DESIGN OF MINIMUM SEEPAGE LOSS CANAL SECTIONS WITH DRAINAGE LAYER AT SHALLOW DEPTH

By Prabhata K. Swamee,[1] Govinda C. Mishra,[2] and Bhagu R. Chahar[3]

ABSTRACT: This paper presents an analytical solution for the quantity of seepage from a rectangular canal underlain by a drainage layer at shallow depth. The solution has been obtained using inverse hodograph and conformal mapping. Using the solution for the rectangular canal and the existing analytical solutions for triangular and trapezoidal canals, simplified algebraic equations for computation of seepage loss from these canals, when the drainage layer lies at finite depth, have been presented, which replace the cumbersome evaluation of complex integrals. Using these seepage loss equations and a general uniform flow equation, simplified equations for the design variables of minimum seepage loss sections have been obtained for each of the three canal shapes by applying a nonlinear optimization technique. The optimal design equations along with the tabulated section shape coefficients provide a convenient method for design of the minimum seepage loss section. A step-by-step design procedure for rectangular and trapezoidal canal sections has been presented.

INTRODUCTION

Canals continue to be major conveyance systems for delivering water for irrigation. The seepage loss from irrigation canals constitutes a substantial percentage of the usable water (Rohwer and Stout 1948). According to the Indian Bureau of Standards (IBS) (1980), the loss of water by seepage from unlined canals in India generally varies from 0.3 to 7.0 m^3/s per 10^6 m^2 of wetted surface. The seepage loss from canals is governed by hydraulic conductivity of the subsoils, canal geometry, location of the water table relative to the canal, and several other factors [International Commission on Irrigation and Drainage (ICID) (1967)].

Analytical solutions for seepage from canals in a homogeneous isotropic porous medium of large depth have been given by Vedernikov (Harr 1962) and Morel-Seytoux (1964). Swamee et al. (2000) gave simplified algebraic equations in explicit form to compute the seepage from triangular, rectangular, and trapezoidal canals. However, in most alluvial plains the soil is stratified. In many cases, highly permeable layers of sand and gravel underlie the top low permeable layer of finite depth. In that case the lower layer of sand and gravel acts as a free drainage layer for the top seepage layer. The seepage from a canal running through such stratified strata is much more than that in homogeneous medium of very large depth. The difference in quantity of seepage becomes appreciable when the drainage layer lies at a depth less than twice the depth of water in the canal. Further, the quantity of seepage becomes very large as the drainage layer approaches the bed of the canal. Bruch (1966) and Bruch and Street (1967a,b) obtained an analytical solution for seepage from a triangular canal in a soil layer of finite depth overlying the drainage layer. Muskat (1946) and Vedernikov (Harr 1962) presented an approximate solution for a trapezoidal canal. El Nimr (1963), Hammad (1960), Garg and Chawla (1970), Sharma and Chawla (1979), and Youngs (1986) dealt with different orientations and positions of the draining layer.

An analytical solution for a rectangular canal has not been reported in the literature. In this study, using inverse hodograph and conformal mapping, an analytical solution has been obtained to compute seepage and loci of phreatic lines for a rectangular canal. Analytical solutions for computing seepage from rectangular, triangular, and trapezoidal canals contain complex integrals involving unknown implicit transformation variables. These solutions hence are not convenient in estimating seepage from the existing canals and in designing canals. In the study, these analytical solutions have been simplified and explicit algebraic equations have been obtained to facilitate easy computation of seepage from triangular, rectangular, and trapezoidal canals when a drainage layer lies at shallow/finite depth.

Canals are lined to check the seepage. But canal lining deteriorates with time; hence, significant seepage losses continue to occur from a lined canal (Wachyan and Rushton 1987). Therefore, seepage loss must be considered in the design of a canal section. Bandini (1966) considered seepage loss in the canal design and compared the economy of lined and unlined canals. Preissmann (1957), Ilyinsky and Kacimov (1984), and Kacimov (1992) had investigated optimal shape and optimal dimensions of a canal from a seepage point of view. Swamee et al. (2000) presented a method for the design of minimum seepage loss sections for canals running through a homogeneous medium of large depth. A design method of canals considering seepage, when a drainage layer or an aquifer lies at finite depth, is not available in the literature. In this investigation, using simplified seepage loss equations and the resistance equation for open channel flow (Swamee 1995), explicit design equations and section shape coefficients for minimum seepage loss sections have been obtained for rectangular, triangular, and trapezoidal canal shapes. An equation to compute the seepage from the optimal seepage sections has also been given.

SEEPAGE FROM RECTANGULAR CANAL

The seepage domain for a rectangular canal, underlain by a drainage layer or an aquifer at a depth d (m), is shown in Fig. 1(a). It is assumed that the water table is below the top of the drainage layer; hence, atmospheric pressure prevails at the bottom of the seepage layer. The inverse hodograph dZ/dW [Fig. 1(c)] and the complex potential W [Fig. 1(d)] for the physical flow domain were drawn following the standard steps (Harr 1962; Polubarinova-Kochina 1962). The dZ/dW-plane and W-plane were mapped onto the lower half of an auxiliary ζ-plane [Fig. 1(e)] using the Schwarz-Christoffel conformal transformation.

Mapping of the dZ/dW-plane onto the ζ-plane resulted in

[1]Prof. of Civ. Engrg., Univ. of Roorkee, Roorkee 247 667, India.
[2]Prof., Water Resour. Devel. Training Ctr., Univ. of Roorkee, Roorkee 247 667, India.
[3]Asst. Prof. in Civ. Engrg., MBM Engrg. Coll., JNV Univ., Jodhpur 342 011, India.

Note. Discussion open until March 1, 2002. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on May 30, 2000; revised March 23, 2001. This paper is part of the ***Journal of Irrigation and Drainage Engineering***, Vol. 127, No. 5, September/October, 2001. ©ASCE, ISSN 0733-9634/01/0005-0287–0294/$8.00 + $.50 per page. Paper No. 22218.

