Flow in Unsaturated Soil

Lecture 3

3.0 Hydraulic Conductivity

The flow of water in a saturated soil was studied by applying Darcy's law in the basic soil mechanics text books. However, the question is: whether the Darcy's law is applicable for studying the flow through unsaturated soils. This question will be addressed here.

As we have seen in the previous lectures that a given soil may be unsaturated in several ways. The natural soil may contain two or more different liquids and air or only liquid and air. The saturation in such soils can be defined with respect to one of the fluids. The soils in nature, generally, contain air and water that are occupied in the pore space of the soil matrix. Therefore, the saturation here is in terms of the available water in the pore matrix. In such soils the air phase and water phase may form continuous phase or be separated by the other phases or soil particles. For the time being we assume that the soil only contains air and water in its pore space.

The fundamental dependent state variable responsible for the water flow through unsaturated soils is total suction, ψ, or total suction head, $\mathbf{h_t}$ (or H). The total driving head can be expressed as:

$$\mathbf{h_t = h_g + h_m + h_o = z + h_m + h_o}$$

where: $\mathbf{h_g}$ is the gravitational head, $\mathbf{h_m}$ is the matric suction head, and $\mathbf{h_o}$ is the osmotic suction head. For most seepage problems, the osmotic suction head is neglected. This total suction head should be used in the Darcy's equation under unsaturated condition.

According to soil physics, the discharge velocity of water is understood to be proportional to the viscosity and density of the permeating fluid through the soil pores. The discharge velocity is high for permeating fluids having higher density and low viscosity, vice versa. Further, the experimental observations, supported by the theoretical analysis, reveal that the discharge velocity is dependent on the pore size and pore size distribution. Therefore, the discharge velocity is :

$$q \propto d^2 \left(\frac{\rho g}{\mu} \right) \qquad \text{-------- (10)}$$

Where: **d** is the pore diameter (m), **ρ** the density of the fluid (kg/m^3), **μ** the dynamic viscosity of the fluid (N.s/m^2).

Combining the Darcy's observation on the dependency of discharge velocity on the hydraulic gradient, the above expression can be written as;

$$q = -C \left(\frac{d^2 \rho g}{\mu} \right) \nabla H \qquad \text{-----(11)}$$

Where: **C** is the dimensionless constant related to the geometry of the soil pores, **H** is the total head, and **∇H** is the hydraulic head.

Therefore, the hydraulic conductivity is the proportionality constant in the Darcy's equation which can be expressed as:

$$k = \left(Cd^2\right)\left(\frac{\rho g}{\mu}\right) \quad \text{-------(12)}$$

Where: $\mathbf{Cd^2}$ is termed as the intrinsic permeability or permeability, denoted by **K**, often used to differentiate the material properties from the fluid properties against the pore geometry. Permeability has the units of m^2 and is dependent only on the pore size, pore geometry, and the pore size distribution. The permeability varies from 10-7 m^2 for the gravel to 10-20 m^2 for the fine clay. It can be easily verified using the above equations that the conductivity of water is several times higher than the air under the same applied gradient and for the same pore geometry. It can readily be recalled from the earlier discussion on the influence of the state variables on the density and viscosity of the fluid. One can easily verify the influence of such state variables on the hydraulic conductivity of air and water through unsaturated soils. Other than these material constants, chemical and electrical pore fluid characteristics also strongly influence the flow behavior in unsaturated clay soils. Such characteristics can also alter the fabric of the clay. The aforementioned discussion on the hydraulic conductivity is applicable to both saturated and unsaturated soil system. However, the constant related to the geometry of the soil pores will be strongly influenced by the tortuous paths formed due to the occluded/entrapped air in the pore space of the unsaturated soils.

