Flow Through Saturated Soil

Lecture 5

11.0 WELLS

Wells form the most important mode of groundwater extraction from an aquifer. While wells are used in a number of different applications, they find extensive use in water supply and irrigation engineering practice.

Consider the water in an unconfined aquifer being pumped at a constant rate from a well. Prior to the pumping, the water level in the well indicates the static water table. A lowering of this water level takes place on pumping. If the aquifer is homogeneous and isotropic and the water table horizontal initially, due to the radial flow into the well through the aquifer the water table assumes a conical shape called *cone of depression*. The drop in the water table elevation at any point from its previous static level is called *drawdown*. The areal extent of the cone of depression is called *area of influence* and its radial extent *radius of influence* (Fig. 2.14). At constant rate of pumping, the drawdown curve develops gradually with time due to the withdrawal of water from storage. This phase is called an *unsteady flow* as the water table elevation at a given location near the well changes with time. On prolonged pumping, an equilibrium state is reached between the rate of pumping and the rate of inflow of groundwater from the outer edges of the zone of influence. The drawdown surface attains a constant position with respect to time when the well is known to operate under *steady-flow* conditions. As soon as the pumping is stopped, the depleted storage in the cone of depression is made good by groundwater inflow into the zone of influence. There is a gradual accumulation of storage till the original (static) level is reached. This stage is called *recuperation* or *recovery* and is an unsteady phenomenon. Recuperation time depends upon the aquifer characteristics.

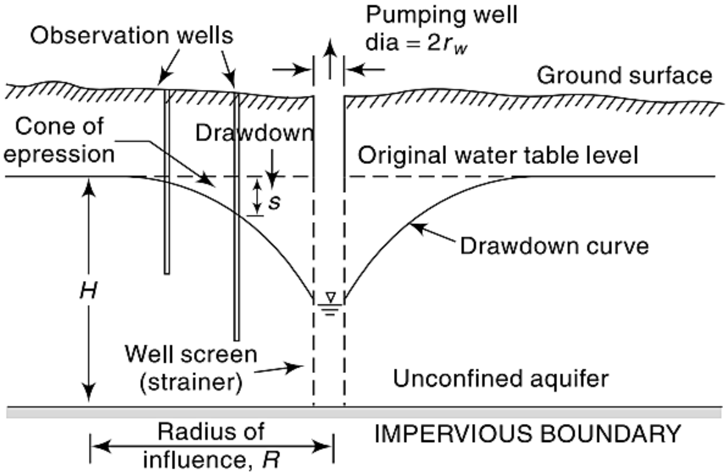

Fig. 2.14 : Well Operating in an Unconfined Aquifer, (definition sketch)

Changes similar to the above take place to a pumping well in a confined aquifer also but with the difference that it is the piezometric surface instead of the water table that undergoes drawdown with the development of the cone of depression. In confined aquifers with considerable piezometric head, the recovery into the well takes place at a very rapid rate.

11.1 Steady Flow Into A Well

Steady state groundwater problems are relatively simpler. Expressions for the steady state radial flow into a well under both confined and unconfined aquifer conditions are presented below.

Confined Flow

Figure 2.15 shows a well completely penetrating a horizontal confined aquifer of thickness B. Consider the well to be discharging a steady flow, Q. The original piezometric head (static head) was H and the drawdown due to pumping is indicated in Fig. 2.15. The piezometric head at the pumping well is h_w and the drawdown s_w.