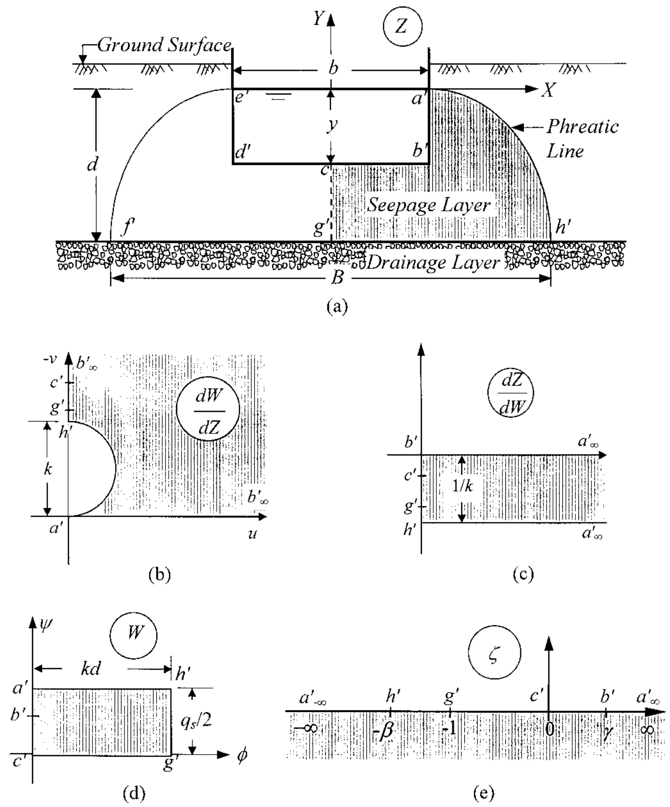

FIG. 1. Seepage from Rectangular Canal Underlain by Drainage Layer: (a) Physical Plane; (b) Hodograph Plane; (c) Inverse Hodograph Plane; (d) Complex Potential Plane; (e) Auxiliary Plane

$$\frac{dZ}{dW} = C_1 \int_0^{\zeta} \frac{dt}{\sqrt{(t-\gamma)(t+\beta)}} + C_2 \tag{1}$$

where β and γ = transformation variables; t = dummy variable; and C_1 and C_2 = constants. The corresponding values at points b' ($\zeta = \gamma$; $dZ/dW = 0$) and h' ($\zeta = -\beta$; $dZ/dW = -i/k$) were used in (1) to find C_1 and C_2. After substituting C_1 and C_2, (1) became

$$\frac{dZ}{dW} = \frac{1}{\pi k} \int_{\gamma}^{\zeta} \frac{dt}{\sqrt{(t-\gamma)(\beta+t)}} \tag{2}$$

The W-plane mapping onto the ζ-plane gave

$$W = C_3 \int_0^{\zeta} \frac{dt}{\sqrt{t(1+t)(\beta+t)}} + C_4 \tag{3}$$

The constants C_3 and C_4 were determined using the values at points c' ($\zeta = 0$; $W = 0$) and g' ($\zeta = -1$; $W = kd$). After substituting C_3 and C_4, (3) was expressed

$$W = i\,\frac{kd\sqrt{\beta}}{2K(1/\sqrt{\beta})} \int_0^{\zeta} \frac{dt}{\sqrt{t(1+t)(\beta+t)}} \tag{4}$$

where $K(1/\sqrt{\beta})$ = complete elliptical integral of the first kind with modulus $1/\sqrt{\beta}$. The elliptical integral of the first kind is defined by Byrd and Friedman (1971) as

$$F(\kappa, \varphi) = \int_0^{\sin\varphi} \frac{dt}{\sqrt{(1-t^2)(1-\kappa^2 t^2)}} = \int_0^{\varphi} \frac{d\tau}{\sqrt{1-\kappa^2\sin^2\tau}}$$

with modulus κ and amplitude φ. When $\varphi = \pi/2$, the integral is said to be complete; i.e., $F(\kappa, \pi/2) = K(\kappa)$.

Using the values at the point h' ($\zeta = -\beta$; $W = kd + iq_s/2$) in (4), and equating the imaginary parts of the resulting expression, led to

$$q_s = 2kd\,\frac{K(\sqrt{(\beta-1)/\beta})}{K(1/\sqrt{\beta})} \tag{5}$$

where q_s = seepage discharge per unit length of the canal (m^2/s).

Because $dZ/d\zeta = (dZ/dW)(dW/d\zeta)$; substitution of dZ/dW from (2) and $dW/d\zeta$ from (4) into it resulted in

$$\frac{dZ}{d\zeta} = \frac{id\sqrt{\beta}}{2\pi K(1/\sqrt{\beta})}\left(\int_{\gamma}^{\zeta} \frac{dt}{\sqrt{(t-\gamma)(\beta+t)}}\right)\frac{1}{\sqrt{\zeta(1+\zeta)(\beta+\zeta)}} \tag{6}$$

Integrating (6) and applying the condition at c' ($\zeta = 0$; $Z = -iy$)

$$Z = -iy + \frac{id\sqrt{\beta}}{2\pi K(1/\sqrt{\beta})}\int_0^{\zeta}\left(\int_{\gamma}^{t}\frac{d\tau}{\sqrt{(\tau-\gamma)(\beta+\tau)}}\right) \cdot \frac{dt}{\sqrt{t(1+t)(\beta+t)}} \tag{7}$$

where τ and t = dummy variables. Eq. (7) defines the physical domain of the seepage flow $a'b'c'g'h'a'$. For example, along the bed of canal $c'b'$ ($0 \le \zeta \le \gamma$), (7) became

$$Z = -iy + \frac{d\sqrt{\beta}}{2K(1/\sqrt{\beta})}\left(\int_0^{\zeta}\frac{dt}{\sqrt{t(1+t)(\beta+t)}} - \frac{2}{\pi}\int_0^{\zeta}\frac{\tan^{-1}\sqrt{(\beta+t)/(\gamma-t)}}{\sqrt{t(1+t)(\beta+t)}}\,dt\right) \tag{8}$$

At the corner of canal b' ($\zeta = \gamma$, $Z = b/2 - iy$); (8) reduced to

$$b = \frac{2d}{K(1/\sqrt{\beta})}\left(F\left(\sqrt{\frac{\beta-1}{\beta}}, \sin^{-1}\sqrt{\frac{\gamma}{1+\gamma}}\right) - \frac{\sqrt{\beta}}{\pi}\int_0^{\gamma}\frac{\tan^{-1}\sqrt{(\beta+t)/(\gamma-t)}}{\sqrt{t(1+t)(\beta+t)}}\,dt\right) \tag{9}$$

where $F(\sqrt{(\beta-1)/\beta}, \sin^{-1}\sqrt{\gamma/(\gamma+1)})$ = incomplete elliptical integral of the first kind with modulus $\sqrt{(\beta-1)/\beta}$ and amplitude $\sin^{-1}\sqrt{\gamma/(\gamma+1)}$.

