Optimizing Cutting Speed:

- The mathematical formulas that have been derived to determine optimal cutting speed for a machining operation, allow the optimal cutting speed to be calculated for either of two objectives: (1) maximum production rate, or (2) minimum unit cost.

1- Maximizing Production Rate

- Minimizing cutting time per unit is equivalent to maximizing production rate.
- In turning, there are three time elements that contribute to the total production cycle time for one part:

a. **Part handling time T_h**: This is the time the operator spends loading the part into the machine tool at the beginning of the production cycle and unloading the part after machining is completed. Any additional time required to reposition the tool for the start of the next cycle should also be included here.

b. **Machining time T_m**: This is the time the tool is actually engaged in machining during the cycle.

c. **Tool change time T_t**: At the end of the tool life, the tool must be changed, which takes time. This time must be apportioned over the number of parts cut during the tool life. Let n_p= the number of pieces cut in one tool life (the number of pieces cut with one cutting edge until the tool is changed); thus, the tool change time per part = T_t / n_p.

- The total time per unit product for the operation cycle will be (T_c):

$$T_c = T_h + T_m + \frac{T_t}{n_p}$$

T_c: Production cycle time per piece, min.

- The relationship between cutting speed and required time is explained as in figure (4-35).

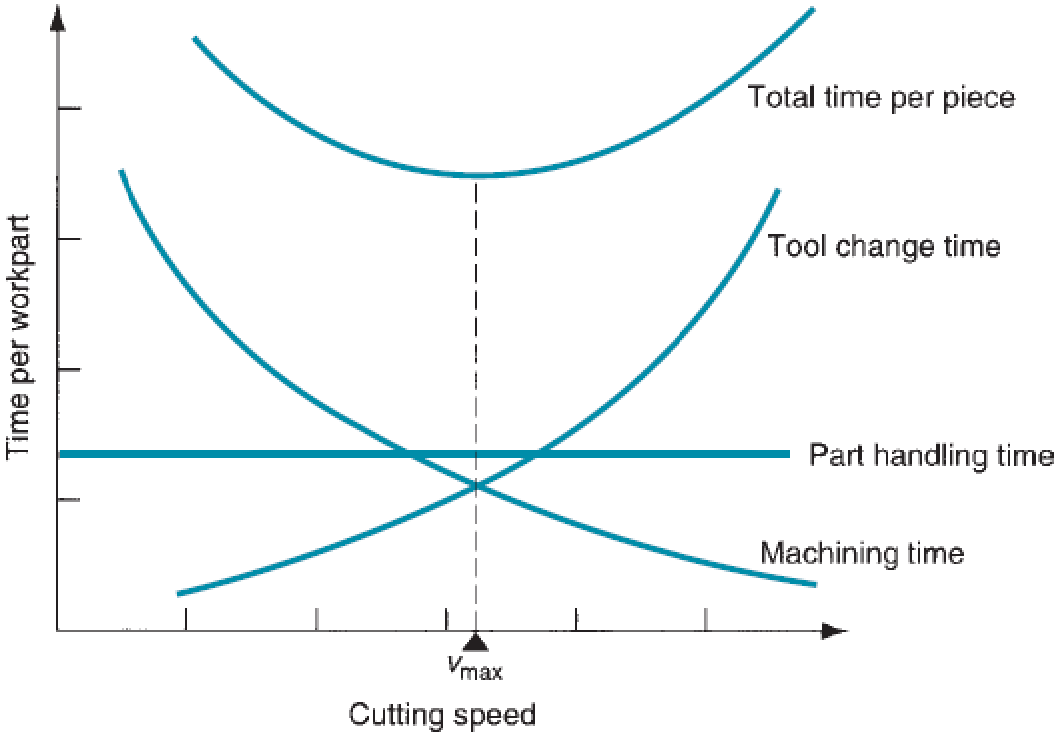

Figure (4-35) Time elements in a machining cycle plotted as a function of cutting speed. Total cycle time per piece is minimized at a certain value of cutting speed. This is the speed for maximum production rate.

