Flow Through Saturated Soil

Lecture 2

7.0 STRATIFICATION

Sometimes the aquifers may be stratified, with different permeabilities in each strata. Two kinds of flow situations are possible in such a case.

(i) When the flow is parallel to the stratification as in Fig. 2.5 (a) equivalent permeability K_e of the entire aquifer of thickness $B = \sum_1^n B_i$ is

$$K_e = \frac{\sum_1^n K_i B_i}{\sum_1^n B_i} \quad \text{------------(2.10)}$$

The transmissivity of the formation is

$$T = K_e \Sigma B_i = \sum_1^n K_i B_i$$

(ii) When the flow is normal to the stratification as in Fig. 2.5 (b) the equivalent permeability K_e of the aquifer of length

$$L = \sum_1^n L_i \quad \text{is}$$

$$K_e = \frac{\sum_1^n L_i}{\sum_1^n (L_i / K_i)} \quad \text{------------(2.11)}$$

(Note that in this case L is the length of seepage and the thickness B of the aquifer does not come into picture in calculating the equivalent permeability)
The transmissivity of the aquifer is $T = K_e \cdot B$

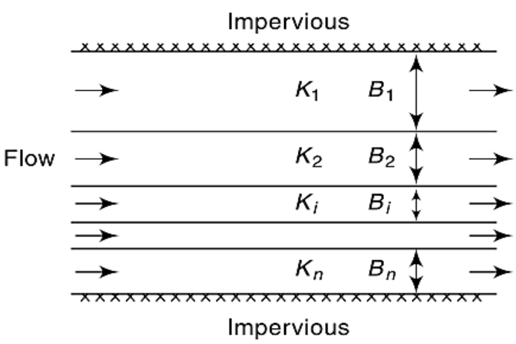

Fig. 2.5(a) Flow Parallel to Stratification

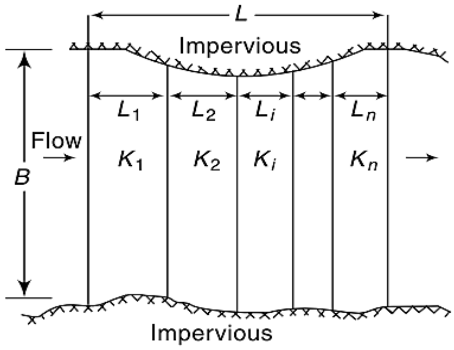

Fig. 2.5(b) Flow Normal to Stratification

EXAMPLE 2.1 : *At a certain point in an unconfined aquifer of 3 km² area, the water table was at an elevation of 102.00 m. Due to natural recharge in a wet season, its level rose to 103.20 m. A volume of 1.5 Mm³ of water was then pumped out of the aquifer causing the water table to reach a level of 101.20 m. Assuming the water table in the entire aquifer to respond in a similar way, estimate (a) the specific yield of the aquifer and (b) the volume of recharge during the wet season.*

SOLUTION:

(a) Volume pumped out = area × drop in water table × specified yield S_y

$$1.5 \times 10^6 = 3 \times 10^6 \times (103.20 - 101.20) \times S_y$$

$$S_y = 0.25$$

(b) Recharge volume $= 0.25 \times (103.20 - 102.00) \times 3 \times 10^6 = 0.9 \text{ Mm}^3$

EXAMPLE 2.2 : *A field test for permeability consists in observing the time required for a tracer to travel between two observation wells. A tracer was found to take 10 h to travel between two wells 50 m apart when the difference in the water-surface elevation in them was 0.5 m. The mean particle size of the aquifer was 2 mm and the porosity of the medium 0.3. If $v = 0.01$ cm²/s estimate (a) the coefficient of permeability and intrinsic permeability of the aquifer and (b) the Reynolds number of the flow.*

SOLUTION:

(a) The tracer records the actual velocity of water

$$V_a = \frac{50 \times 100}{10 \times 60 \times 60} = 0.139 \text{ cm/s}$$

Discharge velocity $V = n\ V_a = 0.3 \times 0.139 = 0.0417$cm/s

Hydraulic gradient $i = \frac{0.50}{50} = 1 \times 10^{-2}$

Coefficient of permeability $K = \frac{4.17 \times 10^{-2}}{1 \times 10^{-2}} = 4.17$ cm/s

Intrinsic permeability, $K_0 = \frac{Kv}{g} = \frac{4.17 \times 0.01}{981} = 4.25 \times 10^{-5}\ \text{cm}^2$