Similarly, (7) at the points g' ($\zeta = -1$; $Z = -id$) and h' ($\zeta = -\beta$; $Z = -id + B/2$) gave the relations for water depth in the canal y (m) and seepage width at the drainage layer B (m), respectively, as follows:

$$y = \frac{d\sqrt{\beta}}{\pi K(1/\sqrt{\beta})}\int_0^{1}\frac{\tan^{-1}\sqrt{(\beta-t)/(\gamma+t)}}{\sqrt{t(1-t)(\beta-t)}}\,dt \tag{10}$$

$$B = \frac{2d}{K(1/\sqrt{\beta})}\left(K\left(\sqrt{\frac{\beta-1}{\beta}}\right) - \frac{\sqrt{\beta}}{\pi}\int_1^{\beta}\frac{\tan^{-1}\sqrt{(\beta-t)/(\gamma+t)}}{\sqrt{t(t-1)(\beta-t)}}\,dt\right) \tag{11}$$

Finally the equation of the phreatic line $a'h'$ ($-\infty < \zeta < -\beta$) was given by

$$Z = \frac{b}{2} + \frac{id\sqrt{\beta}}{2\pi K(1/\sqrt{\beta})}\int_{-\infty}^{\zeta}\left(\int_{\gamma}^{t}\frac{d\tau}{\sqrt{(\tau-\gamma)(\beta+\tau)}}\right)\frac{dt}{\sqrt{t(1+t)(\beta+t)}} \tag{12}$$

Manipulating further, (12) reduced to

$$Z = \frac{b}{2} + \frac{d\sqrt{\beta}}{2K(1/\sqrt{\beta})}\left(\frac{2}{\pi}\int_0^{-1/\zeta}\frac{\tan^{-1}\sqrt{(1-\beta t)/(1+\gamma t)}}{\sqrt{t(1-t)(1-\beta t)}}\,dt - i\int_0^{-1/\zeta}\frac{dt}{\sqrt{t(1-t)(1-\beta t)}}\right) \tag{13}$$

Simultaneous solution of (5), (9), and (10) gives the seepage discharge from the rectangular canal. Then, the location of the phreatic line can be determined using (13). These equations involve complicated integrals with unknown implicit transformation variables. These integrals were evaluated using Romberg integration (Churchhouse 1981; Press et al. 1992) after converting the improper integrals into proper integrals.

Based on the analytical solution presented above for a rectangular channel and available analytical solutions for triangular and trapezoidal canals, simplified functions for computation of seepage from these canals were obtained using numerical methods.

SIMPLIFIED EXPRESSIONS FOR SEEPAGE LOSS

Rectangular Section

The seepage from a rectangular canal was given by (5), in which β was obtained by simultaneous solution of (9) and (10) in the following form:

$$\frac{d}{y} = \frac{\pi}{\sqrt{\beta}} \frac{K(1/\sqrt{\beta})}{\displaystyle\int_0^1 \frac{\tan^{-1}\sqrt{(\beta - \tau)/(\gamma + \tau)}}{\sqrt{\tau(1 - \tau)(\beta - \tau)}}\, d\tau} \tag{14}$$

$$\frac{b}{y} = \left\{ \left[\frac{\pi}{\sqrt{\beta}} F\left(\sqrt{\frac{\beta - 1}{\beta}}, \sin^{-1}\sqrt{\frac{\gamma}{1 + \gamma}} \right) - \int_0^{\gamma} \frac{\tan^{-1}\sqrt{(\beta + \tau)/(\gamma - \tau)}}{\sqrt{\tau(1 + \tau)(\beta + \tau)}}\, d\tau \right] \Big/ 0.5 \int_0^1 \frac{\tan^{-1}\sqrt{(\beta - \tau)/(\gamma + \tau)}}{\sqrt{\tau(1 - \tau)(\beta - \tau)}}\, d\tau \right\} \tag{15}$$

Using (14) and (15) for a given b/y and d/y, the unknown transformation variables β and γ can be obtained by a trial-and-error procedure. But (14) and (15) are highly implicit nonlinear in β and γ; hence, the trial-and-error method is not convenient and accurate for the present problem. Therefore, for assumed values of β and γ, (14) and (15) were used to find b/y and d/y. Further, substituting β and d/y in (5), the quantity of seepage was obtained. Repeating this process, q_s/ky was obtained for a large number of β ($1 < \beta < \infty$) and γ ($0 < \gamma < \infty$) such that b/y and d/y are in the ranges $0 \leq b/y \leq 1{,}000$ and $1 < d/y \leq 1{,}000$. The resulting values may be used to prepare three set of graphs: (1) q_s/ky versus β for different γ; (2) d/y versus β for different γ; and (3) b/y versus β for different γ. Making use of these graphs, β and γ were eliminated to obtain variation in the seepage loss with bed width for different depths of the drainage layer and plotted in Fig. 2.

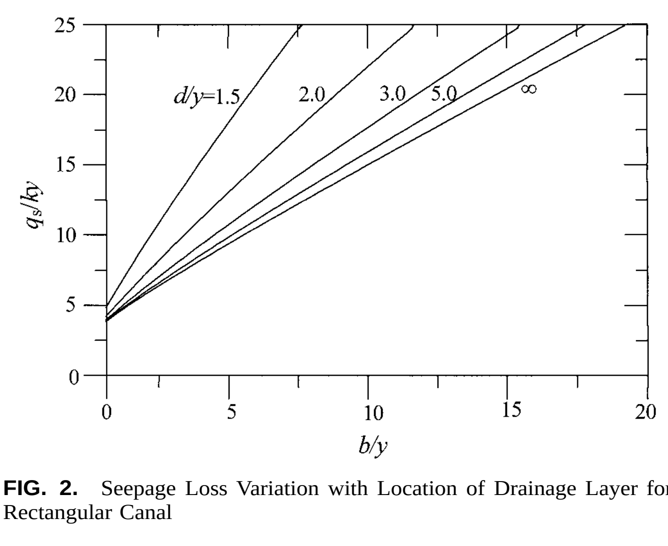

FIG. 2. Seepage Loss Variation with Location of Drainage Layer for Rectangular Canal

