Karnaugh Map (K-Map)

A Karnaugh map consists of a grid of squares, each square representing one canonical minterm combination of the variables or their inverse.
This map provides a systematic method of simplifying a Boolean function to produce the simplest sum of products expression.

- Arranged with squares representing minterms which differ by only one variable to be adjacent both vertically and horizontally.

- Squares on one edge of the map are regarded as adjacent to those on the opposite edge.

-The expression to be minimized should generally be in sum-of-product form.

-The function is 'mapped' onto the Karnaugh map by marking a 1 in those squares corresponding to the terms in the expression to be simplified.

- If two or more pairs are also adjacent, these can also be combined using the same theorem.

-The minimization procedure consists of recognizing multiple pairs in terms of 2, 4 or 8 cells.

Karnaugh Map Format

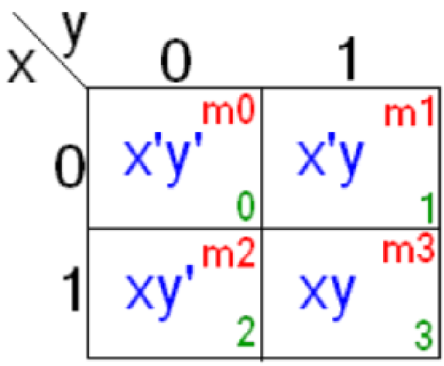
For **n** variables we have 2^n combination , each combination is contained in a **Karnaugh cell.**

For 2 Variables X, Y:

In two variable map, there are 4 minterms hence the map consists of four squares, one for each minterm.

$2^2 = 4$ Products $\longrightarrow$ 4 cells (x'y', x'y, xy' , xy)

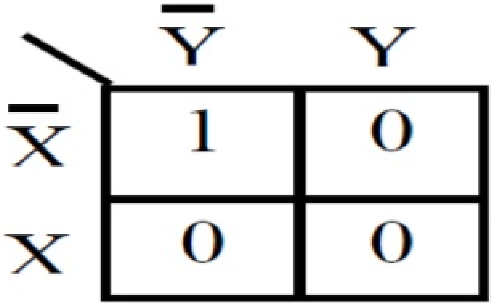
X	Y	Minterms	
0	0	x'y'	m0
0	1	x'y	m1
1	0	xy'	m2
1	1	xy	m3



The Karnaugh Map is filled in by putting (1) in each cell that leads to (1) output. (0) is placed in all the other cells .

EXAMPLE

F = X'Y'

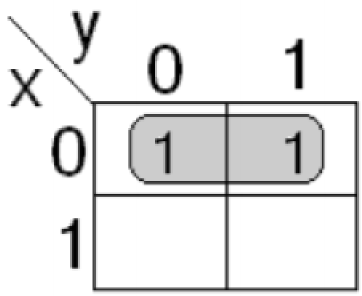


EXAMPLE:

Simplify the following Boolean Function by using Karnaugh Map:

F(x,y) = x'y + x'y'

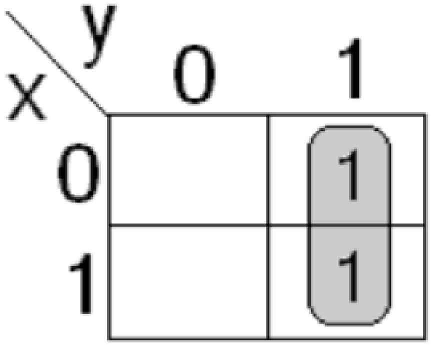
F(x,y) = x'



EXAMPLE:

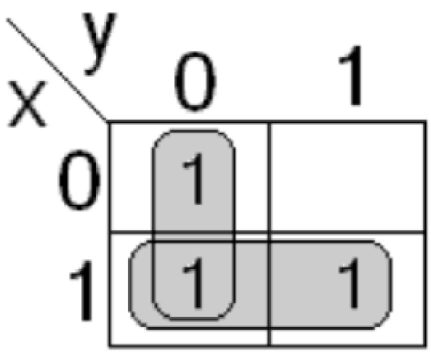
Simplify the following Boolean Function by using Karnaugh Map:
F(x,y) = xy + x'y

F(x,y) = y



EXAMPLE:
Simplify the following Boolean Function by using Karnaugh Map:
F(x,y) = x'y' + xy' +xy

F(x,y) = x + y'



For 3 Variables X, Y, Z: $2^3 = 8$ product Terms → 8 cells.

A three variable map is shown below, note that the minterns are arranged, not in binary sequence, the characteristic of this sequence is that the only one bit changed from (1 to 0) or from (0 to 1) in the listing sequence.

x \ yz	00	01	11	10
0	m0 x'y'z' 0	m1 x'y'z 1	m3 x'yz 3	m2 x'yz' 2
1	m4 xy'z' 4	m5 xy'z 5	m7 xyz 7	m6 xyz' 6

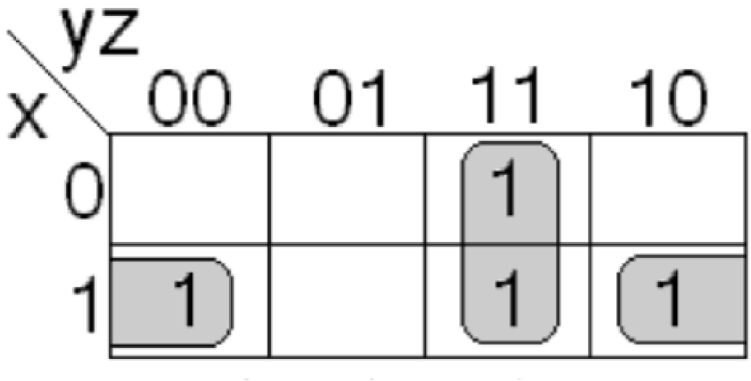
The basic property proposed by the adjacent squares that only two adjacent squares in the map differ by only one variable which is primed in one square and unprimed in the other. For example m6 and m7 hence, the sum of two minterms in the adjacent square can be simplified to a simple AND term consisting of only two literals

m6+m7 = xyz' + xyz = xy (z' + z) = xy . 1 = xy

EXAMPLE:

Simplify the following Boolean Function by using Karnaugh Map:
F(x,y, z) = ∑ (3, 4, 6, 7)

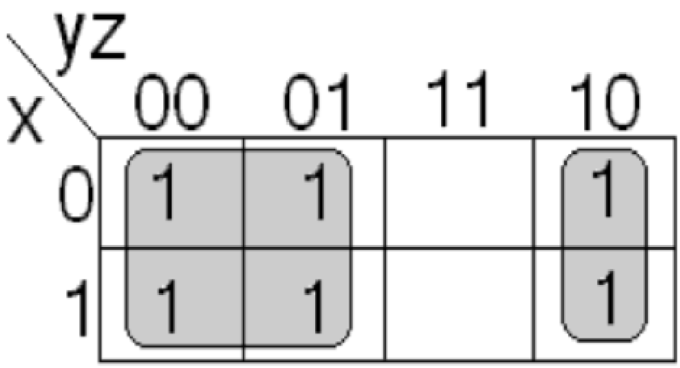
F(x,y) = xz' + yz



EXAMPLE:

Simplify the following Boolean Function by using Karnaugh Map:
F(x,y, z) = ∑ (0, 1, 2, 4, 5, 6)

F(x,y) = y' + yz'



For 4 Variables W, X, Y, Z: 2^4 = 16 product Terms→ 16 cells.

The combinations of adjacent squares that is useful during the simplification process easily determined for inspection of the 4- variable map.

wx \ yz	00	01	11	10
00	m0 w'x'y'z' 0	m1 w'x'y'z 1	m3 w'x'yz 3	m2 w'x'yz' 2
01	m4 w'xy'z' 4	m5 w'xy'z 5	m7 w'xyz 7	m6 w'xyz' 6
11	m12 wxy'z' 12	m13 wxy'z 13	m15 wxyz 15	m14 wxyz' 14
10	m8 wx'y'z' 8	m9 wx'y'z 9	m11 wx'yz 11	m10 wx'yz' 10

EXAMPLE:

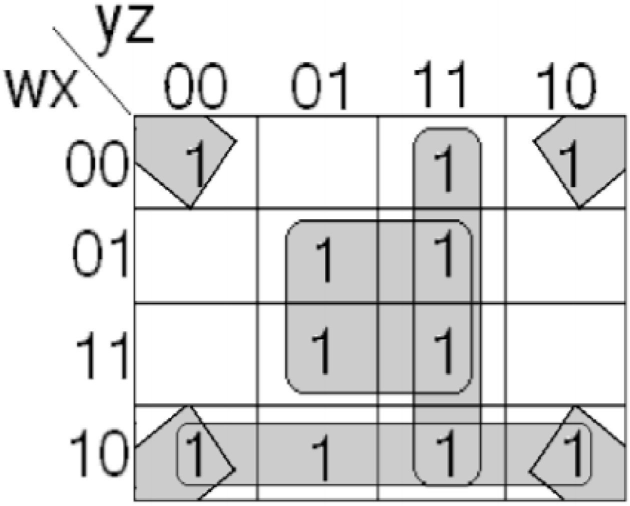
Simplify the following Boolean Function by using Karnaugh Map:

F(w, x,y, z) = ∑ (0,1, 2, 4, 5, 6, 8, 9, 12, 13, 14)

wx \ yz	00	01	11	10
00	1	1		1
01	1	1		1
11	1	1		1
10	1	1		

F(w, x,y, z) = y' + w'z' + xz'

EXAMPLE:
Simplify the following Boolean Function by using Karnaugh Map:
F(w, x,y, z) = ∑ (0, 2, 3, 5, 7, 8, 9, 10, 11, 13, 15)



$F(w,x,y,z) = wx' + yz + xz + x'z'$

Don't care Conditions:

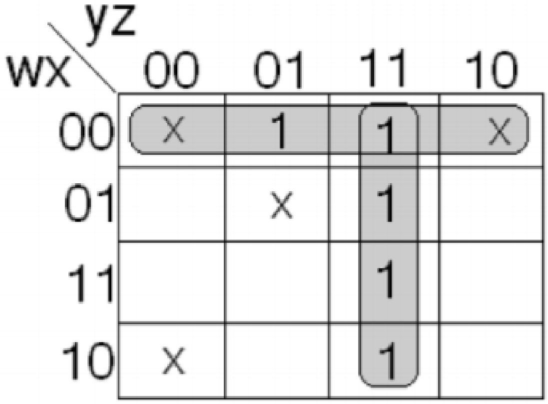
The don't care conditions can be used on a map to provide further simplification of the function. To distinguish the don't care conditions form 1's and o's, an X will be used. When choosing the adjacent square to simplify the function in a map, the X's may be assumed to be either (0 or 1) whichever gives the simple expression.

EXAMPLE

Simplify the following Boolean Function by using Karnaugh Map:

F(w, x,y, z) = ∑ (1, 3, 7, 11, 15)

Which has don't care conditions d(w, x,y, z) = ∑ (0, 2, 5, 8)



$F(w,x,y,z) = w'x' + yz$