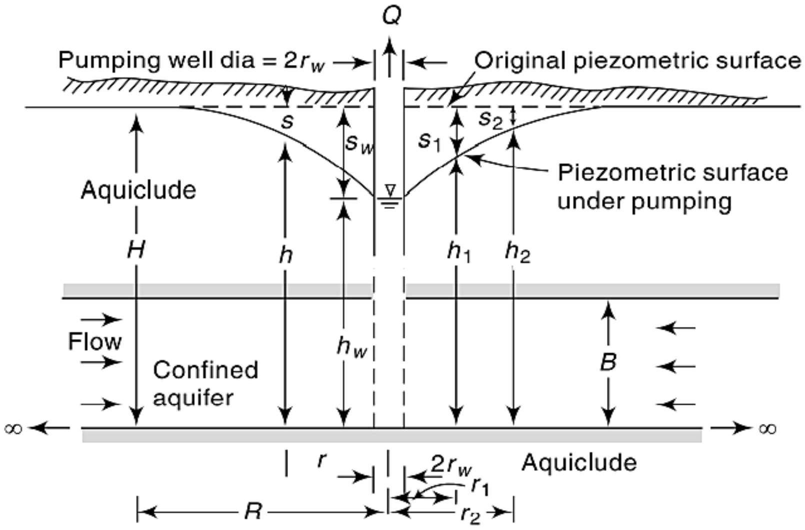

Fig. 2.15 : Well Operating in a Confined Aquifer

At a radial distance r from the well, if h is the piezometric head, the velocity of flow by Darcy's law is

$$V_r = K\frac{dh}{dr}$$

The cylindrical surface through which this velocity occurs is $2\pi r B$. Hence by equating the discharge entering this surface to the well discharge,

$$Q = (2\,\pi r\,B)\left(K\frac{dh}{dr}\right) \qquad \frac{Q}{2\,\pi\,KB}\frac{dr}{r} = dh$$

Integrating between limits r_1 and r_2 with the corresponding piezometric heads being h_1 and h_2 respectively,

$$\frac{Q}{2\,\pi\,KB}\ln\frac{r_2}{r_1} = (h_2 - h_1)$$

or $$Q = \frac{2\,\pi\,KB\,(h_2 - h_1)}{\ln\frac{r_2}{r_1}} \qquad \text{-----------------(2.46)}$$

This is the equilibrium equation for the steady flow in a confined aquifer. This equation is popularly known as *Thiem's equation.*

If the drawdown s_1 and s_2 at the observation wells are known, then by noting that

$$s_1 = H - h_1,\ s_2 = H - h_2 \text{ and } KB = T$$

Equation (2.46) will read as

$$Q = \frac{2\pi T(s_1 - s_2)}{\ln \frac{r_2}{r_1}} \qquad \text{------------ (2.47)}$$

Further, at the edge of the zone of influence, $s = 0$, $r_2 = R$ and $h_2 = H$; at the well wall $r_1 = r_w$, $h_1 = h_w$ and $s_1 = s_w$. Equation (2.47) would then be

$$Q = \frac{2\pi T s_w}{\ln R/r_w} \qquad \text{------------------ (2.48)}$$

Equation (2.47) or (2.48) can be used to estimate T, and hence K, from pumping tests. For the use of the equilibrium equation, Eq. (2.46) or its alternative forms, it is necessary that the assumption of complete penetration of the well into the aquifer and steady state of flow are satisfied.

EXAMPLE : *A 30-cm diameter well completely penetrates a confined aquifer of permeability 45 m/day. The length of the strainer is 20 m. Under steady state of pumping the drawdown at the well was found to be 3.0 m and the radius of influence was 300 m. Calculate the discharge.*

SOLUTION: In this problem, referring to Fig. 2.15 ,

$r_w = 0.15$ m $\quad R = 300$ m $\quad s_w = 3.0$ m $\quad B = 20$ m

$K = 45/(60 \times 60 \times 24) = 5.208 \times 10^{-4}$ m/s

$T = KB = 10.416 \times 10^{-3}$ m^2/s

By Eq. (2.48) :

$$Q = \frac{2\pi T s_w}{\ln R/r_w} = \frac{2\pi \times 10.416 \times 10^{-3} \times 3}{\ln \frac{300}{0.15}} = 0.02583 \text{ m}^3\text{/s} = 25.83 \text{ lps} = 1550 \text{ lpm}$$

UNCONFINED FLOW

Consider a steady flow from a well completely penetrating an unconfined aquifer. In this case because of the presence of a curved free surface, the streamlines are not strictly radial straight lines. While a streamline at the free surface will be curved, the one at the bottom of the aquifer will be a horizontal line, both converging to the well. To obtain a simple solution *Dupit's assumptions* as indicated in Sec. 10.0are made. In the present case these are:

- For small inclinations of the free surface, the streamlines can be assumed to be horizontal and the equipotentials are thus vertical.
- The hydraulic gradient is equal to the slope of the free surface and does not vary with depth. This assumption is satisfactory in most of the flow regions except in the immediate neighbourhood of the well.

