$$= 0.01189 \times 420\left(1 - \frac{1}{2} \times 0.01189 \times 23.53\right) = 4.295\ MPa$$

$$\emptyset Mn = 0.9 \times 4.295 \times 300 \times 440^2 = 224.52\ KN.m$$

$$Self\ weight\ of\ beam = 0.3 \times 0.5 \times 1 \times 24 = 3.6\ KN/m$$

$$M_{Dl} = \frac{3.6 \times 6^2}{8} = 16.2\ KN.m$$

$$Mu = 1.2\ MDL + 1.6\ MLL$$

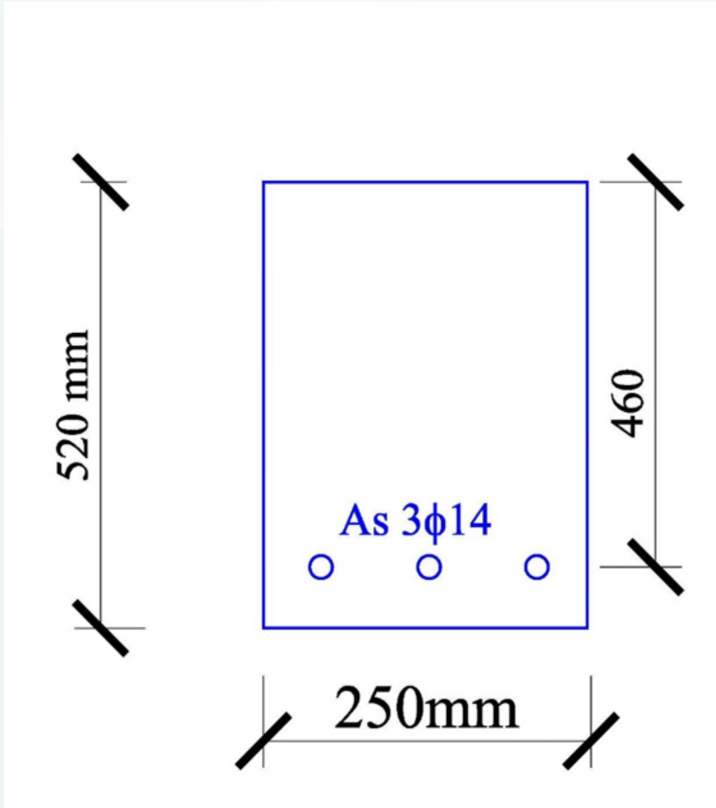
$$224.52 = 1.2 \times 16.2 + 1.6 \times M_{LL} = 128.175$$

$$M_{LL} = 128.175 = \frac{W_l \times 6^2}{8}$$

$$W_{LL} = 28.48\ KN/m$$

H.W

Example (5) : Check the design Adequacy of section below, factored moment Mu= 50 kN.m , using, f_c'=25 MPa, fy = 280 MPa



Example (6) :Determine the design moment strength of section shown below , Given f'c=28 mPa and fy=420 MPa and check the specification of the section according to ACI Code.

Solution:

$$As = 3 \times \pi \frac{25^2}{4} = 1470\, mm^2$$

$$\rho = \frac{As}{effective\ area} = \frac{As}{bd - 150 \times 100}$$

$$= \frac{As}{300 \times 500 - 150 \times 100}$$

$$Effective\ area = 300 \times 500 - 100 \times 150 = 135000\, mm^2$$

$$\rho = \frac{1470}{135000} = 0.01089$$

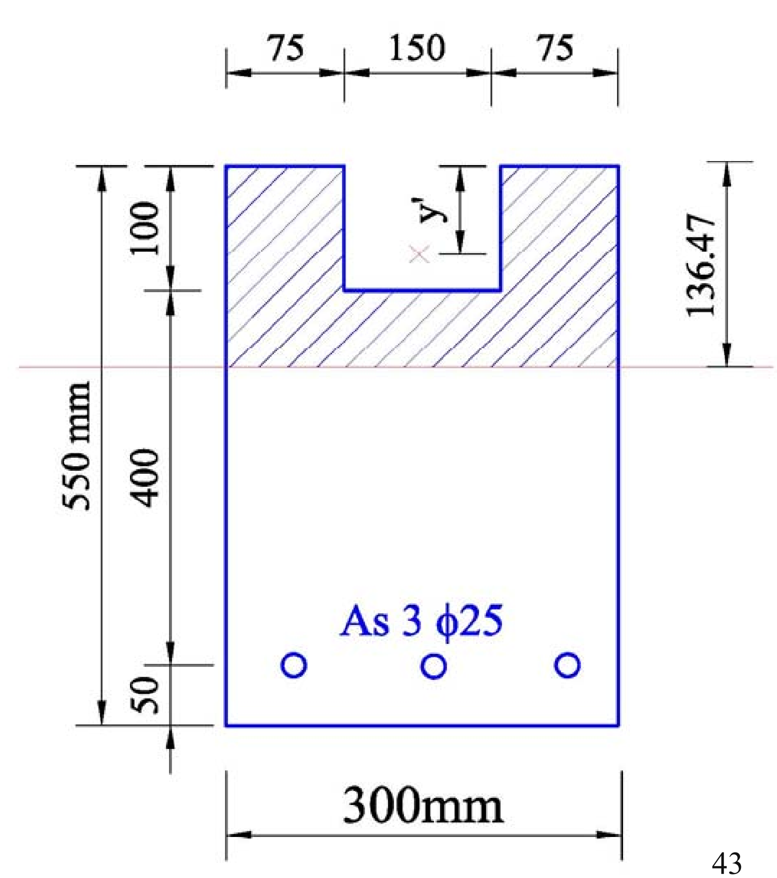
$$\rho b = \frac{\beta 1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right)$$

$dt = d$=500 mm

$$m = \frac{fy}{0.85\, f'c} = \frac{420}{0.85 \times 28} = 17.65$$

$$\rho b = \frac{0.85}{17.65}\left(\frac{600}{600 + 420}\right) \times 17.65 = 0.0283$$

$$\rho_{max} = \left(\frac{0.003 + 420/200000}{0.003 + 0.005}\right) \times 0.0283 = 0.018041$$



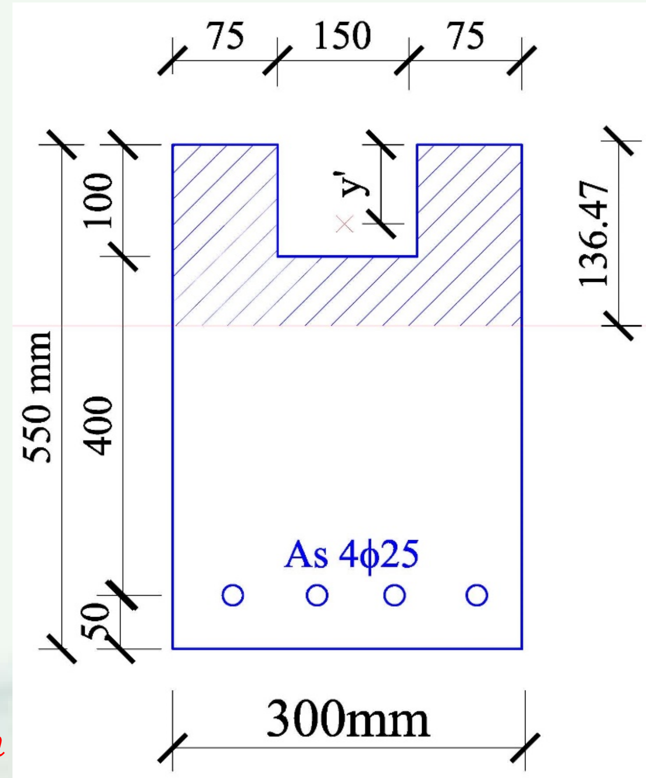
$$\rho_{min} = \left(\frac{1.4}{fy}\right) = \left(\frac{1.4}{420}\right) = 0.00333 \qquad (\,where\ f'c < 31MPa$$

$$\rho_{min} = 0.00333 < \rho = 0.01089 < \rho_{max} = 0.01804$$

$Tension\ Controlled \qquad \emptyset = 0.9$

Assume stress block depth = $a = 100\ mm$

Compression area $A_c = a \times b - 100 \times 150$



$C = T$

$0.85\ f'c\ Ac = As\ fy$

$$A_c = \left(\frac{1470 \times 420}{0.85 \times 28}\right) = 25941\ mm^2$$

$$A_c = a \times b - 100 \times 150 = a \times 300 - 150 \times 100$$

$a = 136.47mm > 100mm$

$$y' = \frac{300 \times 136.47 \times \left(\frac{136.47}{2}\right) - 150 \times 100 \times \left(\frac{100}{2}\right)}{300 \times 136.47 - 150 \times 100} = 78.78\ mm$$

The Moment Arm between C and T is :

$d - y' = 500 - 78.78 = 421.22\ mm$

$$\emptyset Mn = \emptyset Asfy\ (\,d - y')$$
$$= 0.9 \times (1470 \times 420 \times (500 - 78.78) = 234.06\ KN.m$$

Reinforced Concrete Design

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LOGO

Rectangular section with compression reinforcement (Double Reinforced section)

Introduction

In concrete sections proportioned to resist the bending moments resulting from external loading on a structural member, the internal moment is equal to or greater than the external moment, but a concrete section of a given width and effective depth has a minimum capacity when ρ_{max} is used. If the external factored moment is greater than the design moment strength, more compressive and tensile reinforcement must be added.

Compression reinforcement is used when a section is limited to specific dimensions due to architectural reasons, such as a need for limited headroom in multistory buildings. Another advantage of compression reinforcement is that long-time deflection is reduced. A third use of bars in the compression zone is to hold stirrups, which are used to resist shear forces.

Two cases of doubly reinforced concrete sections will be considered, depending on whether compression steel yields or does not yield.

1- When Compression Steel Yields

Internal moment can be divided into two moments, as shown in Fig. below. Let Mu_1 be the moment produced by the concrete compressive force and an equivalent tension force in steel, As_1, acting as a basic section. Then Mu_2 is the additional moment produced by the compressive force in compression steel $A\,s'$ and the tension force in the additional tensile steel, As_2, acting as a steel section.

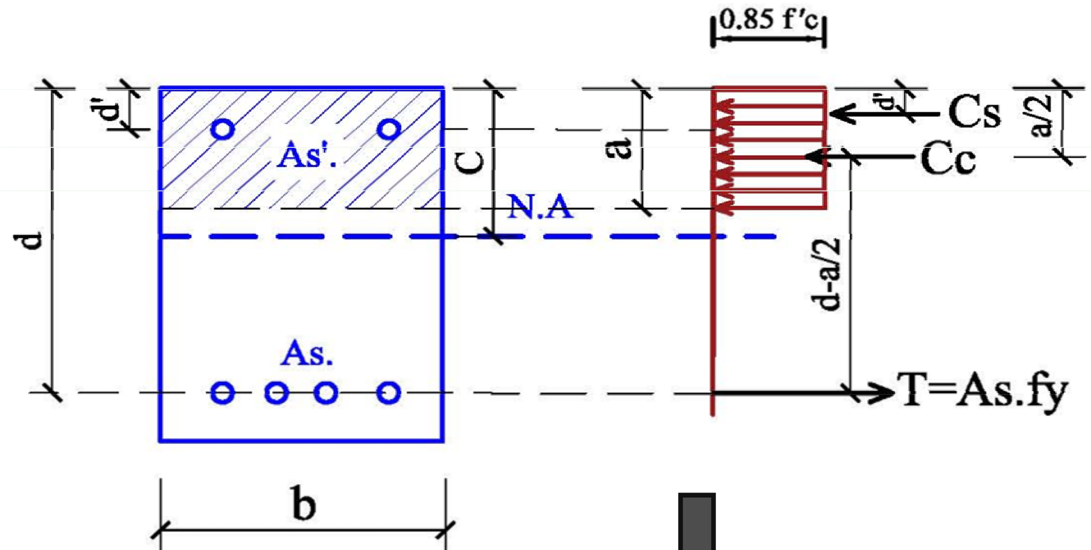
The moment M_{u1} is that of a singly reinforced concrete basic section,