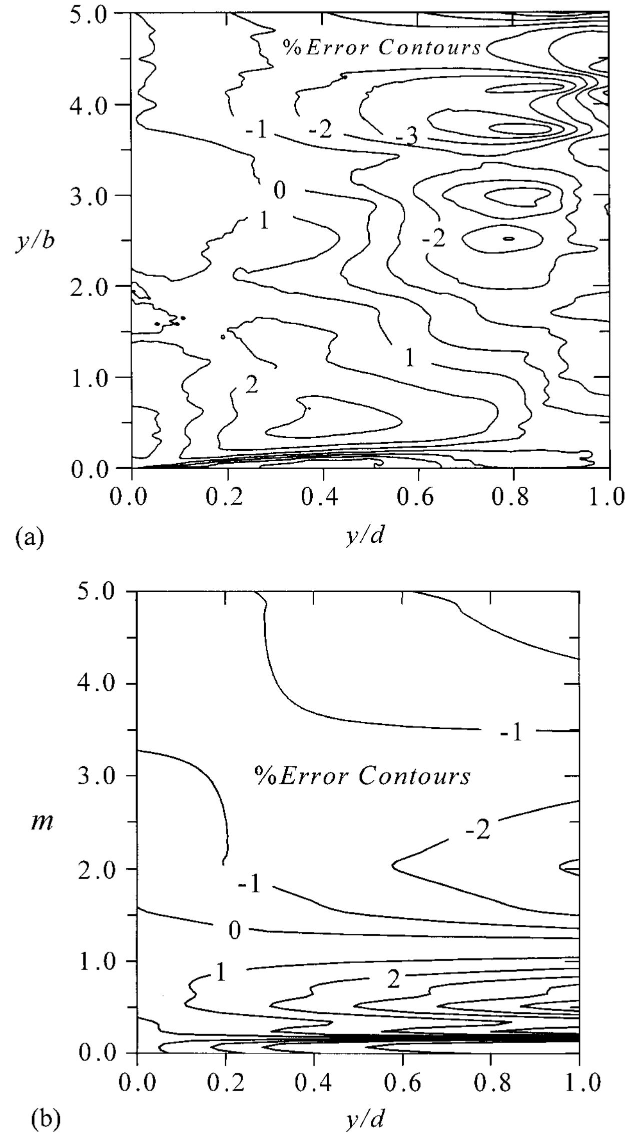

FIG. 3. Error Diagram: (a) Rectangular Canal; (b) Triangular Canal

Using q_s/ky, b/y, and d/y obtained as described above, the following equation was fitted by minimization of errors (Chahar 2000):

$$q_s = ky \left\{ \left(\frac{2.5(b/y)^{0.84} + 0.45}{(d/y - 1)^{0.69}} \right)^{2.38} + [(4\pi - \pi^2)^{0.77} + (b/y)^{0.77}]^{3.094} \right\}^{0.42} \tag{16}$$

Fig. 3(a) depicts the errors involved in (16). The involved error in the practical range ($0.1 \leq y/b \leq 2$) is <3.0%. As $d/y \to \infty$, (16) reduces to

$$q_s = ky((4\pi - \pi^2)^{0.77} + (b/y)^{0.77})^{1.3} \tag{17}$$

which gives q_s from a rectangular canal when the drainage layer or water table is at a very large depth (Chahar 2000; Swamee et al. 2000).

Triangular Section

Using an inverse hodograph and conformal transformation, Bruch (1966) gave the following expression for the quantity

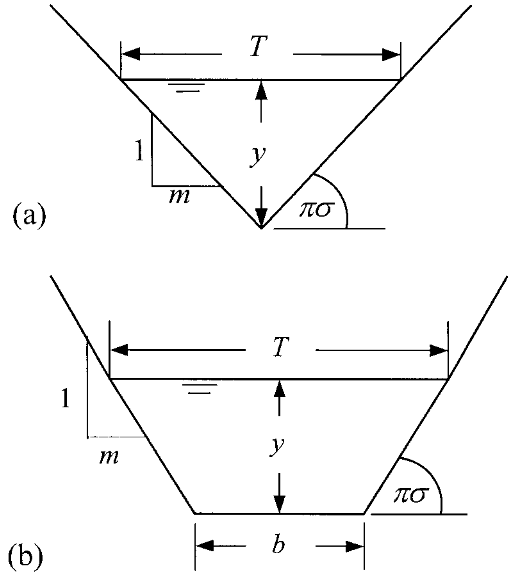

FIG. 4. Canal Sections: (a) Triangular Section; (b) Trapezoidal Section

of seepage from a triangular channel [Fig. 4(a)] in a soil layer of finite depth overlying a drainage layer:

$$q_s = 2kd\,\frac{K(1/\sqrt{\beta})}{K(\sqrt{(\beta - 1)/\beta})} \tag{18}$$

where the transformation variable β was given by

$$1 - \frac{d}{y} = \text{Im}\left\{(m + i)\left[\int_0^{-\beta}\left(\int_0^t \frac{d\tau}{\tau^{1-\sigma}(1 + \tau)^{0.5+\sigma}\sqrt{\tau + \beta}}\right)\cdot\frac{dt}{\sqrt{t(1 + t)(t + \beta)}}\Bigg/\int_0^{-1}\left(\int_0^t \frac{d\tau}{\tau^{1-\sigma}(1 + \tau)^{0.5+\sigma}\sqrt{\tau + \beta}}\right)\cdot\frac{dt}{\sqrt{t(1 + t)(t + \beta)}}\right]\right\} \tag{19}$$

where m = side slope (dimensionless); $\sigma = (1/\pi)\cot^{-1}(m)$; and Im = imaginary part.

The value of β in (19) was obtained by the Fibonacci search method (Bazaraa and Shetty 1979) for a given set of m and d/y. Using β and d/y in (18), q_s/ky was obtained. Repeating the process, q_s/ky was obtained for a large number of m and d/y lying in the range $0 \le m \le 100$ and $1 < d/y \le 1{,}000$. Using these computations, the following equation for q_s was fitted:

$$q_s = ky\left\{\left(\frac{1.81m^{1.18} + 2.1}{(d/y - 1)^{0.26}}\right)^{9.35} + ((4\pi - \pi^2)^{1.3} + (2m)^{1.3})^{7.2}\right\}^{0.107} \tag{20}$$

The errors involved in (20) are shown in Fig. 3(b). The errors are <2.0% in the practical range ($0.5 < m < 5$). As $d/y \to \infty$, (20) reduces to the following equation for seepage for the infinite depth case (Swamee et al. 2000):

$$q_s = ky((4\pi - \pi^2)^{1.3} + (2m)^{1.3})^{0.77} \tag{21}$$

Trapezoidal Section

Using the Zhukovsky function and conformal mapping, Muskat (1946) and Vedernikov (Harr 1962) gave the following approximate solution for the seepage from a trapezoidal canal [Fig. 4(b)]:

$$q_s = 2kd\,\frac{K(\beta)}{K(\sqrt{1 - \beta^2})} \tag{22}$$

where the transformation variable β was given by the simultaneous solution of the following equations:

$$\frac{d}{y} = \frac{mK(\sqrt{1 - \beta^2})}{K(\beta) - F(\beta, \sin^{-1}\gamma)} \tag{23}$$

$$\frac{b}{y} = \frac{2mF(\beta, \sin^{-1}\gamma)}{K(\beta) - F(\beta, \sin^{-1}\gamma)} - \frac{2K(\gamma)}{K(\sqrt{1 - \gamma^2})} \tag{24}$$