Consider the well of radius r_w penetrating completely an extensive unconfined horizontal aquifer as shown in Fig. 2.16 . The well is pumping a discharge Q. At any radial distance r, the velocity of radial flow into the well is

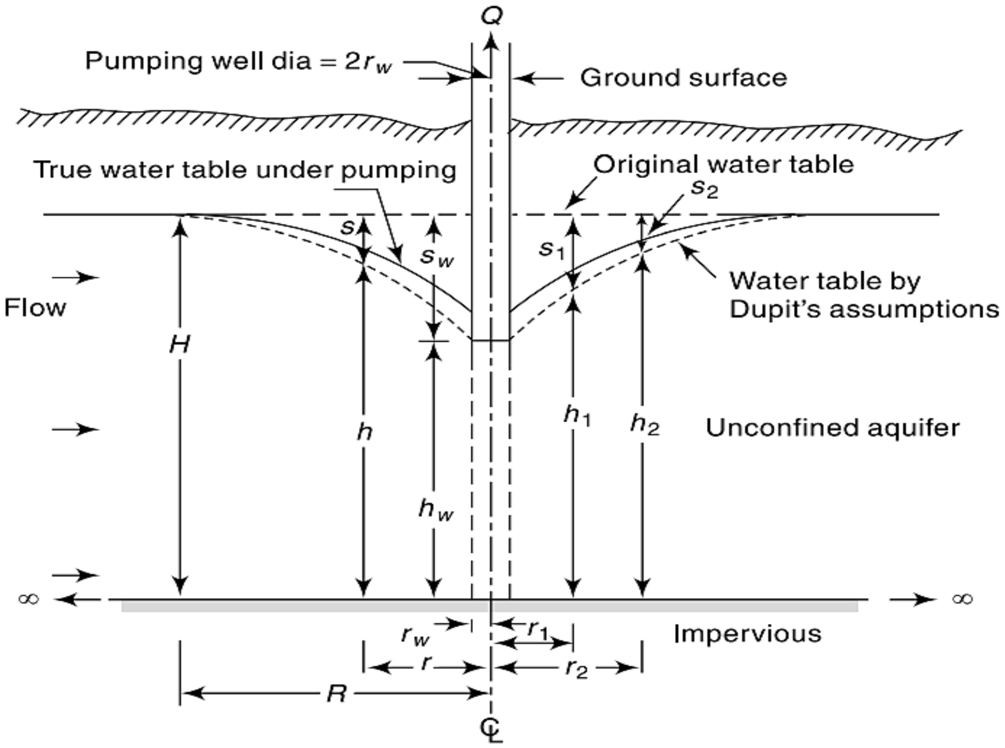

Fig. 2.16 : Radial Flow to a Well in an Unconfined Aquifer

$$V_r = K\frac{dh}{dr}$$

where h is the height of the water table above the aquifer bed at that location. For steady flow, by continuity

$$Q = (2\,\pi r\,h)V_r = 2\,\pi r\,K\,h\,\frac{dh}{dr}$$

or $$\frac{Q}{2\,\pi K}\frac{dr}{r} = h\,dh$$

Integrating between limits r_1 and r_2 where the water-table depths are h_1 and h_2 respectively and on rearranging

$$Q = \frac{\pi K\,(h_2^2 - h_1^2)}{\ln\frac{r_2}{r_1}} \qquad \text{------------------ (2.49)}$$

This is the equilibrium equation for a well in an unconfined aquifer. As at the edge of the zone of influence of radius R, H = saturated thickness of the aquifer, Eq. (2.49) can be written as

$$Q = \frac{\pi K\,(H^2 - h_w^2)}{\ln\frac{R}{r_w}} \qquad \text{--------------------- (2.50)}$$

where h_w = depth of water in the pumping well of radius r_w.

Equations (2.49) and (2.50) can be used to estimate satisfactorily the discharge and permeability of the aquifer by using field data. Calculations of the water-table profile by Eq. (2.49), however, will not be accurate near the well because of Dupit's assumptions. The water-table surface calculated by Eq. (2.49) which involved Dupit's assumption will be lower than the actual surface. The departure will be appreciable in the immediate neighbourhood of the well (Fig. 2.16). In general, values of R in the range 300 to 500 m can be assumed depending on the type of aquifer and operating conditions of a well. As the logarithm of R is used in the calculation of discharge, a small error in R will not seriously affect the estimation of Q. It should be noted that it takes a relatively long time of pumping to achieve a steady state in a well in an unconfined aquifer. The recovery after the cessation of pumping is also slow compared to the response of an artesian well which is relatively fast.

