Chapter One

Basic Concepts and Units

1.1 Modern Electron Theory:

It's states that all matter (solid, liquid, gaseous) consists of very small particles called molecules; which are made up rather small particles known as atoms.

An atom is the smallest part of chemical element that retains the chemical characteristics of that element.

The atoms consists of the following parts

1) Electrons: they have positive charge
2) Protons: they have negative charge
3) Neutron: they have no any charge (neutral)

Protons and Neutrons are exists in a central core called nucleus, while the electrons spin around the nucleus in one or more elliptical orbits

Mass of proton =1836 times of electron.

1.2 Electric Charge:

Electric charge is a characteristic property of many subatomic particles.

In physics, a **neutral particle** is a particle with no electric charge(no. of electrons=no of protons).

When some of electrons removed from neutral body, then it will charged positively.

While some of electrons are supplied to a neutral body, it will charged negatively.

Coulomb is the unit of charge

1Coulomb=charge on 6.28×10^{16}

Charge of electron $=\frac{1}{6.28*10^{16}} = 1.6 * 10^{-19}\ C$

example(1): How much charge is represented by 4,600 electrons?

Solution:

Each electron has −1.602 × 10−19 C. Hence 4,600 electrons will have

−1.602 × 10−19 C/electron × 4,600 electrons =−7.369 × 10−16 C

1.3 Electric Current:

Electric current is a flow of electric charge through a conductive medium.

In some type material which called conductors, electrons can move freely and randomly from one atom to another

A flow of positive charges gives the same *electric* current, and has the same effect in a circuit, as an equal flow of negative charges in the opposite direction. Since current can be the flow of either positive or negative charges, or both, a convention for the direction of current which is independent of the type of charge carriers is needed. **The direction of *conventional current* is defined arbitrarily to be the direction of the flow of positive charges.**

In metals, which make up the wires and other conductors in most electrical circuits, the positive charges are immobile, and the charge carriers are electrons. Because the electron carries negative charge, the *electron* motion in a metal conductor is in the direction opposite to that of conventional (or *electric*) current.

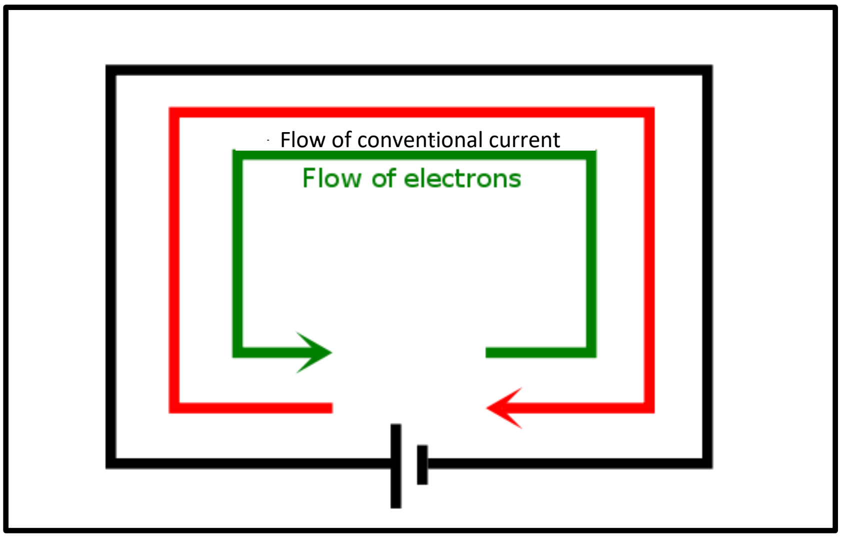


Fig.1 The direction of flow of conventional current

The strength of electric current I is the rate of change of electric charge in time

$$I = \frac{dQ}{dt}$$

Where:

I : current in Ampere (A)

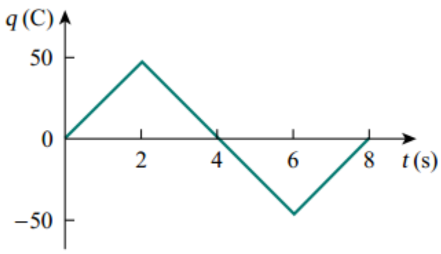
Q: charge in Coulomb (C)

t: time in seconds (sec)

example(2): flow of charge that passes throw a given reference point is shown below sketch the current wave form

1) for $0 \leq t < 2$ sec

$$I = \frac{\Delta Q}{\Delta t} = \frac{Q_2 - Q_1}{t_2 - t_1}$$



$$= \frac{50-0}{2-0} = 25 \text{ A}$$

2) for $2 \leq t < 6$ sec

$$I = \frac{\Delta Q}{\Delta t} = \frac{Q_2 - Q_1}{t_2 - t_1}$$

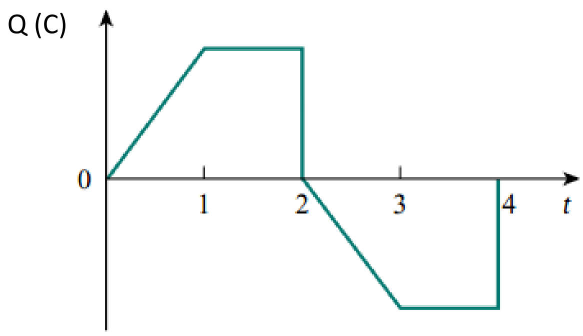
$$= \frac{-50-50}{6-2} = -25 \text{ A}$$

3) for $6 \leq t < 8$ sec

$$\backslash = \frac{\Delta Q}{\Delta t} = \frac{Q_2 - Q_1}{t_2 - t_1}$$

$$= \frac{0-50}{8-6} = 25 \text{ A}$$

H.W: flow of charge that passes throw a given reference point is shown below sketch the current wave form



The Conditions of Contineous Current Flow are:

1) a closed circuit around which the electron may travel along
2) there must be a source which cause the current.

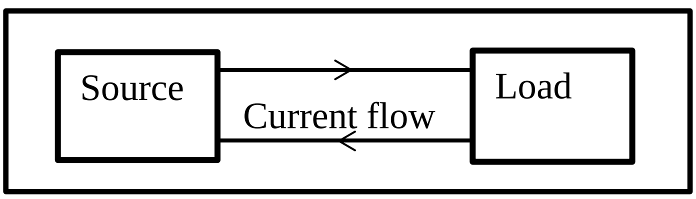


Fig.2 Simple circuit

The three essential parts of any electrical circuit are: a **power source** (like a battery or generator), a **load** (like a light lamp or motor) and **transmission system** (wires) to join them together. See Fig. 3.

A fourth part, that is always a very important thing to have in most circuits, is a **control device** such as a switch, circuit breaker or fuse.

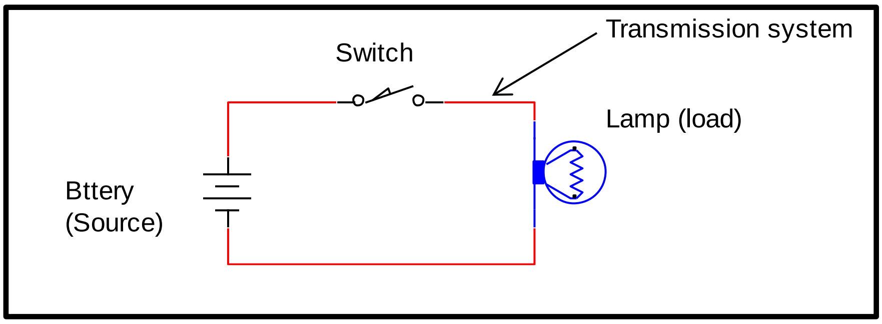


Fig.4 An example of basic circuit.

1.4 Electric Potential, Electric Potential Difference, and Electromotive Force:

Electric Potential: Is the work required by an electric field to move electric charges. Its unit is the volt. Also called *voltage*

Electric Potential= $\frac{work}{charge}$ $\frac{Jouls}{Coulomb}(Volt)$

Electric Potential Difference: It's the difference in the potential between two charged bodies. Or the work that has to be done in transferring unit positive charge from one point to the other.

Electromotive Force (emf): The energy per unit charge that is converted reversibly from chemical, mechanical, or other forms of energy into electrical energy in a battery or dynamo.

1.5 Electrical Power:

Electric power, like mechanical power, is the rate of doing work, measured in watts, and represented by the letter *P*.

If there is a q coulombs of charge moves as a result of potential difference V, the work (w) done will be

$$w = Vq \quad (Joules)$$

And the power (p) will be:

$$p = \frac{dw}{dt} = v\frac{dq}{dt} = vi \quad (watt)$$

1.6 The Principle of Ohm's law:

Ohm's law states that the current (I) through a conductor between two points is directly proportional to the potential difference (V) across the two points. Introducing the constant of proportionality,

$$\frac{V}{I} = constant = R$$

Where R is the resistance of the conductor measured in ohm (Ω).

The interchangeably of the equation may represented by a triangle, where V (voltage) is placed on the top section, the I (current) is placed to the left section, and the R (resistance) is placed to the right. The line that divides the left and right sections indicate multiplication, and the divider between the top and bottom sections indicates division (hence the division bar).