Using a process similar to that described for the rectangular canal, q_s/ky was obtained for a large number of m, b/y, and d/y lying in the range $0.5 \le m \le 20$; $0.5 \le b/y \le 50$; and $1 < d/y \le 100$. Using the data so obtained for the trapezoidal section for the above ranges, and the data for the triangular (b = 0) and the rectangular (m = 0) sections obtained earlier, the following equation was fitted:

$$q_s = ky\left\{\left((1.81(m^{1.3} + 1.432(b/y)^{0.93})^{0.9} + \frac{b + 100my}{2.22b + 47.62my + 1.57bm^5}\right)^{p_1}\left(\frac{d}{y} - 1\right)^{-p_1p_3} + (((4\pi - \pi^2)^{1.3} + (2m)^{1.3})^{0.77p_2} + (b/y)^{p_2})^{p_1/p_2}\right\}^{1/p_1} \tag{25}$$

where

$$p_1 = \frac{2.38b + 7.48my}{b + 0.8my};\quad p_2 = \frac{1 + 0.6m}{1.3 + 0.6m};\quad p_3 = \frac{0.318b + 0.26my}{0.461b + my}$$

When the drainage layer is located at large depth, (25) becomes the following particular equation for the trapezoidal section (Swamee et al. 2000):

$$q_s = ky[((4\pi - \pi^2)^{1.3} + (2m)^{1.3})^{(0.77+0.462m)/(1.3+0.6m)} + (b/y)^{(1+0.6m)/(1.3+0.6m)}]^{(1.3+0.6m)/(1+0.6m)} \tag{26}$$

The errors involved in (25) are <6% in the practical range ($0.5 < m < 5$ and $0.5 < b/y < 10$) for the trapezoidal section. The higher errors occur at the extreme points; e.g., $m = 0.5$, $b/y = 1.3$; $m = 1.0$, $b/y = 0.5$; and $m = 2.0$, $b/y = 10.0$. This much error may be admissible in this type of problem, where uncertainty lies in fixing hydraulic conductivity and depth of the drainage layer.

An analysis of the seepage data for all three canal shapes indicated that the aquifer/drainage layer could be assumed at an infinite depth when $d \ge T + 3y$, where T = top width of the canal at the water surface (m).

MINIMUM SEEPAGE LOSS SECTION

A rigid boundary canal is designed for the condition of uniform flow. Swamee (1994, 1995) gave the following uniform flow equation:

$$V = -2.457\sqrt{gRS_0}\,\ln\left(\frac{\varepsilon}{12R} + \frac{0.221\nu}{R\sqrt{gRS_0}}\right) \tag{27}$$

where V = average flow velocity (m/s); g = gravitational acceleration (m/s^2); R = hydraulic radius (m) defined as the ratio of the flow area A (m^2) to the flow perimeter P (m); S_0 = longitudinal canal bed slope (dimensionless); ε = average roughness height of the canal lining (m); and ν = kinematic viscosity of water (m^2/s). Using the continuity equation and (27), the discharge Q (m^3/s) was obtained

$$Q = AV = -2.457A\sqrt{gRS_0}\,\ln\left(\frac{\varepsilon}{12R} + \frac{0.221\nu}{R\sqrt{gRS_0}}\right) \tag{28}$$

Optimization Algorithm

The depth of flow in a uniform flow is the normal depth y_n. In such a case, the seepage loss is determined by replacing y

with y_n in (16), (20), and (25). Thus, the problem of determination of the minimum seepage loss section was reduced to

$$\text{minimize } q_s \tag{29}$$

$$\text{subject to } Q + 2.457A\sqrt{gRS_0}\ \ln\left(\frac{\varepsilon}{12R} + \frac{0.221\nu}{R\sqrt{gRS_0}}\right) = 0 \tag{30}$$

Adopting a length scale λ (m)

$$\lambda = (Q/\sqrt{gS_0})^{0.4} \tag{31}$$

the following nondimensional variables were obtained:

$$\varepsilon_* = \varepsilon/\lambda; \quad \nu_* = \nu\lambda/Q; \quad y_{n*} = y_n/\lambda; \quad b_* = b/\lambda \qquad (32a\text{–}d)$$

$$d_* = d/\lambda; \quad A_* = A/\lambda^2; \quad R_* = R/\lambda; \quad q_{s*} = q_s/(k\lambda) \qquad (32e\text{–}h)$$

where subscript * denotes the corresponding nondimensional parameter.

Using (29), (30), and (32), the problem of determination of optimal canal section shape in nondimensional form was reduced to

$$\text{minimize } q_{s*} \tag{33}$$

$$\text{subject to } \Phi = 1 + 2.457A_*\sqrt{R_*}\ \ln\left(\frac{\varepsilon_*}{12R_*} + \frac{0.221\nu_*}{R_*^{1.5}}\right) = 0 \tag{34}$$

where Φ = equality constraint function.

As the objective function (33) and the constraint (34) are nonlinear, an analytical solution with its proof of uniqueness and globality at the optimum is very difficult. A practical solution to such a problem was obtained through a numerical method with multiple starts and testing convergence. The following paragraph describes the optimization algorithm used for this purpose.

The constrained optimization problem was converted into an unconstrained optimization problem using the penalty function (Fox 1971). The auxiliary function Ψ for the unconstrained optimization problem was expressed

$$\Psi = q_{s*} + p\Phi^2 \tag{35}$$

where p = penalty parameter. Adopting small p initially, (35) was minimized using a grid or lattice search method (Burley 1974) to find the design variables. Increasing p 10-fold, the minimization was carried through various cycles until the optimization results stabilized. Alternatively, Powel's conjugate direction search method (Himmelblau 1972; Avriel 1976) can be used for minimizing (35).

Optimal Design Equations

The optimization algorithm was applied on triangular, rectangular, and trapezoidal canal sections for a number of input variables varying in the ranges

$$10^{-6} \le \varepsilon_* \le 10^{-3}; \quad 10^{-7} \le \nu_* \le 10^{-5} \qquad (36a,b)$$

$$0.01 \le d_* < \infty \qquad (36c)$$

Analysis of a large number of optimal sections so obtained for triangular, rectangular, and trapezoidal canal sections resulted in the following generalized empirical equations for all three types of canal sections:

$$m^* = k_{ms}\left[1 + \left(\frac{t_{md}L}{d}\right)^{r_{md}}\right]^{s_{md}} \qquad (37a)$$

$$b^* = k_{bs}\left[1 + \left(\frac{t_{bd}L}{d}\right)^{r_{bd}}\right]^{s_{bd}} L \qquad (37b)$$

$$y_n^* = k_{ys}\left[1 + \left(\frac{t_{yd}L}{d}\right)^{r_{yd}}\right]^{-s_{yd}} L \qquad (37c)$$

where superscript * indicates optimality; k_{fs} and t_{fs} = section shape coefficients; r_{fs} and s_{fs} = exponents; and L = length scale (m) given by

$$L = \lambda(\varepsilon_* + 8\nu_*)^{0.04} \tag{38}$$

The first subscripts m, b, and y denote side slope, bed width, and normal depth, respectively, and the second subscripts s and d denote seepage loss cases corresponding to the drainage layer at infinite depth and finite depth, respectively. The optimal section shape coefficients and exponents are listed in Table 1.

