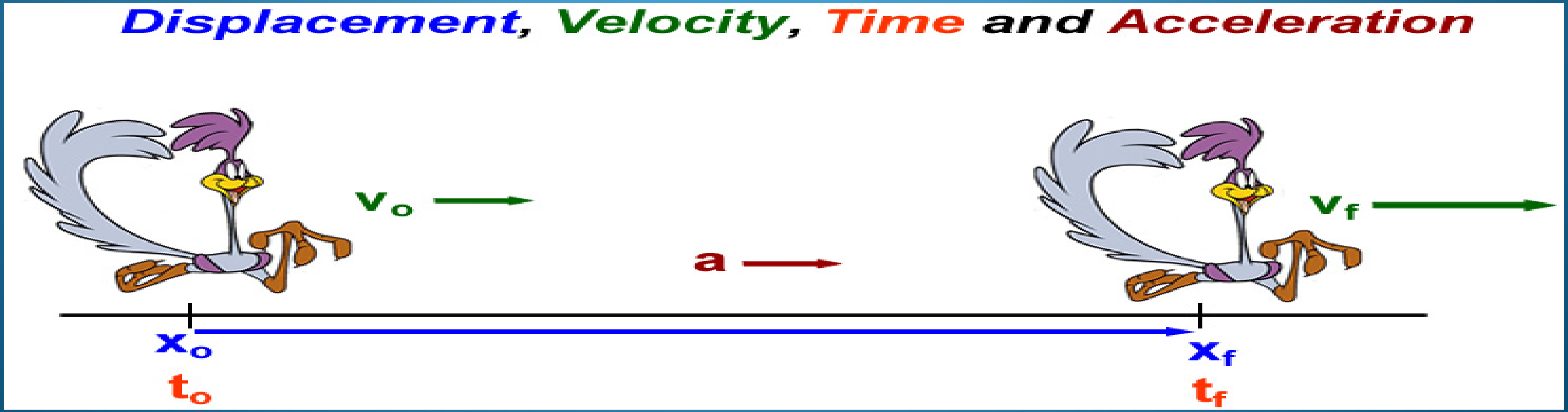
Chapter two

Motion kinematics



Motion kinematics

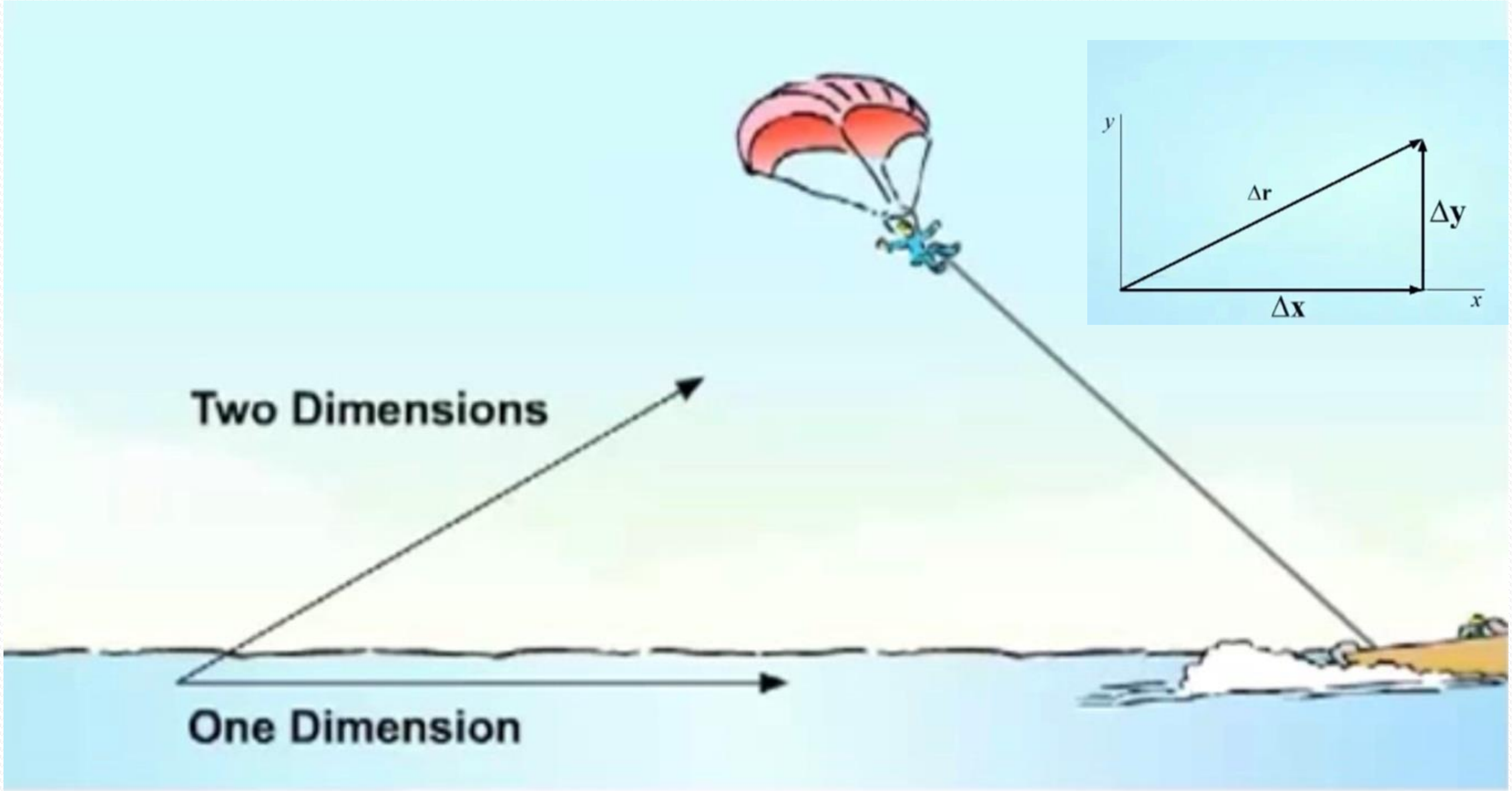
Two - Dimensional Motion

Lecture fourth

Motion kinematics

- Position vector and displacement vector
- Average velocity and Instantaneous velocity
- The Average and Instantaneous Acceleration
- One -dimensional
- Free fall
- Motion in two dimensions
- Projectile motion

The Kinematics of a particle Moving in Two Dimensions

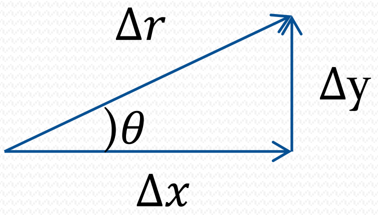


The Kinematics of a particle Moving in Two Dimensions

$$\Delta r = \Delta x + \Delta y$$

Magnitude

$$|\Delta r| = \sqrt{\Delta x^2 + \Delta y^2}$$



components

$$\Delta x = \Delta r \cos\theta$$

$$\Delta y = \Delta r \sin\theta$$

$$\tan\theta = \frac{\Delta y}{\Delta x}$$

Motion in Two Dimensions

درسنا في الفصل السابق الحركة في بعد واحد اي عندما يتحرك الجسم في خط مستقيم على محور xاو ان يسقط الجسم سقوطا حرا في محورy,سندرس الان حركة الجسم في بعدين اي في كل من x,y مثل حركة المقذوفات حيث يكون للإزاحة والسرعة مركبتان في اتجاه المحور xوالمحور y.

Motion in two dimensions like the motion of projectiles and satellites and the motion of charged particles in electric fields. Here we shall treat the motion in plane with constant acceleration.