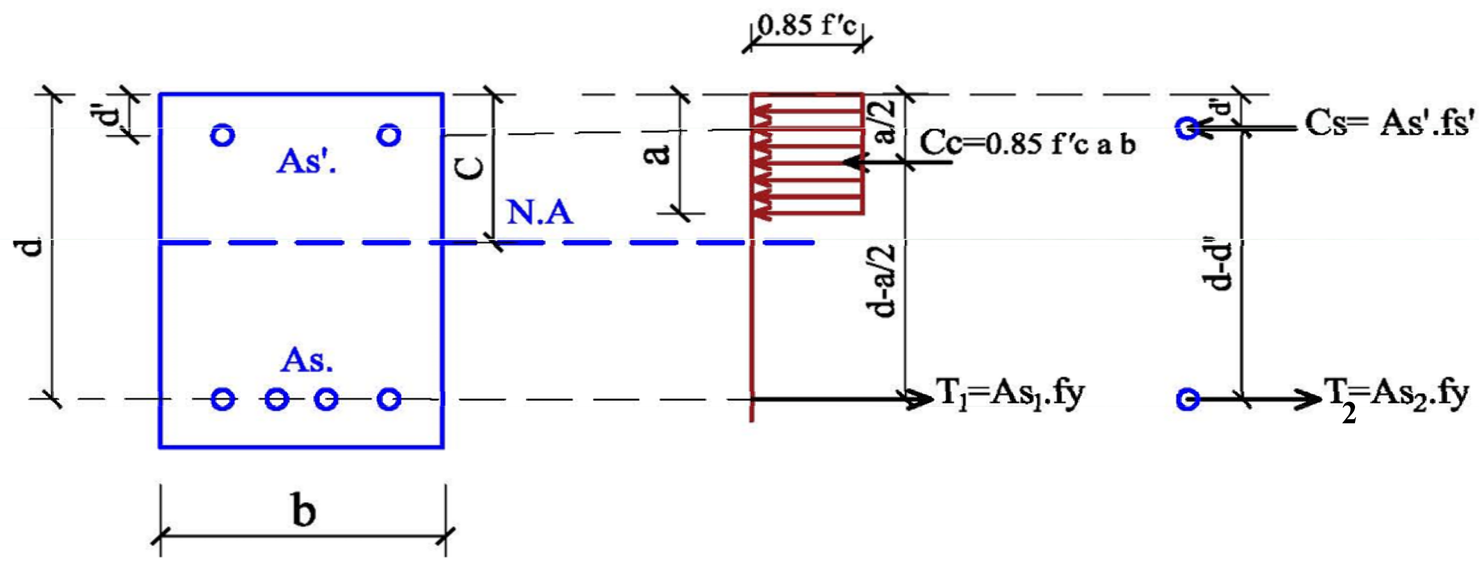
$$T_1 = Cc$$

$$As_1 fy \ = 0.85\ f_c'\ b\ a$$

$$a = \frac{As_1 fy}{0.85\ f_c'\ b}$$

$$\emptyset Mn = \emptyset As_1 fy\ \left(d - \frac{a}{2}\right)$$


0.85 f'c
d'
As'.
C
a
d'
Cs
a/2
Cc
N.A
d
d-a/2
As.
T=As.fy
b


0.85 f'c
d'
As'.
C
a
a/2
Cc=0.85 f'c a b
d'
Cs= As'.fs'
N.A
d
d-a/2
d-d''
As.
T1=As1.fy
T2=As2.fy
b

$\emptyset M_1 = \emptyset As_1 fy\left(d - \frac{a}{2}\right)$

$\emptyset M_2 = \emptyset As_2 fy(d - d')$

For $\emptyset M_1$: −

$\rho_1 = \dfrac{As_1}{bd}$ $\quad less\ or\ equal\ \ \rho_{\max}\ for\ singl\ reinforcement\ section\ under\ tension\ control$

And

$fs' = fy$ then

$\emptyset M_2 = \emptyset As_2 fy(d - d')$

Or :

T_2=Cs

$As_2 . fy = As' fy \longrightarrow As' = As_2$

$\emptyset Mn = \emptyset Mn_1 + \emptyset Mn_2$

$As = As_1 + As_2 \longrightarrow As_1 = As - As'$

$$a = \frac{As_1 fy}{0.85 f'c\, b} = \frac{(As - As')fy}{0.85 f'c\, b}$$

$$\emptyset Mn = \emptyset\left[(As - As') \times fy\left(d - \frac{a}{2}\right) + As' fy(d - d')\right]$$

$$\rho_1 = \rho - \rho' \le \rho_{max} = \left(\frac{0.003 + fy/Es}{0.003 + \varepsilon_t}\right)\rho_b$$

and when $\rho_1 = \rho - \rho' \leq \rho_{max\ t}$ *then the failure case will be at transiton region*
And $\emptyset$ will be less than 0.9 for M_{u1} and $\emptyset = 0.9\ for\ M_{u2}$, so:

$$\emptyset Mn = \left[\emptyset(As - As') \times fy\left(d - \frac{a}{2}\right) + 0.9As'fy(\,d - d')\right]$$

Noted that: $(As - As') \leq \rho_{max\ t}\, b\, d$
In the compression zone, the force in the compression steel is $Cs = A's(fy - 0.85f'c)$,taking into account the area of concrete displaced by A′s. In this case,

$T = C$

$As\ fy = C_c + C_s$

$As\ fy = 0.85\ f_c'\ a\ b + As'(fy - 0.85\ f_c')$

$As\ fy - As'fy + 0.85\ f_c'As' = 0.85\ f_c'a\ b$ $\quad$ $where\ \ C_c = As_1 fy = 0.85\ f_c'a\ b$ *(for the basic section)*

Divided by (b d) fy :

$$\rho - \rho'\left(1 - 0.85\frac{f_c'}{fy}\right) = \rho_1 \qquad where:\ \rho_1 \leq \left(\frac{As_1}{b\ d}\right)$$

Therefore,

$$\rho_1 = \rho - \rho'\left(1 - 0.85\frac{f_c'}{fy}\right) \leq \rho_{max} = \left(\frac{0.003 + fy/Es}{0.008}\right)\rho_b$$

This Eq. is more accurate than previous Eq.it is quite practical to use both equations to check the condition for maximum steel ratio in rectangular sections when compression steel yields.

The maximum total tensile steel ratio, ρ ,that can be used in a rectangular section when compression steel yields is as follows:

$$Max\ \rho \ = \ \rho_{\max} + \rho'$$

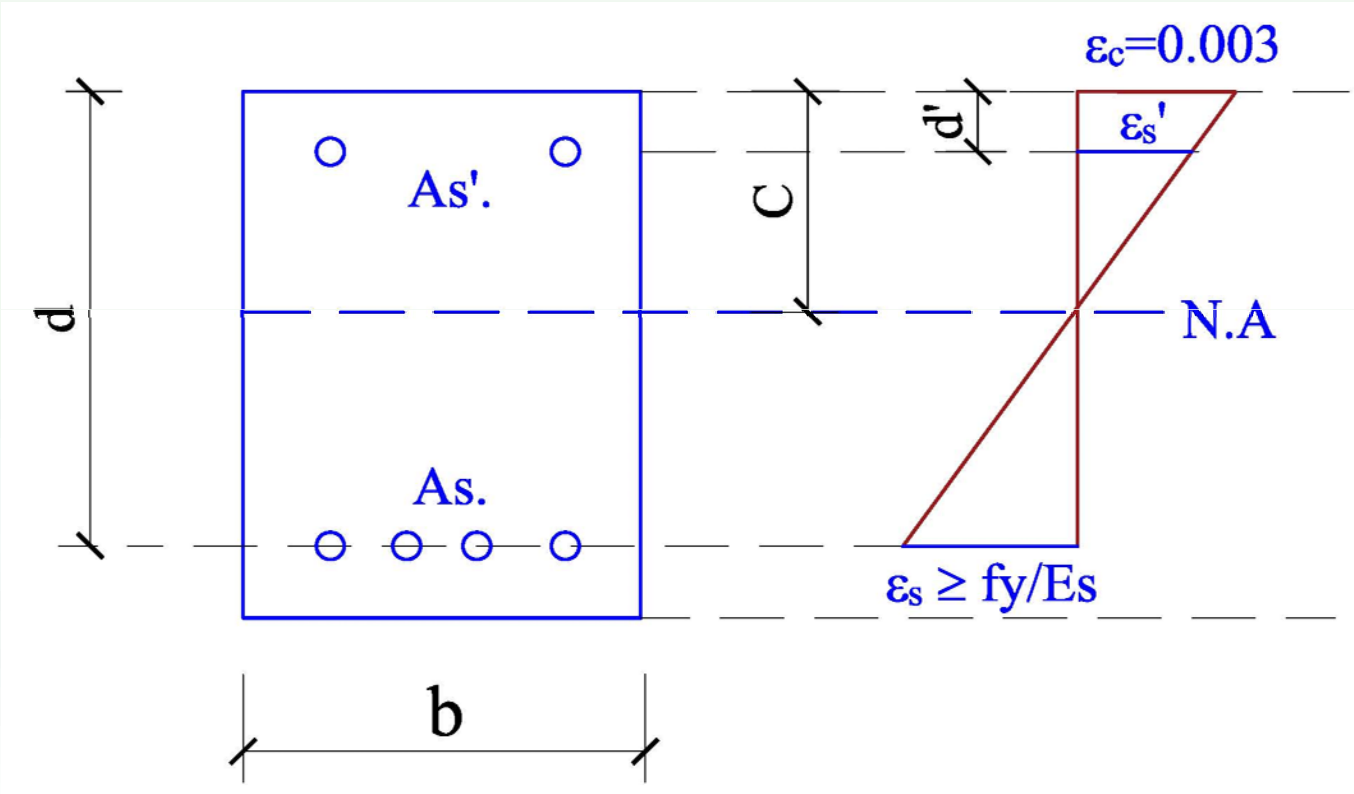
where $\rho\ max$ is maximum tensile steel ratio for the basic singly reinforced tension controlled concrete section. This means that maximum total tensile steel area that can be used in a rectangular section when compression steel yield is as follows:

$$Max\ As = b\ d\ (\rho_{\max} + \ \rho')$$

In the preceding equations, it is assumed that compression steel yields. To investigate this condition, refer to the strain diagram in Fig. Below. If compression steel yields, then :

$$\varepsilon_s' \geq \varepsilon_y = \frac{fy}{E_s}$$

$$\frac{c}{d'} = \frac{0.003}{0.003 - \frac{fy}{Es}} = \frac{600}{600 - fy} \quad \Rightarrow \quad C = \left(\frac{600}{600 - fy}\right) d'$$



$$as\ known:$$
$$As_1\, fy\ = 0.85\, f'c\ a\ b$$
$$As_1 = As - As'\ \ and\ \ \rho_1 = \rho - \rho'$$