For a given set of data, a direct optimization procedure [minimize (33) subject to constraint (34)] can be adopted for the design of minimum seepage loss irrigation canal sections. This requires a considerable amount of programming and computation work. On the other hand, using optimal design equations [(37a–c)] along with tabulated section shape coefficients and exponents (Table 1), the optimal design variables can be obtained in a single step computation. For the designed section, (28) can be used to obtain the average flow velocity. This velocity should be greater than the nonsilting velocity but less than the limiting velocity V_L. The limiting velocity depends on the lining material. Table 2 lists the limiting velocities for different types of linings (IBS 1982). If $V > V_L$, a superior lining material should be selected. The quantity of seepage from the designed section can be obtained using (16), (20), or (25), depending upon the type of section; however, the following equation closely approximated the seepage from a minimum seepage section:

$$q_s^* = k_{qs}\left[1 + \left(\frac{t_{qd}L}{d}\right)^{r_{qd}}\right]^{s_{qd}} kL \tag{39}$$

TABLE 1. Coefficients and Exponents for Minimum Seepage Canal Sections

Entity	Coefficients or exponents	Section Shape: Triangular	Rectangular	Trapezoidal
Side slope	k_{ms}	1.2445	—	0.5984
	t_{md}	0.4826	—	1.5156
	r_{md}	3.1847	—	3.5709
	s_{md}	0.8232	—	0.1077
Bed width	k_{bs}	—	0.7986	0.5447
	t_{bd}	—	0.4922	0.6042
	r_{bd}	—	2.5897	2.8821
	s_{bd}	—	0.5571	0.5129
Normal depth	k_{ys}	0.4518	0.3178	0.3309
	t_{yd}	0.5161	0.6293	0.6253
	r_{yd}	3.1465	2.8715	2.5376
	s_{yd}	0.3177	0.3125	0.3608
Seepage loss	k_{qs}	2.0015	2.0399	1.9227
	t_{qd}	0.4385	0.4417	0.4372
	r_{qd}	2.8994	2.2473	2.0318
	s_{qd}	0.7238	0.5807	0.6551

TABLE 2. Limiting Velocities

Lining material	Limiting velocity (m/s)
Boulder	1.0–1.5
Brunt clay tile	1.5–2.0
Concrete tile	2.0–2.5
Concrete	2.5–3.0

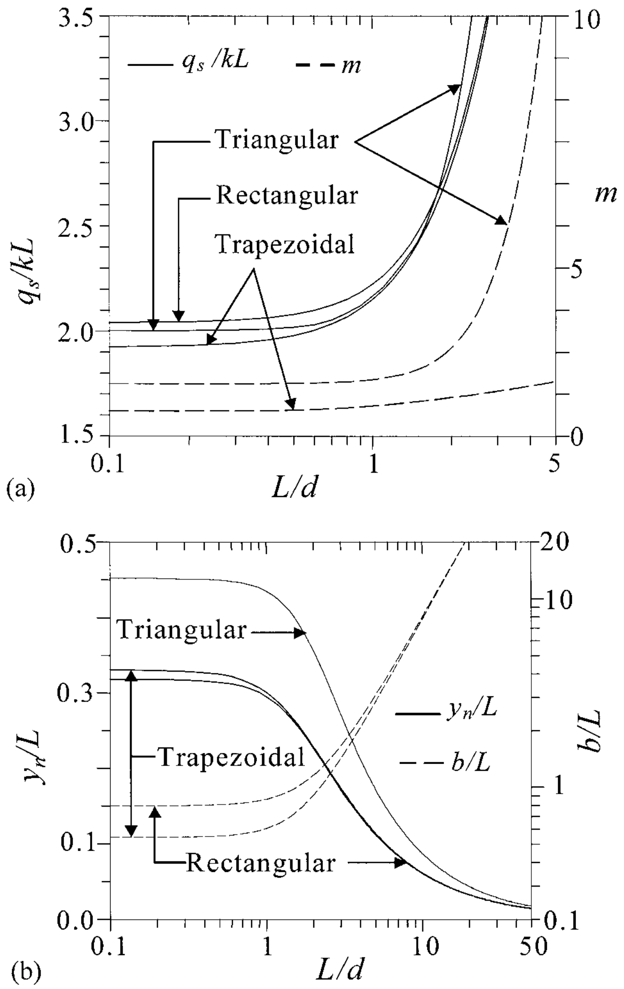

FIG. 5. Properties of Minimum Seepage Loss Canal Sections: (a) Seepage Loss and Side Slope; (b) Normal Depth and Bed Width

where the first subscript q in coefficients and exponents denotes seepage discharge. For a large d/L ratio, (37) and (39) reduce to corresponding equations (Swamee et al. 2000) for the minimum seepage loss section when the water table is at a large depth; i.e.

$$m^* = k_{ms}; \quad b^* = k_{bs}L; \quad y_n^* = k_{ys}L; \quad q_s^* = k_{qs}kL \qquad (40a\text{–}d)$$

The behavior of the optimal design equations [(37a–c) and (39)] was plotted in Fig. 5. A perusal of Fig. 5 reveals that the minimum seepage section becomes wider and shallower as the canal bed approaches the drainage layer. The minimum seepage section becomes impractical ($T > 20y_n$) for $d < 0.22L$. For $L > d$, the optimal canal dimensions are hypersensitive. For $d < 0.01L$, the optimal section results in a meaningless strip ($T > 12{,}000y_n$). On the other hand, the optimal section approaches its counterpart corresponding to the water table or drainage layer at very large depth for $d \geq 3L$. A minimum seepage loss trapezoidal section has the least seepage compared to the optimal triangular and rectangular sections. For $L > 1.8d$, a triangular canal loses the highest seepage, whereas the seepage from an optimal rectangular canal approaches the seepage from an optimal trapezoidal canal.

The optimal section design equations [(37a–c)] are independent of k, which shows that the optimal canal dimensions do not depend on the hydraulic conductivity of the seepage layer through which a canal passes. This is so because, for 2D seepage in the vertical plane, k disappears from the governing Laplace's equation and q_s becomes a linear function of k.

DESIGN EXAMPLES

Example 1

Design a minimum seepage loss concrete-lined rectangular canal section for carrying a discharge of 50 m^3/s on a longitudinal slope of 0.0004. The canal passes through a stratum underlain by a highly pervious layer at a depth of 5 m.