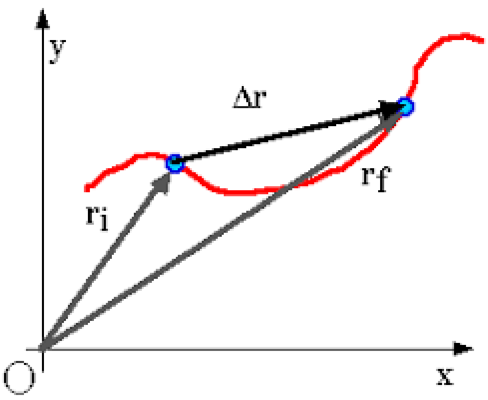
In order to study motion in two dimensions , we need to use the concepts of vectors.

Position , velocity , and acceleration in two dimensions are determined by not only specifying their magnitude ,but also their direction.

Displacement vector in 2D

Assume that the magnitude and direction of acceleration remain unchanged during the motion .

The position vector for particle moving in two dimensions (x y plane) can be written as



$$r = x_i \ \ + y_j$$

Where x , y and r change with time as the particle moves

The displacement vector $\Delta\vec{r}$ for a particle

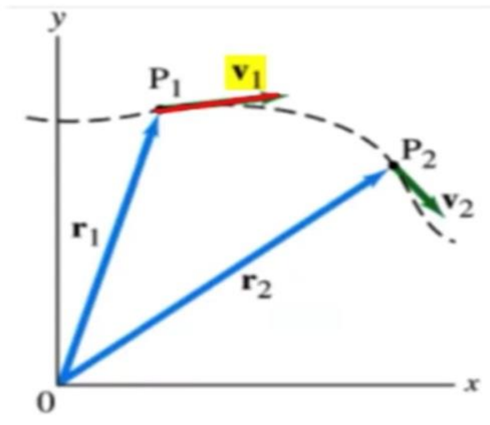
$$\Delta\vec{r} = r_f - r_i$$

يتحرك جسم على المسار المنحنى في البعدين x ,y يحدد المتجه r_1 موضع الجسم عند الزمن t_1 ويحدد المتجة r_2 موضعه عند الزمن t_2 وتكون الازاحة ممثلة بالمتجه $\Delta\vec{r}$

The Velocity Vector in 2D

The direction of the instantaneous velocity $\vec{v}$ of a particle is tangent to the path at the particles position.

The velocity of the particle as function Of time is given by :



$$\vec{v}=\frac{d\vec{r}}{dt}=\frac{dx}{dt}\,\text{i}+\frac{dy}{dt}\,\text{j} \longrightarrow \vec{v}=v_x\text{i}+v_y\text{j}$$

We define the average velocity $\overrightarrow{v_{avg}}$ of particle during the time interval Δt as the displacement of the particle divided by the time interval:

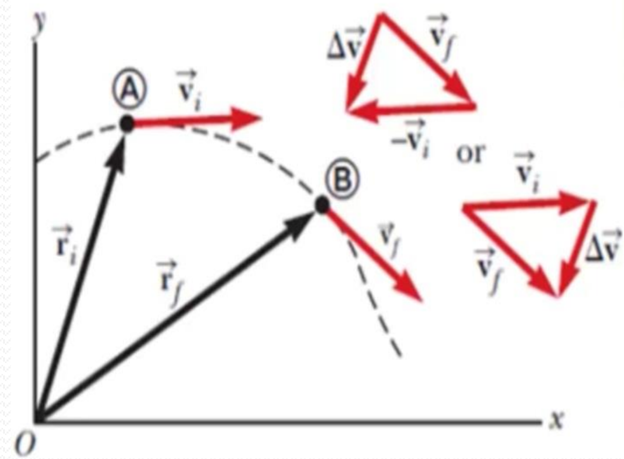
$$\overrightarrow{v_{avg}}=\frac{\Delta\vec{r}}{\Delta t}$$

Because displacement is a vector quantity and the time interval is a positive scalar quantity ,

We conclude that the average velocity is a vector quantity directed along $\overrightarrow{\Delta r}$.

