Reinforced Concrete Design

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LOGO

Chapter I

Introduction

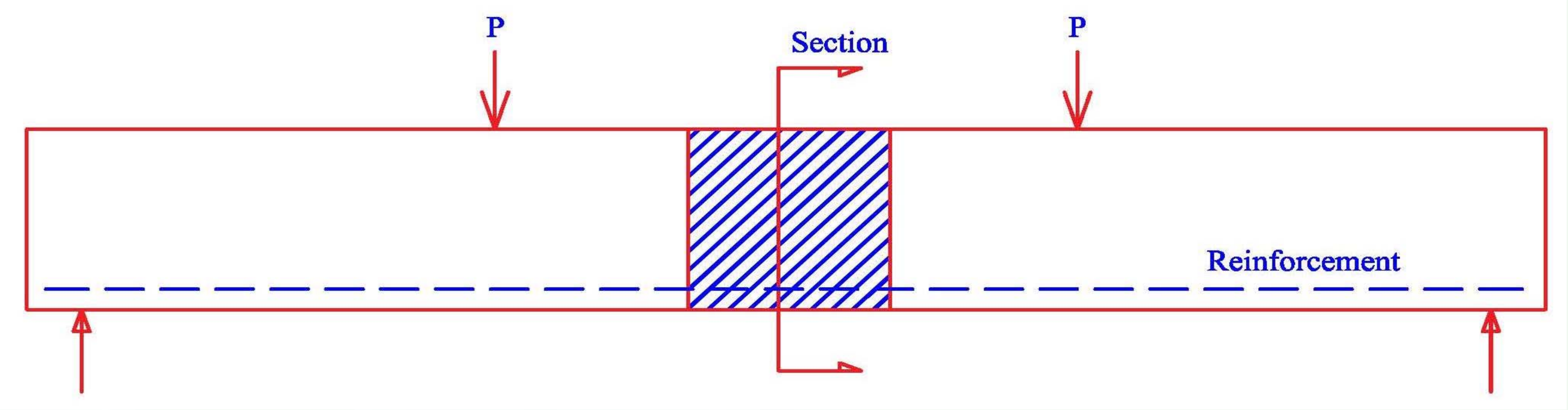
Beam Analysis -Working Stress Method

FLEXURAL ANALYSIS OF REINFORCED CONCRETE BEAMS

BEHAVIOR OF SIMPLY SUPPORTED REINFORCED CONCRETE BEAM LOADED TO FAILURE

BEAM LOADED TO FAILURE

Concrete being weakest in tension, a concrete beam under an assumed working load will definitely crack at the tension side, and the beam will collapse if tensile reinforcement is not provided. Concrete cracks occur at a loading stage when its maximum tensile stress reaches the modulus of rupture of concrete. Therefore, steel bars are used to increase the moment capacity of the beam; the steel bars resist the tensile force, and the concrete resists the compressive force.



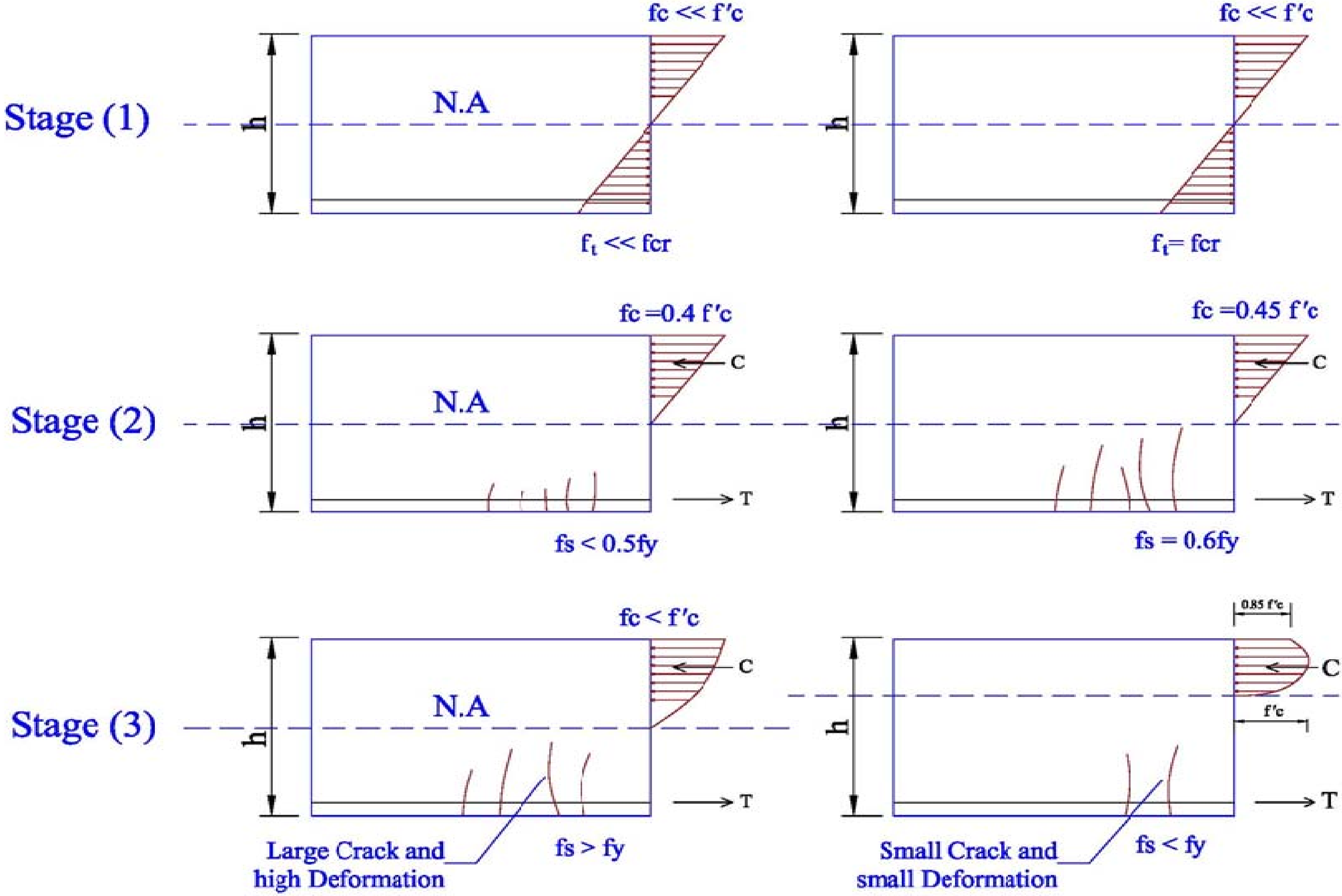
By consider any reinforced concrete beam carry an incrementally accumulative increase load as shown below.

The beam will pass through three stress stages which are:

Stage 1: Elastic Un-cracked Stage: The applied load on beam less than the load which cause cracking.