- The number of pieces per tool n_p is also a function of speed.

$$n_p = \frac{T}{T_m}$$

T: tool life, min/tool;

T_m: machining time per part, min/piece.

- Both T and T_m are function of speed then:

$$vT^n = C \quad \text{(Taylor tool life equation)} \quad \text{then} \quad T = \left(\frac{C}{v}\right)^{1/n}$$

$$T_m = \frac{\pi D_o L}{vf}$$

v: cutting speed (m/min),

f: feed (m, mm, ft),

D_o: original w.p. diameter,

L: w.p. length,

n, *C*: are parameters whose values depend on feed, depth of cut, work material, tooling (material in particular), and the tool life criterion used.

Thus,

$$n_p = \frac{\left(\frac{C}{v}\right)^{1/n}}{\frac{\pi D_o L}{vf}} = \frac{vf\,\frac{C^{1/n}}{v^{1/n}}}{\pi D_o L} = \frac{vfC^{1/n}}{\pi D_o L v^{1/n}} = \frac{fC^{1/n}}{\pi D_o L v^{(1-n/n)}}$$

This will cause the T_t /n_p term in production time equation to increase as cutting speed increases.

Thus,

$$T_c = T_h + \frac{\pi D_o L}{vf} + \frac{T_t\left(\pi D_o L v^{(1-n/n)}\right)}{fC^{1/n}}$$

The cycle time per piece is a minimum at the cutting speed at which the derivative of T_c equation relative to v is zero:

$$\frac{dT_c}{dv} = 0$$

Then,

$$\frac{dT_c}{dv} = 0 = 0 + \frac{\pi D_o L}{f}\frac{(-1)}{v^2} + \frac{T_t \pi D_o L}{fC^{1/n}}\left(\frac{1-n}{n}\right)v^{(1-2n)/n}$$

Then,

$$v_{max} = \frac{C}{\left[T_t\left(\frac{1-n}{n}\right)\right]^n}$$

v_{max} is expressed in m/min (ft/min).

The corresponding tool life for maximum production rate is:

$$\therefore T_{max} = T_t\left(\frac{1-n}{n}\right)$$

2- Minimizing Cost per Unit

- For minimum cost per unit, the speed that minimizes production cost per piece for the operation is determined.
- To derive the equations for this case, we begin with the four cost components that determine total cost of producing one part during a turning operation:

1. **Cost of part handling time:** this is the cost of the time the operator spends loading and unloading the part. Let **C_o = the cost rate (e.g., \$/min)** for the operator and machine. Thus **the cost of part handling time = C_oT_h.**
2. **Cost of machining time:** this is the cost of the time the tool is engaged in machining. Using **C_o** again to represent the cost per minute of the operator and machine tool, **the cutting time cost = C_oT_m.**
3. **Cost of tool change time:** the **cost of tool change time per part = C_oT_t/n_p.**
4. **Tooling cost:** in addition to the tool change time, the tool itself has a cost that must be added to the total operation cost. This cost is the cost per cutting edge **C_t**, divided by the number of pieces machined with that cutting edge n_p.
   Thus, tool cost per w.p. is given by C_t/n_p.

<u>Note:</u> Tooling cost depends on tooling situations:
A. for disposable inserts (e.g., cemented carbide inserts), the tool cost is:

$$C_t = \frac{P_t}{n_e}$$

C_t: cost per cutting edge, \$/tool life.
P_t: price of the insert, \$/insert.
n_e: number of cutting edges per insert.
For triangular inserts that used one side have three edges/insert.
For triangular inserts that used both sides of the insert have six edges/insert.

B. for re-grindable tooling (e.g., high-speed steel solid shank tools, brazed carbide tools), the tool cost is:

$$C_t = \frac{P_t}{n_e} + T_gC_g$$

C_t: cost per tool life, \$/tool life
P_t: purchase price of the solid shank tool or brazed insert, \$/tool.
n_e: number of times the tool can be grind before it can no longer be used (5 to 10 times for roughing tools and 10 to 20 times for finishing tools).
T_g: time to grind or regrind the tool, min/tool life.
C_g: grinder's rate, \$/min.