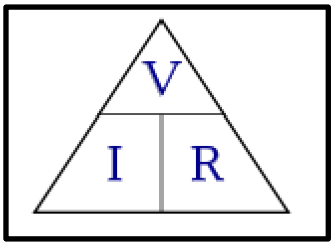


Fig.5 Ohm's triangle

if we plot voltage on the x-axis of a graph and current on the y-axis of the graph, we will get a straight-line. The gradient of the straight-line graph is related to the resistance of the conductor.

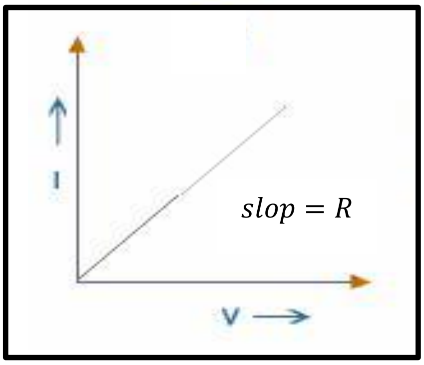


Fig.6 Graphical representation of Ohm's law

1.7 The SI Units:

In 1960 the Conference Generale des Poids et Mesures (CGPM), which is the international authority on the metric system, accepted a universal, practical system of units and gave it the name Le Systeme International d'Unites with the abbreviation SI. Since then, this most modern and simplest form of the metric system was introduced throughout the world.

The International System of Units consists of a set of units together with a set of prefixes. The units are divided into two classes—base units and derived units. There are seven base units, each representing, by convention, different kinds of physical quantities. see Table 1.

Table 1. the SI base units

Physical Quantity	Name of Unit	Abbreviation
Mass	Kilogram	kg
Length	Meter	m
Time	Second	s
Electric current	Ampere	A
Temperature	Kelvin	K

Since, in practice, one often needs to describe quantities that occur in large multiples or small fractions of a unit, standard prefixes are used to denote powers of 10 of SI (and derived) units. These prefixes are listed in Table 2.

Table 2. Standard prefixes.

Prefix	Symbol	Power
Pico	P	10^{-12}
nano	N	10^{-9}
micro	µ	10^{-6}
milli	M	10^{-3}
centi	C	10^{-2}
kilo	K	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}

The derived units are:

1. Area (A) m^2
2. Volume (V) m^3
3. Velocity (v) m/s
4. Acceleration (a) m/s^2
5. Angular velocity (ω) rad/sec
6. Work or energy (w) Joule
   w=F.*l* *(N.m) or* $(kg.m^2/s^2)$
7. Force F (Newton) or $(kg.m^2/s)$
8. Torque *T (N.m)*
9. *Power P (watt) or (J/s)*
10. *Potential v (Volt) or (J/C)*
11. *Charge Q (C)or (A.s)*
12. *Resistance (R) (Ω) or (V/A)*
13. *Kilowatthour (kwh)*= is a unit of energy which is used commercially

1 kwh =1000 w.h

= 1000 60 w.min*

*=1000*60*60 w.s*

$=3.6*10^{6}$ *J= 3.6 MJ*

14.Efficiency (η) (unitless)

$$\eta = \frac{output\ power}{input\ power} \times 100\%$$

15.Horse power hp =746w

Chapter Two

Analysis of Direct Current Circuits

2.1 Introduction:

An electric circuit is a closed path or combination of paths through which current can flow. Fig 2.1 shows a simple direct current circuit. The direct current (DC) starts from the positive terminal of the battery and comes back to the starting point via the load.

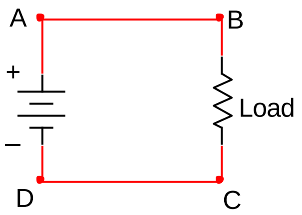


Fig 2.1 simple dc circuit

Electric Network: A combination of various electric elements, connected in any manner is called an electric network.

Node: is a junction in a circuit where two or more circuit elements are connected together.
Loop. It is a close path in a circuit in which no element or node is encountered more than once.

Mesh. It is a loop that contains no other loop within it.

For example, the circuit of Fig. 2.2 (a) has six nodes, three loops and two meshes whereas the circuit of Fig. 2.2 (b) has four branches, two nodes, six loops and three meshes.

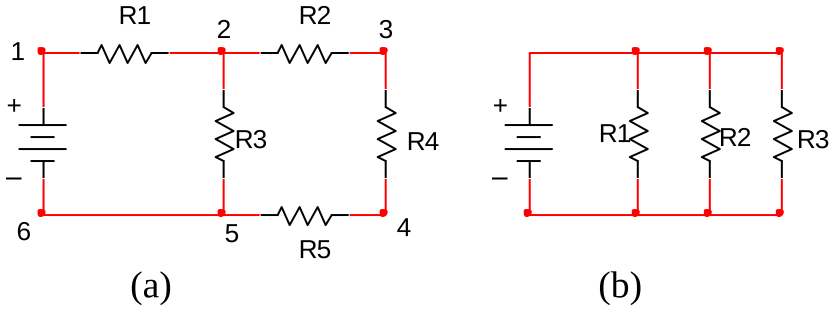
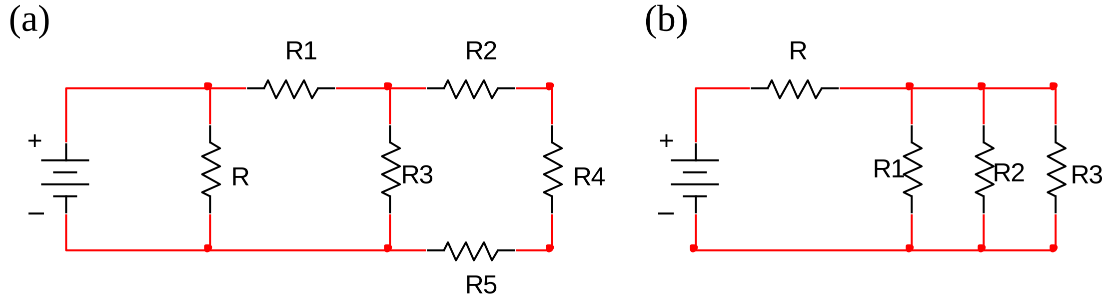
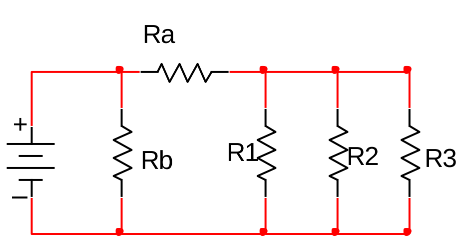


Fig 2.2

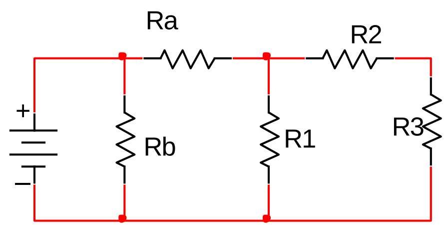
Example 2.1 find the numbers of nodes, meshes, loops in the following networks



(b)



(d)



* There are two general approaches to network analysis :

(i) Direct Method

Here, the network is left in its original form while determining its different voltages and currents. Such methods are usually restricted to fairly simple circuits and include Kirchhoff's laws, Loop analysis, Nodal analysis, superposition theorem, etc.

(ii) Network Reduction Method

Here, the original network is converted into a much simpler equivalent circuit for rapid calculation of different quantities. This method can be applied to simple as well as complicated networks. Examples of this method are : Delta/Star and Star/Delta conversions, Thevenin's theorem and Norton's Theorem etc.

2.2 Kirchhoff's Laws *

These laws are more comprehensive than Ohm's law and are

used for solving electrical networks which may not be readily solved

by the latter. Kirchhoff's laws, two in number, are particularly useful

(a) in determining the equivalent resistance of a complicated net-work of conductors and (b) for calculating the currents flowing in the

various conductors. The two-laws are :

1) **Kirchhoff's Point Law or Current Law (KCL)**

It states as follows :

in any electrical network, the algebraic sum of the currents meeting at a point (or junction or node) is zero.

Put in another way, it simply means that the total current leaving a junction is equal to the total current entering that junction. It is obviously true because there is no accumulation of charge at the junction of the network.

