Standard Forms: Each term may have any number of variables. Requires fewer operators than canonical forms.

Sum terms: single variable or logical sum of several variables such as (A, B, (x+y, (A+C')).

Product terms: single variable or logical product of several variables such as (x, y, AB', CD').

Note: the expression x+y'z (not sum terms nor product terms)

Sum of Product (SOP):

Is a Boolean expression containing AND terms, called product terms of one or more literals each. The sum denotes the ORing of these terms. An example of a function expressed as a sum of product is

F1= y'+xy+x'yz

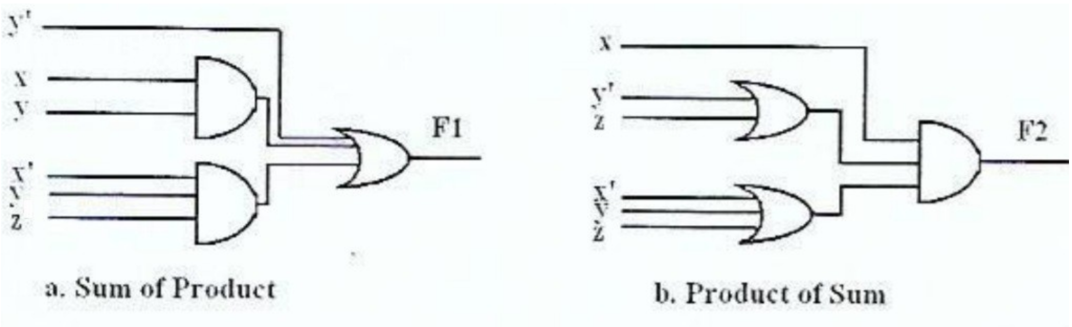
The expression contains three product terms (y' one literal, xy two literals and x'yz' three literals). Their sum is in effect an OR operation. The logic diagram if a sum-of-product expression consist of a group of AND gates followed by a single OR gate.

Products of Sum (POS):

Is a Boolean expression containing OR terms, called sum terms of one or more literals each. The product denotes the ANDing of these terms. An example of a function expressed as a product of sum is

F2=x(y'+z)(x'+y+z)

The expression contains three sum terms (x one literal, y'+z two literals and x'+y+z three literals). The product is an AND operation. The logic diagram if a product-of- sum expression consist of a group of OR gates followed by a single AND gate.



Two-level implementation

Note: a Boolean function may be expressed in nonstandard form.

EXAMPLE: $F=(AB+CD)(A'B'+C'D')$ neither in sum of product nor product of sum. It can be change to the standard form using the distributive law.

$F= ABA'B' + ABC'D' + CDA'B' + CDC'D'$

$= 0 + ABC'D' + CDA'B' + 0$

$F = ABC'D' + CDA'B'$ sum of product.

EXAMPLE: From the given truth table express F as a sum of minterms then simplify as a sum of product .

Solution: the function equal to 1 in (2, 3, 6).

So $F= x'yz'+x'yz+xyz'$ (Sum of Minterms)

Simplification of F:

$F= \underline{x'yz'}+\underline{x'yz}+xyz'$

$= x'y(z'+z)+xyz'$ (distributive law)

$= x'y.1+xyz'$ (complement definition)

$= x'y+xyz'$ (identity element)

$= y(x'+xz')$ (distributive law)

$= y(x'+x)(x'+z')$ (distributive law)

$=y.1.(x'+z')$ (complement definition)

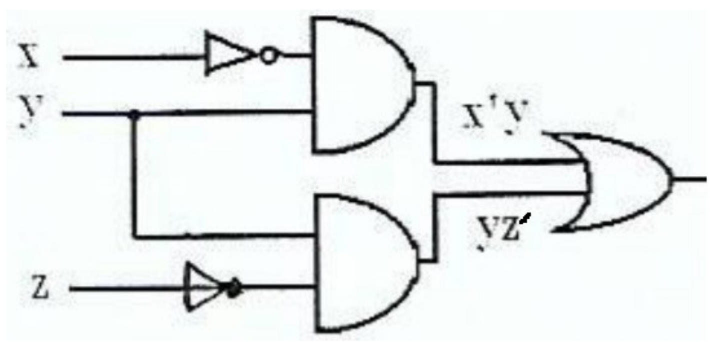
$= y(x'+z')$ (identity element)

$= x'y+yz'$ (distributive law)

Given			
x	y	z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

Logic Circuit of F=

$x'y+yz'$



EXAMPLE: From the given truth table express F as a product of maxterms then simplify as a product of sum.

Solution: the function equal to **0** in (0, 1, 4, 5, 7).

So F=(x+y+z)(x+y+z')(x'+y+z)(x'+y+z')(x'+y'+z')

(Product of Maxterms)

Given			
x	y	z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

EXAMPLE: Express the Boolean function F=A+B'C in a sum of minterms.

Solution: the function F has three variables A, B and C, it is in SOP standard form the first product term (A) missing two variable (B,C); therefore

A= A(B+B') = AB + AB'

This terms still missing one variable C):

A = AB(C+C') + AB'(C+C')

= ABC + ABC' +AB'C +AB'C'

The second term (B'C) missing one variable (A):

B'C = B'C(A+A') = B'CA + B'CA'

rerange the variable alphabetically

B'C = AB'C + A'B'C

Combining all terms we have:

F=A + B'C

= ABC + ABC' + <u>AB'C</u> +AB'C' + <u>AB'C</u> + A'B'C

Since (x+x=x) we can eliminate one of the underlined term

F = A'B'C + AB'C' + AB'C + ABC' + ABC

$= m_1 + m_4 + m_5 + m_6 + m_7$

F(A,B,C) ∑(1, 4, 5, 6, 7)

Conversion to product of maxterms:

To express function in its product of maxtermes form by:

a. Expanding the expression in to product of OR terms (POS) using distributive law.

b. Each OR term is inspected to see if it contains all the variables. If it missing one or more variables, it is ORed with an expression (xx'), where x is one of the missing variable.

EXAMPLE: Express the Boolean function F=xy + x'z in a product of maxterms.

First: convert the function into OR terms (POS) by using distributive law:

F= (xy + x')(xy + z)

= (x' + x)(x' + y)(z + x)(z + y)

F= (x' + y)(z + x)(z + y)

The function has three variables x, y and z. each OR term is missing one variable; therefore:

(x'+y) = (x'+y) +zz' = (x'+y+z) (x'+y+z')

(z + x) = (z + x) + yy' = (z + x +y) (z + x +y') = (x+y+z) (x+y'+z)

(z + y) = (z + y) + xx' = (z + y + x) (z + y + x') = (x+y+z) (x'+y+z)

Combining all terms and removing all those that appear more than once, we finally obtain:

$F = (x+y+z)\ (x+y'+z)\ (x'+y+z)\ (x'+y+z')$

$= M_0, M_2, M_4, M_5$

$F(x, y, z) = \prod (0, 2, 4, 5)$

Conversions between Canonical forms:

To convert from on canonical form to another: interchanging the symbols $\prod$ and $\sum$ then list those numbers missing in the original form. In order to find the missing terms, one must realize the total number of minterms and maxterms is 2^n, where n is the number of binary variables in the function.

EXAMPLE:

Convert the Boolean function $F(x, y, z) = \sum(0, 2, 4, 5)$ to the other canonical form.

The number of variables is three (x,y, z) therefore the total numbers is in range $(0....2^3 - 1) = (0...7)$ therefore, $F(x,y,z) = \prod (1, 3, 6, 7)$