Approximate Equations If the drawdown at the pumping well $s_w = (H - h_w)$ is small relative to H, then

$$H^2 - h_w^2 = (H + h_w)(H - h_w) \approx 2\,H\,s_w$$

Noting that $T = KH$, Eq. (2.50) can be written as

$$Q = \frac{2\pi T\, s_w}{\ln \dfrac{R}{r_w}} \qquad \text{------------------ (2.50 a)}$$

which is the same as Eq. (2.48). Similarly Eq. (2.49) can be written in terms of $s_1 = (H - h_1)$ and $s_2 = (H - h_2)$ as

$$Q = \frac{2\pi T (s_1 - s_2)}{\ln \dfrac{r_2}{r_1}} \qquad \text{--------------- (2.49 a)}$$

Equations (2.49 a) and (2.50 a) are approximate equations to be used only when Eq. (2.49) or (2.50) cannot be used for lack of data. Equation (2.50 a) over estimates the discharge by $[1/2\ (H/s_w - 1)]$ % when compared to Eq. (2.50).

EXAMPLE : *A 30-cm well completely penetrates an unconfined aquifer of saturated depth 40 m. After a long period of pumping at a steady rate of 1500 lpm, the drawdown in two observation wells 25 and 75 m from the pumping well were found to be 3.5 and 2.0 m respectively. Determine the transmissivity of the aquifer. What is the drawdown at the pumping well?*

Solution:

(a) $$Q = \frac{1500 \times 10^{-3}}{60} = 0.025\ \text{m}^3/\text{s}$$

$h_2 = 40.0 - 2.0 = 38.0$ $\qquad r_2 = 75$ m

$h_1 = 40.0 - 3.5 = 36.5$ m $\qquad r_1 = 25$ m

From Eq. (2.49),

$$Q = \frac{\pi K (h_2^2 - h_1^2)}{\ln \dfrac{r_2}{r_1}}$$

$$0.025 = \frac{\pi K [(38)^2 - (36.5)^2]}{\ln \dfrac{75}{25}}$$

$$K = 7.823 \times 10^{-5}\ \text{m/s}$$

$$T = KH = 7.823 \times 10^{-5} \times 40 = 3.13 \times 10^{-3}\ \text{m}^2/\text{s}$$

(b) At the pumping well, $r_w = 0.15$ m

$$Q = \frac{\pi K (H_1^2 - h_w^2)}{\ln \frac{r_1}{r_w}}$$

$$0.025 = \frac{\pi \times 7.823 \times 10^{-5} [(36.5)^2 - h_w^2]}{\ln \frac{25}{0.15}}$$

$$h_w^2 = 811.84 \quad \text{and} \quad h_w = 28.49 \text{ m}$$

Drawdown at the well, $s_w = 11.51$m

11.2 Open Wells

Open wells (also known as *dug wells*) are extensively used for drinking water supply in rural communities and in small farming operations. They are best suited for shallow and low yielding aquifers. In hard rocks the cross sections are circular or rectangular in shape. They are generally sunk to a depth of about 10 m and are lined wherever loose over burden is encountered. The flow into the well is through joints, fissures and such other openings and is usually at the bottom/lower portions of the well. In unconsolidated formations (e.g. alluvial soils) the wells are usually dug to a depth of about 10 m below water table, circular in cross section and lined. The water entry into these wells is from the bottom. These wells tap water in unconfined aquifers.

When the water in an open well is pumped out, the water level inside the well is lowered. The difference in the water table elevation and the water level inside the well is known as *depression head*. The flow discharge into the well (Q) is proportional to the depression head (H), and is expressed as

$$Q = K_0 H \qquad \text{---------------- (2.51)}$$

where the proportionality constant K_0 depends on the characteristic of the aquifer and the area of the well. Also, since K_0 represents discharge per unit drawdown it is called as *specific capacity* of the well. There is a *critical depression head* for a well beyond which any higher depression head would cause dislodging of soil particles by the high flow velocities. The discharge corresponding to the critical head is called as *critical or maximum yield*. Allowing a factor of safety (normally 2.5 to 3.0) a *working head* is specified and the corresponding yield from the well is known as *safe yield*.

