Complement of a Function:

The complement of any function F is $\overline{F}$ or F' its value can be obtained by interchange the (1's to 0's) and (0's to 1's) in the value of F. The complement of a function may be obtained algebraically through Demerger's theory.

$(A + B + C + D)' = A'B'C'D'$

$(ABCD)' = A' + B' + C' + D'$

EXAMPLE1:

Find F' of $F1 = (x yz')$ and $F2 = (x + y'z)$

Solution:

$\overline{F1} = (xyz')' = x' + y' + z'' = x' + y' + z$

$\overline{F2} = (x + y'z)' = x'.(y'z)' = x'.(y'' + z') = x'.(y + z')$

EXAMPLE2:

Find the complement of the following function: $F1 = x'yz' + x'y'z$

Solution:

$\overline{F1} = (x'yz' + x'y'z)'$

$= (x'yz')' . (x'y'z)'$

$= (x+y'+z) . (x+y+z')$

EXAMPLE3:

Find the complement of the following function:

$F1 = (x+y'+z').(x'+y+z).(x'+y'+z')$

Solution:

$\overline{F1} = ((x+y'+z').(x'+y+z).(x'+y'+z'))'$

$=(x+y'+z')' + (x'+y+z)' + (x'+y'+z')'$

$= (x'yz) + (xy'z') + (xyz)$

Canonical and Standard Forms:

A Canonical forms (Sum of Minterms or Product of Maxterms) are used to obtain the function from the given truth table.
In general, the unique algebraic expression for any Boolean function can be obtained from its truth table by using an OR operator to combined all minterms for which the function is equal to 1.

A minterm, denoted as m_i, where $0 \leq i < 2^n$, is a product (AND) of the **n** variables (**that includes all input variables**) in which each variable is complemented if the value assigned to it is 0, and uncomplemented if it is 1.

1-minterms = minterms for which the function F = 1.

0-minterms = minterms for which the function F = 0.

A **maxterm**, denoted as M_i, where $0 \leq i < 2^n$, is a sum (OR) of the **n** variables (literals) in which each variable is complemented if the value assigned to it is 1, and uncomplemented if it is 0.

Literal: variable or its complement : ***A*** , ***A'*** , ***B*** , ***B'*** , ***C*** , ***C'***
1-maxterms = maxterms for which the function F = 1.
0-maxterms = maxterms for which the function F = 0.

			Minterms		Maxterms	
X	Y	Z	Term	Deaignation	Term	Deaignation
0	0	0	x'y'z'	m0	x+y+z	M0
0	0	1	x'y'z	m1	x+y+z'	M1
0	1	0	x'yz'	m2	x+y'+z	M2
0	1	1	x'yz	m3	x+y'+z'	M3
1	0	0	xy'z'	m4	x'+y+z	M4
1	0	1	xy'z	m5	x'+y+z'	M5
1	1	0	xyz'	m6	x'+y'+z	M6
1	1	1	xyz	m7	x'+y'+z'	M7

Sum of Minterms:

Any Boolean function can be expressed as a sum (OR) of its **1-minterms**.
A shorthand notation: **F**(list of variables) = Σ(list of 1-minterm indices)

EXAMPLE: From the given truth table express F, F' as a sum of minterm.

x	y	z	Minterms	F	F'
0	0	0	$m_0=x'\,y'\,z'$	0	1
0	0	1	$m_1=x'\,y'\,z$	0	1
0	1	0	$m_2=x'\,y\,z'$	0	1
0	1	1	$m_3=x'\,y\,z$	1	0
1	0	0	$m_4=x\,y'\,z'$	0	1
1	0	1	$m_5=x\,y'\,z$	1	0
1	1	0	$m_6=x\,y\,z'$	1	0
1	1	1	$m_7=x\,y\,z$	1	0

$F = x'\,y\,z + x\,y'\,z + x\,y\,z' + x\,y\,z$
$= m_3 + m_5 + m_6 + m_7$

or
$F(x, y, z) = \Sigma(3, 5, 6, 7)$

-The inverse of the function can be expressed as a sum (OR) of its **0- minterms**. A shorthand notation: **F'**(list of variables) = **Σ** (list of 0-minterm indices)

$F' = x'\,y'\,z' + x'\,y'\,z + x'\,y\,z' + x\,y'\,z'$
$= m_0 + m_1 + m_2 + m_4$

Or
$F'(x, y, z) = \Sigma(0, 1, 2, 4)$

Products of Maxterms:

Any Boolean function can be expressed as a product (AND) of its **0-maxterms**. A shorthand notation: **F**(list of variables) = **Π**(list of 0-maxterm indices)

EXAMPLE: From the given truth table express F, F' as a product of maxterms.

x	y	z	Maxterms	F	F'
0	0	0	$M_0 = x + y + z$	0	1
0	0	1	$M_1 = x + y + z'$	0	1
0	1	0	$M_2 = x + y' + z$	0	1
0	1	1	$M_3 = x + y' + z'$	1	0
1	0	0	$M_4 = x' + y + z$	0	1
1	0	1	$M_5 = x' + y + z'$	1	0
1	1	0	$M_6 = x' + y' + z$	1	0
1	1	1	$M_7 = x' + y' + z'$	1	0

$F = (x+y+z) \bullet (x+y+z') \bullet (x+y'+z) \bullet (x'+y+z)$

$= M_0 \bullet M_1 \bullet M_2 \bullet M_4$

Or

$F(x, y, z) = \Pi(0, 1, 2, 4)$

-The inverse of the function can be expressed as a product (AND) of its **1-maxterms**. A shorthand notation: **F'**(list of variables) = **Π**(list of 1-maxterm indices).

$F' = (x+y'+z') \bullet (x'+y+z') \bullet (x'+y'+z) \bullet (x'+y'+z')$

$= M_3 \bullet M_5 \bullet M_6 \bullet M_7$

Or

$F'(x, y, z) = \Pi(3, 5, 6, 7)$