Vectors (Applications 1)

Example: Find the plane through P_o (2, 4, 5) perpendicular to the line;

$x=5+t$, $y = 1 + 3t$, $z = 4t$

Solution:

The line $x=5+t$, $y = 1 + 3t$, $z = 4t$ has a vector

$v = i + 3j + 4k$

The line vector is parallel to the same line.

The required plane is perpendicular to the given line. Therefore, it is perpendicular to its vector.

The standard equation of the plane is;

$n1\ (x - x1) + n2\ (y - y1) + n3\ (z - z1) = 0$

Where;

$(x1, y1, z1)$ a point in the plane. In this problem we have Po $(2, 4, 5)$

n1, n2 and n3 are coefficient of the normal vector

Normal vector

$v = n1\ i + n2\ j + n3\ k$

Then,

$(x - 2) + 3\ (y - 4) + 4\ (z - 5) = 0$

$x - 2 + 3\ y - 12 + 4\ z - 20 = 0$

$x - 3y + 4\ z = 34$

Example: Find the plane containing the intersecting lines.

L1: x = – 1 + t , y = 2 + t , z = 1 – t ; $-\infty < t < \infty$

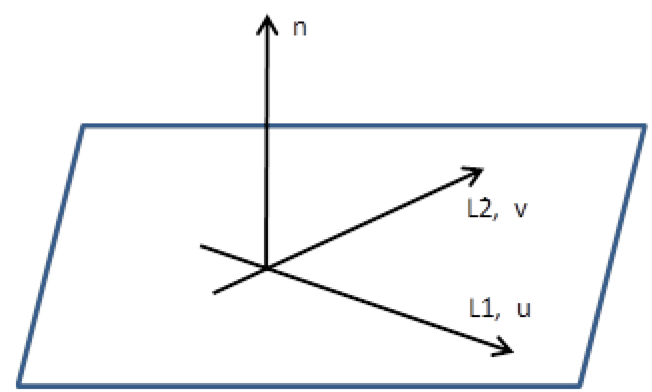
L2: x = 1 – 4s , y = 1 + 2 s , z = 2 – 2 s ; $-\infty < s < \infty$

Solution:

L1 has a vector u = i + j – k

L2 has a vector v = – 4 i + 2 j – 2 k

$$n = u \times v = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ -4 & 2 & -2 \end{vmatrix} = 6j + 6k$$



Choose any of the points from either L1 or L2

Use (-1, 2 , 1) from L1

The standard equation of the plane is;

$n1\,(x - x1) + n2\,(y - y1) + n3\,(z - z1) = 0$

Then

$0\,(x + 1) + 6\,(y - 2) + 6\,(z - 1) = 0$

$6y - 12 + 6z - 6 = 0$

$6y + 6z = 18$

$y + z = 3$

Example: Find the distance from the plane $x + 2y + 6z = 1$ to the plane $x + 2y + 6z = 10$.

Solution:

Distance between point and plane = $\left|\overrightarrow{PS} \cdot \frac{n}{|n|}\right|$

P point in the plane

S the point that required to find its distance from the plane.

n the normal vector to the plane

Select P from the first plane by setting $y = 0$ and $z = 0$ then $x = 1$ yields $P(1, 0, 0)$

$n = i + 2j + 6k$ (normal to the first plane)

Select S from the second plane by setting y = 0 and z = 0 then

x = 10 yields S (10, 0, 0)

$\overrightarrow{PS} = 9\text{i}$

D = $\left|\overrightarrow{PS} \cdot \frac{n}{|n|}\right| = = \left|9\, i \cdot \frac{i + 2j + 6k}{\sqrt{41}}\right| = \frac{9}{\sqrt{41}}$