Consider the case of a few conductors meeting at a point A as in Fig. 2.2 (a). Some conductors have currents leading to point A, whereas some have currents leading away from point A. Assuming the incoming currents to be positive and the outgoing currents negative, we have

$I_1 + (-I_2) + (-I_3) + (+I_4) + (-I_5) = 0$

- or $I_1 + I_4 - I_2 - I_3 - I_5 = 0$ or $I_1 + I_4 = I_2 + I_3 + I_5$
- or incoming currents = outgoing currents
  Similarly, in Fig. 2.2 (b) for node A
  $+I + (-I_1) + (-I_2) + (-I_3) + (-I_4) = 0$ or $I = I_1 + I_2 + I_3 + I_4$

We can express the above conclusion thus : $\Sigma I = 0$ ...at a junction

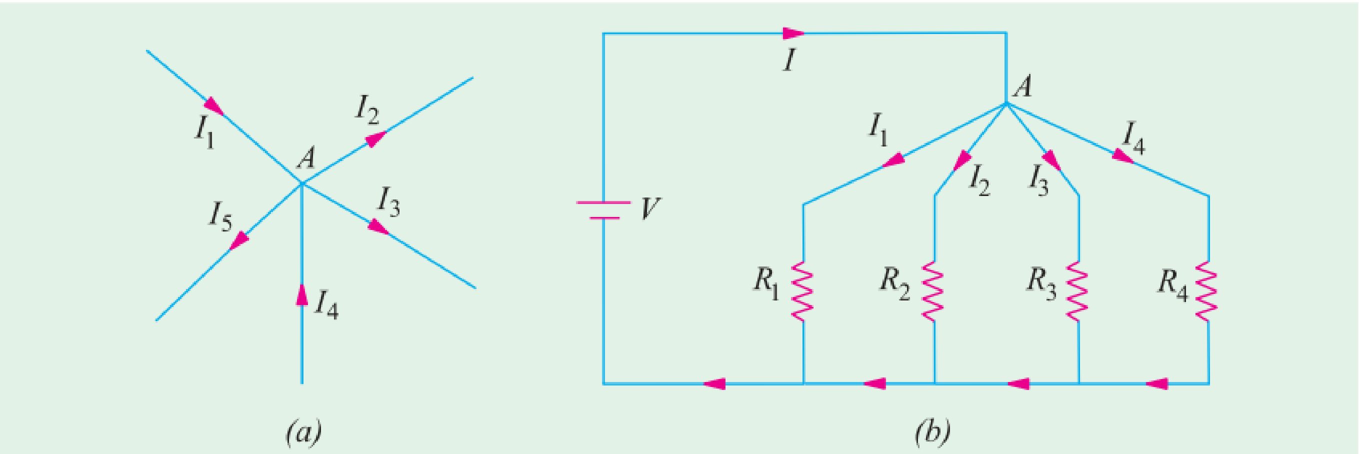


Fig 2.2

2) Kirchhoff's Mesh Law or Voltage Law (KVL)

It states as follows :
The algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the e.m.fs. in that path is zero.

- In other words, $\Sigma I_R + \Sigma$ e.m.f $= 0$ ...round a mesh
- It should be noted that algebraic sum is the sum which takes into account the polarities of the voltage drops.

2.2.1 **Determination of Voltage Sign**

In applying Kirchhoff's laws to specific problems, particular attention should be paid to the algebraic signs of voltage drops and e.m.fs., otherwise results will come out to be wrong. Following sign conventions is suggested :

(a) Sign of Battery E.M.F.

If we go from the −ve terminal of a battery to its +ve terminal (Fig. 2.3), and hence this voltage should be given a + ve sign, and vice versa.

- It should be noted that algebraic sum is the sum which takes into account the polarities of the voltage drops.

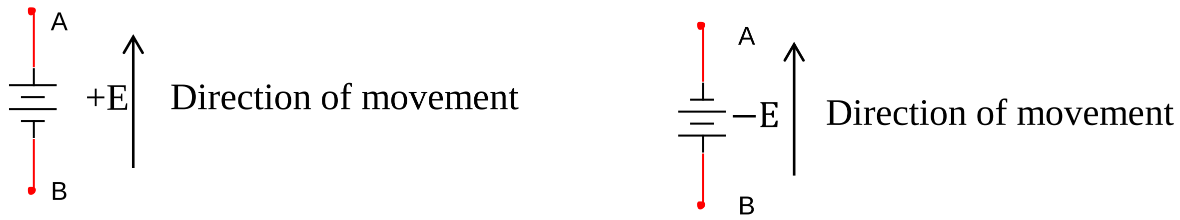


Fig 2.3

(b) Sign of IR Drop

Now, take the case of a resistor (Fig 2.4). If we go through a resistor in the same direction as the current, then there is a fall in potential because current flows from a higher to a lower potential. Hence, this voltage fall should be taken −ve. However, if we go in a direction opposite to that of the current, then there is a rise in voltage. Hence, this voltage rise should be given a positive sign. It is clear that the sign of voltage drop across a resistor depends on the direction of current through that resistor but is independent of the polarity of any other source of e.m.f. in the circuit
under consideration.

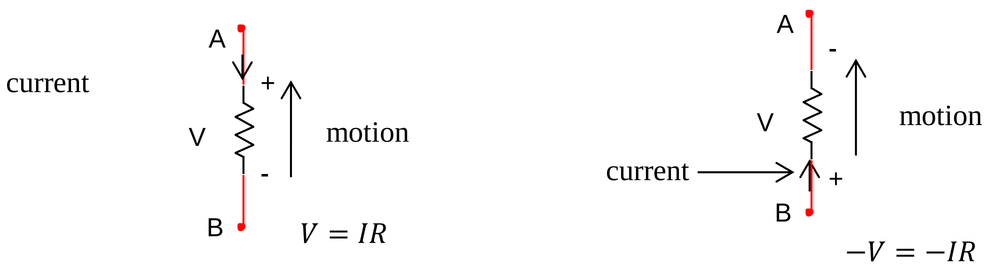


Fig 2.4

Consider the closed path ABCDA in Fig. 2.5. As we travel around the mesh in the clockwise direction, different voltage drops will have the following
signs :

I_1R_2 is − ve

I_2R_2 is − ve

I_3R_3 is + ve

I_4R_4 is − ve

E_2 is − ve

E_1 is + ve

Using Kirchhoff's voltage law, we get

$- I_1R_1 - I_2R_2 - I_3R_3 - I_4R_4 - E_2 + E_1 = 0$

or $I_1R_1 + I_2R_2 - I_3R_3 + I_4R_4 = E_1 - E_2$

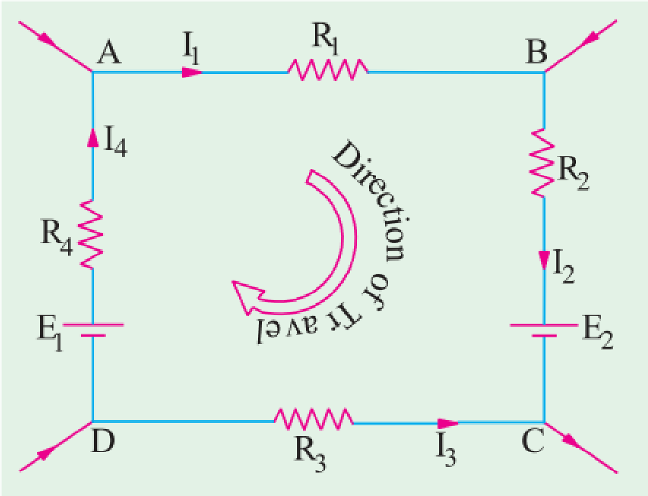


Fig 2.5

Example 2.1 What is the voltage V_s across the open switch in the circuit of Fig. 2.6 ?

Solution. We will apply KVL to find Vs. Starting from point A in the clockwise direction and
using the sign convention given in Art. 2.3, we have

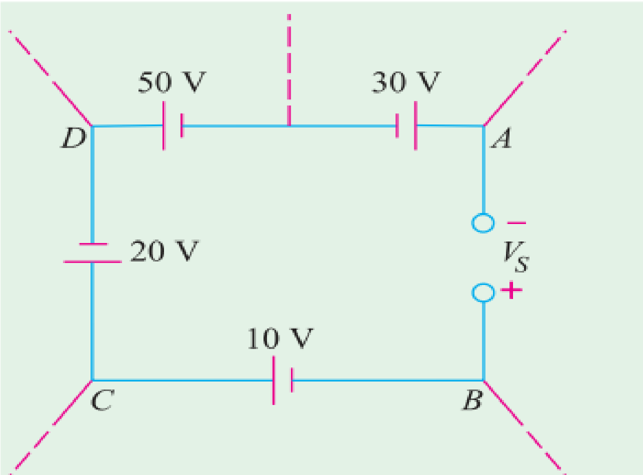


Fig 2.6

$$+V_s + 10 - 20 - 50 + 30 = 0$$

$$\therefore V_s = 30 \text{ V}$$

Example 2.2 Find the unknown voltage V_1 in the circuit of Fig. 2.7.

Solution. Initially, one may not be clear regarding the solution of this question. One may think of Kirchhoff's laws or mesh analysis etc. But a little thought will show that the question can be solved by the simple application of Kirchhoff's voltage law. Taking the outer closed loop ABCDEFA and applying KVL to it, we get

$-16 \times 3 - 4 \times 2 + 40 - V_1 = 0$;

$V_1 = -16$ V

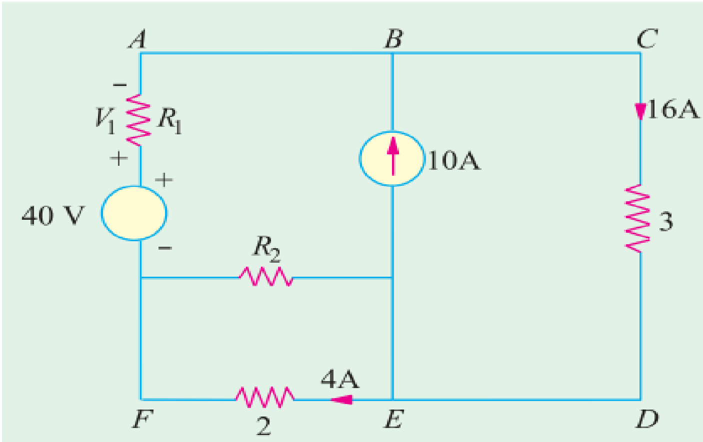


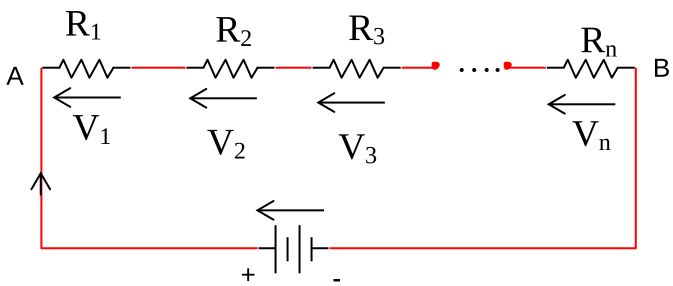
Fig 2.7

2.3 type of DC circuits:

DC circuit can be classified as:

1) series circuits
2) parallel circuits
3) series-parallel circuits

2.3.1 Series Circuits



$$V_1 + V_2 + V_3 + \cdots + V_n = E$$

$$I_1R_1 + I_2R_2 + I_3R_3 + \cdots + I_nR_n = E$$

$$I_1 = I_2 = \cdots = I$$

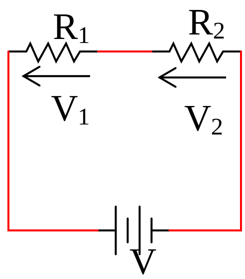
$$E = I(R_1 + R_2 + \cdots + R_3) = IR_{eq}$$

$$R_{eq} = (R_1 + R_2 + \cdots + R_3)$$

- Special case of series circuits (voltage divider)

$$I = \frac{V}{R_1+R_2}$$

$$V_1 = IR_1 = \frac{V}{R_1+R_2}R_1$$



Or

$$V_1 = V\frac{R_1}{R_1+R_2}$$

Similarly

$$V_2 = V\frac{R_2}{R_1+R_2}$$

In general for n resistances in series

$$V_x = V\frac{R_x}{R_T}$$

The equivalent resistance actually the resistance "seen" by the battery as it "looks" into the series combination of elements as shown in Fig. 2.8.

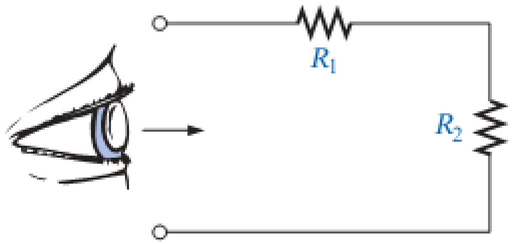


Fig 2.8

* Two elements are in series if

1. They have only one terminal in common (i.e., one terminal of the 1st element is connected to only one terminal of the other).
2. The common point between the two elements is not connected to another current-carrying element. Look to Fig 2.9

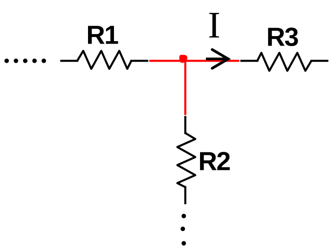
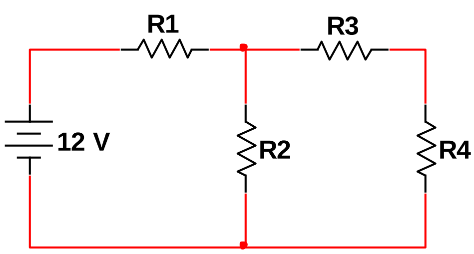
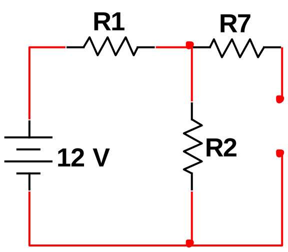
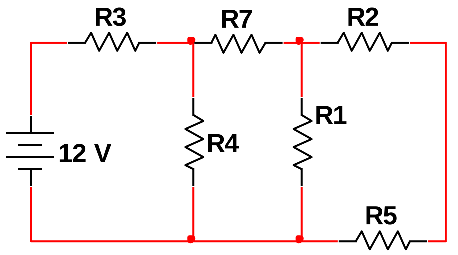


Fig 2.9

Example 2.3: In which one of the following circuits R_1 and R_2 are in series?







Example 2.4: a. Find the total resistance for the series circuit of Fig. 2.10
b. Calculate the source current I_s.
c. Determine the voltages V_1, V_2, and V_3.
d. Calculate the power dissipated by R_1, R_2, and R_3.
e. Determine the power delivered by the source, and compare it to the sum of the power levels of part (d).

Solutions:

a. $R_T = R_1 + R_2 + R_3 = 2 + 1 + 5 = 8\Omega$

b. $I_s = \frac{E}{R_T}$ =2.5 A

c. $V_1 = IR_1 = (2.5)(2\ ) = 5$ V

$V_2 = IR_2 = (2.5)(1) = 2.5$ V

$V_3 = IR_3 = (2.5)(5) = 12.5$ V

d. $P_1 = V_1 I_1 = (5)(2.5) = 12.5$ W

$P_2 = I_2^2\, R_2 = (2.5)^2\,(1) = 6.25\, W$

$$P_3 = \frac{V_3}{R_3} = \frac{(12.5\,)^2}{5} = 31.25\, W$$

e. $P_{del} = EI = (20\,)(2.5\,) = 50\, W$

$$P_{del} = P_1 + P_2 + P_3 = 31.25 + 12.5 + 6.25 = 50\ W$$

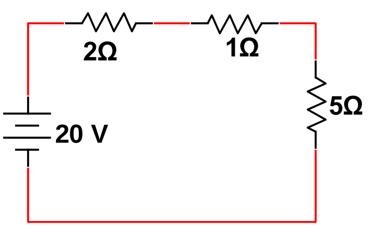


Fig 2.10

Example 2.5: Find the total resistance and current I for each circuit of Fig.2.11

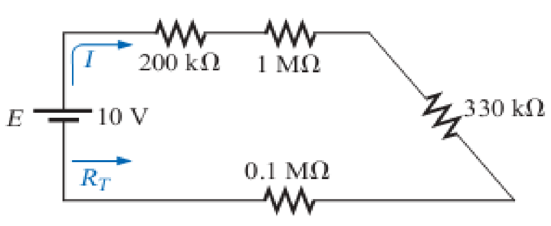


Fig 2.11

Example 2.6: Using Kirchhoff 's Current Law and (ı's Law, find the magnitude and polarity
of voltage V in Fig. 2.12. Directions of the two current sources are as shown.

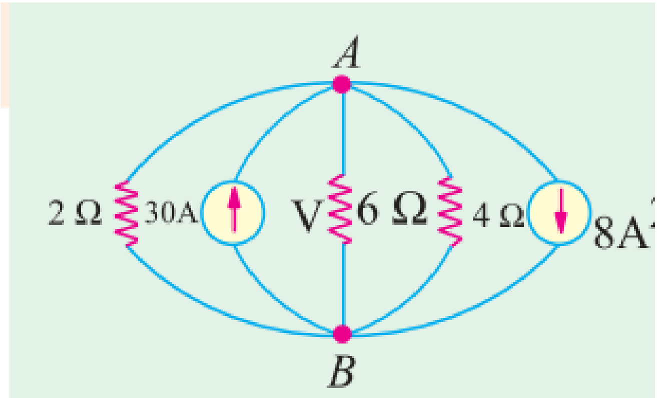


Fig 2.12

Solution:

Let us **arbitrarily** choose the directions of I_1, I_2 and I_3 and polarity of V as shown in Fig. 13.

Applying KCL to node A, we have

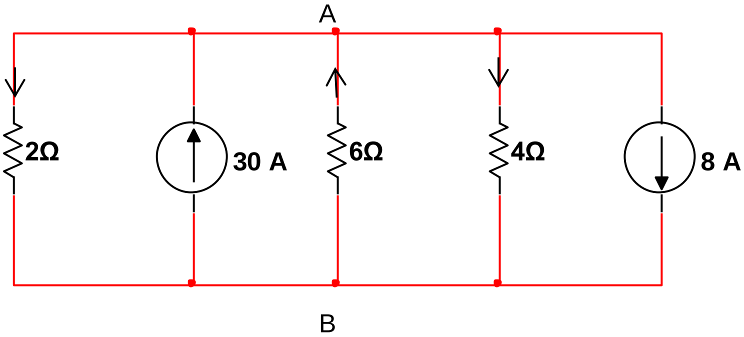


Fig 2.13

$- I_1 + 30 + I_2 - I_3 - 8 = 0$

or $I_1 - I_2 + I_3 = 22$ ...(i)

Applying Ohm's law to the three resistive branches in Fig. 2.13, we have

$I_1 = \frac{V}{2}, I_3 = \frac{V}{4}, I_2 = -\frac{V}{6}$ (Please note the − ve sign.)

Substituting these values in (i) above, we get

$\frac{V}{2} + \frac{V}{4} + \frac{V}{6} = 22 \Rightarrow$ V = 24 V

$\therefore I_1 = \frac{V}{2} = \frac{24}{2} = 12\ A, I_2 = -\frac{24}{6} = -4\ A, I_3 = \frac{24}{4} = 6\ A$

The negative sign of I_2 indicates that actual direction of its flow is opposite to that shown in Fig.

2.13. Actually, I_2, flows from A to B and not from B to A as shown.
Incidentally, it may be noted that all currents are outgoing except 30A which is an incoming current.