Stage 2: Elastic Cracked Stage: The applied load makes the bottom fiber stress equal to modulus of rupture of concrete *fr*. Entire concrete section was effective, steel bar at tension side has same strain as surrounding concrete. At this stage before develop any effective cracks the section is under service stresses

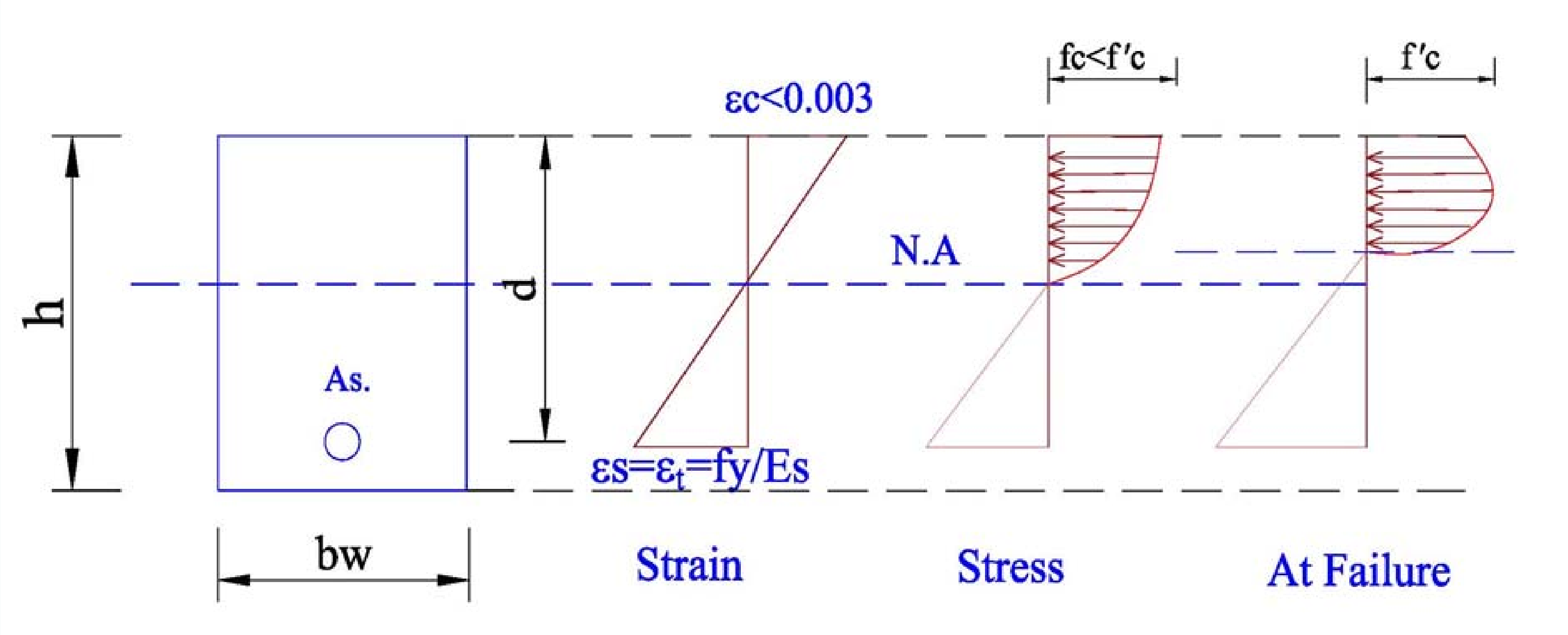
Stage 3: This stage includes two : (a): Inelastic Cracking Stage : The tensile strength of the concrete exceeds the rupture *fr* and cracks develop. The neutral axis shifts upward and cracks extend to neutral axis. Concrete loses tensile strength and steel starts working effectively and resists the entire tensile load. (b): Ultimate Strength Stage: The reinforcement yields. Followed by the failure Stage and the material stresses will be exceed its corresponding capacity.


Stage (1)
fc << f′c
N.A
h
f_t << fcr
fc << f′c
f_t= fcr
Stage (2)
fc =0.4 f′c
C
N.A
T
fs < 0.5fy
fc =0.45 f′c
fs = 0.6fy
Stage (3)
fc < f′c
N.A
Large Crack and
high Deformation
fs > fy
0.85 f′c
f′c
Small Crack and
small Deformation
fs < fy

TYPES OF FLEXURAL FAILURE

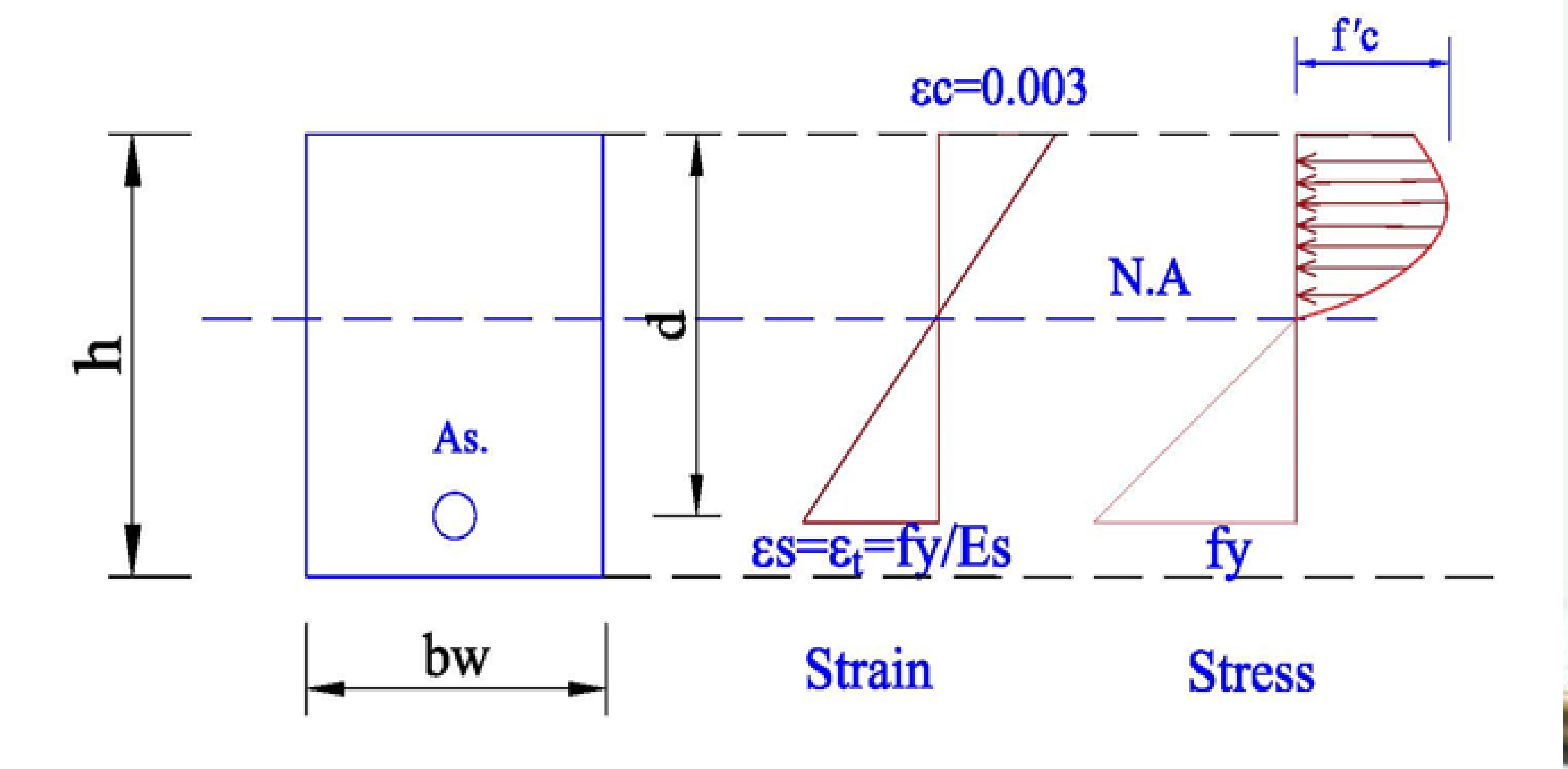
Three types of flexural failure of a structural member can be expected depending on the percentage of steel used in the section.