The Acceleration Vector in 2D

As a particle moves from one point to Another along some path, its instantaneous Velocity vector changes from $\overrightarrow{v_i}$ at time t_i to $\overrightarrow{v_f}$ at time t_f. Knowing the velocity at these points allow Us to determine the average acceleration of the particle.



The average acceleration $\overrightarrow{a_{avg}}$ of a particle is defined As the change in its instantaneous velocity vector $\Delta\vec{v}$ Divided by the time interval Δt during which that change occurs.

$$\overrightarrow{v_{avg}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\overrightarrow{v_f} - \overrightarrow{v_i}}{t_f - t_i}$$

The acceleration of the particle as a function of time is given by

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\,\hat{i} + \frac{dv_y}{dt}\,\hat{j}$$

$$\vec{a} = a_x\,\hat{i} + a_y\,\hat{j}$$

One Dimension

Position: x

Displacement: Δx

Velocity : displacement per unit time .sign is equal to the sign of the displacement Δx.

Acceleration :change in velocityΔv *per* unit time .sign is equal to

The sign of the velocity deference Δv.

Two Dimension

Position vector : r

Displacement vector : Δr

Velocity vector: change in the position vector per unit time. The direction is equal to the direction of displacement vectorΔr .

Acceleration vector :change in velocity vector per unit time .The direction is equal to the direction of the velocity deference vector Δv.

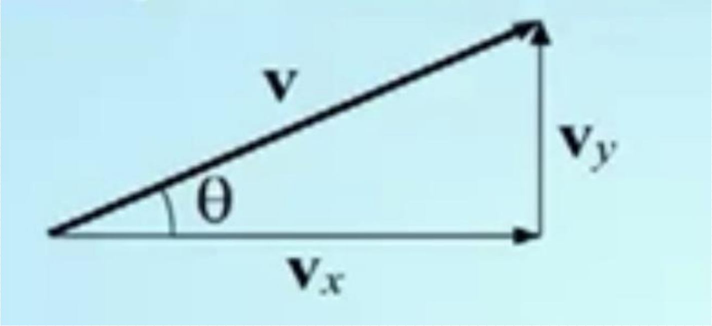
Motion in Two Dimensions

The motion in two dimensions can be modeled as two independent motions in each of the two perpendicular directions associated with the x and y axes. That is ,any influence in direction does not affect the motion in the x direction and vice versa.

عند التعامل مع مسائل الحركة في بعدين علينا ان نتعامل مع مركبات الحركة كلا على حدا

Motion in Two Dimensions

Velocity in two Dimensions



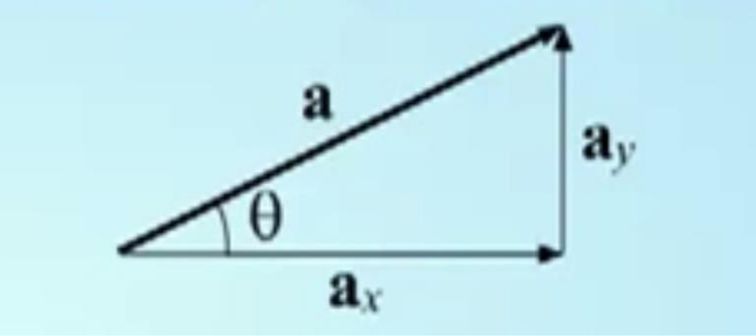
$v_x = v\cos\theta$

$v_y = v\sin\theta$

$|v| = \sqrt{{v_x}^2 + {v_y}^2}$

$\tan\theta = \frac{v_y}{v_x}$

Acceleration in Two Dimensions



$a_x = a\cos\theta$

$a_y = a\sin\theta$

$|a| = \sqrt{{a_x}^2 + {a_y}^2}$

$\tan\theta = \frac{a_y}{a_x}$

kinematics Equations With Constant Acceleration

- Constant acceleration.
- Constant acceleration equations hold in each dimension

$$v_x = v_{0x} + a_x t$$

$$v_y = v_{0y} + a_y t$$

$$x - x_0 = v_{0x} t + \tfrac{1}{2} a_x t^2$$

$$y - y_0 = v_{0y} t + \tfrac{1}{2} a_y t^2$$

$$v_x{}^2 = v_{0x}{}^2 + 2a_x (x - x_0)$$

$$v_y{}^2 = v_{0y}{}^2 + 2a_y (y - y_0)$$

Example(1)

A ball is projected horizontally with a velocity v_0 of magnitude $5m/s$ .Find its position and velocity after $0.25s$.