$(As - As')fy \;= 0.85\, f_c'\, a\, b \qquad devided\ by\ (bd)$

$(\rho - \rho')fy = 0.85\, f_c'\, a\, b$

$$\rho - \rho' = 0.85\,\frac{f_c'}{fy}\left(\frac{a}{d}\right)$$

$$a = \beta_1 C = \beta_1\left(\frac{600}{600 - fy}\right)d'$$

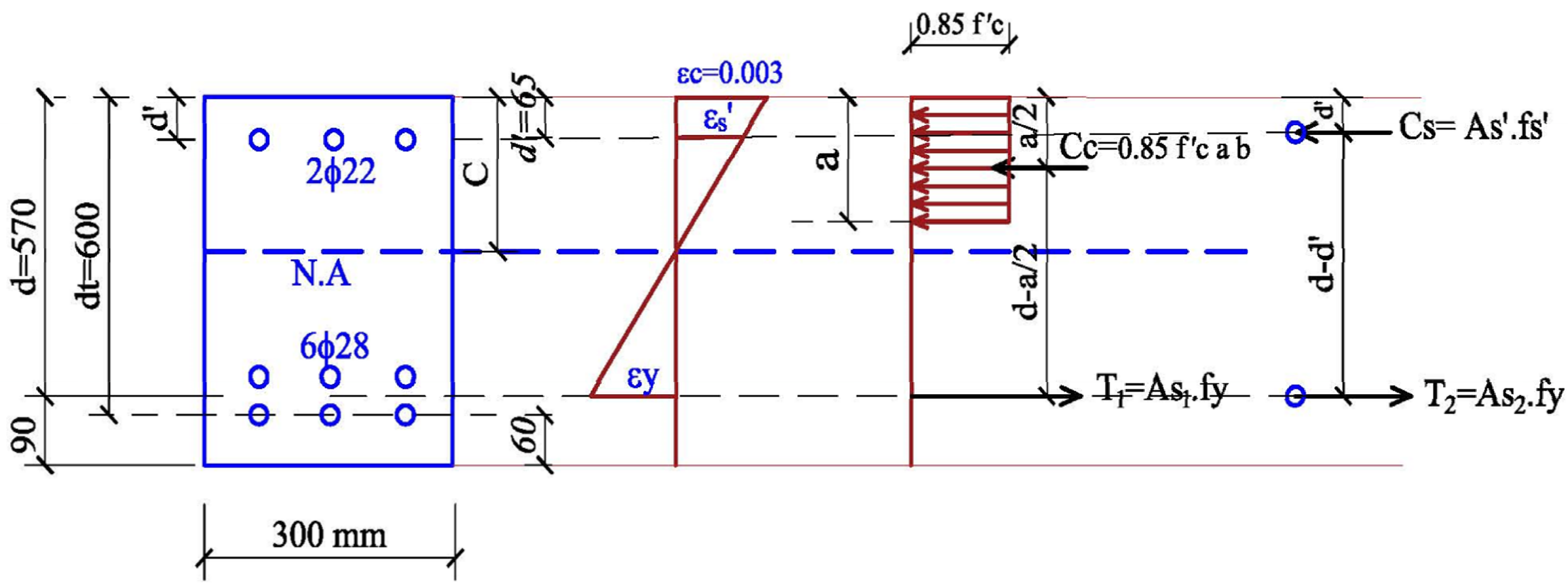
$$\rho - \rho' = 0.85\beta_1\left(\frac{f_c'}{fy}\right)\left(\frac{d}{d'}\right)\left(\frac{600}{600 - fy}\right)$$

$$\boxed{\rho - \rho' \geq \frac{\beta_1\, d'}{m\, d}\left(\frac{600}{600 - fy}\right)}$$

$where:$

$\rho - \rho'$ is the steel ratio for the single reinforced basic section $= \dfrac{As_1}{bd} = \dfrac{(As - As')}{bd}$

Example (7) : A rectangular beam section have a width of 300 mm, and an effective depth $d = 570$ mm to centroid of tension steel . Tension steel consist of $6\emptyset\, 28\, mm$ in two layers. Compression reinforcement consist of $2\emptyset\, 22\, mm$, and $d' = 50$ mm as shown below . Calculate the design moment strength of the beam , Given f_c'=28 MPa and $fy = 420\ MPa$.



Solution : Check if the compression steel yields :

$$\rho = \left(\frac{As}{bd}\right) = \left(\frac{6 \times 28^2 \times \frac{\pi}{4}}{300 \times 570}\right) = 0.02158$$

$$\rho' = \left(\frac{As'}{bd}\right) = \left(\frac{2 \times 22^2 \times \frac{\pi}{4}}{300 \times 570}\right) = 0.00444$$

1- *Check*

$$\rho - \rho' \geq \left(\frac{\beta 1 d'}{m\, d}\right)\left(\frac{600}{600 - fy}\right)$$

$$\beta_1 = 0.85\,, \qquad \mathrm{m} = \frac{fy}{0.85\, f_c'} = \frac{420}{0.85 \times 28} = 17.65$$

$$0.01714 \geq \left(\frac{0.85 \times 50}{17.65 \times 570}\right)\left(\frac{600}{600 - 420}\right) = 0.01408$$

$$Then: \; fs' = fy \qquad (O.K.)$$

$$2 - \; Check \quad \rho - \rho' \leq \rho_{max}$$

$$\rho_b = \frac{\beta 1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right) = \frac{0.85}{17.65}\left(\frac{600}{600 + 420}\right)\left(\frac{600}{570}\right) = 0.0298 \text{ or } \rho_{max} = \left(\frac{3\beta 1}{8\, m}\right)\left(\frac{dt}{d}\right)$$

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.003 + \varepsilon t}\right)\rho_b = \left(\frac{0.003 + \frac{420}{200000}}{0.003 + 0.005}\right) = 0.6375\rho_b = 0.019 \text{ or } \rho_{max} = \left(\frac{3\beta 1}{8\, m}\right)\left(\frac{dt}{d}\right) = 0.019$$

$$\rho - \rho' = 0.01714 \leq \rho_{max} = 0.0190$$

Tension Controlled -Section so: $\phi = 0.9$

3- Calculate ϕ Mn

$$\emptyset Mn = \emptyset\left[(As - As') \times fy\left(d - \frac{a}{2}\right) + As' fy(\,d - d')\right]$$

$$a = \frac{As_1 fy}{0.85 f'c\; b} = \frac{(\,As - As')fy}{0.85 f'c\; b} = \frac{(\,3690 - 760) \times 420}{0.85 \times 28 \times 300} = 172.35\; mm$$

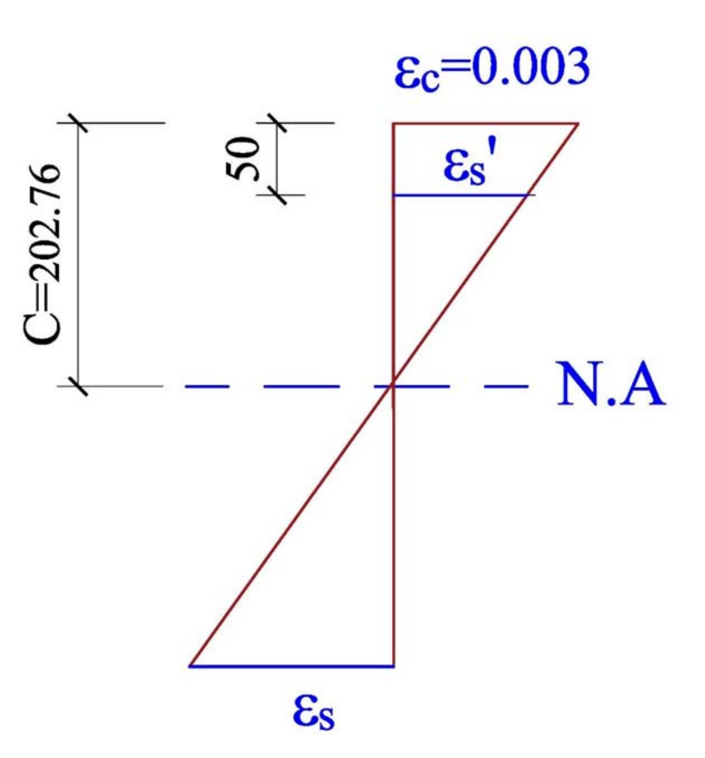
$$\emptyset Mn = 0.9\left[(3690 - 760) \times 420 \times \left(570 - \frac{172.3}{2}\right) + 760 \times 420 \times (570 - 50)\right] = 685.3\; kN.m$$

4- Another way to check the yield in compression steel

$$c = \frac{a}{\beta_1} = \frac{172.35}{0.85} = 202.76\; mm$$

$$\frac{\varepsilon s'}{\varepsilon c} = \frac{c - d'}{c}$$

$$\varepsilon s' = \frac{202.76 - 50}{202.76} \times 0.003 = 0.00226 > \varepsilon_y = 0.002$$



5- Check ε_t

$$\varepsilon_t = \left(\frac{d_t - c}{c}\right) \times \varepsilon_c = \frac{600 - 202.76}{202.76} \times 0.003 = 0.005877 > 0.005$$

6- Check The Maximum Tension steel Area for this section :

$$Max\ As = \ (\rho_{max} + \ \rho')\ bd = (0.0190 + 0.00444) \times 300 \times 570 = 4008\; mm^2$$

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Rectangular section with compression reinforcement (Double Reinforced section-II)

Steel Compression Dose Not Yield ($f'_s < fy$)

As was explained earlier, if the formula not checked,

$$\rho - \rho' \geq \left(\frac{\beta_1 d'}{m\ d}\right)\left(\frac{600}{600 - fy}\right) \dots\dots\dots\dots$$

Then compression steel does not yield. This indicates that if $\rho - \rho'$ is greater than the value of the right-hand side in above eq., So the solution can be done depend on static analysis . The stress in compression steel can be calculated in two method :

1- From Internal Forces Balance

2- direct method

3- indirect method (Iterative method)

2- direct method

$$A\ a^2 - Ba - C = 0$$

$$A = 1,$$

$$B = m\ d\left(\rho - \frac{600}{fy}\rho'\right)$$

$$C = \frac{600}{fy}\beta_1\ m\ d\ d'\rho'$$

$$a = \frac{1}{2}\left[B + \sqrt{B^2 + 4\ A\ C}\ \right] \qquad \text{and} \qquad C = \frac{a}{\beta_1}$$

Then Stress can be calculated :

$$fs' = 600\left(\frac{c-d'}{c}\right) \le fy$$

2- indirect method (Iterative method)

$claculate$ (a) $value$ for double reinforced section (DRRS):

$$a = \frac{As\,fy - As'\,fs'}{0.85\,f'c\,b} \qquad assume \qquad fs' = fy$$

$find$ a and c= a/β_1

$f'si = 600\left(\frac{c-d\prime}{c}\right) \le fy$, Compare this value $f'si$ with first one ($f's$)

If its not same then re-calculate (a) using $f'si$ and continue until obtain approximately equal $f's$ in last two step. After obtain $f's$ then can calculate the Cs and Cc

Cc= $As\,fy - A's\,fs'$ $\qquad where$: $\quad Cs = A's\,fs'$

$$\emptyset Mn = \emptyset\left[Cc\left(d-\frac{a}{2}\right) + Cs(\,d-d')\right]$$

$When\ f's < fy$
$then\ the\ maximum\ steel\ area\ in\ tension\ zone\ for\ rectangular\ section\ can\ be\ found$

$$Max\ As = \left(\rho_{\max} b\, d + A's\,\frac{fs'}{fy}\right)$$
$$= \left(\rho_{\max} + \rho'\frac{fs'}{fy}\right) bd$$
$$Max\ \rho = \frac{\max As}{bd} \le (\rho_{max} + \rho'\frac{fs'}{fy})$$

$$(\rho - \rho'\frac{fs'}{fy}) \le \rho_{max}$$

ρ_{max}: *maximum steel ratio for single beam section under tension controlled*

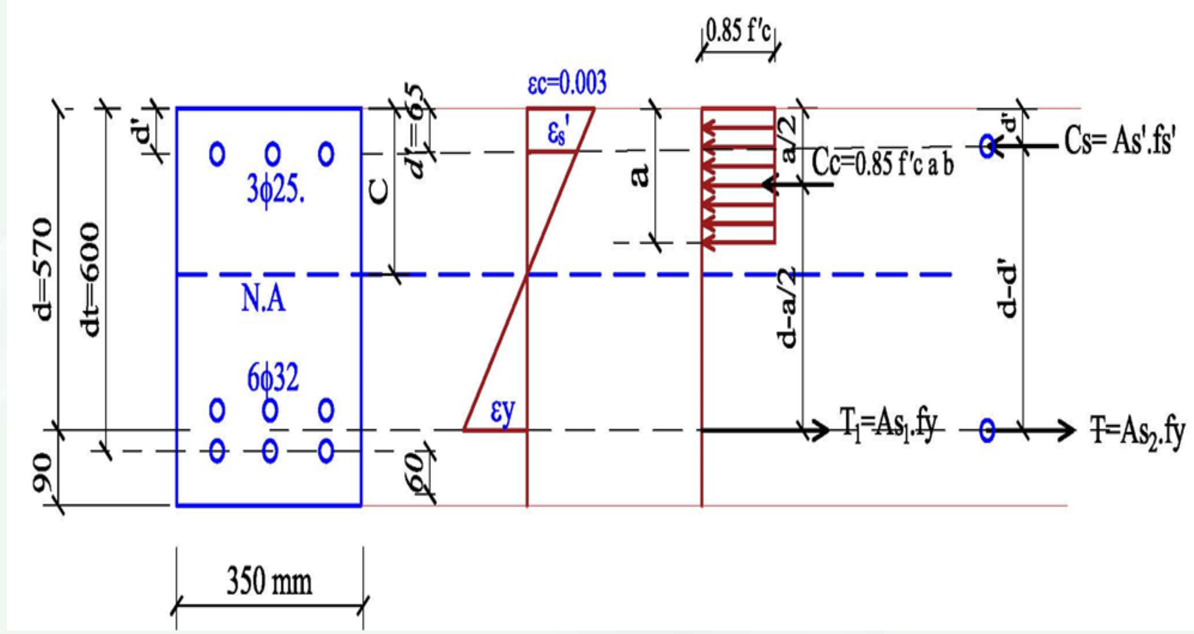
$$a = \frac{As\,fy - A's\,fs'}{0.85\,f'c\,b}$$