The foregoing discussion signifies that the ratio of flux (q) to the hydraulic gradient (∇H) is non-linear under unsaturated conditions. Therefore, the hydraulic conductivity depends on the volumetric water content or the soil matric suction. The plot of flux versus hydraulic gradient is obtained as shown in Fig. 10. It results a family of straight lines passing through the origin. Each line represents a straight line having a slope equal to the hydraulic conductivity at the indicated moisture content, θ_I, as shown in the figure. The hydraulic conductivity is no longer constant and is dependent on the volumetric water content.

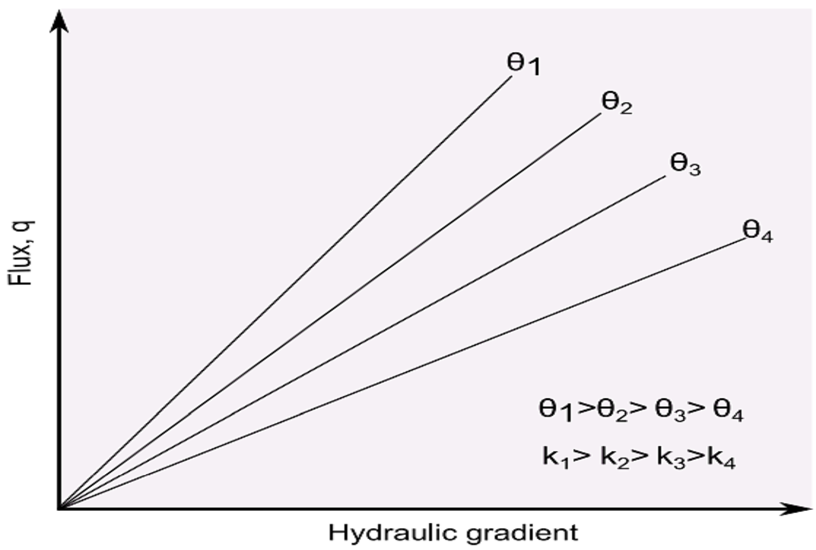

Fig. 10. Relationship between flux and hydraulic gradient in unsaturated soils

Therefore, functional relationships are used in Darcy's law to represent hydraulic conductivity dependency on matric suction for the flow through unsaturated soils. The modified form of Darcy's law under unsaturated conditions is given as:

$$q = K(\theta)\nabla H$$ -------(13)

Where: **K (θ**) is the functional form of hydraulic conductivity. Therefore, the nature of the soil and amount of soil water content influence the hydraulic conductivity in unsaturated soils. As the water content is reduced in the initially saturated soil, the air enters through a largest pore of the soil matrix. Hence, the effective flow channels are reduced which causes the reduction in unsaturated hydraulic conductivity as depicted in Fig. 11. Since the contribution to conductivity per unit cross-sectional area depends on the square of the pore radius, conductivity decreases much more rapidly than the amount of water in the soil, indicating a sudden drop in the conductivity in Fig. 11 after the air-entry value. Moreover, the contribution of larger pores having a radius, r, is better than the combination of smaller pores equal to the same radius as the viscosity effect is large. Therefore, as the larger pores get emptied first, the conductivity decreases exponentially. Further, reduction in water causes discontinuity of the flow paths in the nearly dry soil where the water is present in the form of occluded bubbles. Therefore, the conductivity is close to zero at this water content as shown in the figure. The Fig. 11 represents the general characteristic curve for K-function.

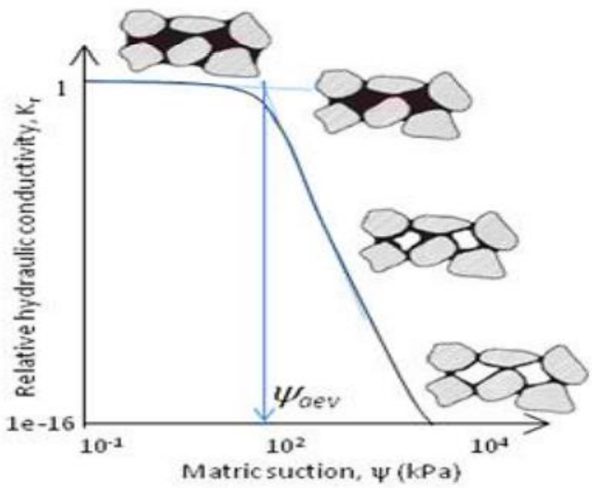

Fig. 11. Generalized hydraulic conductivity function

Several approaches can be used to obtain the permeability function [k (θ)] of an unsaturated soil from its soil-water characteristic curve. Compared with macroscopic and empirical models, the most rigorous approach is through the use of a statistical model given by the following expression:

$$k(\theta) = \frac{T_s^2}{2\rho_w g\mu}\frac{n^2}{m^2}\sum_{i=1}^{l}\frac{2(l-i)-1}{(u_a-u_w)_i^2}$$ -----(14)

Where: $\mathbf{T_s}$ = surface tension of water; $\boldsymbol{\rho_w}$ = density of water; **μ** = dynamic viscosity of water; **n** = porosity of soil; and **m** (= θs/Δθw) = total number of intervals; **l** (=θw/Δθw) = number of intervals corresponding to θw; and **(ua - uw)i** = matric suction corresponding to the midpoint of the ith interval of the soil-water characteristic curve.

4.0 Flow through unsaturated soils

The flow of water in soils is governed by the total head, as mentioned earlier, which is expressed as :

$$H = h_g + h_m + h_o$$

Where: h_g is the gravitational head, h_m the matric suction head, and h_o the osmotic suction head. In the absence of gravitational head (horizontal flow) and the presence of solute in soils, matric suction head controls the flow of water in soils. Flow through soils can be either steady state or unsteady state (transient) depending on the type of soil and the boundary conditions.

4.1 Steady-state flow

Steady-state flow is a time invariant flow as shown in the Fig. 12.

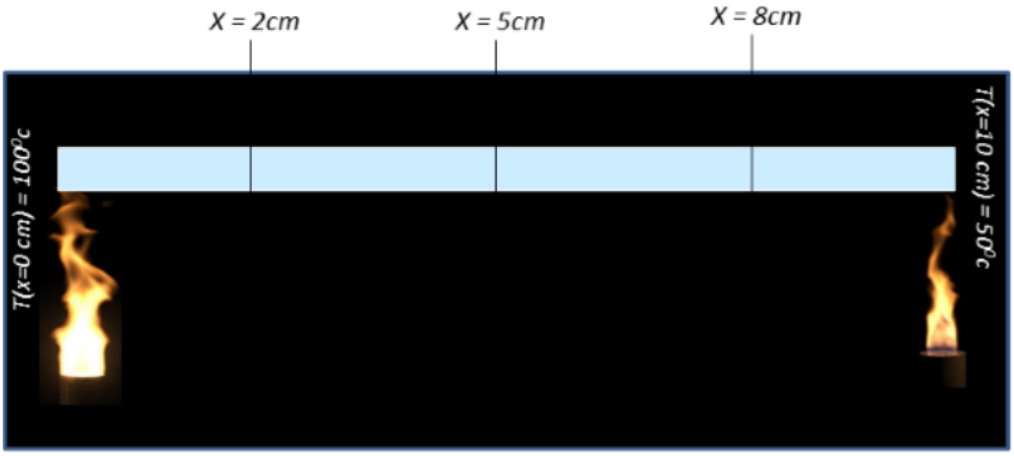

Fig. 12. Conceptual illustration describing the steady state heat conduction in solids

A metallic object is heated at two ends by maintaining two different, but constant temperatures. Conduction of heat in the object is governed by Fourier's law. The heat distribution in the object can be measured by sensors (thermometers) placed along the length of the object as shown in figure #. It can be observed that the heat distribution with space becomes constant and time-independent after certain time. This state is called the steady state. Similarly, the steady state and transient flows take place in saturated soils.