1 .Steel may reach its yield strength before the concrete reaches its maximum strength. In this case, the failure is due to the yielding of steel reaching a high strain equal to or greater than 0.005. The section contains a relatively small amount of steel and is called a tension-controlled section.

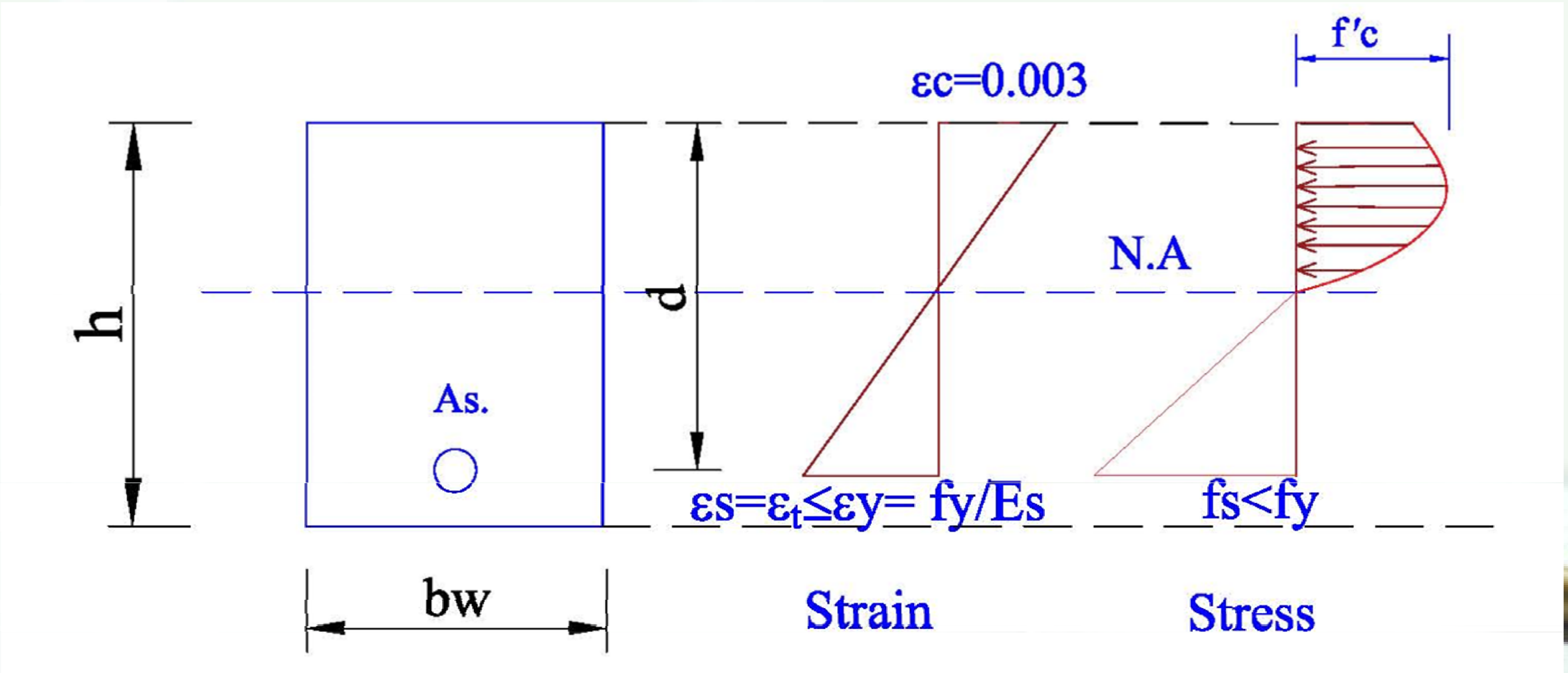


Tension-controlled section.

2 .Steel may reach its yield strength at the same time as concrete reaches its ultimate strength. The section is called a Balanced section



3 .Concrete may fail before the yield of steel, due to the presence of a high percentage of steel in the section. In this case, the concrete strength and its maximum strain of 0.003 are reached, but the steel stress is less than the yield strength, that is, fs is less than fy. The strain in the steel is equal to or less than 0.002. This section is called a compression-controlled section



Analysis and Design Methods of Reinforced Concrete Structure Working Stress Method (WSM)

Stresses are computed in both the concrete and steel using principles of mechanics that include consideration of composite behavior

Actual Stresses < Allowable Stresses

Ultimate design method (UDM)

The Strength of members is computed at ultimate capacity Load Factors are applied to the loads Internal forces are computed from the factored loads

Required Strength < Actual Strength

Working Stress Method (WSM)

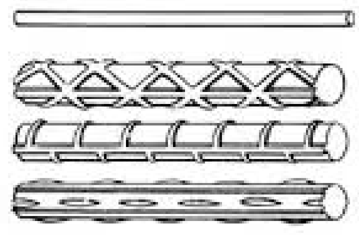
Basic assumptions for design applicable to flexural and compression members are as follows:

(1) Plane section before bending remains plane after bending.

(2) The tensile stress of concrete is neglected unless otherwise mentioned.

(3) The strain-stress relation for concrete as well as for steel reinforcement is linear.

(4) Perfect bond between steel and concrete.



plain and deformed steel bars

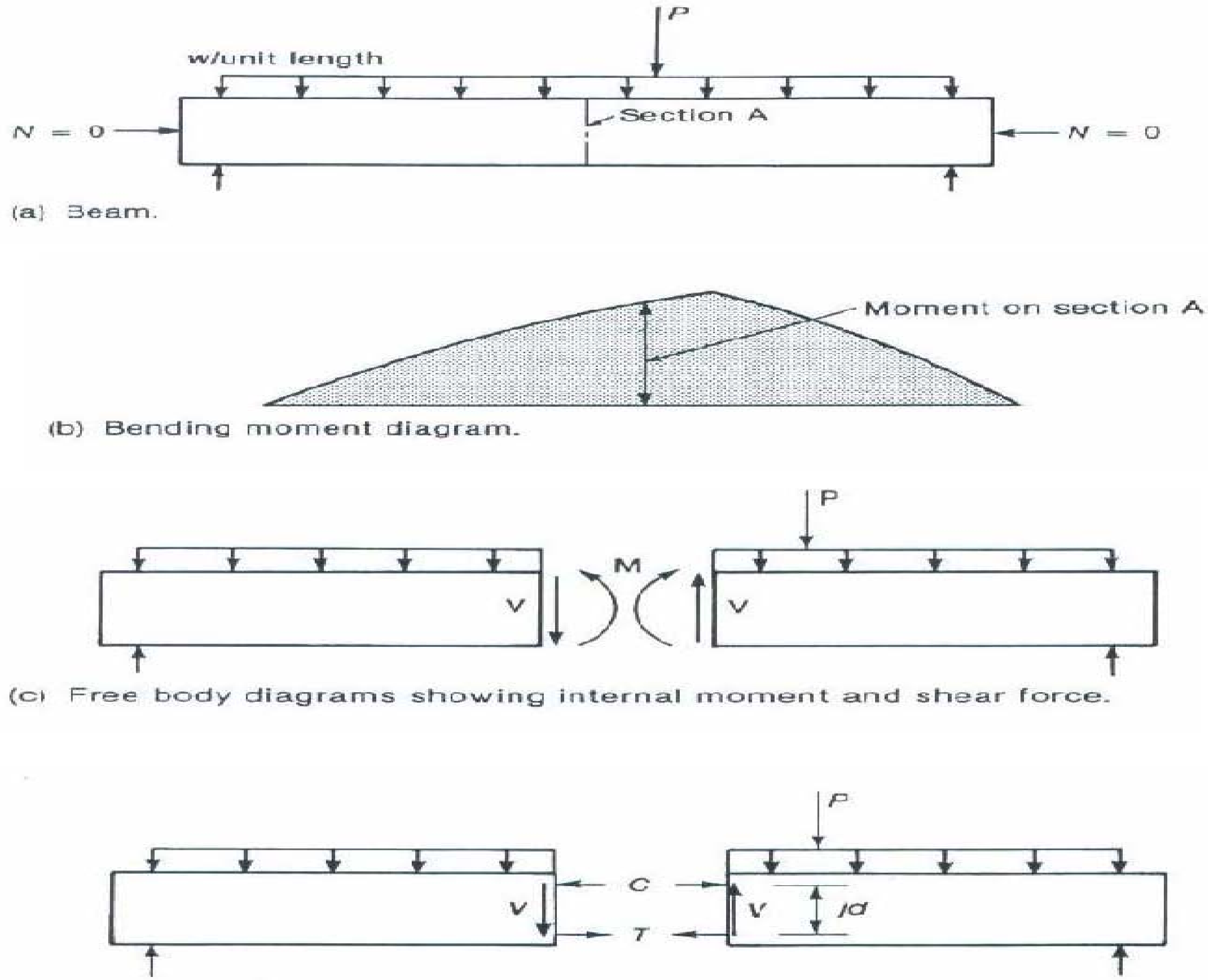
Loading Stages: Un-cracked section and Cracked section and Permissible Stresses