Example 2.7: Determine the current I and the voltage V for the network of Fig. 2.14.

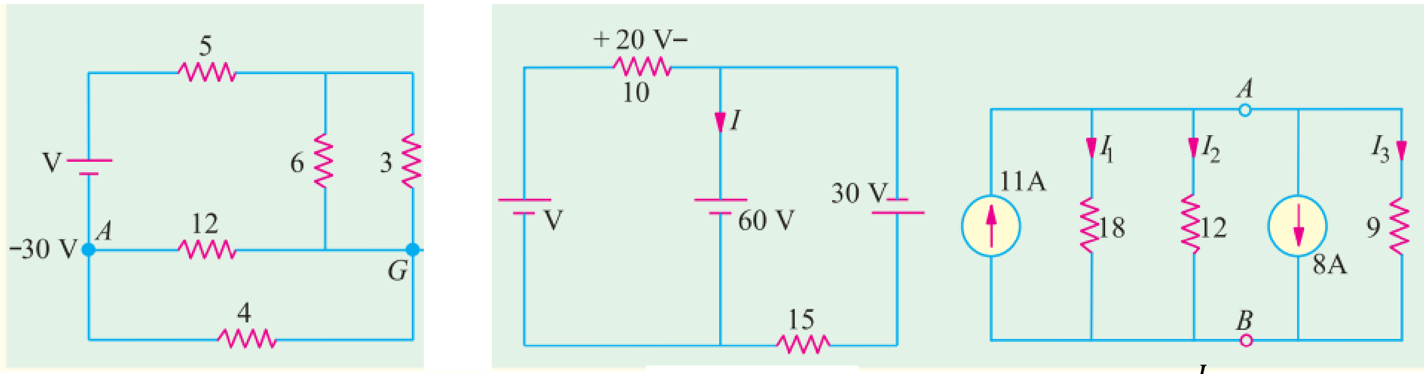
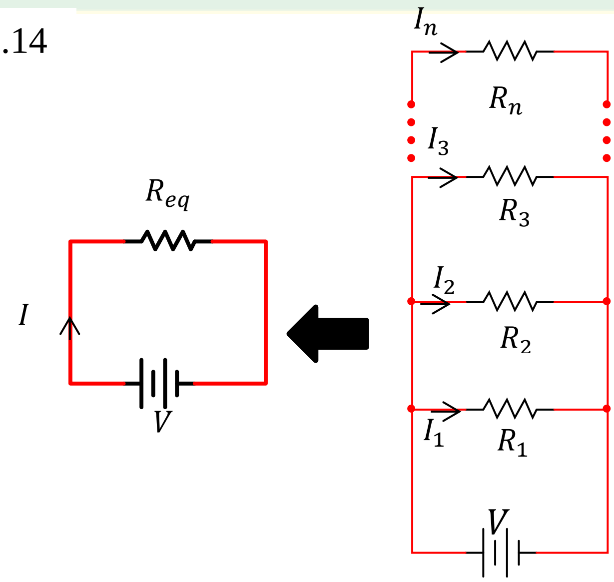


Fig 2.14

2.3.2 **Parallel circuits**:

$$I_1 = \frac{V}{R_1},\ I_2 = \frac{V}{R_2},\ I_3 = \frac{V}{R_3},\ \ldots..,\ I_n = \frac{V}{R_n}$$



$$I = I_1 + I_2 + I_3 + \cdots + I_n$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \cdots + \frac{V}{R_n}$$

$$= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}\right)$$

$$\therefore \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

Or

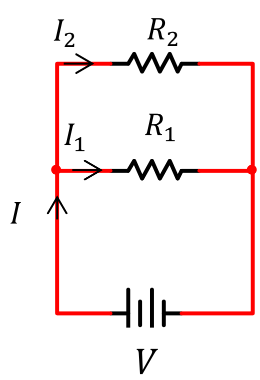
$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_n$$

Where G: is the conductance measured by ℧ (moh) or (Siemens) S.

- Special case of parallel circuit (current divider)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\therefore R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$



$$I_1 = \frac{V}{R_1},\ I_2 = \frac{V}{R_2},\qquad I = \frac{V}{R_{eq}}$$

$$V = I R_{eq}$$

$$I_1 R_1 = I \frac{R_1 R_2}{R_1 + R_2}$$

$$\therefore\ I_1 = I\frac{R_2}{R_1 + R_2}$$

Example 2.8: Determine the total conductance and resistance for the parallel network of Fig. 2.15

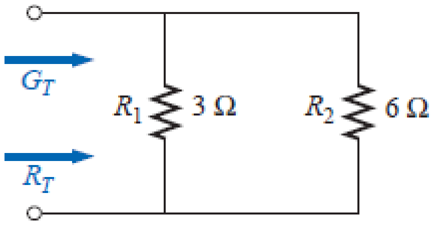


Fig 2.15

Solution:

$$G_T = G_1 + G_2 = \frac{1}{3\ \Omega} + \frac{1}{6\ \Omega} = 0.333\ \text{S} + 0.167\ \text{S} = \mathbf{0.5\ S}$$

and $$R_T = \frac{1}{G_T} = \frac{1}{0.5\ \text{S}} = \mathbf{2\ \Omega}$$

Example 2.9: Determine the total resistance for the network of Fig. 2.16.

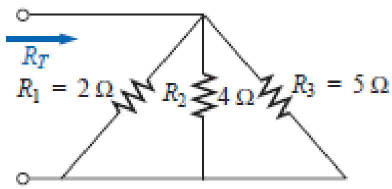
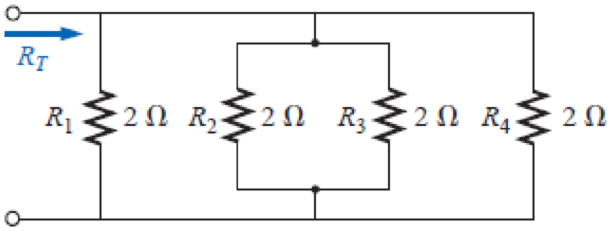
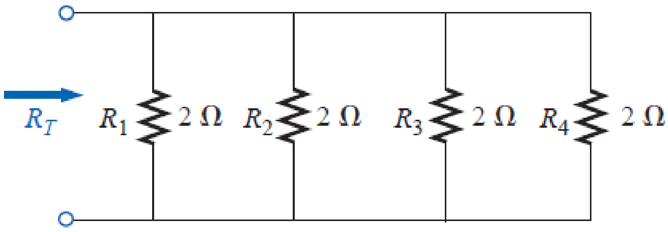


Fig 2.16

H.w: Determine the total resistance for the network of the following Figures:





Example 2.9: For the parallel network of Fig. 2.17:

a. Calculate R_T.
b. Determine I_s.
c. Calculate I_1 and I_2, and demonstrate that I_s, I_1, and I_2.
d. Determine the power to each resistive load.
e. Determine the power delivered by the source, and compare it to the total power dissipated by the resistive elements.

Solutions:

a. $R_{eq} = \frac{R_1R_2}{R_1+R_2} = 6\Omega$

b. $I_s = \frac{E}{R_{eq}} = 4.5\ A$

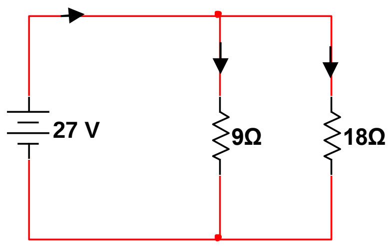


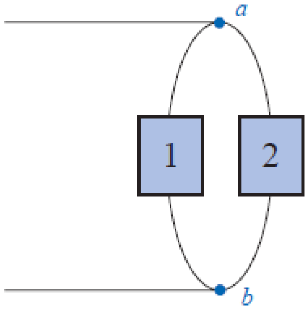
Fig 2.17

c. $I_1 = I\frac{R_2}{R_1+R_2} = \boldsymbol{3A}$

$$I_2 = I\frac{R_1}{R_1 + R_2} = \mathbf{1.5\ A}$$

d. $P_1 = V_1 I_1 =$
$P_1 = V_1 I_1$

- ***Two elements, branches, or networks are in* parallel *if they have two points in common as shown below .***



Example 2.10: for the network shown in Fig 2.18 calculate V_{ab}

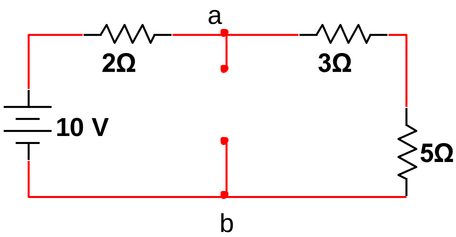


Fig 2.18

2.3.3 Series Parallel Circuits:

Series-parallel networks *are networks that contain both series and parallel circuit configurations.* For many single-source, series-parallel networks, the analysis is one that works back to the source, determines the source current, and then finds its way to the desired unknown. In Fig. 2.19(a), for instance, the voltage V_4 is desired. The absence of a single series or parallel path to V_4 from the source immediately reveals that the methods introduced in the last two chapters cannot be applied here. First, series and parallel elements must be combined to establish the reduced circuit of Fig. 2.19(b). Then series elements are combined to form the simplest of configurations in Fig. 2.19 (c). The source current can now be determined using Ohm's law, and we can proceed back through the network as shown in Fig. 2.19 (d). The voltage V_2 can be determined and then the original network can be redrawn, as shown in Fig. 2.19 (e). Since V_2 is now known, the voltage divider rule can be used to find the desired voltage $V4$. Because of the similarities between the networks of Figs. 2.19 (a) and 2.19 (e), and between 2.19 (b) and 2.19 (d), the networks drawn during the reduction phase are often used for the return path.