4.1.1 Steady-state up-flow

Can the steady state flow take place in unsaturated soils? The answer to this question is affirmative. Steady state flows can take place in unsaturated soils when a constant matric suction head values are maintained at the boundaries . The steady state flow can takes place in a partially saturated soil column when the flow boundaries are time invariant. Similarly, it is also possible to maintain a constant water content ($\theta < \theta_s$) in unsaturated soils by applying low hydraulic gradient across the soil sample.

Application of small gradients can drive the water flow through the thick water films in the pore space, but the gradients may not be sufficient to drive the air pockets out of the system (saturation). Therefore, a constant (time-independent) flows take place in unsaturated soils with water contents less that the saturation.

The head distribution in steady state is linear which can be easily verified. The Darcy's law for steady flow through saturated soils can be written as :

$$q = -k_s i = -k_s \frac{dh}{dx} \quad \text{------(15)}$$

The head distribution can be obtained by rearranging the terms and integrating along the flow length

$$q\int_0^x dx = -k_s \int_0^h dh \quad \text{-------(16)}$$

Therefore,

$$h = -\left(q/k_s \right) x \quad \text{-----------(17)}$$

4.1.2 Rate of capillary rise

The rate of capillary rise can be appreciated after going through the flow through unsaturated soils in this module. Terzaghi in his infamous work (1943) provided a simple relationship for rate of capillary rise in soils. Terzaghi assumed that Darcy's law is valid for describing the steady flow through unsaturated soils which can be expressed as:

$$q = -k_s i \quad \text{-------(18)}$$

where the hydraulic conductivity of the wetting front is assumed to be described by ks. Further, the gradient is assumed to be ($h_c - z / z$) where hc is the capillary rise.

Therefore, the discharge flux becomes :

$$q = -k_s \left(\frac{h_c - z}{z} \right) \quad \text{------(19)}$$

Or :

$$n\frac{dz}{dt} = -k_s\left(\frac{h_c - z}{z}\right)$$ --------(20)

solving the equation for z results,

$$-\int\left(\frac{z}{h_c - z}\right)dz = \frac{k_s}{n}\int dt$$ -------(21)

which can be simplified to :

$$-\int dz + h_c\int\left(\frac{dz}{h_c - z}\right) = \frac{k_s}{n}\int dt$$ ------(22)

Therefore,

$$-z - h_c \ln(h_c - z) = \frac{k_s t}{n} + c$$ ------(23)

where : **c** is the constant of integration. The constant can be obtained by substituting the initial condition (i.e., t = 0 and z = 0) which is:

$$t = \left(\frac{nh_c}{k_s t}\right)\left[\ln\left(\frac{h_c}{h_c - z}\right) - \frac{z}{h_c}\right]$$ ------- (24)

4.1.3 Vertical steady state flow

The driving force for vertical flows in unsaturated soils is the combination of gravity and the suction head. The gravity, therefore, has a subsequent influence on the spatial distribution of the suction head. In case of one-dimensional vertical flow the governing flow equation can be written as:

$$q = -k\left(\frac{dh_m}{dz} + 1\right)$$ -------(25)

Where: **z** is the spatial distance. The suction head distribution can be obtained by rearranging the terms as:

$$dz = -\frac{dh_m}{1 + q/k}$$ --------(26)

substitution of the boundary conditions: hm = 0 at z = 0 and hm = h at z = Z gives :

$$Z = -\int_0^{h_m} \frac{1}{1 + q/k(\theta)} dh_m(\theta)$$ -------(27)

which can be solved to obtain the profiles of the matric suction head, h_m, or total head if the steady flux q, the soil-water characteristic curve, and the hydraulic conductivity function are known. The solution can be obtained numerically if we write the above equation in discrete form as shown in the following equation :

$$z = -\sum_{j=1}^{n} \frac{\Delta h_m(\theta_i)}{1 + q/k(\theta_i)}$$ -------(28)

where : **n** is the number of discrete data points selected from the SWCC and hydraulic conductivity function.