RECUPERATION TEST The specific capacity K_0 of a well is determined from the recuperation test described below.

Let the well be pumped at a constant rate Q till a drawdown H_1 is obtained. The pump is now stopped and the well is allowed to recuperate. The water depth in the well is measured at various time intervals t starting from the stopping of the well. Referring to Fig. 2.17 ,

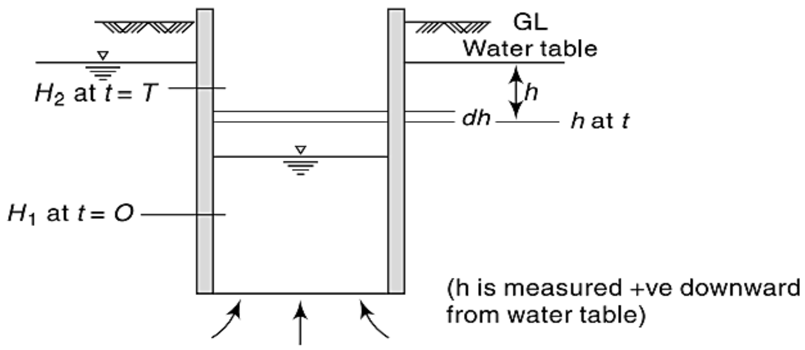

Fig. 2.17: Recuperation Test for Open well

H_1 = drawdown at the start of recuperation, $t = 0$

H_2 = drawdown at a time, $t = T_r$

h = drawdown at any time t

Δh = decrease in drawdown on time Δt

At any time t, the flow into the well $Q = K_0 h$

In a time interval Δt causing a small change Δh in the water level,

$$Q \cdot \Delta t = K_0 h \cdot \Delta t = -A \cdot \Delta h$$

where A is the area of the well. In differential form

$$dt = -\frac{A}{K_0}\frac{dh}{h}$$

Integrating for a time interval T_r

$$\int_0^{T_r} dt = -\frac{A}{K_0}\int_{H_1}^{H_2}\frac{dh}{h}$$

$$T_r = \frac{A}{K_0}\ln\frac{H_1}{H_2} \qquad \text{------------- (2.52)}$$

or

$$\frac{K_0}{A} = \frac{1}{T_r}\ln\frac{H_1}{H_2} \qquad \text{------------- (2.52 a)}$$

The term $\frac{K_0}{A} = K_s$ represents *specific capacity per unit well area* of the aquifer and is essentially a property of the aquifer. Knowing H_1, H_2 and the recuperation time T_r for reaching H_2 from H_1, and the specific capacity per unit well area is calculated by Eq. (2.52 a).

Usually the K_s of an aquifer, determined by recuperation tests on one or more wells, is used in designing further dug wells in that aquifer. However, when such information is not available the following approximate values of K_s, given by Marriot, are often used.

Type of sub-soil	**Value of K_s in units of h^{-1}**
Clay	0.25
Fine sand	0.50
Coarse sand	1.00

The yield Q from an open well under a depression head H is obtained as

$$Q = K_s AH \qquad \text{------------ (2.51 a)}$$

For dug wells with masonry sidewalls, it is usual to assume the flow is entirely from the bottom and as such A in Eq. (2.51a) represents the bottom area of the well.

EXAMPLE : *During the recuperation test of a 4.0 m open well a recuperation of the depression head from 2.5 m to 1.25 m was found to take place in 90 minutes. Determine the (i) specific capacity per unit well area and (ii) yield of the well for a safe drawdown of 2.5 m (iii) What would be the yield from a well of 5.0 m diameter for a drawdown of 2.25 m?*