And : $$\emptyset Mn = \emptyset\left[(Asfy - As'fs')\left(d - \frac{a}{2}\right) + As'fs'\,(d - d')\right]$$

Example (8) : Determine the design moment strength of the section shown below , using f'c= 35 Mpa , fy = 420 Mpa. $As = 6\emptyset\, 32\, mm$ (two layer) and $A's = 3\emptyset\, 25\, mm$.

Solution

1-Calculate ρ *and* ρ'



$$\rho = \left(\frac{AS}{bd}\right) = \frac{6 \times 804}{350 \times 570} = 0.02418$$

$$\rho' = \left(\frac{A's}{bd}\right) = \frac{3 \times 490}{350 \times 570} = 0.007368$$

$$m = \frac{fy}{0.85\, f_c'} = 14.12$$

$\beta 1 = 0.8$ for f'c = 35 mPa

$$\rho - \rho' \geq \left(\frac{\beta 1 d'}{m\, d}\right)\left(\frac{600}{600 - fy}\right)$$

$$= 0.02417 - 0.007368 \geq \left(\frac{0.8 \times 65}{14.12 \times 570}\right) \times \left(\frac{600}{600 - 420}\right) = 0.016812 \leq 0.0215 \qquad (\,not\ checked)$$

$\therefore \quad fs' < fy$

$$\rho_b = \frac{\beta 1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right) = \frac{0.8}{14.12}\left(\frac{600}{600 + 420}\right)\left(\frac{600}{570}\right) = 0.03508$$

$$\rho_{max} = \left(\frac{0.003 + fy/Es}{0.003 + \varepsilon t}\right)\rho b = \left(\frac{0.003 + 420/200000}{0.003 + 0.005}\right) \times 0.03508 = 0.0224$$

$\rho - \rho' < \rho_{\ max}$ Tension controlled ϕ=0.9

3- calculate $\emptyset Mn$ $(internal\ section\ analysis)$

$Cc = 0.85\ f'c\ a\ b$ $a = \beta_1 \times C = 0.8\ C$

$Cc = 0.85\ \times 35\ \times (\ 0.8\ C) \times 350 = 8330\ \text{C}$ N

$Cs = A's\ (\ fs'\ - 0.85\ f'c\ )$

$$fs' = 600\left(\frac{c-d'}{c}\right) = 600\left(\frac{c-65}{c}\right)$$

$$Therefore: \quad Cs = 1470 \times \left(600 \times \left(\frac{c-65}{c}\right) - 0.85 \times 35\right)$$

$$= 882000\left(\frac{c-65}{c}\right) - 43732.5$$

$T = T_1 + T_2 = (\ As_1 + As_2)fy = As \times fy = 6 \times 804 \times 420 = 2026080\ N$

4- Internal Forces

$T = Cc + Cs$

$$2026082 = 8330\ C + 882000 \times \left(\frac{c-65}{c}\right) - 43732.5$$

$2026080C\ = 8330C^2 + 882000\ C - 65 \times 882000 - 43732.50C$

$8330C^2 - 1187812.5\ C - 57330000 = 0$

C= 180.68 mm

$\mathrm{a} = \beta_1 \times \mathrm{C} = 0.8 \times 180.68 = 144.54$ mm

Or

2- Using Direct Method

$A\,a^2 - Ba - C = 0$

$A = 1,$

$$B = m\,d\left(\rho - \frac{600}{fy}\rho'\right)$$

$$C = \frac{600}{fy}\beta 1\; m\;d\;d'\rho'$$

$$a = \frac{1}{2}\left[B + \sqrt{B^2 + 4\,A\,C}\right]$$

$$a = \frac{1}{2}\left[\left(m\,d\,\left(\rho - \frac{600}{fy}\rho'\right)\right) + \sqrt{\left[m\,d\,\left(\rho - \frac{600}{fy}\rho'\right)\right]^2 + 4\;\times 1.0 \times \frac{600}{fy}\beta_1\,m\,d\,d'\rho'}\right], \qquad C = \frac{a}{\beta}$$

$$= \frac{1}{2}\left[\left(14.12\; * 570\;\left(0.02418 - \frac{600}{420} * 0.007368\right)\right) + \sqrt{\left[14.12\; * 570\;\left(0.02418 - \frac{600}{420} * 0.007368\right)\right]^2 + 4 * \left(\frac{600}{420} * 0.8 * 14.12 * 570 * 65 * 0.007368\right)}\right]$$

$$a = 141.11\,mm \quad and \quad c = \frac{a}{\beta_1} = 176.39mm$$

Or Using

3-Indirect Method (iterative method)

both method dose not subtract the term (0.85 f'c)

-Calculate *(a)*

$$a = \frac{As\,fy - A's\,fs'}{0.85\,f'c\,b} \qquad where\ fs' = fy$$

$$a = \frac{4824 \times 420 - 1470 \times 420}{0.85\ \times 35 \times 350} = 135.29\,mm$$

$$c = \frac{a}{\beta 1} = \frac{135.29}{0.8} = 169.11mm$$

$$f's = 600\left(\frac{c - d'}{c}\right) = 600\left(\frac{169.11 - 65}{169.11}\right) = 369.33\ Mpa$$

$$a = \frac{4824 \times 420 - 1470 \times 369.33}{0.85\ \times 35 \times 350} = 142.44\,mm$$

$$c = \frac{a}{\beta 1} = \frac{142.44}{0.8} = 178.04\,mm$$

$$f's = 600\left(\frac{c - d'}{c}\right) = 600\left(\frac{178.04 - 65}{178.04}\right) = 381\ MPa$$

$$a = \frac{4824 \times 420 - 1470 \times 381}{0.85\ \times 35 \times 350} = 140.79\,mm$$

$$c = \frac{a}{\beta_1} = \frac{140.79}{0.8} = 176\,mm$$

$$f's = 600\left(\frac{c - d'}{c}\right) = 600\left(\frac{176 - 65}{178.04}\right) = 378.4\ MPa \qquad almost\ last\ two\ value\ are\ equal$$

$$\frac{4824 \times 420 - 1470 \times 378}{0.85 \ \times 35 \times 350} = 141.21\,mm$$

$$c = \frac{a}{\beta_1} = \frac{141.21}{0.8} = 176.5\,mm$$

5- Calculate fs’ , Cc and Cs

$$f's = 600\left(\frac{c - d'}{c}\right) = 600\left(\frac{176.5 - 65}{176.5}\right) = 379.1\,MPa$$

$$Cc = 0.85 \times \ 35 \ \times (\,0.8\,C) \times 350 = 8330\ \mathrm{C} \ \ = 8330 \times 176.5 = 1470245\ \mathrm{N}$$

$$Cs = A's\,(\,fs' \ - 0.85\,f'c\,) = 1470(379.1 - 0.85 \times 35) = 513544\,N$$

6- Calculate $\emptyset Mn$

$$\emptyset Mn = \emptyset\left[Cc\left(d - \frac{a}{2}\right) + Cs(\,d - d')\right] = 0.9\left[1470245\left(570 - \frac{141.21}{2}\right) + 513544(\,570 - 65)\right]$$

= 894222 065 N.m

= 894.22 KN.m

7- Check that $\rho - \rho' \frac{fs'}{fy} \leq \rho_{max}$

$$0.02418 - 0.007368 \times \left(\frac{379.1}{420}\right) = 0.01753 < \rho_{max} = 0.02236 \qquad (\text{O.K.})$$

The maximum total tension steel can be used in this is calculated by :

$$Max\ As = \left(\rho_{\ max} + \rho' \frac{fs'}{fy}\right) b\ d$$

$$= \left(0.02236 + 0.007368 \times \frac{379.1}{420}\right) \times 350 \times 570 = 5787\ mm^2$$

8- Let Check εt as follow:

$C = 176.5mm \quad , \quad d_t = 600\ mm$

$$\varepsilon_t = \left(\frac{dt - c}{c}\right) \times 0.003 = \frac{600 - 176.5}{176.5} = 0.0072 > 0.005 \qquad O.K\ tension\ controll$$

Thank You……..

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Analysis of T-and I-sections

ANALYSIS OF T-AND I-SECTIONS

It is normal to cast concrete slabs and beams together, producing a monolithic structure. Slabs have smaller thicknesses than beams. Under bending stresses, those parts of the slab on either side of the beam will be subjected to compressive stresses, depending on the position of these parts relative to the top fibers and relative to their distances from the beam. The part of the slab acting with the beam is called the flange, and it is indicated in Fig. below a by area $b*h_f$. The rest of the section confining the area $(h-h_f)\ b_w$ is called the stem, or web.

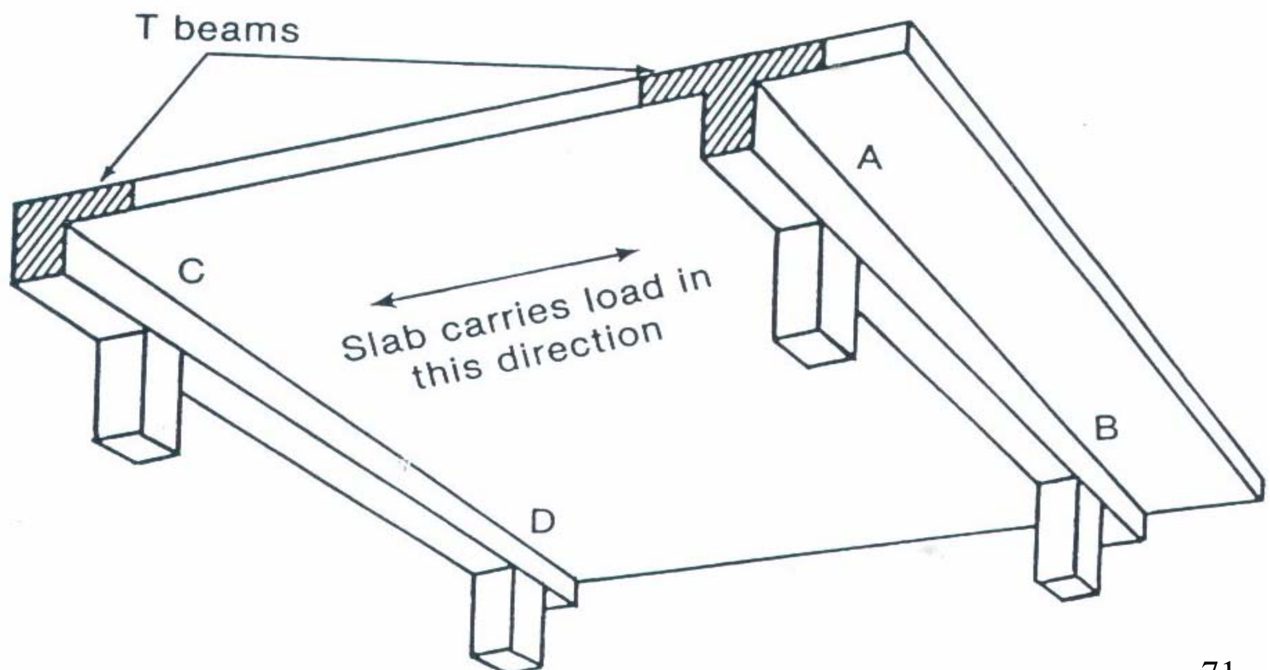
In an I-section there are two flanges, a compression flange, which is actually effective, and a tension flange, which is ineffective because it lies below the neutral axis and is thus neglected completely. Therefore, the analysis and design of an I-beam is similar to that of a T-beam.