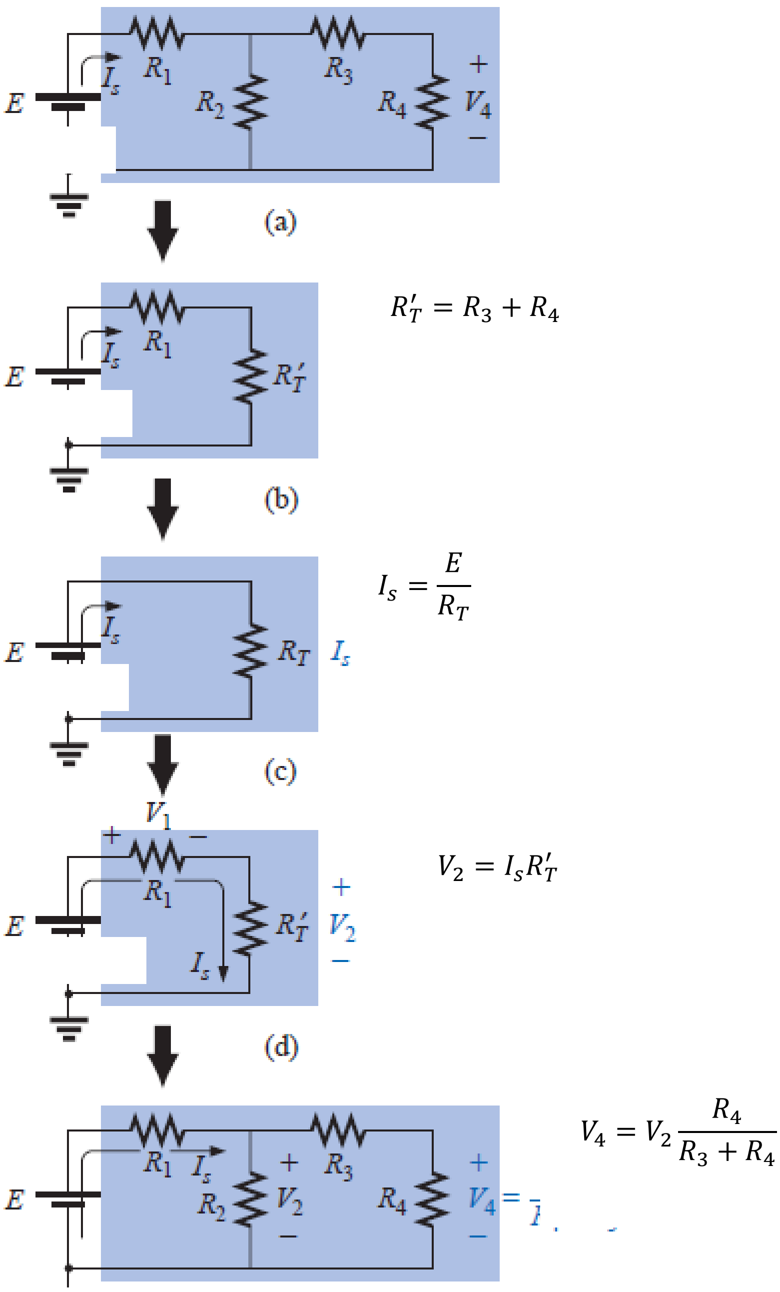
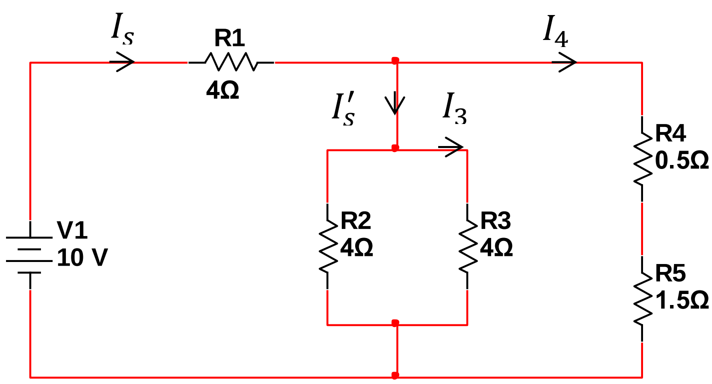


Fig 2.19

Example 2.11: find I_s, R_T, I_3, and I_4 for the network shown in Fig 2.20

Fig 2.20



Solution:

$$R_A = 4\Omega$$

$$R_B = R_2||R_3 = \frac{4*4}{4+4} = 2\Omega$$

$$R_C = R_4 + R_5 = 0.5 + 1.5 = 2\Omega$$

$$R_B||R_C = \frac{2}{2} = 1\Omega$$

$$R_T = R_A + R_B||R_C = 4 + 1 = 5\Omega$$

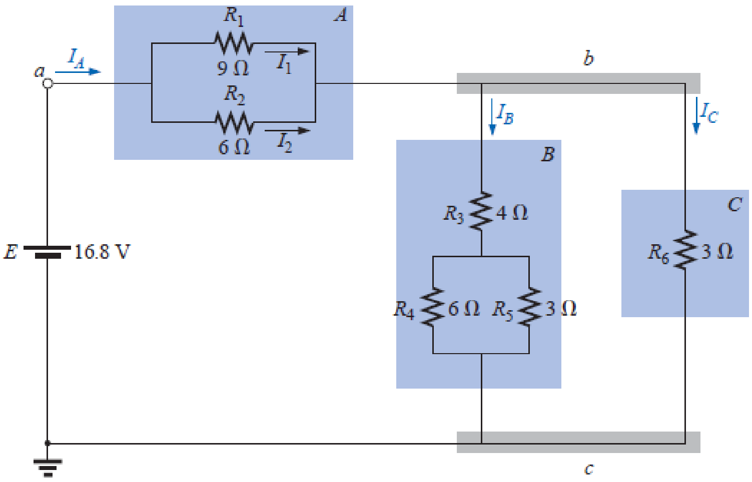
$$I_s = \frac{E}{R_T} = \frac{10}{5} = 2A$$

$$I_s' = I_s \frac{R_C}{R_C + R_B} = 2 * \frac{2}{2+2} = 1A$$

$$I_3 = I_s' \frac{R_2}{R_2 + R_3} =$$

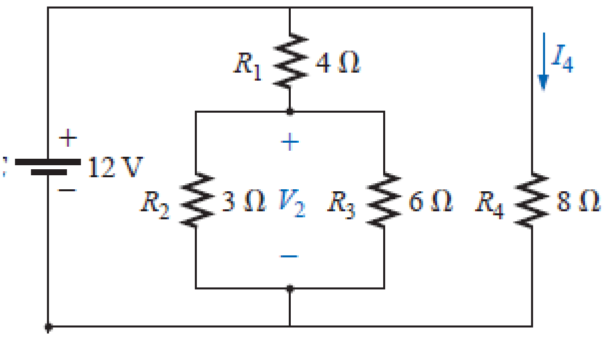
$$I_4 = I_s - I_s'$$

H.w: find the value of all currents shown in and the voltage across each resistances Fig 2.12



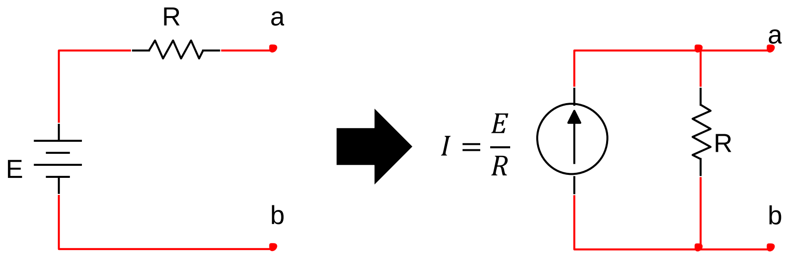
H.w: Find the current I_4 and the voltage V_2 for the network of Fig. 2.13.

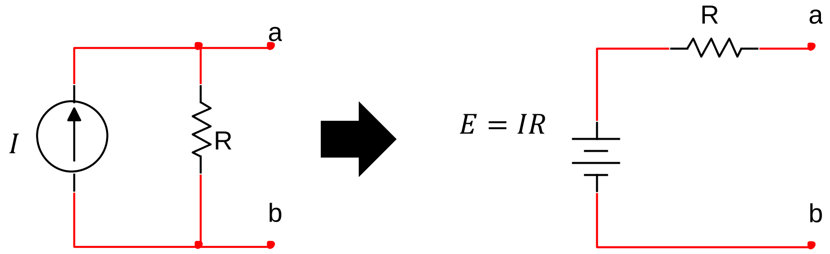
Fig 2.13



2.4 **SOURCE CONVERSIONS**

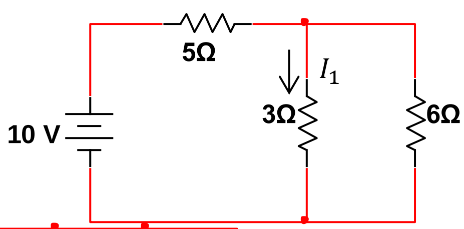
Any voltage source with a series resistance can be transformed to a current source with a parallel resistance and vice versa.