SOLUTION: $A = \frac{\pi}{4} \times (4.0)^2 = 12.566 \text{ m}^2$

From Eq. (2.52a), $\frac{K_0}{A} = \frac{1}{T_r} \ln \frac{H_1}{H_2}$

Here $T_r = 90$ min $= 1.50$ h, $H_1 = 2.5$ m, and $H_2 = 1.25$ m

(i) $K_s = \frac{K_0}{A} = \frac{1}{1.5} \ln \frac{2.5}{1.25} = 0.462 \; h^{-1}$

(ii) $Q = K_s \cdot A \cdot H = 0.462 \times 12.566 \times 2.5 = 14.52 \text{ m}^3\text{/h}$

(iii) $A_2 = \frac{\pi}{4} \times (5.0)^2 = 19.635$

$Q = K_s \times A_2 \times H_2 = 0.462 \times 19.635 \times 2.25 = 20.415 \text{ m}^3\text{/h}$

11.3 Unsteady Flow In A Confined Aquifer

When a well in a confined aquifer starts discharging, the water from the aquifer is released resulting in the formation of a cone of depression of the piezometric surface. This cone gradually expands with time till an equilibrium is attained. The flow configuration from the start of pumping till the attainment of equilibrium is in unsteady regime and is described by Eq. (2.26) .

In polar coordinates, Eq. (2.26), to represent the radial flow into a well, takes the form

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t} \qquad \text{---------- (2.53)}$$

Making the same assumptions as used in the derivation of the equilibrium formula (Eq. 2.46), Thies (1935) obtained the solution of this equation as

$$s = (H - h) = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-u}}{u}\,du \qquad \text{---------- (2.54)}$$

where $s = H - h$ = drawdown at a point distance r from the pumping well, H = initial constant piezometric head, Q = constant rate of discharge, T = transmissibility of the aquifer, u = a parameter = $r^2 S/4Tt$, S = storage coefficient and t = time from start of pumping. The integral on the right hand side is called the *well function,* $W(u)$, and is given by

$$W(u) = \int_u^\infty \frac{e^{-u}}{u}\,du = -0.577216 - \ln u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!}\ldots \qquad \text{------- (2.55)}$$

Table of $W(u)$ are available in literature (e.g. Refs. 1, 9 and 10). Values of $W(u)$ can be easily calculated by the series (Eq. 2.55) to the required number of significant digits which rarely exceed 4. For small values of $u(u \leq 0.01)$, only the first two terms of the series are adequate.

For small values of u ($u \leq 0.01$), Jacob (1946, 1950) showed that the calculations can be considerably simplified by considering only the first two terms of the series of $W(u)$, (Eq. 2.55). This assumption leads Eq. (2.54) to be expressed as

$$s = \frac{Q}{4\pi T}\left[-0.5772 - \ln\frac{r^2 S}{4Tt}\right]$$

i.e.
$$s = \frac{Q}{4\pi T}\ln\left[\frac{2.2\,Tt}{r^2 S}\right] \qquad \text{---------- (2.56)}$$

If s_1 and s_2 are drawdowns at times t_1 and t_2,

$$(s_2 - s_1) = \frac{Q}{4\pi T}\ln\frac{t_2}{t_1} \qquad \text{---------- (2.57)}$$

If the drawdown s is plotted against time t on a semi-log paper, the plot will be a straight line for large values of time. The slope of this line enables the storage coefficient S to be determined. From Eq. (2.54), when $s = 0$,

$$\frac{2.25\,T\,t_0}{r^2 S} = 1$$

or
$$S = \frac{2.25\,T\,t_0}{r^2} \qquad \text{----------- (2.58)}$$

in which t_0 = time corresponding to "zero" drawdown obtained by extrapolating the straight-line portion of the semi-log curve of s *vs* t ----------. It is important to remember that the above approximate method proposed by Jacob assumes u to be very small.

DRAWDOWN TEST Equations (2.56 and 2.57) relating drawdown s with time t and aquifer properties is used to evaluate formation constants S and T through pumping test. The method is known as drawdown test.

Procedure: An observation well at a distance r from the production well is selected. The pumping is started and the discharge is maintained at a constant value (Q) throughout the test. Values of the drawdown s are read at the observation well at various times, t. The time intervals between successive readings could progressively increase to cut down on the number of observations. The pumping is continued till nearly steady state conditions are reached. This may take about 12 to 36 hours depending on the aquifer characteristics. The best values of S and T are obtained from Eqs. 2.56 and 2.57 through semi-log plot of s against time t.

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