Load factors for all types of loads are taken to be unity for this design method. Permissible stresses are defined as characteristic strength divided by factor of safety.

The factor of safety is not unique values either for concrete or for steel; therefore, the permissible stresses at service load must not exceed the following :

- \- Flexural Extreme fiber stress in compression : $0.45 f'c$
- \- Tensile stress in reinforcement: $0.5 fy$
- \- Modular Ratio $n= Es/Ec$
- \- Transformer section : Substitute steel area with (n As) of fictitious concrete
- \- Location of Neutral axis depends on weather we are analyzing or designing a section

The beam is a structural member used to support the internal moments and shears

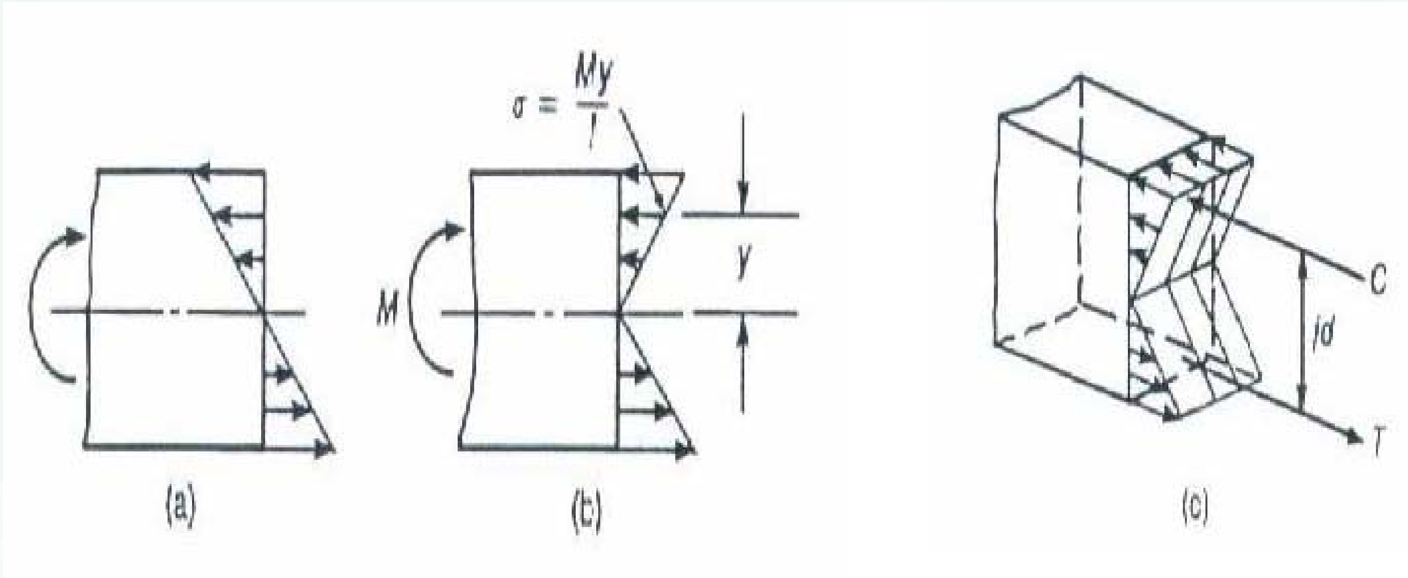


(a) Beam.

(b) Bending moment diagram.

(c) Free body diagrams showing internal moment and shear force.

(d) Free body diagrams showing internal moment as a compression–tension force couple.



The stress in the block is defined as:

$$\sigma = \frac{M \times y}{I} \quad \textbf{(for homogenous section)}$$

Under the action of transverse loads on a beam strains, normal stresses and internal forces developed on a cross section are as shown below :

1-Stage 1: Before Cracking (Uneconomical).
2- Stage 2: After Cracking (Service Stage).
3- Stage 3: Ultimate (Failure).

1- Un-cracked Section:

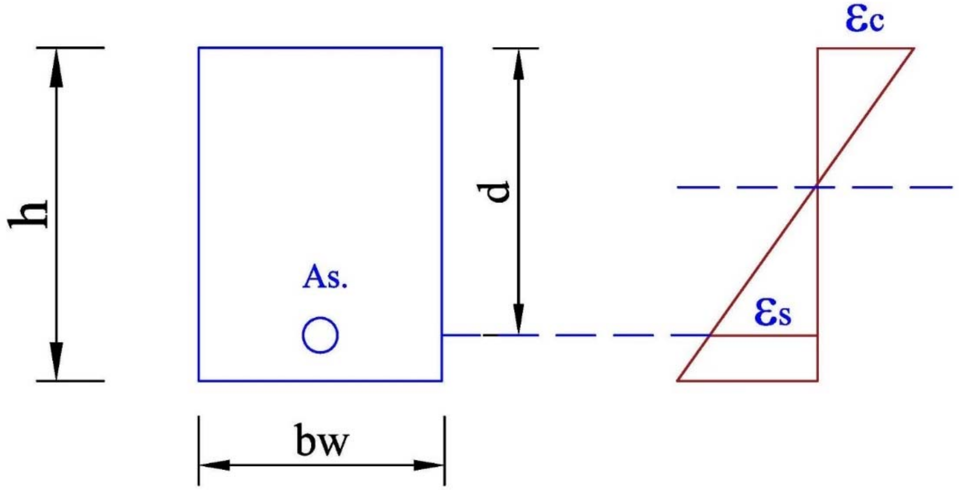
Assuming perfect bond between steel and concrete, we have:

$\varepsilon_s = \varepsilon_c$

$f_s\, Es = E_c\, fc$

$$f_s = \frac{E_s}{E_c} fc = nfc$$

$\text{Tensile Force} = As\, fs = As.nfc$



$A_{eq} = At = A_c + n\,As$

A_{eq}: *Equivalent Area*

Ac: *Concrete Area*

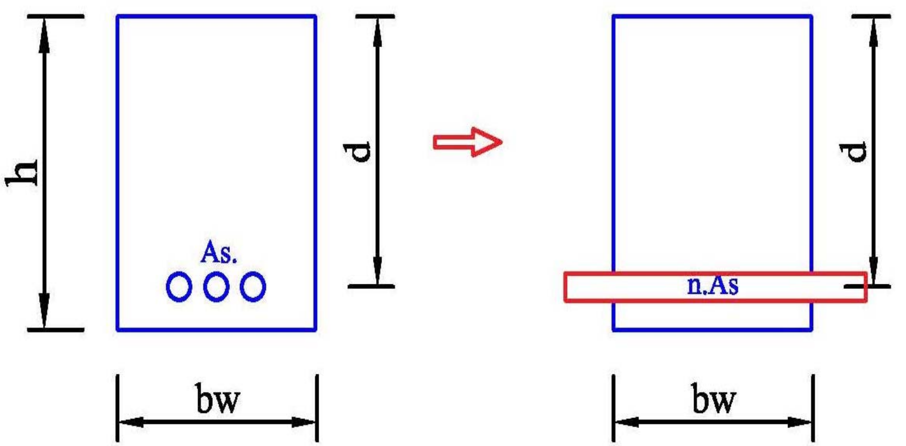
As: *Steel Area*

n: *Modular Ratio*

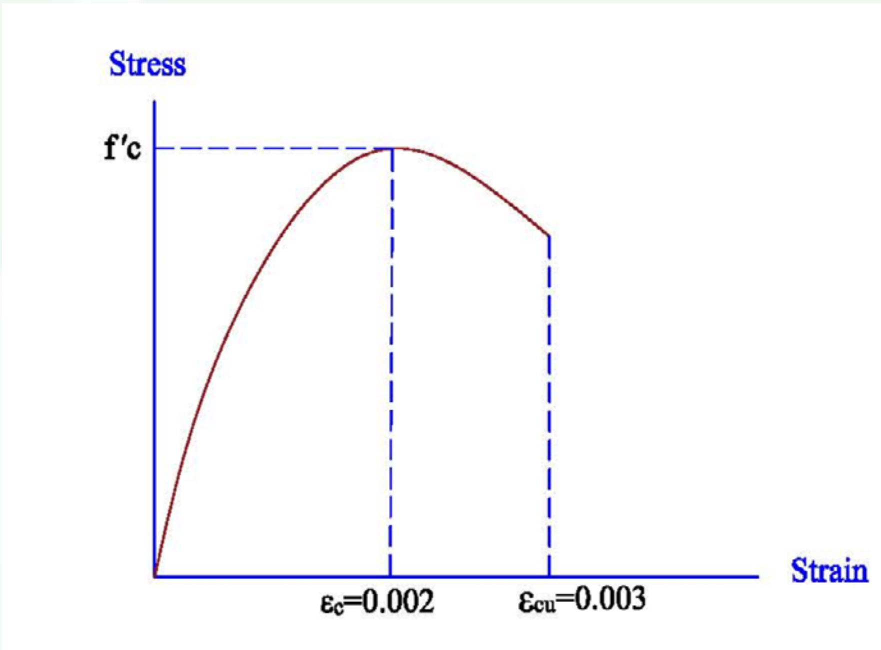
Permissible Stress:

Concrete $= \mathbf{0.45}\, \acute{f}_c$

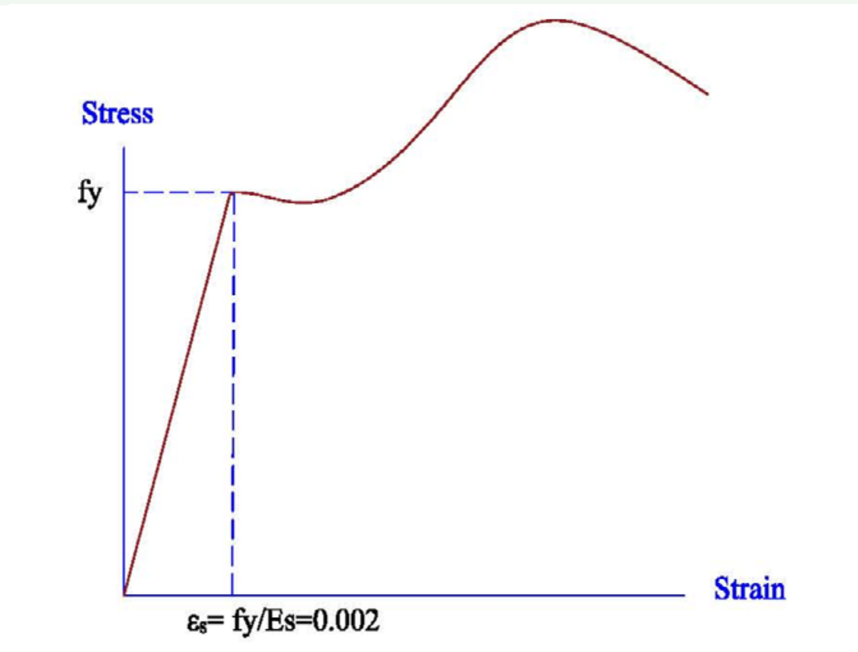
Steel$= \mathbf{0.5} f_y$



Original Section   Equivalent Section



Concrete



Steel

Stress-Strain Relationship of concrete and steel

Homogenous section & under bending:

$$f_c = \frac{M.C}{I}$$

$$f_s = n\, f_c$$

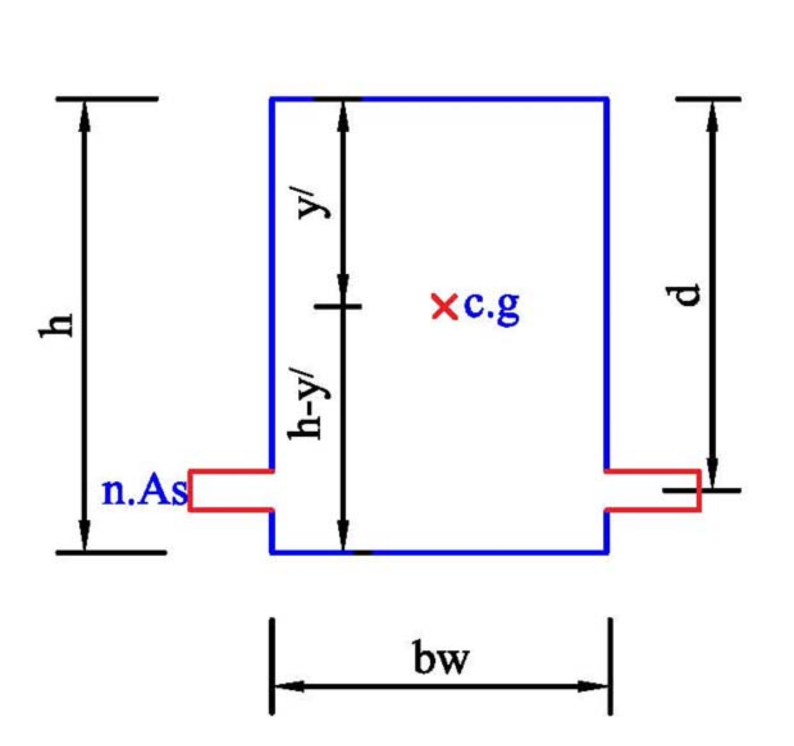
Transformer section:

$$1 - At = (Ac - As) + nA_s = Ac + (n-1)A_s$$

$$2 - \acute{y} = \frac{A_c \times \frac{h}{2} + (n-1)A_s \times d}{A_t}$$

$$3 - I = \frac{b\,h^3}{12} + Ac(\acute{y} - \frac{h}{2})^2 + (n-1)A_s (d - \acute{y})^2$$

$$where\ At = b\,h + (n-1)A_s$$



Equivalent Section

Stress:

$$f_c = \frac{M.\acute{y}}{I} \qquad Top\ fiber$$

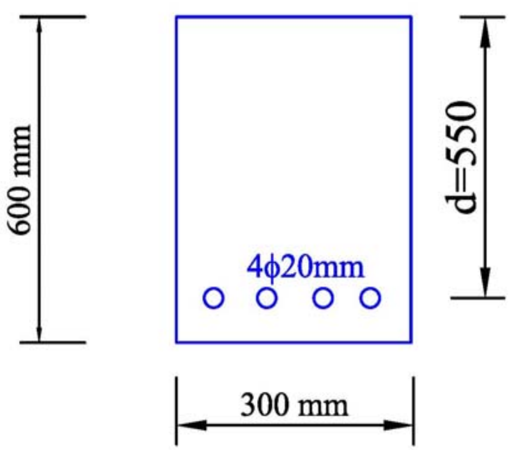
$$f_c = \frac{M\,(h - \acute{y})}{I} \qquad Bottom\ fiber$$

$$f_s = nf_c = \frac{M.(h - \acute{y} - cover)}{I} \qquad at\ steel\ fiber$$

Example (1):

Determine the crack moment for the section shown below , and the stresses. Es= 200000 Mpa

$f'c = 28\ Mpa\ , fy = 413\ Mpa\ , b = 300\ , h = 600, concrete\ cover = 50mm$



Solution:

$$f_c = \frac{M.C}{I}$$

$$As = 4 \times 202 \times \pi/4 = 1256\ mm^2$$

$$y' = \frac{bh^2/2 + (n-1)A_s.d}{b.h + (n-1)A_s}$$

$$n = \frac{E_S}{E_c}$$

$$E_c = 4700\sqrt{f'c} = 4700 \times \sqrt{28} = 24870\ \text{MPa}$$

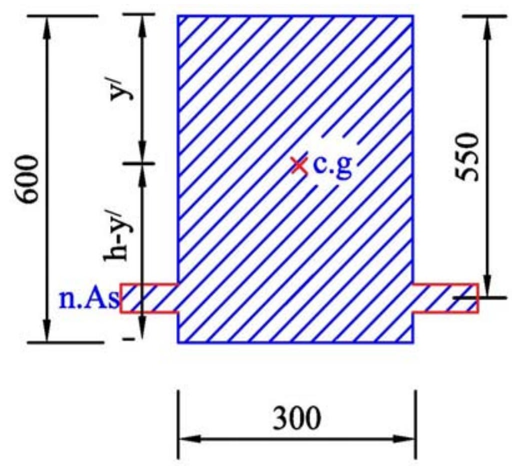
$$n = \frac{Es}{E_c} = \frac{200000}{24870} = \cong 8$$

$$y' = \frac{(300 \times 600 \times \frac{600}{2}) + (8-1) \times 1256 \times (600-50)}{300 \times 600 + (8-1) \times 1256} = 311.63\ mm$$

$$I_{gr} = \frac{bh^3}{12} + bh(y' - \frac{h}{2})^2 + (n-1)As(d-y')^2$$

$$= \frac{300 \times 600^3}{12} + 300 \times 600(311.63 - \frac{600}{2})^2 + (8-1) \times 1256(550 - 311.63)^2$$

$$= 59.232 \times 10^8\ mm^4$$



Equivalent Section

$y_{bottom} = y_b = h - y' = 600 - 311.63 = 288.37\ mm$
$y_{top} = y_t = y' = 311.63\ mm$
$y_{steel} = y_b - \ cover = 288.37 - 50 = \ 238.37\ mm$

From ACI code $\quad f_{cr} = 0.625\sqrt{f'c} = 0.625\sqrt{28} = 3.31 MPa$

ft bottom fiber :

$$f_{cr} = \frac{M_{cr} \times y_b}{I_{gr}}$$

$$3.31 \ = \frac{M_{cr} \times 288.37}{59.232 \times 10^8} \quad \Longrightarrow \quad Mcr = 67.99 \times 10^6\ N.mm$$

$or \quad M_{cr}$= 67.99 KN.m

$$f_c \text{ top fiber} = f_{ct} = \frac{M_{cr} \times y_t}{I_{gr}}$$

$$= \frac{M_{cr} \times 311.63}{59.232 \times 10^8} = 3.58\ Mpa < f'c = 28\ MPa$$

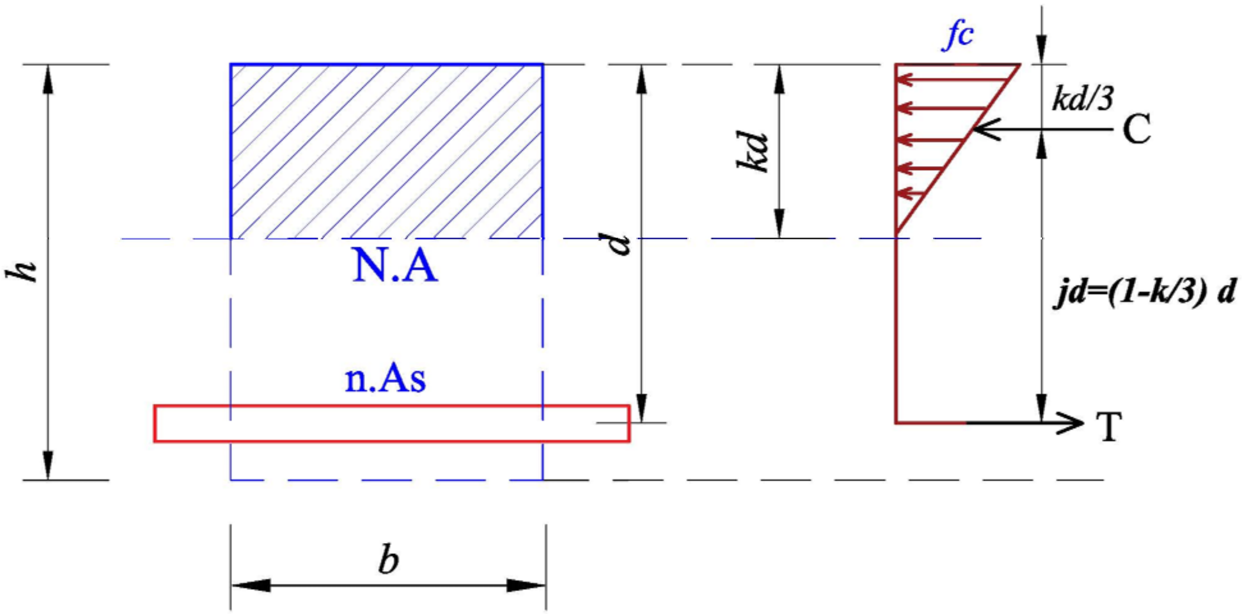
$$f_s = nf_c \ = n \times \frac{M_{cr} \times (y_b - cover)}{I_{gr}} = 7.99 \times \frac{67.99 \times 10^6 (238.37)}{59.232 \times 10^8}$$

$$= \ 21.86\ MPa \ << \ fy \ = \ 413\ MPa$$

2-Cracked Section

$f_t > fr, \quad fc \le 0.45f'_c \text{ and } fs < 0.5fy$

Assume the crack goes all the way to the N.A and will use the transformed section