Floor systems with slabs and beams are placed in monolithic pour.

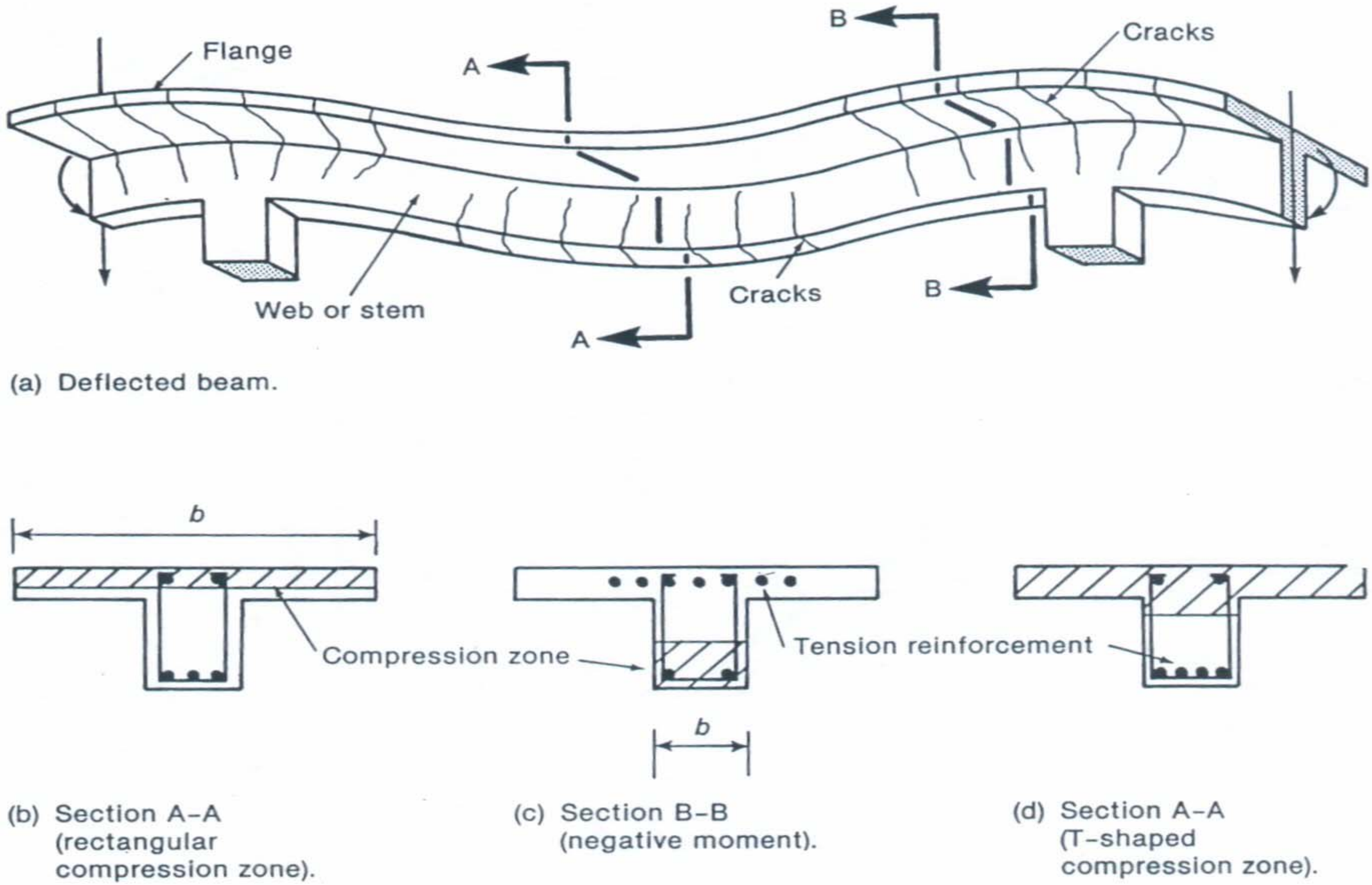
Slab acts as a top flange to the beam;

1- T-beams

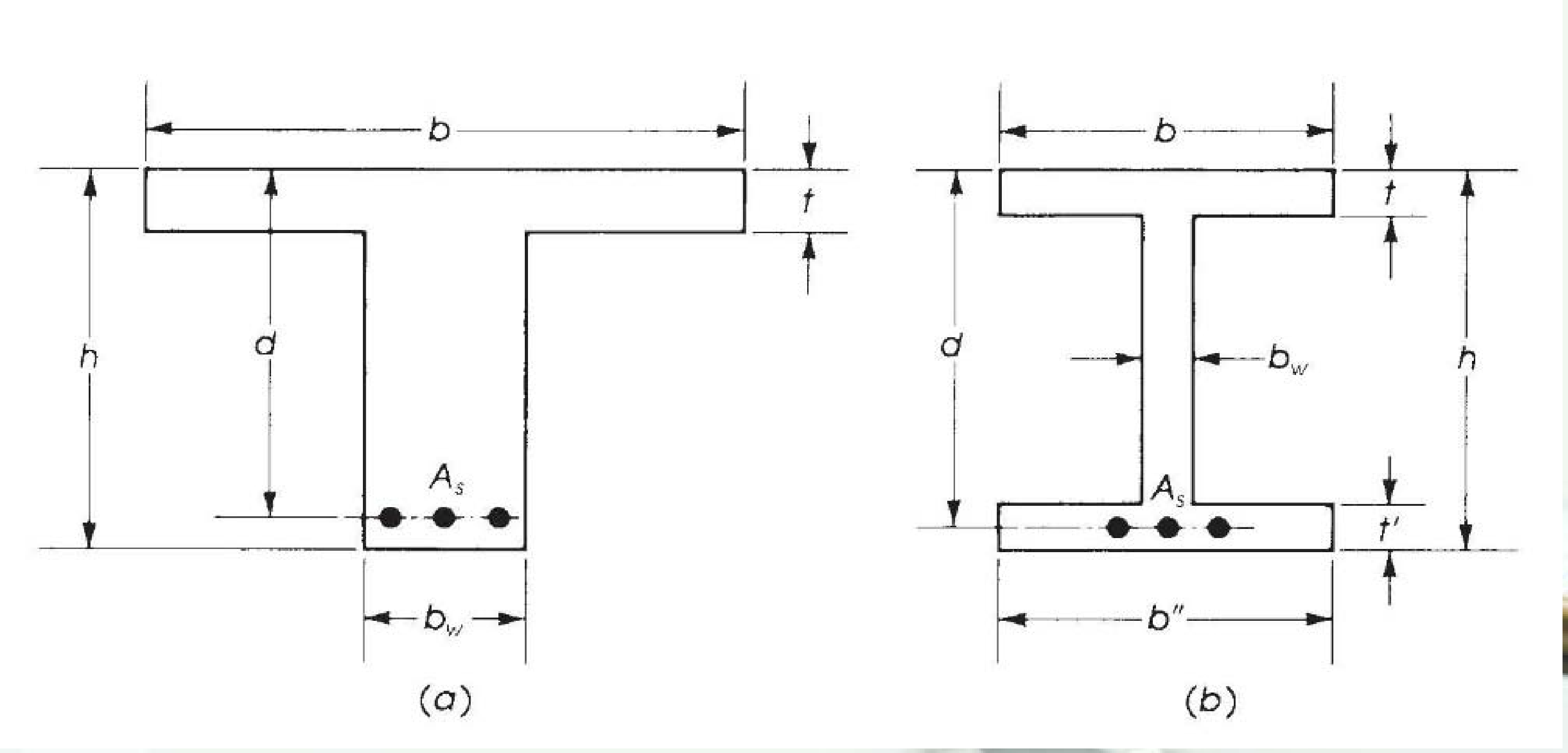
2- Inverted L(Spandrel) Beams.



Positive and Negative Moment Regions in a T-beam

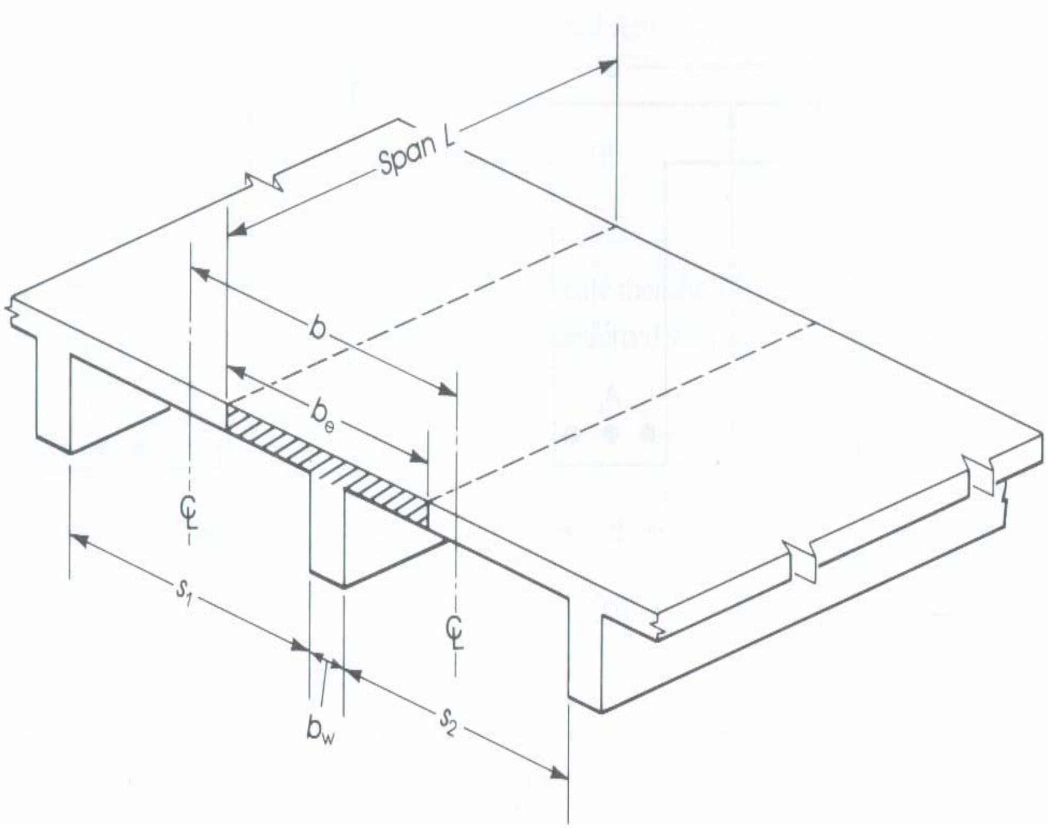


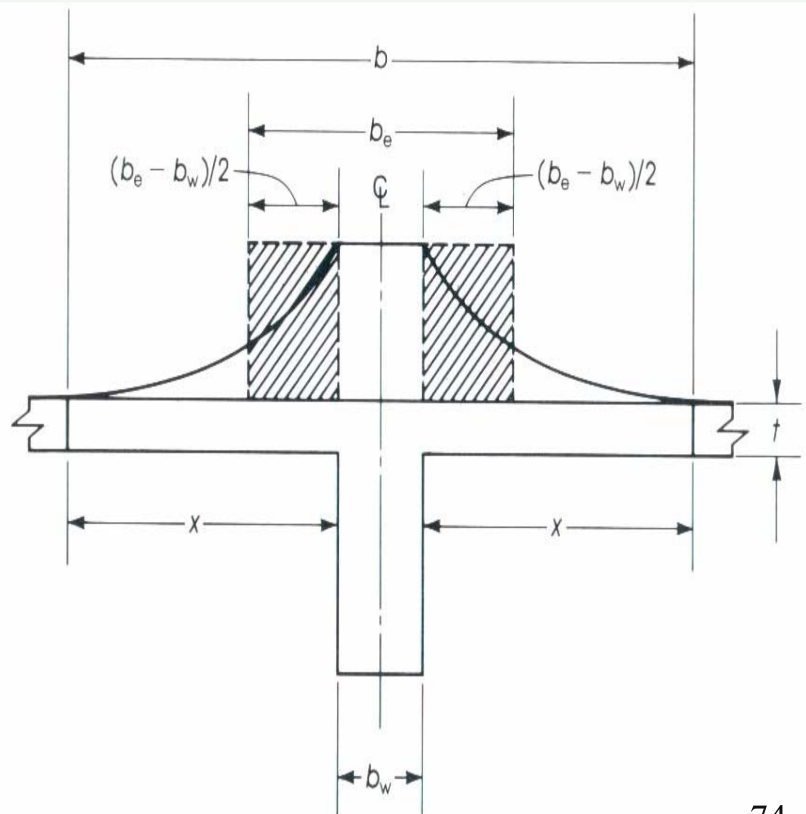
If the neutral axis falls within the slab depth analyze the beam as a rectangular beam, otherwise as a T-beam.