Example 1 Design Steps

For the design, $g = 9.79$ m/s^2, $\nu = 1.007 \times 10^{-6}$ m^2/s (water at 20°C), and $\varepsilon = 1$ mm (concrete lining) are adopted.

Using (31), $\lambda = 14.488$ m; (32a), $\varepsilon_* = 6.902 \times 10^{-5}$; (32$b$), $\nu_* = 2.918 \times 10^{-7}$; (32$e$), $d_* = 0.345$; and (38), $L = 9.889$ m.

Using Table 1, the section shape coefficients and exponents for a rectangular section are, for bed width, $k_{bs} = 0.7986$, $t_{bd} = 0.4922$, $r_{bd} = 2.5897$, and $s_{bd} = 0.5571$; for normal depth, $k_{ys} = 0.3178$, $t_{yd} = 0.6293$, $r_{yd} = 2.8715$, and $s_{yd} = 0.3125$; and, for seepage, $k_{qs} = 2.0399$, $t_{qd} = 0.4417$, $r_{qd} = 2.2473$, and $s_{qd} = 0.5807$.

Assuming a drainage layer at large depth [(40b,c)], give $b^* = 0.7986 \times 9.889 = 7.897$ m and $y_n^* = 0.3178 \times 9.889 = 3.143$ m. Thus, $A = b^* \times y_n^* = 24.823$ m^2. Further, assuming that the lining is cracked and $k = 10^{-6}$ m/s, (40d) yields $q_s = 2.0399 \times 10^{-6} \times 9.890 = 2.0175 \times 10^{-5}$ m^2/s whereas (16) results in actual seepage loss $q_s = 10^{-6} \times 3.143 \times 10.059 = 3.1616 \times 10^{-5}$ m^2/s.

Using (37b), $b^* = 0.7986 \times 1.4436 \times 9.889 = 11.400$ m, and using (37c), $y_n^* = 0.3178 \times 0.7189 \times 9.889 = 2.259$ m. Therefore, $A = 11.400 \times 2.259 = 25.769$ m^2 and $V = 50/25.769 = 1.940$ m/s, which is within the permissible limit (Table 2). Using (39), the seepage loss $q_s = 2.0399 \times 1.3786 \times 10^{-6} \times 9.889 = 2.7813 \times 10^{-5}$ m^2/s.

The design shows that the optimal section is influenced very much by the presence of a drainage layer at the shallow depth. The optimal section for the drainage layer at shallow depth results in 28.11% less y_n, 44.36% more b, and 12.03% less seepage than the optimal section considering a drainage layer at a large depth.

Example 2

Design a trapezoidal canal section for $Q = 250$ m/s, $S = 0.0001$, and $d = 7.5$ m.

Example 2 Design Steps

Following the steps similar to the rectangular section, $\lambda = 36.393$ m, $\varepsilon_* = 2.748 \times 10^{-5}$, $\nu_* = 1.466 \times 10^{-7}$, $d_* = 0.206$; and $L = 23.950$ m.

The section shape coefficients from Table 1 are, for side slope, $k_{ms} = 0.5984$, $t_{md} = 1.5156$, $r_{md} = 3.5709$, and $s_{md} = 0.1077$; for bed width, $k_{bs} = 0.5446$, $t_{bd} = 0.6042$, $r_{bd} = 2.8821$, and $s_{bd} = 0.5129$; for normal depth, $k_{ys} = 0.3309$, $t_{yd} = 0.6253$, $r_{yd} = 2.5376$, and $s_{yd} = 0.3608$; and for seepage, $k_{qs} = 1.9227$, $t_{qd} = 0.4372$, $r_{qd} = 2.0318$, and $s_{qd} = 0.6551$.

Using (37a–c), $m = 1.098$, $b^* = 37.042$ m, and $y_n^* = 3.972$ m. Therefore, $A = 164.454$ m^2 and $V = 250/164.454 = 1.520$ m/s, which is safe. Using (39) with $k = 10^{-6}$ m/s, $q_s = 1.9227 \times 2.0406 \times 10^{-6} \times 23.954 = 9.3983 \times 10^{-5}$ m^2/s. Assuming the drainage layer at large depth, the canal dimensions are $m = 0.598$, $b^* = 13.045$ m, and $y_n^* = 7.926$ m.

CONCLUSIONS

An analytical solution for the seepage from a rectangular canal when a pervious layer lies at a finite depth has been obtained by use of inverse hodograph and conformal mapping. The analytical solution for the rectangular canal and the existing analytical solutions for triangular and trapezoidal canals involve complicated integrals with unknown implicit transformation variables. These solutions have been expressed as a simple algebraic function of canal geometry and depth of the drainage layer. These simplified expressions for computing seepage losses from triangular, rectangular, and trapezoidal canals replace approximately the cumbersome evaluation of seepage by analytical methods. Optimal design equations and section shape coefficients for all three canal shapes have been obtained to facilitate design of minimum seepage loss canals. The method overcomes the complexity in design of the minimum seepage loss canal section by a constrained nonlinear optimization technique. The optimal design equations show that the minimum seepage section becomes very wide and shallow as the canal bed approaches the drainage layer. On the other hand, the minimum seepage section attains $m = 1.244$ for a triangular canal, ratio of bed width to normal depth = 2.513 for a rectangular canal, and $m = 0.598$ and ratio of bed width to normal depth = 1.646 for a trapezoidal canal when $d \geq T + 3y_n$. The optimization method can be extended to investigate the robustness of the optimal shape subject to small perturbations in the constraint function (i.e., selecting the constraint as a Manning equation or other uniform flow formula and then comparing the results). The optimal canal dimensions do not depend on the hydraulic conductivity of the seepage layer through which a canal passes. The proposed design method is very simple, as demonstrated by the design examples.

REFERENCES

Avriel, M. (1976). *Nonlinear programming analysis and methods*, Prentice-Hall, Englewood Cliffs, N.J.

Bandini, A. (1966). ''Economic problem of irrigation canals: Seepage losses.'' *J. Irrig. and Drain. Div.*, ASCE, 92(4), 35–57.

Bazaraa, M. S., and Shetty, C. M. (1979). *Nonlinear programming: Theory and algorithms*, Wiley, New York.

Bruch, J. C. (1966). ''Studies of free surface flow and two dimensional dispersion in porous media.'' PhD thesis, Dept. Civ. Engrg., Stanford University, Stanford, Calif.

Bruch, J. C., and Street, R. L. (1967a). ''Seepage from an array of triangular channels.'' *J. Engrg. Mech. Div.*, ASCE, 93(3), 63–82.