- The total cost per unit product for the machining cycle (C_c):

$$C_c = C_o T_h + C_o T_m + \frac{C_o T_t}{n_p} + \frac{C_t}{n_p}$$

C_c is a function of cutting speed v. The relationships for the individual terms and total cost as a function of cutting speed are shown in Figure (4-36).

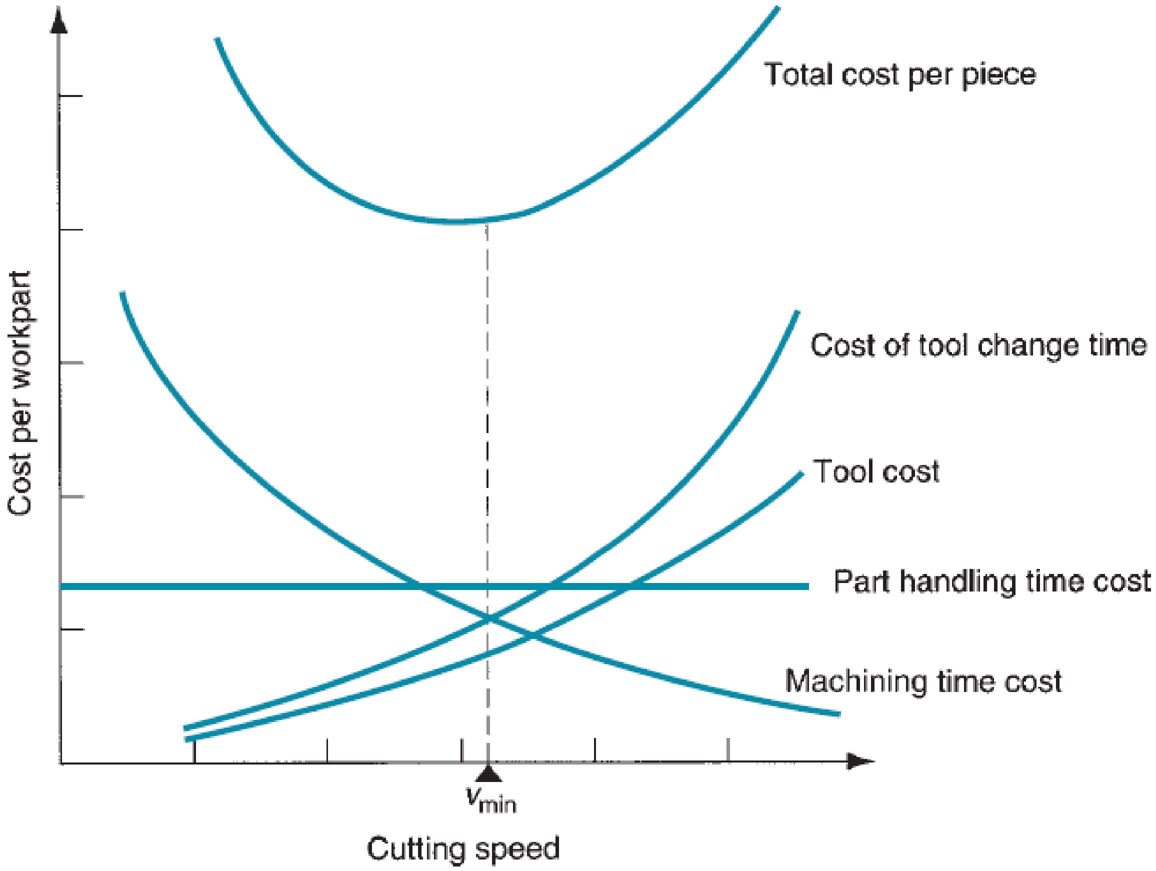

Figure (4-36) Cost components in a machining operation plotted as a function of cutting speed. Total cost per piece is minimized at a certain value of cutting speed. This is the speed for minimum cost per piece.