Effective width (b_e)

b_e is width that is stressed uniformly to give the same compression force actually developed in compression zone of width $b_{(actual)}$





1-From ACI 318, 2014 Section 6.3.2.1

T Beam Flange:

$$be \leq \frac{L}{4}$$

$$be \leq 16\ h_f + bw$$

$$be \leq b \qquad \text{(clear distance to next web)}$$

2-From ACI 318 2014 Section 6.3.2.1

Inverted L Shape Flange

$$be \leq \frac{L}{12} + bw$$

$$be \leq 6\ h_f + bw$$

$$be \leq b = bw + 0.5 \times \text{(clear distance to next web)}$$

3-From ACI 318 2014 Section 6.3.2.2

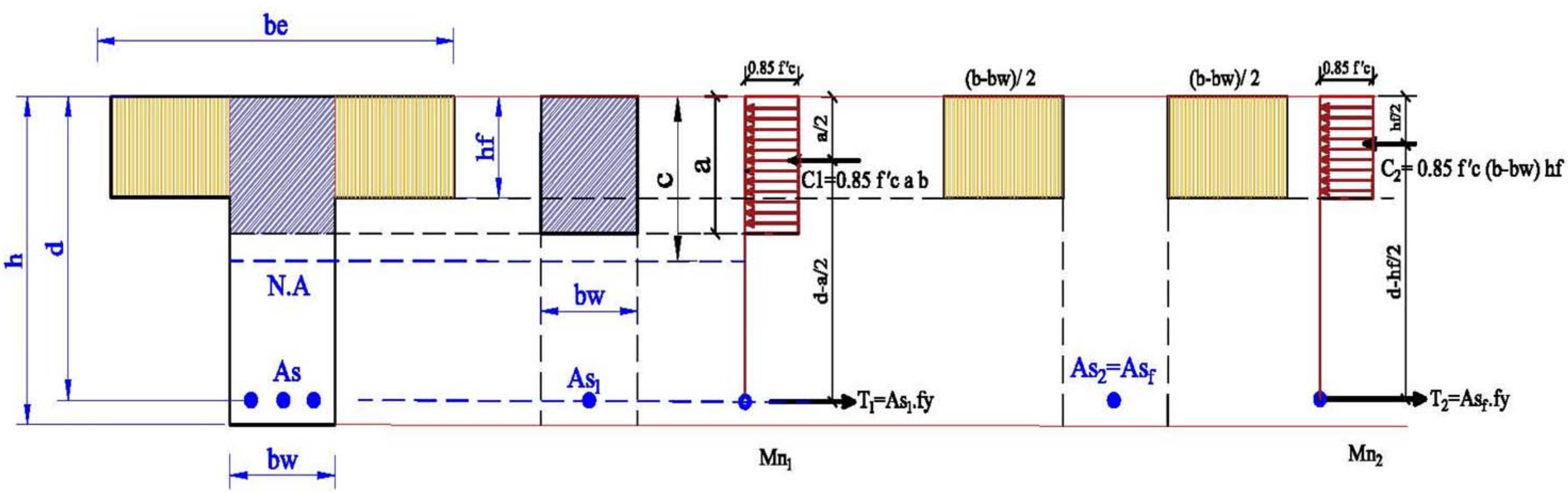
Isolated T-Beams

$$h_f \geq \frac{b_w}{2}$$

$$be \geq 4\ bw$$

The analysis of a T-section is similar to that of a doubly reinforced concrete section, considering an area of concrete (be−bw)*t as equivalent to the compression steel area A′s. The analysis is divided into two parts, as shown in Fig. below.

1. A singly reinforced rectangular basic section , bw *d ,and steel reinforcement As_1. The compressive force, C1, is equal to 0.85f′c a bw, the tensile force, T_1, is equal to $As_1 fy$, and the moment arm is equal to (d−a/2).
2. A section that consists of the concrete over hanging flange sides $2\times[(be-bw)\ h_f]/2$ developing the additional compressive force (when multiplied by 0.85f′c) and a moment arm equal to d−hf/2. If A_{sf} is the area of tension steel that will develop a force equal to the compressive strength of the overhanging flanges, then



$As_f\, fy = 0.85f'c(be - bw)h_f$

$$As_f = \frac{0.85f'c\, hf\ (be - bw)}{fy}$$

The total steel used in the T-section As is equal to $As_1 + Asf$, or:

$As_1 = As - As_f$

The T-section is in equilibrium, so $C_1 = T_1$, $C_2 = T_2$, and $\mathrm{C} = C_1 + C_2$ *and* $T = T_1 + T_2$.

Considering equation $C_1 = T_1$ for the basic section, then

$As_1\, fy = 0.85f'c\, a\, b_w \quad or \ \left(As - As_f\right)fy = 0.85f'c\, a\, b_w$ therefore,

$$a = \frac{(As - As_f)\, fy}{0.85f'c\, bw}$$

Note that $\boldsymbol{bw}$ is used to calculate a. The factored moment capacity of the section is the sum of the two moments Mu_1 and Mu_2:

$\emptyset Mn = Mu_1 + Mu_2$

$$Mu_1 = \emptyset As_1\, fy\left(d - \frac{a}{2}\right) = \emptyset(As - Asf)fy(d - \frac{a}{2})$$

$As_1 = (As - Asf)$ and

$$a = \frac{(As - As_f)\, fy}{0.85f'c\, bw}$$

$$Mu_2 = \emptyset Asf\ fy\ (d - \frac{h_f}{2})$$

$$\emptyset\ Mn = \emptyset[(As - Asf)fy(d - \frac{a}{2}) + Asf\ fy\ (d - \frac{h_f}{2})]$$

Considering the web section $bw \times d$, the net tensile strain (NTS), εt, can be calculated from a, c, and dt as follows:

If $c = \frac{a}{\beta_1}$ and $d_t = h - 62.5$, then $\varepsilon_t = 0.003(d_t - c)/c$. For tension-controlled section in the web, $\varepsilon t \geq 0.005$. The design moment strength of a T-section or I- section can be calculated from the preceding equation above .It is necessary to check the following:

1.The total tension steel ratio relative to the web effective area is equal to or greater than ρ min:

$$\rho_w = \frac{As}{bw\ d} \geq \rho_{min}$$

$$\rho_{min} = \frac{0.25\sqrt{f'c}}{fy} \geq \frac{1.4}{fy}$$

2. Also, check that the NTS is equal to or greater than 0.005 for tension-controlled sections.

3.The maximum tension steel (Max As) in a T-section must be equal to or greater than the steel ratio used, As, for tension-controlled sections, with Ø =0.9.

$$Max\ As = As_f\ (Flange) + \rho_{max}\ (bw\ d)\ (web)$$

$$Max\ As = \left(\frac{1}{fy}\right)[0.85f'c\ h_f(b - bw)\,] + \rho_{max}(b_w\ d)$$

In steel ratios , relative to the web only, divide by bw d:

$$\rho_w = \left(\frac{As}{b_w\, d}\right) \leq \left(\rho_{max} + \frac{A_{sf}}{b_w\, d}\right)$$

$$\rho_w - \rho_f \leq \rho_{max} \text{ (web)}$$

where ρ_{max} *is the maximum steel ratio for the basic singly reinforced web section and* $\rho_f = \frac{As_f}{bw\, d}$.

Analysis of T-and I-sections

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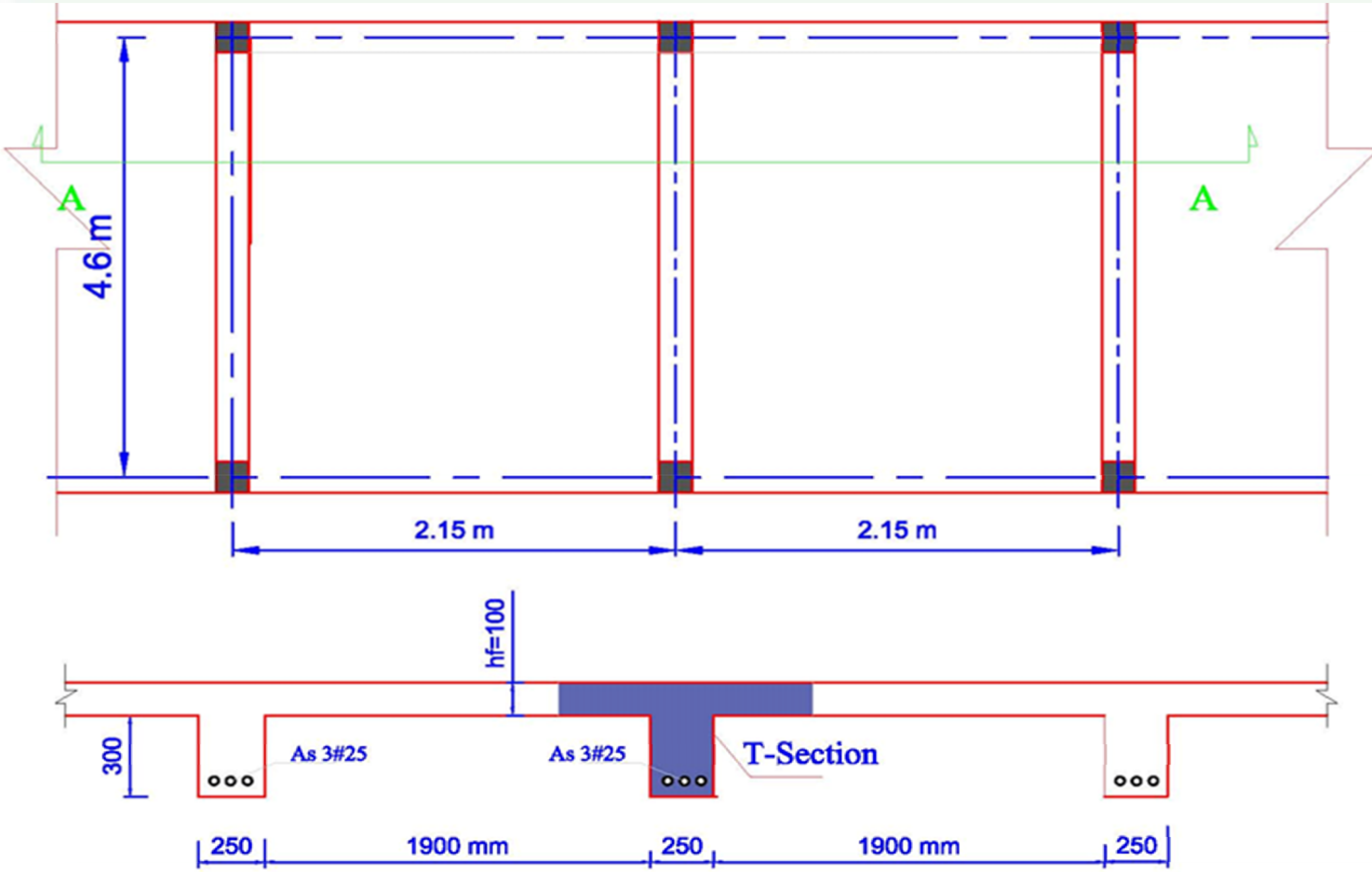
Examples -Analysis of T Sections

Example (10) :A series of reinforced concrete beams spaced at , 2.15 m on centers have a simply supported span of 4.6 m. The beams support a reinforced concrete floor slab 100 mm thick. The dimensions and reinforcement of the beams are shown in Fig. below .Using $f'c$=21 MPa and fy=420 MPa , determine the design moment strength of a typical interior beam.