Bruch, J. C., and Street, R. L. (1967b). ''Free surface flow in porous media.'' *J. Irrig. and Drain. Div.*, ASCE, 93(3), 125–145.

Byrd, P. F., and Friedman, M. D. (1971). *Handbook of elliptic integrals for engineers and scientists*, Springer, Berlin.

Burley, D. M. (1974). *Studies in optimization*, International Text Book Co., Beds, U.K.

Chahar, B. R. (2000). ''Optimal design of channel sections considering seepage and evaporation losses.'' PhD thesis, Dept. of Civ. Engrg., University of Roorkee, Roorkee, India.

Churchhouse, R. F. (1981). *Numerical methods. Handbook of applied mathematics*, Vol. 3, Wiley, New York.

El Nimr, A. (1963). ''Seepage from parallel trapezoidal channels.'' *J. Engrg. Mech. Div.*, ASCE, 89(4), 1–11.

Fox, R. L. (1971). *Optimization methods for engineering design*, Addison-Wesley, Reading, Mass.

Garg, S. P., and Chawla, A. S. (1970). ''Seepage from trapezoidal channels.'' *J. Hydr. Div.*, ASCE, 96(6), 1261–1282.

Hammad, H. Y. (1960). ''Seepage losses from parallel channels.'' *J. Engrg. Mech. Div.*, ASCE, 86(4), 42–50.

Harr, M. E. (1962). *Groundwater and seepage*, McGraw-Hill, New York.

Himmelblau, D. M. (1972). *Applied nonlinear programming*, McGraw-Hill, New York.

Ilyinsky, N. B., and Kacimov, A. R. (1984). ''Seepage-limitation optimization of the shape of an irrigation channel by the inverse boundary value problem method.'' *J. Fluid Dyn.*, 19(4), 404–410.

Indian Bureau of Standards (IBS). (1980). ''Measurement of seepage losses from canals.'' *IS:9452*, Parts 1 and 2, New Delhi.

Indian Bureau of Standards (IBS). (1982). ''Criteria for design of lined canals and guidelines for selection of type of lining.'' *IS:10430*, New Delhi.

International Commission on Irrigation and Drainage (ICID). (1967). ''Controlling seepage losses from irrigation canals.'' *Worldwide survey*, New Delhi.

Kacimov, A. R. (1992). ''Seepage optimization for trapezoidal channel.'' *J. Irrig. and Drain. Engrg.*, ASCE, 118(4), 520–526.

Morel-Seytoux, H. J. (1964). ''Domain variations in channel seepage flow.'' *J. Hydr. Div.*, ASCE, 90(2), 55–79.

Muskat, M. (1946). *Flow of homogeneous fluids through porous media*, J. W. Edwards Brothers, Inc., Ann Arbor, Mich.

Polubarinova-Kochina, P. Ya. (1962). *Theory of ground water movement*, Princeton University Press, Princeton, N.J.

Preissmann, A. (1957). ''A propos de la filtration au-dessous des canaux.'' *Houille Blanche*, 12, 181–188 (in French).

Press, W. H., Teukolsky, S. A., Vetterling, W. T., and Flannery, B. P. (1992). *Numerical recipes in FORTRAN*, Cambridge University Press, Cambridge, U.K.

Rohwer, C., and Stout, O. V. P. (1948). ''Seepage losses from irrigation canals.'' *Tech. Bull. 38*, Colorado Agricultural Experiment Station, Fort Collins, Colo.

Sharma, H. D., and Chawla, A. S. (1979). ''Canal seepage with boundary at finite depth.'' *J. Hydr. Div.*, ASCE, 105(7), 877–897.

Swamee, P. K. (1994). ''Normal-depth equations for irrigation canals.'' *J. Irrig. and Drain. Engrg.*, ASCE, 120(5), 942–948.

Swamee, P. K. (1995). ''Optimal irrigation canal sections.'' *J. Irrig. and Drain. Engrg.*, ASCE, 121(6), 467–469.

Swamee, P. K., Mishra, G. C., and Chahar, B. R. (2000). ''Design of minimum seepage loss canal sections.'' *J. Irrig. and Drain. Engrg.*, ASCE, 126(1), 28–32.

Wachyan, E., and Rushton, K. R. (1987). ''Water losses from irrigation canals.'' *J. Hydro.*, Amsterdam, 92(3-4), 275–288.

Youngs, E. G. (1986). ''Water-table heights and discharge rates with artesian flow to interceptor land drains.'' *J. Hydro.*, Amsterdam, 87(3-4), 255–266.

NOTATION

The following symbols are used in this paper:

- A = flow area of canal (m^2);
- B = seepage width at drainage layer (m);
- b = bed width of canal (m);
- d = depth of drainage layer/aquifer (m);
- $F(\kappa, \varphi)$ = incomplete elliptical integral of first kind (dimensionless);
- g = gravitational acceleration (m/s^2);
- $K(\kappa)$ = complete elliptical integral of first kind (dimensionless);
- k = hydraulic conductivity (m/s);
- k_{fs}, t_{fs} = section shape coefficients for subscripts f and s (dimensionless);
- L = length scale (m);
- m = side slope of canal (dimensionless);
- p = penalty parameter (dimensionless);
- p_1, p_2, p_3 = exponents (dimensionless);
- Q = discharge (m^3/s);
- q_s = seepage discharge per unit length of canal (m^2/s);
- R = hydraulic radius (m);
- r_{fs}, s_{fs} = exponents for subscripts f and s (dimensionless);
- S_0 = bed slope of canal (dimensionless);
- T = width of canal at water surface (m);
- V = average velocity (m/s);
- V_L = limiting velocity (m/s);
- W = $\phi + i\psi$ complex potential (m^2/s);
- X = real axis of complex plane (m);
- Y = imaginary axis of complex plane (m);
- y = water depth in canal (m);
- y_n = normal depth of flow in canal (m);
- Z = $X + iY$ complex plane variable (m);
- β, γ = transformation variable (dimensionless);
- ε = average roughness height of canal lining (m);
- ζ = complex variable in auxiliary plane (dimensionless);
- κ = modulus of elliptical integral (dimensionless);
- λ = length scale (m);
- ν = kinematic viscosity (m^2/s);
- τ, t = dummy variable (dimensionless).
- Φ = equality constraint (dimensionless);
- ϕ = velocity potential (m^2/s);
- φ = amplitude of elliptical integral (dimensionless);

Ψ = augmented function (dimensionless); and
ψ = stream function (m^2/s).

Subscripts

b = bed width;
d = seepage case for drainage layer at shallow depth;
m = side slope;
q = seepage discharge;
s = seepage case for drainage layer at large depth;
y = normal depth; and
$*$ = nondimensional.

Superscript

$*$ = optimal.