Since $9.87 \times 10^{-9}\ \text{cm}^2 = 1$ darcy

$K_0 = 4307$ darcys

(b) Reynolds number $\mathbf{Re} = \frac{Vd_a}{v}$

Taking d_a = mean particle size = 2 mm

$$\mathbf{Re} = \frac{0.0417 \times 2}{10} \times \frac{1}{0.01} = 0.834.$$

8.0 COMPRESSIBILITY OF AQUIFER

In confined aquifers the total pressure at any point due to overburden is borne by the combined action of the pore pressure and intergranular pressure. The compressibility of the aquifer and also that of the pore water causes a readjustment of these pressures whenever there is a change in storage and thus have an important bearing on the storage characteristics of the aquifer. In this section a relation is developed between a defined storage coefficient and the various compressibility parameters.

Consider an elemental volume $\Delta V = (\Delta x \Delta y)\ \Delta Z = \Delta A\ \Delta Z$ of a compressible aquifer as shown in Fig. 2.7. A cartesian coordinate system with the Z-axis pointing vertically upwards is adopted. Further the following three compressible aquifer assumptions are made:

- The elemental volume is constrained in lateral directions and undergoes change of length in the z-direction only, i.e. ΔA is constant.
- The pore water is compressible
- The solid grains of the aquifer are incompressible but the pore structure is compressible.

By defining the reciprocal of the bulk modulus of elasticity of water as *compressibility of water* β, it is written as

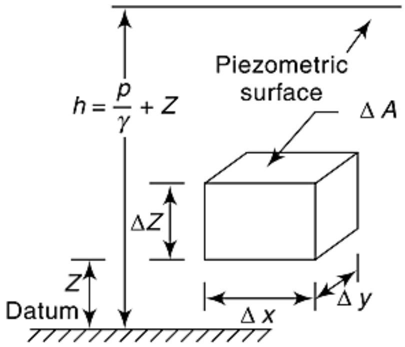

Fig. 2.7 : Volume element of a compressible aquifer

$$\beta = -\frac{(d\,\Delta V_w)}{\Delta V_w}/dp \quad \text{--------(2.12)}$$

where ΔV_w = volume of water in the chosen element of aquifer, and p = pressure. By conservation of mass

$$\rho \cdot \Delta V_w = \text{constant, where } \rho = \text{density of water.}$$

Thus $\rho\, d(\Delta V_w) + \Delta V_w\, d\rho = 0$

Substituting this relationship in Eq. (9.12),

$$\beta = d\rho/(\rho\, dp) \quad \text{--------------------(2.13)}$$

or $d\rho = \rho\beta dp$ -------------------- (2.13a)

Similarly by considering the reciprocal of the bulk modulus of elasticity of the pore-space skeleton as the *compressibility of the pores,* α, it is expressed as

$$\alpha = \frac{d(\Delta V)/\Delta V}{d\sigma_z} \quad \text{-------------- (2.14)}$$

in which σ_z = intergranular pressure. Since $\Delta V = \Delta A \cdot \Delta Z$ with ΔA = constant,

$$\alpha = -\frac{d(\Delta Z)/\Delta Z}{d\sigma_z} \quad \text{------------ (2.15)}$$

The total overburden pressure $w = p + \sigma_z$ = constant.

Thus $dp = -d\,\sigma_z$, which when substituted in Eq.(2.15) gives

$$d(\Delta Z) = \alpha(\Delta Z)\, dp \quad \text{------------------------(2.16)}$$

As the volume of solids ΔV_s in the elemental volume is constant,

$$\Delta V_s = (1-n)\,\Delta A \cdot \Delta Z = \text{constant}$$

$$d(\Delta V_s) = (1-n)\, d(\Delta Z) - \Delta Z \cdot dn = 0$$

where n = porosity of the aquifer. Using this relationship in Eq. (2.16) ,

$$dn = \alpha(1-n)\, dp \quad \text{-----------------------(2.17)}$$

Now, the mass of water in the element of volume ΔV, is

$$\Delta M = \rho n\, \Delta A\, \Delta Z$$

or $$d(\Delta M) = \Delta V\left[n\, d\rho + \rho\, dn + \rho n \frac{d(\Delta Z)}{\Delta Z}\right]$$

i.e. $$\frac{d(\Delta M)}{\rho \Delta V} = n\frac{d\rho}{\rho} + dn + n\frac{d(\Delta Z)}{\Delta Z}$$

Substituting from Eqs (2.13), (2.17) and (2.15) for the terms in the right-hand side respectively

$$\frac{d(\Delta M)}{\rho \Delta V} = n\beta\, dp + \alpha(1-n)\, dp + n\,\alpha\, dp = (n\beta + \alpha)\, dp$$
$$= \gamma(n\beta + \alpha)\, dh = S_s dh \qquad \text{------------------------ (2.18)}$$

where $S_s = \gamma(n\,\beta + \alpha)$ and h = piezometric head $= z + \dfrac{\rho}{\gamma}$ and $\gamma = \rho g$ = weight of unit volume of water.