Solution

The initial angle is 0.The initial vertical velocity component is therefore 0.

The horizontal velocity component equal the initial velocity and is constant.

$x = v_0$ t =$5 \times 0.25 = 1.25m$

$y = -\frac{1}{2}gt^2 = -0.306m$

The distance of the projectile is given by $r = \sqrt{x^2 + y^2}$ =$1.29m$

The component of velocity are $v_x = v_{0x} + a_x t$ → $v_x = v_0 = 5m/s$

$v_y = v_{0y} + a_y t$ → $v_y = -gt = -2.45m/s$

The resultant velocity is given by $v = \sqrt{{v_x}^2 + {v_y}^2}$ = 5.57m/s

The angle θ is given $\theta = \tan^{-1}(\frac{-2.45)}{5}) = -26.1°$

Example(2)

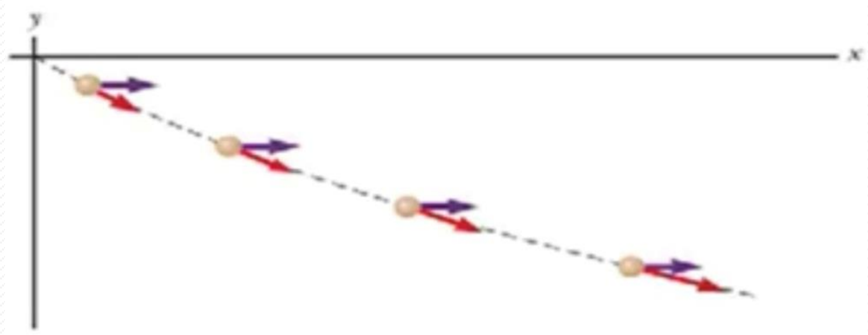
Particle moves in the x y plane , starting from the origin at $t = 0$ with an initial velocity having an x-component of $20m/s$ and a y-component of $-15m/s$.The particle experiences an acceleration in the x direction , given by $a_x = 4.0m/s^2$.

A) Determine the total velocity vector at any time.

B) Calculate the velocity and speed of the particle at $t = 5.0s$ and the angle the velocity vector makes with the x axis.

Solution

A)



نلاحظ من مركبات السرعة الابتدائية ان الجسم يتحرك في بعدين بحيث ان المركبة Xللسرعة تتأثر بعجلة ثابتة مع الزمن , في حين ان المركبة Yتبقى ثابتة مع الزمن لعدم وجود تعجيل في هدا الاتجاه .

نلاحظ من الشكل الموضح ان متجه السرعة يزداد مع الزمن.

v_{xi}=20m/s $v_{yi} = -15m/s$ $a_x = 4\, {}^{m}/_{s^2}$ and a_y=o

$$v = v_0 + a\,t$$

$$v = (v_{x0} + a_x t)\, i + (v_{y0} + a_y t)\, j$$

$$v = (20 + 4t)i + (-15 + 0t)j$$

$$v = (20 + 4\, t)i \; - 15\, j$$

Notice that the x component of velocity increases in time while the y component remains .

B)

لا يجاد متجه سرعة الجسم بعد مرور 5 ثواني نعوض في المعادلة:

$$v = (20 + 4\, t)i \; - 15\, j$$

$$v = (20 + 4 \times 5)i \; - 15\, j = (40i \; - 15\, j)mls$$

$$\theta = \tan^{-1}(\frac{-15}{40}) \; = -21°$$

The negative sign for the angle θ indicates that the velocity vector is directed at angle of 21° below the positive x axes.