C_c can be rewritten in terms of v to yield:

$$C_c = C_o T_h + C_o \frac{\pi D_o L}{vf} + \frac{(C_o T_t + C_t)\left(\pi D_o L v^{(1-n/n)}\right)}{f C^{1/n}}$$

The minimum cost per piece for the operation can be determined by taking the derivative of C_c with respect to v, setting it to zero, and solving for v_{min}:

$$\frac{dC_c}{dv} = 0$$

Then,

$$\frac{dC_c}{dv} = 0 = 0 + C_o \frac{\pi D_o L}{f} \frac{(-1)}{v^2} + \frac{(C_o T_t + C_t)\pi D_o L}{f C^{1/n}} \left(\frac{1-n}{n}\right) v^{(1-2n)/n}$$

Then,

$$\therefore v_{min} = C \left(\left(\frac{n}{1-n}\right) \frac{C_o}{(C_o T_t + C_t)} \right)^n$$

v_{min} is expressed in m/min (ft/min).

The corresponding tool life for minimum cost is given by:

$$\therefore T_{min} = \left(\frac{1-n}{n} \left(\frac{C_o T_t + C_t}{C_o} \right) \right)$$

Example (10):
Suppose a turning operation is to be performed with HSS tooling on mild steel, with Taylor tool life parameters n=0.125, C=70 m/min . Workpart length = 500mm and diameter = 100 mm. Feed = 0.25 mm/rev. Handling time per piece = 5.0 min, and tool change time = 2.0 min. Cost of machine and operator = \$30/hr, and tooling cost = \$3 per cutting edge. Find: (a) cutting speed for maximum production rate, and (b) cutting speed for minimum cost (c) the hourly production rate and cost per piece for the two cutting speeds computed.

Solution:
(a) cutting speed for maximum production rate:

$$v_{max} = \frac{C}{\left[T_t\left(\frac{1-n}{n}\right)\right]^n} = \frac{70}{\left[2\left(\frac{1-0.125}{0.125}\right)\right]^{0.125}} = 50.3\ m/min \quad \textbf{Answer}$$

(b) cutting speed for minimum cost:

C_o=30 \$/hr = 30\$/60=0.5\$/min

C_t=3\$

T_t=2min

$$v_{min} = C \left(\left(\frac{n}{1-n}\right) \frac{C_o}{(C_o T_t + C_t)} \right)^n = 70 \left(\left(\frac{0.125}{1-0.125}\right) \frac{0.5}{(0.5(2)+3)} \right)^{0.125} = 42.3\ m/min \quad \textbf{Answer}$$

(c) <u>For v_{max}=50.3m/min</u>

The production cycle time per piece: $T_c = T_h + T_m + \frac{T_t}{n_p}$

T_c: Production cycle time per piece, min.
T_h=5min, T_t=2min

$$T_m = \frac{\pi D_o L}{vf} = \frac{\pi(0.1)(0.5)}{50.3(0.25(10^{-3}))} = 12.49min$$

$$T = \left(\frac{C}{v}\right)^{1/n} = \left(\frac{70}{50.3}\right)^{1/0.125} = 14.1\ min$$

$n_p = \frac{T}{T_m} = \frac{14.1}{12.49} = 1.13\ number\ of\ pieces\ per\ tool \rightarrow \therefore n_p =$
$1\ \ to\ avoid\ failure\ during\ the\ 2nd\ workpiece$

$$\therefore T_c = T_h + T_m + \frac{T_t}{n_p} = 5 + 12.49 + \frac{2}{1} = 19.49\ min/piece$$

1 hr=60min, ∴ Hourly production rate=60/19.49=**3.1 pieces/hr** <u>**Answer**</u>

$C_c = C_o T_h + C_o T_m + \frac{C_o T_t}{n_p} + \frac{C_t}{n_p} = 0.5(5) + 0.5(12.49) + \frac{0.5(2)}{1} + \frac{3}{1} = \mathbf{12.745}\$/\boldsymbol{piece}$ <u>**Answer**</u>

<u>For v_{min}=42.3m/min</u>

$$T_m = \frac{\pi D_o L}{vf} = \frac{\pi(0.1)(0.5)}{42.3(0.25(10^{-3}))} = 14.85min$$

$$T = \left(\frac{C}{v}\right)^{1/n} = \left(\frac{70}{42.3}\right)^{1/0.125} = 56.24\ min$$