<u>Solution</u>

1.Determine the effective flange width *be*. The effective flange width is the smallest of:

$$be = \frac{L}{4} = \frac{4.6}{4} = 1150\,mm$$

$be = 16\,h_f + bw = 16 \times 100 + 250 = 1850\,mm.$

be = Center to center of adjacent slabs = 2.15 m

Therefore be= 1150 mm

2.Check the depth of the stress block. If the section behaves as a rectangular one, then these stress block lies within the flange. In this case, the width of beam used is equal to 1150 mm.

$$a = \frac{As\,fy}{0.85f'c\;be} = \frac{1470\ \times 420}{0.85 \times 21 \times 1150} = 30.01\ <\ h_f = 100mm$$

therefore ,it is a rectangular section.

3.Check that:

$$\rho_w = \frac{As}{b_w\,d} \geq \rho_{min} = \frac{1.4}{fy} = \frac{1.4}{420} = 0.0033 \qquad for\ f'c\ < 31\,MPa$$

$$\rho_w = \frac{1470}{250 \times 400} = 0.0148 > \rho_{min} = 0.0033$$

$4. Check\ \varepsilon t : a = 30.01\ mm.,C = \frac{30.01}{0.85} = 35.31\ mm, \quad d_t = d = 400\ mm.$

$$\varepsilon_t = \frac{d_t - c}{c}\ \varepsilon_c = \frac{400 - 35.31}{35.31} \times 0.003 = 0.03098 > 0.005 \quad Ok$$

Tension Controlled and $\phi = 0.9$

5. Calculate $\emptyset\ Mn$

$$\emptyset\ Mn = \emptyset Asfy\left(d - \frac{a}{2}\right) = 0.9 \times 1470 \times 420\ (400 - \left(\frac{30.01}{2}\right) = 213.93\ KN.m$$

6.Check that As used is less than or equal to Max As

$$Max\ As = As_f + \rho_{max}(b_w\ d)$$

$$MaxAs = \left(\frac{0.85f'c\ h_f(be - bw)}{fy}\right) + \rho_{max}(b_w\ d)$$

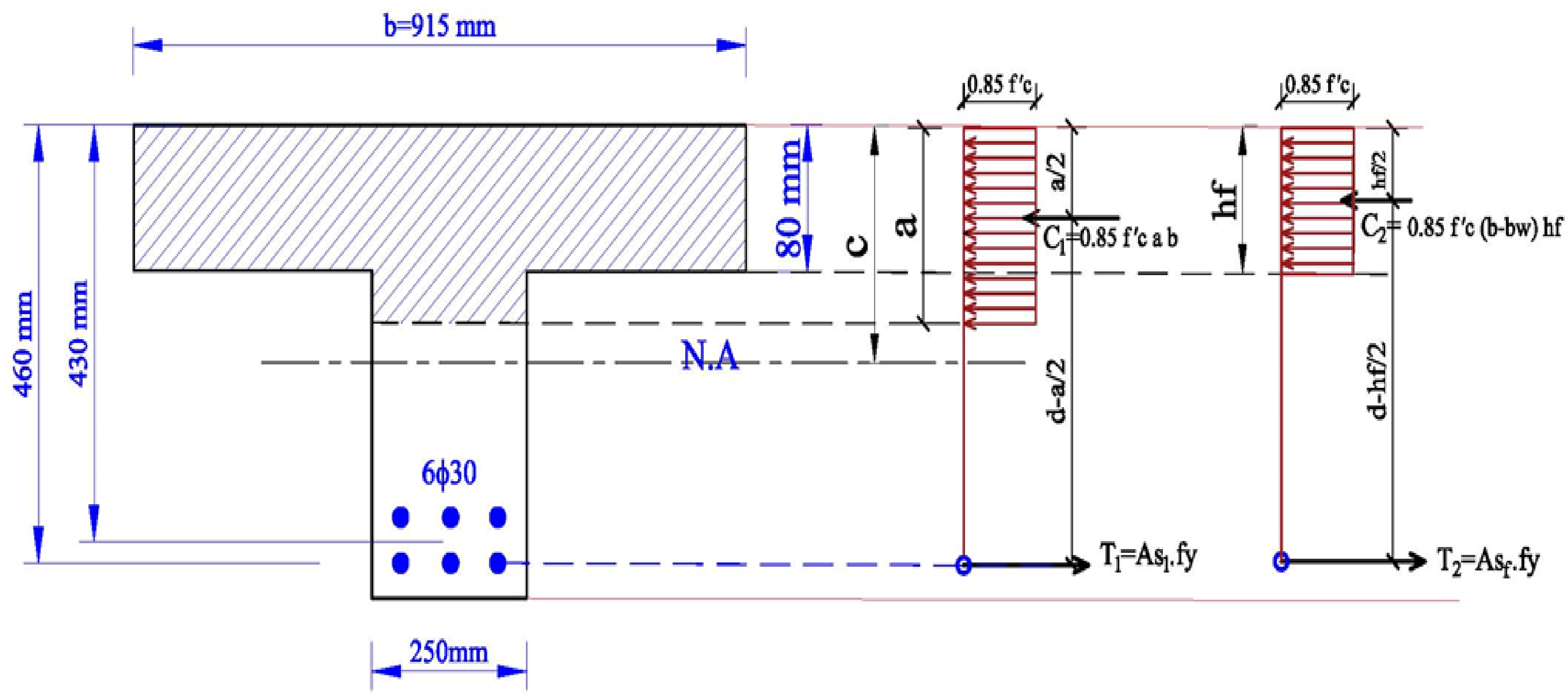
$$\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{d_t}{d}\right) = \frac{0.85}{23.53}\left(\frac{600}{600 + 420}\right)(1) = 0.02125$$

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.008}\right)\rho_b = \left(\frac{0.003 + \frac{420}{200000}}{0.008}\right) \times 0.02125 = 0.01354$$

$$Max\ As = \left(\frac{0.85 \times 21 \times 100(1150 - 250)}{420}\right) + 0.01354\ (250 \times 400) = 5179\ mm^2 > As\ (used)$$

$$= 1470\ mm^2 \quad O.K.$$

Example (11) :Calculate the design Moment strength of T- Section Shown below using f′c=24 MPa and fy=420 MPa , determine the design moment strength of a typical interior beam.



Solution

1- Calculate a :

$$a = \frac{As\,fy}{0.85f'c\;be}$$

$As = \emptyset\ 30 = 706\ mm^2$

$$a = \frac{6 \times 706 \times 420}{0.85 \times 24 \times 915} = 95.31\ mm\ > h_f = 80\ mm$$

Since a $> h_f$, $it\ is\ a\ \ T - \ Section\ analysis$

2- Find As_f

$$As_f = \frac{0.85f'c\ h_f\ (be - bw)}{fy} = \frac{0.85 \times 24 \times 80 \times (915 - 250)}{420} = 2584\ mm^2$$

$$As_1 = As - As_f = 4236 - 2584 = 1652\ mm^2$$

$$a = \frac{As_1\ fy}{0.85f'c\ bw} = \frac{1652 \times 420}{0.85 \times 24 \times 250} = 136.05\ mm$$

$$c = \frac{a}{\beta_1} = \frac{136.05}{0.85} = 160.06\ mm$$

3- Check εt

$d_t = 460\ mm$

$$\varepsilon_t = \frac{d_t - c}{c} \times 0.003 = \frac{460 - 160.06}{160.06} \times 0.003 = 0.005623\ > 0.005\quad OK$$

$\emptyset$=0.9 Tension Failure

4- Check As min.

$$As_{min} = \rho_{\text{min}}\ bw\ d \ \geq \frac{1.4}{fy} b_w\, d \quad where \quad f'c \ \leq 31\, MPa$$

$$As_{min} = 0.0033 \times 250 \times 430 = 357.98\ mm^2$$

$$Max\ As = A_{sf} + \rho_{max}\,(b_w\, d)$$

$$m = \frac{fy}{0.85\, f'c} = \frac{420}{0.85 \times 24} = 20.59$$

$$\rho_{max} = 0.6375\, \rho_b = \frac{\beta_1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right) = 0.6375 \times \frac{0.85}{20.59}\left(\frac{600}{600 + 420}\right)\left(\frac{460}{430}\right) = 0.01651$$

Max As= $2584 + 0.01651 \times 250 \times 430 = 4364.3\ \text{mm}^2$

$As = 4236mm^2 \ < 4364.3\ mm^2$ $\quad O.K$

5. Calculate $\emptyset\ Mn$

$$\emptyset\ Mn = \emptyset\left[(As - Asf)fy\left(d - \frac{a}{2}\right) + Asf\ fy\left(d - \frac{h_f}{2}\right)\right]$$

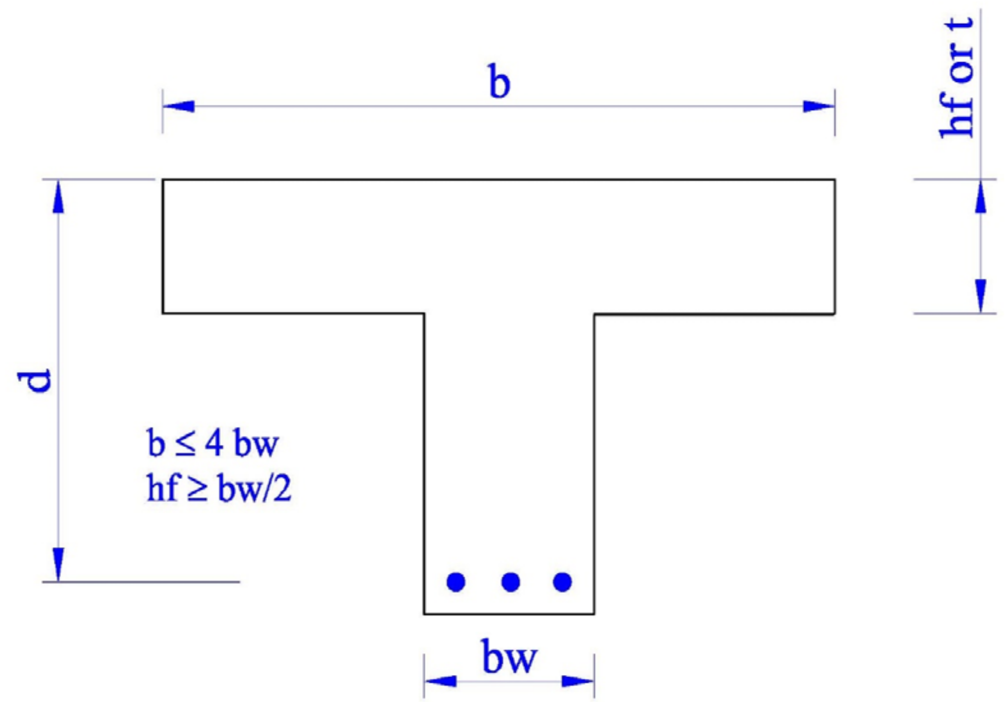
$$= 0.9\ [(4236 - 2584) \times 420\ (430 - \left(\frac{136.05}{2}\right) + 2584 \times 420(430 - \frac{80}{2}) = 606.97\ \ KN.m$$