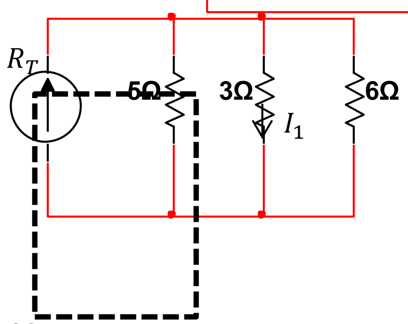


- The direction of the transformed voltage or current source is the same of original current or voltage source as shown below:

Example 1) find the value of I_1 in the following circuit:



$I = \frac{10}{5} = 2$ A

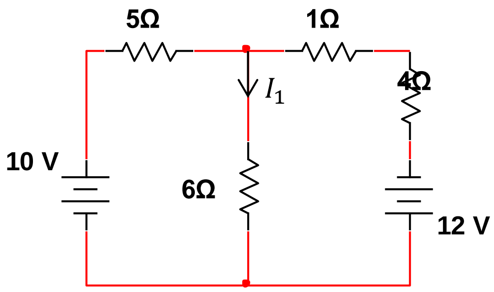


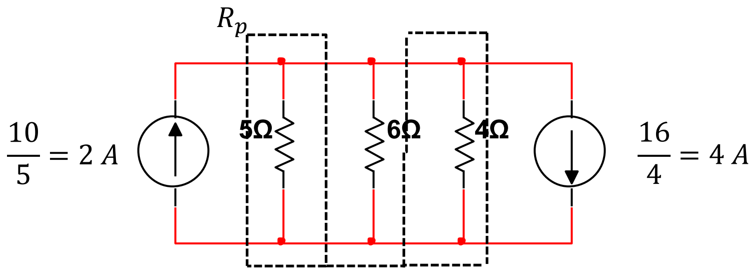
$$R_T = \frac{5 * 3}{5 + 3} = 1.87\ \Omega$$

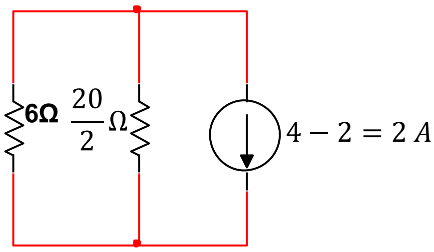
$$I_1 = 2\frac{1.87}{1.87 + 6}$$

- **Only the parallel current sources can be added together and the direction of the resulting current source will be in the same direction of the largest value of the original current sources.**

Example 2) find I_1

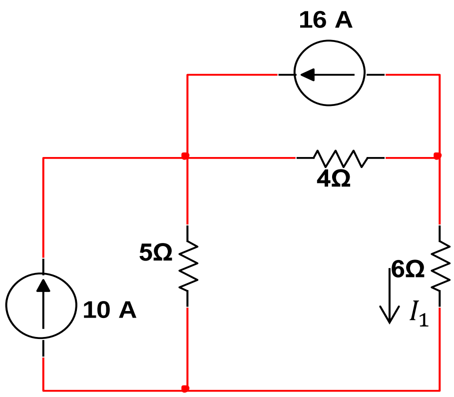




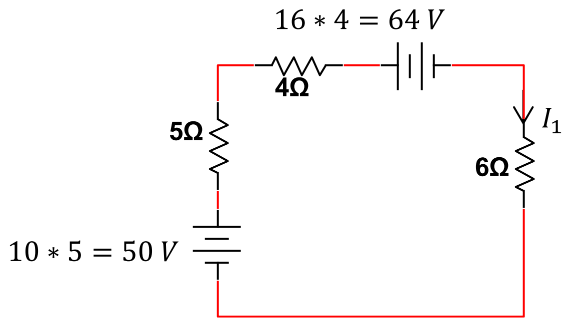


$$I_1 = -2 * \frac{\frac{20}{9}}{\frac{20}{9} + 6} =$$

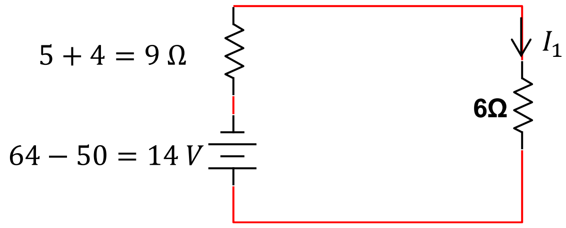
Example 3: find the value of I_1



Solution:

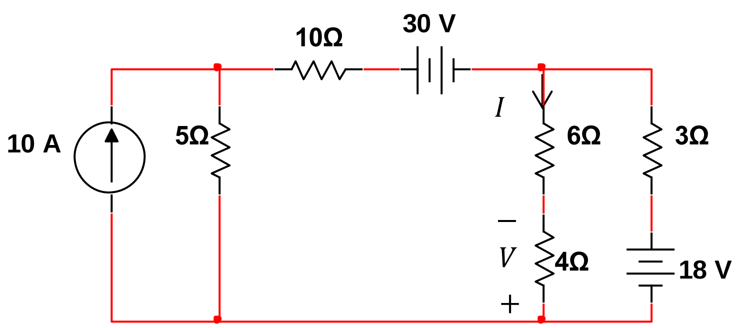


- **Only the voltage sources in series can be added together and the direction of the resulting voltage source will be in the same direction of the largest value of the original voltage sources.**



$$I_1 = \frac{14}{9+6} = \frac{14}{15}\ A$$

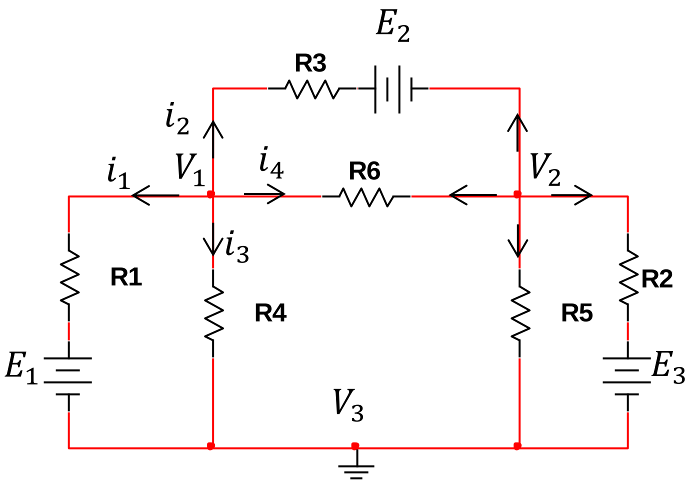
H.W: find I and V:



2.5 Nodal analysis:

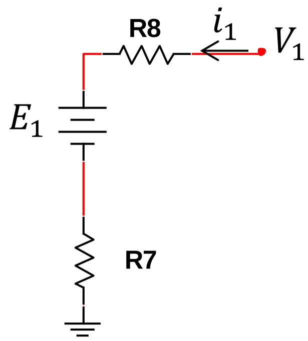
1) Determine the No. of Nodes
2) Choose one of the Nodes as a Reference.
3) Suppose output currents from the remaining Nodes.
4) Apply KCL at these Nodes.
5) Solve the resulting equations to find the voltage of each Node

Note: the resulting values of voltage represent the voltage of each Node measured with respect to the Reference.

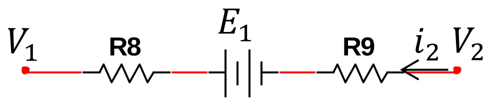


Note: the following figures and equations will help u to write the equations of the currents:

1) $i_1 = \frac{V_1 - E_1 - 0}{R_8 + R_7}$



1) $i_2 = \frac{V_2 + E_1 - V_1}{R_8 + R_9}$



At Node 1

$$i_1 + i_2 + i_3 + i_4 = 0$$

$$\frac{V_1 - E_1}{R_1} + \frac{V_1 - E_3 - V_2}{R_3} + \frac{V_1 - 0}{R_4} + \frac{V_1 - V_2}{R_6} = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_6}\right) V_1 - \left(\frac{1}{R_3} + \frac{1}{R_6}\right) V_2 = \frac{E_1}{R_1} + \frac{E_3}{R_3} \ldots \ldots \ldots \ldots (1)$$

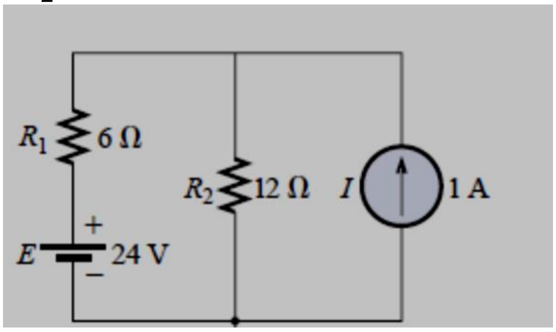
At Node 2:

$$\frac{V_2 - E_2}{R_2} + \frac{V_2 + E_3 - V_1}{R_3} + \frac{V_2 - 0}{R_5} + \frac{V_2 - V_1}{R_6} = 0$$

$$-\left(\frac{1}{R_3}+\frac{1}{R_6}\right)V_1+\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}+\frac{1}{R_6}\right)V_2=\frac{E_2}{R_2}-\frac{E_3}{R_3}\ \ldots\ldots\ldots\ldots(2)$$