$n_p = \frac{T}{T_m} = \frac{56.24}{14.85} = 3.787\ number\ of\ pieces\ per\ tool \rightarrow \therefore n_p =$
$3\ \ to\ avoid\ failure\ during\ the\ 4th\ workpiece$

$$\therefore T_c = T_h + T_m + \frac{T_t}{n_p} = 5 + 14.85 + \frac{2}{3} = 20.52\ min/piece$$

1 hr=60min, ∴ Hourly production rate=60/20.52=**2.9 pieces/hr** <u>**Answer**</u>

$C_c = C_o T_h + C_o T_m + \frac{C_o T_t}{n_p} + \frac{C_t}{n_p} = 0.5(5) + 0.5(14.85) + \frac{0.5(2)}{3} + \frac{3}{3} = \mathbf{11.26}\$/\boldsymbol{piece}$ <u>**Answer**</u>

Example (11):
A high-speed steel tool is used to turn a steel workpart that is 300 mm long and 80 mm in diameter. The parameters in the Taylor equation are: n = 0.13 and C = 75 (m/min) for a feed of 0.4 mm/rev. The operator and machine tool rate = $30/hr, and the tooling cost per cutting edge = $4. It takes 2 min to load and unload the workpart and 3.5 min to change tools. Determine (a) cutting speed for maximum production rate, (b) cutting speed for minimum cost, (c) tool life of cutting in (a) and (b), (d) cycle time and cost per unit of product in (a).

Solution:

a. T_t=3.5min, then $v_{max} = \frac{C}{\left[T_t\left(\frac{1-n}{n}\right)\right]^n} = \frac{75}{\left[3.5\left(\frac{1-0.13}{0.13}\right)\right]^{0.13}} = \mathbf{49.8\ m/min}$ **Answer**

b. C_o=30 $/hr = 30$/60=0.5$/min

C_t=4$

T_t=3.5min

$$v_{min} = C\left(\left(\frac{n}{1-n}\right)\frac{C_o}{(C_oT_t+C_t)}\right)^n = 75\left(\left(\frac{0.13}{1-0.13}\right)\frac{0.5}{(0.5(3.5)+4)}\right)^{0.13} = \mathbf{42.6\ m/min}$$ **Answer**

c. For (a) $T_{max} = T_t\left(\frac{1-n}{n}\right) = 3.5\left(\frac{1-0.13}{0.13}\right) = \mathbf{23.4\ min}$ **Answer**

For (b) $T_{min} = \left(\frac{1-n}{n}\left(\frac{C_oT_t+C_t}{C_o}\right)\right) = \left(\frac{1-0.13}{0.13}\left(\frac{0.5(3.5)+4}{0.5}\right)\right) = \mathbf{76.96\ min}$ **Answer**

d. The production cycle time per piece: $T_c = T_h + T_m + \frac{T_t}{n_p}$

T_h=2min, T_t=3.5min

$$T_m = \frac{\pi D_o L}{v_{max} f} = \frac{\pi(0.08)(0.3)}{49.8(0.4(10^{-3}))} = 3.785 min$$

$T_{max} = 23.4\ min$ (tool life)

$n_p = \frac{T}{T_m} = \frac{23.4}{3.785} = 6.18\ number\ of\ pieces\ per\ tool \rightarrow \therefore n_p = 6\ to\ avoid\ failure\ during\ the\ 7th\ workpiece$

$\therefore T_c = T_h + T_m + \frac{T_t}{n_p} = 2 + 3.785 + \frac{3.5}{6} = \mathbf{6.368\ min/piece}$ **Answer**

$C_c = C_oT_h + C_oT_m + \frac{C_oT_t}{n_p} + \frac{C_t}{n_p} = 0.5(2) + 0.5(3.785) + \frac{0.5(3.5)}{6} + \frac{4}{6} = \mathbf{3.851}\ \$/\boldsymbol{piece}$ **Answer**