To locate N.A. , tension force = compressive force (by def. NA) (Note, for linear stress distribution and with Tensile and compressive forces are equal to

$$C = b\left(\frac{kd}{2}\right)\times f_c \text{ and } T = As \times fs$$

To determine the location of neutral axis, the moment of the tension is about the axis is set equal to the moment of the compression area, which gives:

$$b(kd)\left(\frac{kd}{2}\right) = nAs\,(d - kd) \quad \text{second degree equation}$$

$$where\ rienforcement\ ratio = \rho = \frac{As}{bd}\ or\ As = \rho bd$$

$$b(kd)\left(\frac{kd}{2}\right) - n\rho bd^2(\,1 - k) = 0 \qquad \text{multiply by } \left(\frac{1}{bd^2}\right)$$

$$\left(\frac{k^2}{2}\right) = n\rho(\,1 - k) = 0$$

$$k^2 + 2n\rho k - 2n\rho + (n\rho)^2 - (n\rho)^2 = 0$$

$$(k + 2n\rho)^2 = (n\rho)^2 + 2(n\rho) = 0$$

Then :

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

Taking moments about C gives:

$M = T\,.\,jd = As\, fs\, jd$

where: jd is the internal lever arm between C and T. From the above equation steel stress is

$$\therefore fs = \frac{M}{As\, jd}$$

Or Conversely, taking moment about T gives

$$M = C\, jd = b\,\frac{(kd)}{2} f_c\, jd \;=\; \frac{f_c}{2} kj\, bd^2$$

$$\therefore fc = \frac{2M}{kjbd^2}$$

Where :

$$j = \left(1 - \frac{k}{3}\right)$$

$\boldsymbol{n} = ratio\ of\ modulus\ of\ elasticity\ of\ steel\ to\ that\ of\ concrete = \frac{E_s}{E_c}$

$\boldsymbol{f_c} = compressive\ unit\ stress\ on\ the\ concrete\ at\ the\ surface\ most\ remote$
$from\ the\ neutral\ surface, in\ pound\ per\ square\ inch$

$\boldsymbol{f_s} = tensile\ unit\ stress\ in\ the\ longitudinal\ reinforcement, in\ pound\ per\ square\ inch$

$\boldsymbol{b} = the\ width\ of\ the\ rectangular\ beam, in\ inches.$

$\boldsymbol{d} = the\ effective\ depth\ of\ the\ beam\ in\ inches$

$\boldsymbol{k} = ratio\ of\ distance\ of\ the\ neural\ axis\ of\ the\ cross\ section, from\ extreme$
$fibers\ in\ compression\ to\ the\ effective\ depth\ of\ the\ beam$

$\boldsymbol{kd} = the\ distance\ from\ the\ neutral\ axis\ of\ the\ cross\ section\ to\ the\ extremefibers\ in\ compression$

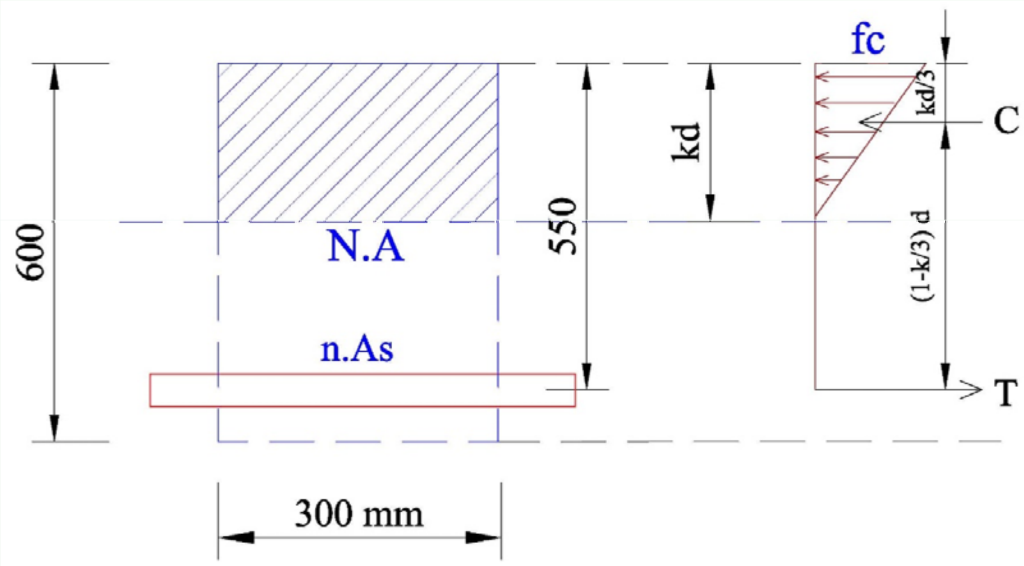
$\boldsymbol{j} = ratio\ of\ the\ distance\ between\ the\ resultant\ of\ the\ compressive\ stresses\ and\ centre\ of\ the$
$tensile\ stresses\ to\ d, the\ effective\ depth\ of\ the\ beam$

$\boldsymbol{jd} = the\ distance\ between\ the\ resultant\ of\ the\ compressive\ stresses\ and\ the\ centre\ of\ the\ tensile$
$stresses. It\ is\ the\ lever\ arm\ of\ the\ resisting\ couple, in\ inches$

$\boldsymbol{\rho} = the\ ratio\ of\ the\ area\ of\ the\ cross\ section\ of\ the\ longitudinal\ steel\ reinforcement$
$to\ the\ effective\ area\ of\ the\ concrete\ beam, \rho \frac{A_s}{bd}$

Example (2):

Determine the stresses in concrete and steel of section (300 x 600 mm) as in Exa. (1) subjected to service moment 100 KN.m and $f'_c = 28\ Mpa,\ fy = 413\ Mpa$, cover =50 mm , As= 4ϕ20 mm, Es= 200000 Mpa ,



Solution :

$M_{cr} = 67.99\ KN.m \qquad (Example - 1)$

$While\ M\ applied = 100\ KN.m > M_{cr}$

$The\ section\ is\ cracked$

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

$$\rho = \frac{A_s}{bd} = \frac{1256}{300 * 550} = 0.007612$$

$n = 7.99$

$$k = \sqrt{2 \times 0.007612 \times 7.99 + (0.007612 \times 7.99)^2} - 0.007612 \times 7.99$$

$= 0.2932$

$C = kd = 0.2932 \times 550 = 161.26mm$

$$I_{cr} = \frac{bc^3}{3} + nA_s(d - c)^2$$

$$= \frac{300 \times 161.26^3}{3} + 7.99 \times 1256\ (550 - 161.26)^2 = 19.359 \times 10^8\ mm^4$$