The term S_s is called *specific storage*. It has the dimensions of $[L^{-1}]$ and represents the volume of water released from storage from a unit volume of aquifer due to a unit decrease in the piezometric head. The numerical value of S_s is very small being of the order of 1×10^{-4} m^{-1}.

By integration of Eq.(2.18) for a confined aquifer of thickness B, a dimensionless *storage coefficient* S can be expressed as

$$S = \gamma(n\beta + \alpha)\, B \qquad \text{--------------- (2.19)}$$

The storage coefficient S (also known as *Storativity)* represents the volume of water released by a column of a confined aquifer of unit cross-sectional area under a unit decrease in the piezometric head. The storage coefficient S and the transmissibility coefficient T are known as the *formation constants* of an aquifer and play very important role in the unsteady flow through the porous media. Typical values of S in confined aquifers lie in the range 5×10^{-5} to 5×10^{-3}. Values of α for some formation material and β for various temperatures are given in Tables 2.3 and 2.4 respectively.

Table 2.3 : Range of α for Some Formation Materials

Material	Bulk modulus of elasticity, E_s (N/cm^2)	Compressibility $\alpha = 1/E_s$ (cm^2/N)
Loose clay	$10^2 - 5 \times 10^2$	$10^{-2} - 2 \times 10^{-3}$
Stiff clay	$10^3 - 10^4$	$10^{-3} - 10^{-4}$
Loose sand	$10^3 - 2 \times 10^3$	$10^{-3} - 5 \times 10^{-4}$
Dense sand	$5 \times 10^3 - 8 \times 10^3$	$2 \times 10^{-4} - 1.25 \times 10^{-4}$
Dense sandy gravel	$10^4 - 2 \times 10^4$	$10^{-4} - 5 \times 10^{-5}$
Fissured and jointed rock	$1.5 \times 10^4 - 3 \times 10^5$	$6.7 \times 10^5 - 3.3 \times 10^{-6}$

Table 2.4 : Values of β for Water at Various Temperatures

Temperature (°C)	Bulk modulus of elasticity, E_w (N/cm^2)	Compressibility $\beta = 1/E_w$ (cm^2/N)
0	2.04×10^5	4.90×10^{-6}
10	2.11×10^5	4.74×10^{-6}
15	2.14×10^5	4.67×10^{-6}
20	2.20×10^5	4.55×10^{-6}
25	2.22×10^5	4.50×10^{-6}
30	2.23×10^5	4.48×10^{-6}
35	2.24×10^5	4.46×10^{-6}

For an unconfined aquifer, the coefficient of storage is given by

$$S = S_y + \gamma(\alpha + n\beta)\, B_s \qquad \text{------------------ (2.20)}$$

where B_s = saturated thickness of the aquifer. However, the second term on the right-hand side is so small relative to S_y that for practical purposes S is considered equal to S_y, i.e. the coefficient of storage is assumed to have the same value as the specific yield for unconfined aquifers.

The elasticity of the aquifer is reflected dramatically in the response of the water levels in the wells drilled in confined aquifers to changes in the atmospheric pressure. Increase in the atmospheric pressure causes an increase in the loading of the aquifer. The change in the pressure is balanced by a partial compression of the water and partial compression of the pore skeleton. An increase in the atmospheric pressure causes a decrease in the water level in the well. Converse is the case with the decrease in pressure. The ratio of the water level change to pressure head change is called *barometric efficiency* (BE) and is given in terms of the compressibility parameters as

$$BE = -\left(\frac{n\beta}{\alpha + n\beta}\right) \qquad \text{------------- (2.21)}$$

The negative sign indicates the opposite nature of the changes in pressure head and water level. Using in Eq (2.21),(Eq. 2.19) $BE = -n\beta/\gamma S B$ and this affords a means of finding S. The barometric efficiency can be expected to be in the range 10–75%. It is apparent that unconfined aquifers have practically no barometric efficiency.

A few other examples of compressibility effects causing water level changes in artesian wells include (i) tidal action in coastal aquifers, (ii) earthquake or underground explosions, and (iii) passing of heavy railway trains.