Dimensions Of Isolated T-shaped Sections

In some cases, isolated beams with the shape of a T-section are used in which additional compression area is provided to increase the compression force capacity of sections. These sections are commonly used as prefabricated units. The ACI Code, Section 6.3.2.2, specifies the size of isolated T-shaped sections as follows:

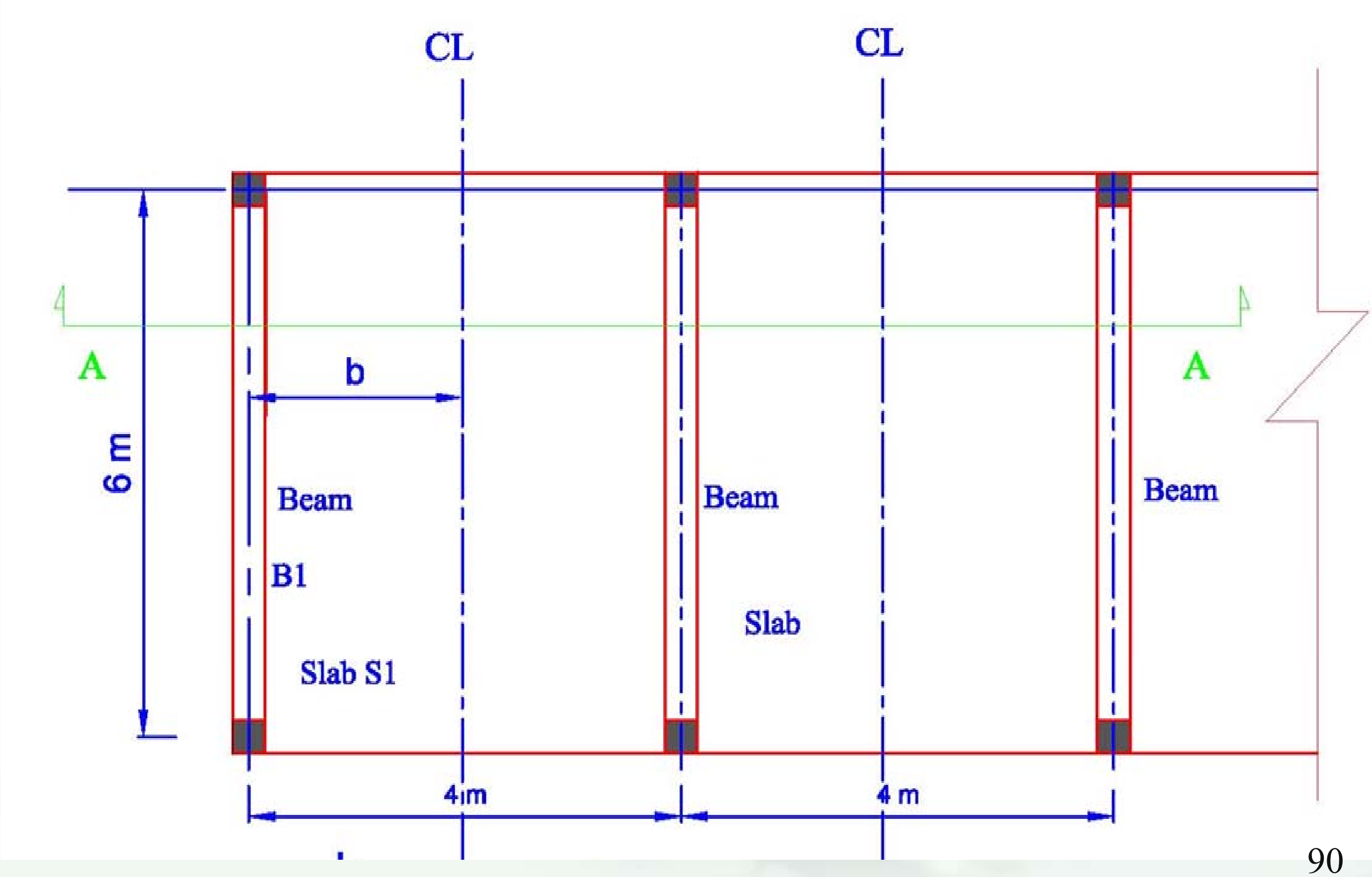
1.Flange thickness, $\boldsymbol{h_f}$, shall be equal to or greater than one-half of the width of the web ,$\boldsymbol{b_w}$.

2.Total Flange width b shall be equal to or less than four times the width of the web, $\boldsymbol{b_w}$.



Inverted L-shaped Sections

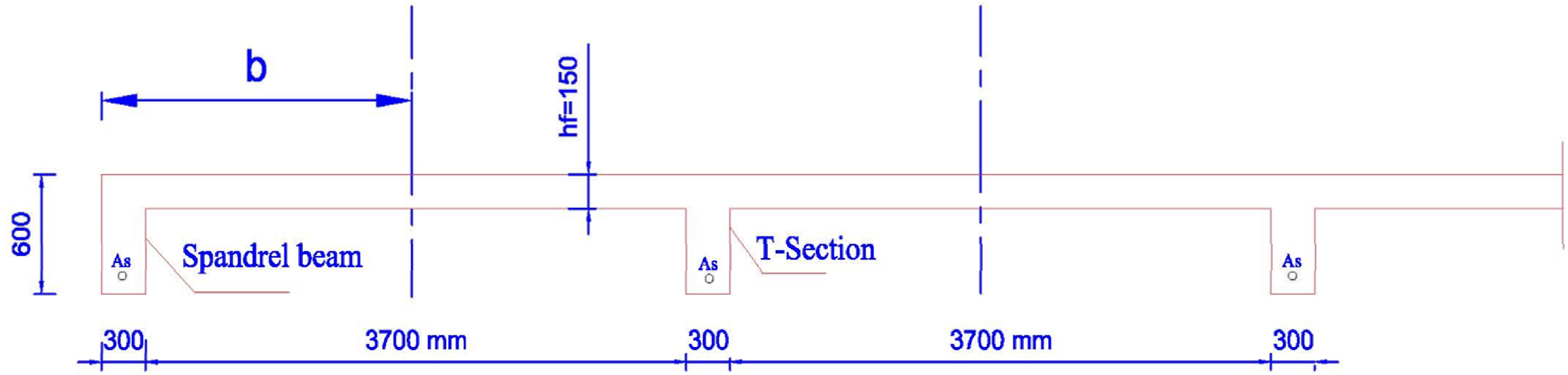
In slab beam girder Floors, the end beam is called a spandrel beam. This type of Floor has part of the slab on one side of the beam and is cast monolithically with the beam. The section is un symmetrical under vertical loading (Fig. shown below). The loads on slab S1 cause torsional moment uniformly distributed on the spandrel beam B1. The over hanging Flange width (b- bw) of a beam with the Flange on one side only is limited by the ACI Code, Section 6.3.2.1, to the smallest of the following:

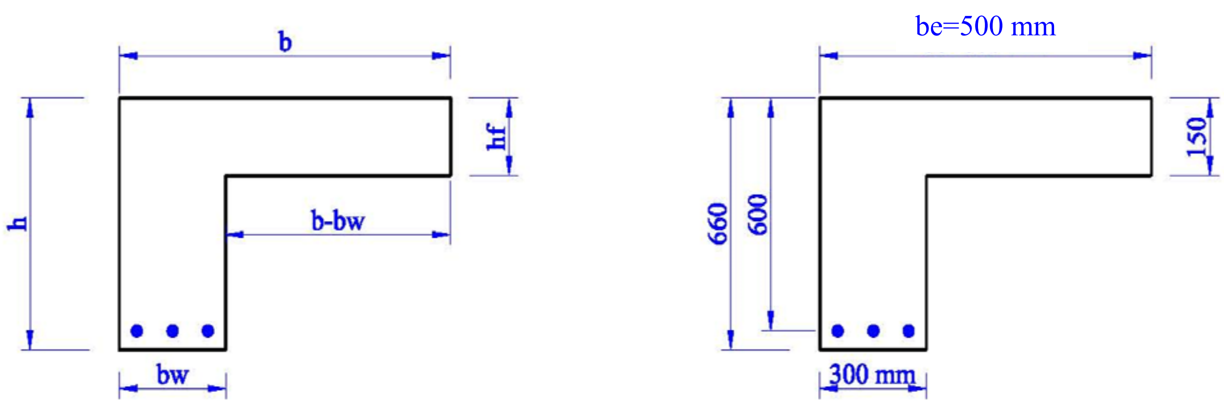


1. $be = \frac{L}{12}$ $\quad be = \frac{6000}{12} = 500\,mm$ $\quad$ (controlled)

2. $be = 6 \times h_f + bw.$ $\quad be = 6 \times 150 + 300 = 1200\,mm$

3. $be = b$ $\quad be = \frac{3700}{2} + 300 = 2150\,mm$





Example (12) : Calculate the design moment strength of the precast concrete section shown below using f′c= 28 MPa and fy=420 MPa .

Solution:

1.The section behaves as a rectangular section with b= 350 mm and $d = 610 - 62.5 = 547.5mm$.

Note that: the width b is that of the section on the compression side.

2. Check that $\rho = As/bd = 5 \times 615/(350 \times 547.5) = 0.01605$

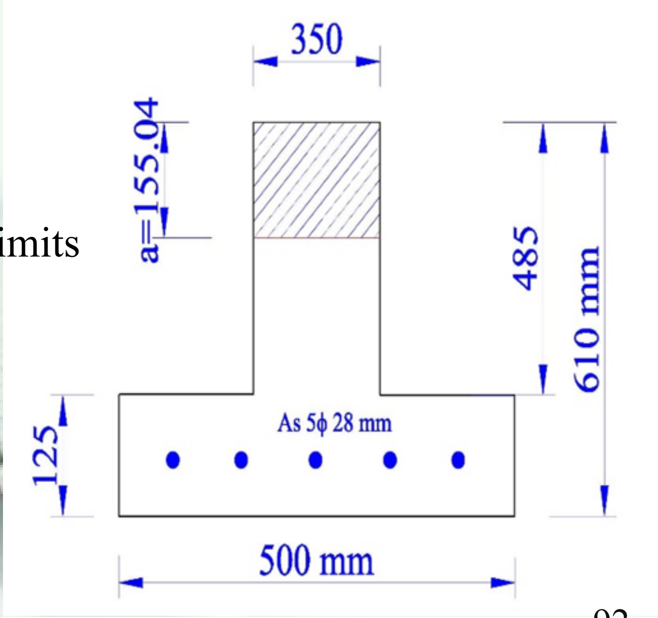
$$\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600+fy}\right)\left(\frac{d_t}{d}\right) = \frac{0.85}{17.65}\left(\frac{600}{600+420}\right)(1) = 0.02834$$

$$\rho_{max} = \left(\frac{0.003+\frac{fy}{Es}}{0.008}\right)\rho_b = \left(\frac{0.003+\frac{420}{200000}}{0.008}\right) \times 0.02834 = 0.01807 > \rho = 0.01605$$

$$\rho_{min}\frac{1.4}{fy} = \frac{1.4}{420} = 0.00333$$

So its tension-controlled sections.

Therefore ϕ=0.9. Also $\rho > \rho_{min}$ min=0.00333.Therefore, ρ is within the limits of a tension-controlled section.



3.Calculatea (a)

$$a = \frac{As\,fy}{0.85f'c\,b} = \frac{5 \times 615 \times 420}{0.85 \times 28 \times 350} = 155.04\,mm$$

$$\emptyset\,Mn = \emptyset As\,fy\left(d - \frac{a}{2}\right) = 0.9 \times 5 \times 615 \times 420 \times \left(547.5 - \frac{155.04}{2}\right)$$

$$= 546.28\,kN.m$$

Thank You……..