$Steel\ stress$:

$$f_s = n \times f_{cs} = n \times \frac{M.(d-c)}{Icr}$$

$$= 7.99 \times \frac{100\ \times\ 10^6\ (550 - 161.26)}{19.359 \times 10^8} = 160.6\ MPa\ \ <\ Fs\ allowable = 0.5\ fy$$

$$= 206.5\ MPa$$

Concrete Stress

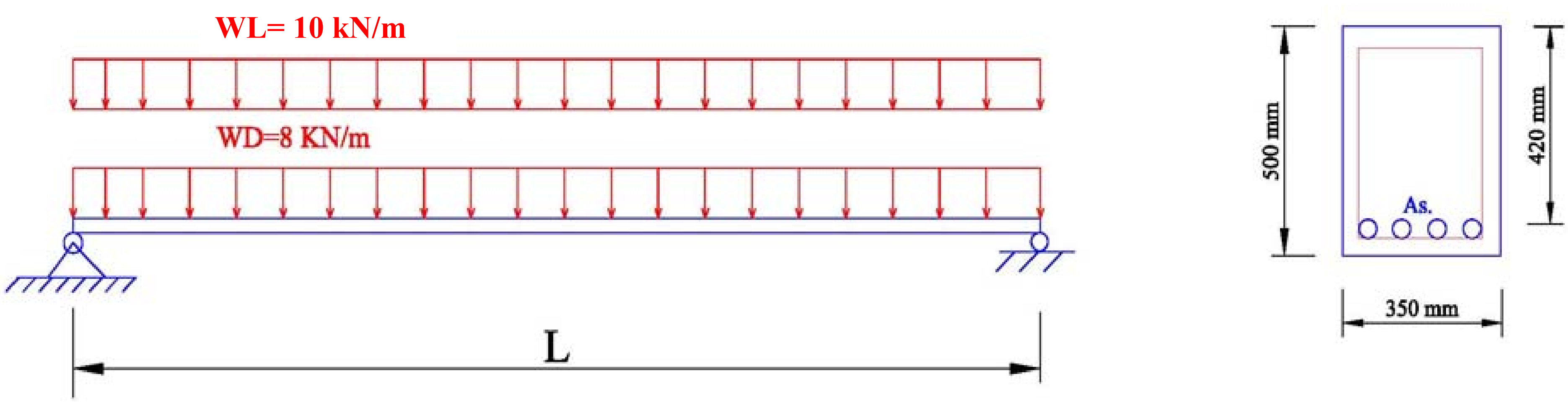
$$f_c = \frac{M.c}{I_{cr}} = \frac{100 \times 10^6 \times\ 161.26}{19.359 \times 10^8} = 8.33\ Mpa < 0.45 \times 28 = 12.6\ Mpa \qquad O.K$$

Example (3): For the simply supported beam shown reinforced by 4ϕ 25 mm bars ($fy = 420\ MPa$) , the concrete strength ($f'c = 21\ MPa$) , evaluate the following :

1- If the span beam = 4 m and dead load = 8 KN/m, live load=10 KN/m check the actual flexural stress in concrete and steel.

2- The length of the beam span that make the concrete in tension face start to crack.

3- The actual stress in concrete and steel if the span of beam = 7m



Solution : First

$$Total\ Load\ = W\ = WD + WL = 8 + 10 = 18\ kN/m^2$$

$$M = \frac{W\ L^2}{8} = \frac{18 \times 4^2}{8} = 36\ KN/mm^2$$

$$n = \frac{E_s}{E_c} = \frac{200000}{4700\sqrt{21}} = 9.22$$

$$A_b = 4 \times \left(\frac{\pi \times 25^2}{4}\right) = 1964\ mm^2$$

Assume $\boldsymbol{f_t = f_r}$

Transformed section area

$$= Ac + (n-1)A_s = 500 \times 350 + (9.22 - 1) \times 1964 = 191144.1\ mm^2$$

$$\acute{y} = \frac{A_c \times \frac{h}{2} + (n-1)A_s \times d}{A_t} = \frac{350 \times 500 \times \frac{500}{2} + (9.22-1) \times 1964 \times 420}{191144.1} = 264.4\ mm$$

$$I = \frac{b\ h^3}{12} + Ac(\acute{y} - \frac{h}{2})^2 + (\ n-1)A_{s\ (}d - \acute{y})^2$$

$$= \frac{350 \times 500^3}{12} + 350 \times 500 \times \left(264.4 - \frac{500}{2}\right)^2 + (9.22 - 1) \times 1964 \times (420 - 264.4)^2$$

$$= 4.076 \times 10^9\ mm^4$$

$$f_c = \frac{M.C}{I}$$

Compression fiber:

$$f_c = \frac{36 \times 10^6 \times 264.4}{4.076 \times 10^9} = 2.33\ MPa$$

Allowable stress in compression $= 0.45f'_c$

$F_c = 0.45 \times 21 = 9.45\ mPa$

$\therefore f_c < f'_c$ O.K

For Tension bottom fiber:

$$f_t = \frac{M.C}{I} = \frac{36 \times 10^6 \times (500 - 264.4)}{4.076 \times 10^9} = 2.08\ MPa$$

$f_r = 0.62\sqrt{f'_c} = 0.62\sqrt{21} = 2.84$ Mpa

$\therefore f_r > f_t$ the assumption is correct and the section is not cracked

$$f_s = nfc = n \times \frac{M.C}{I} = 9.22 \times \frac{36 \times 10^6 \times (420 - 264.4)}{4.076 \times 10^9} = 12.67\ MPa$$

$F_s = 0.5 \times fy = 0.5 \times 420 = 210\ MPa$

$\therefore f_s < F_s$

Second: to make concrete start to crack put the concrete tension stress at the extreme fiber equal to concrete stress at rupture

$(f_r = f_t = 2.84\ mPa)$

$$f_t = \frac{M_{cr}(h - c)}{I}$$

$$2.84 = \frac{M_{cr}(500 - 264.4)}{4.076 \times 10^9}$$

$M_{cr} = 49.12\ kN.m$

$$M_{cr} = \frac{W\,L^2}{8} = \frac{18 \times L^2}{8}$$

$$49.12 = \frac{18 \times L^2}{8} \qquad \therefore L = 4.67\ m$$

$$\textit{Third}: M = \frac{W\,L^2}{8} = \frac{18 \times 7^2}{8} = 110.25\ kN.m$$

$since\ the\ moment\ M = 110.25\ kN.m > M_{cr} = 49.12\ KN.m$ $\therefore$ The concrete section is cracked

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

$$\rho = \frac{A_s}{b\ d} = \frac{1964}{350 \times 420} = 0.0134$$

$$\rho n = 0.0134 \times 9.22 = 0.124$$

$$k = \sqrt{2 \times 0.124 + (0.124)^2} - 0.124 = 0.389$$

$$kd = 0.389 \times 420 = 163.46\ mm$$

$$j = 1 - \frac{k}{3} = 0.87$$

$$jd = 365.54\ mm$$

$$f_c = \frac{2M}{kjbd^2} = \frac{2 \times 110.25 \times 10^6}{0.389 \times 0.87 \times 350 \times (420)^2} = 10.55\ MPa$$

concrete allowable compression stress $F_c = 0.45f'_c = 0.45 \times 21 = 9.45\ mPa$

$\therefore f_c > f'_c$ the concrete behavior is not in elastic range .

$$f_s = \frac{M}{A_S\,jd} = \frac{110.25 \times 10^6}{1694 \times 365.54} = 153.57\ mPa$$

Allowable steel stress = 210 MPa $\therefore f_s > F_s$ the steel stress with in limits (OK)

Thank You………………….