Solving (1) & (2) we can get the value of voltages of node 1 & 2

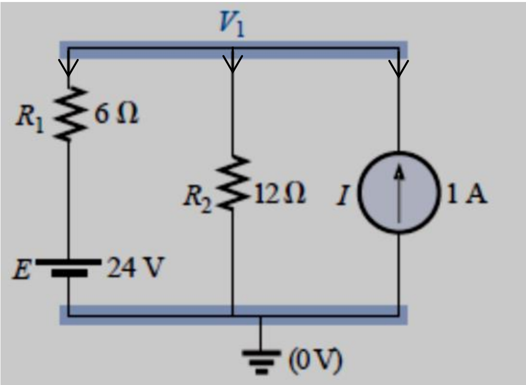
Example 1): using Nodal analysis find the current of R_2



Solution:

The network has two nodes, as shown in the previous Figure The lower node is defined as the reference node

$$\frac{V_1-E}{R_1}+\frac{V_1-0}{R_2}-1=0$$



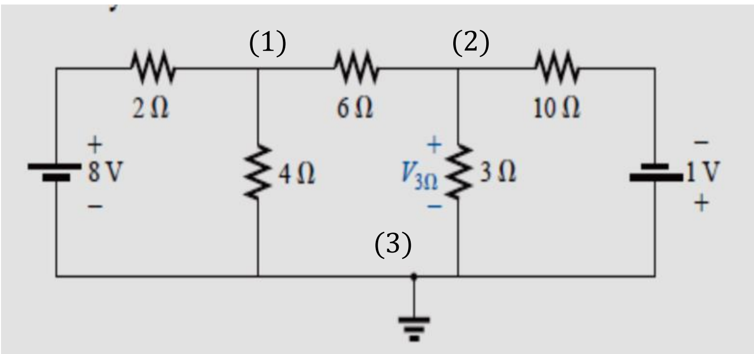
$$\left(\frac{1}{6}+\frac{1}{12}\right)V_1=\frac{24}{6}+1\ \ldots\ldots\ldots\ldots(1)$$

$$\therefore\ V_1=5*\frac{12}{3}=20\ V$$

Example 2): Find the voltage across the 3Ω resistor of the following circuit:

Solution:

Node (3) will be the reference node



At node 1)

$$\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}\right)V_1-\left(\frac{1}{6}\right)V_2=\frac{8}{2}$$

at node 2)

$$-\left(\frac{1}{6}\right)V_1-\left(\frac{1}{6}+\frac{1}{3}+\frac{1}{10}\right)V_2=-\frac{1}{10}$$

$$\frac{11}{12}V_1-\frac{1}{6}V_2=4$$

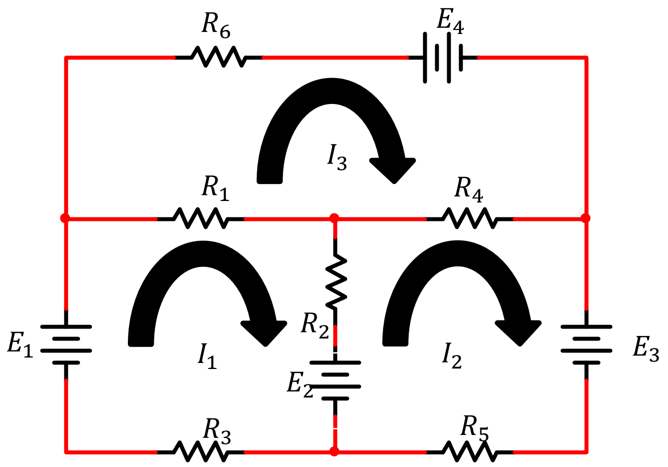
$$-\frac{1}{6}V_1+\frac{3}{5}V_2=-0.1$$

$$11V_1-\ 2V_2=+48 \quad \dots\dots\dots\dots(1)$$
$$-5V_1+18V_2=-3 \quad \dots.(2)$$

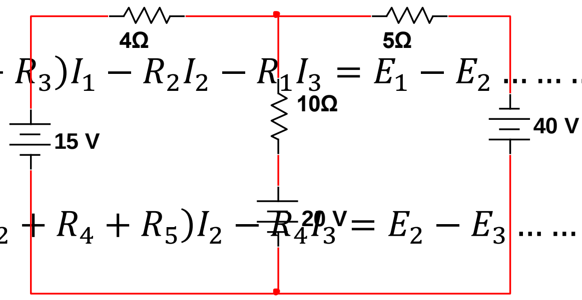
$$V_2=V_{3\Omega}=\frac{\begin{vmatrix}11 & 48\\ -5 & -3\end{vmatrix}}{\begin{vmatrix}11 & -2\\ -5 & 18\end{vmatrix}}=\frac{-33+240}{198-10}=\frac{207}{188}=\mathbf{1.101\ V}$$

2.6 Mesh Analysis:

1) suppose a rotating current around each mesh in clock wise direction.
2) Write Kirchhoff's voltage for each mesh.
3) Solve the resulting equations.



For loop (1)



$$(R_1 + R_2 + R_3)I_1 - R_2 I_2 - R_1 I_3 = E_1 - E_2 \dots\dots\dots (1)$$

For loop (2)

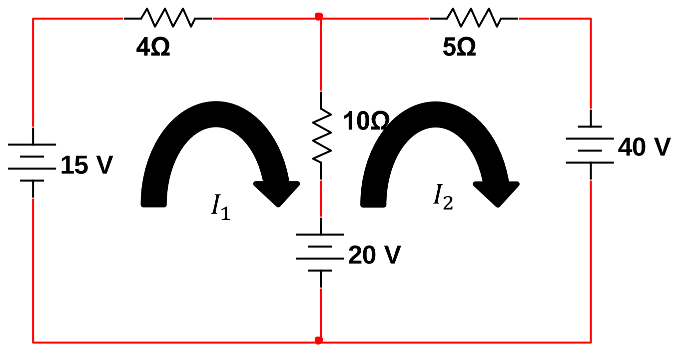
$$-R_2 I_1 + (R_2 + R_4 + R_5)I_2 - R_4 I_3 = E_2 - E_3 \dots\dots\dots (2)$$

For loop (3)

$$-R_1 I_1 - R_4 I_2 + (R_1 + R_4 + R_6)I_3 = -E_4 \dots\dots\dots (3)$$

Example: Using mesh analysis find the voltage across 10 Ω resistor for the following network:

Solution:



For loop (1)

$$(4 + 10)I_1 - 10I_2 = 15 - 20$$

$$14I_1 - 10I_2 = -5 \dots\dots\dots (1)$$

For loop (2)

$$-10I_1 + (10+5)I_2 = 20 + 40$$

$$-10I_1 + 15I_2 = 60 \ldots\ldots\ldots (2)$$

$$\Delta = \begin{vmatrix} 14 & -10 \\ -10 & 15 \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} -5 & -10 \\ 60 & 15 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} 14 & -5 \\ -10 & 60 \end{vmatrix}$$

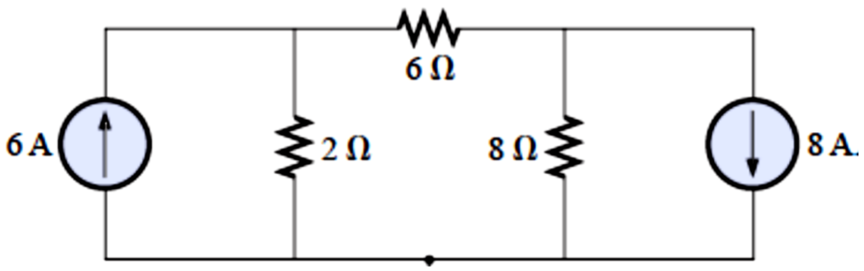
$$I_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} -5 & -10 \\ 60 & 15 \end{vmatrix}}{\begin{vmatrix} 14 & -10 \\ -10 & 15 \end{vmatrix}} = \frac{-5*(15) - (60*-10)}{14*15 - (-10*-10)} = 4.773\ A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 14 & -5 \\ -10 & 60 \end{vmatrix}}{\begin{vmatrix} 14 & -10 \\ -10 & 15 \end{vmatrix}} = 7.182A$$

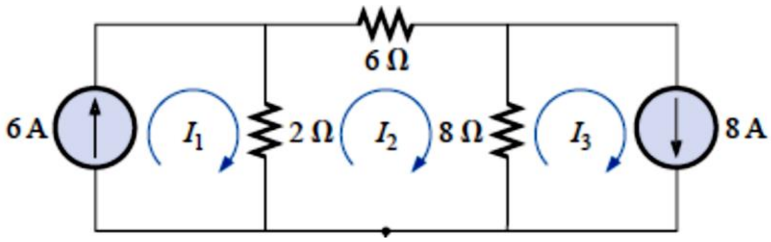
$$I_{10\Omega} = I_2 - I_1 = 7.182 - 4.773 = 2.409\ A$$

$$V_{10\Omega} = 2.409 * 10 = 24.09\ V$$

Example: Using mesh analysis, determine the currents for the following network



Solution:



For loop (1):

$$I_1 = 6\ A \ldots\ldots\ldots (1)$$

For loop (2)

$$-2I_1 + (2+6+8)I_2 - 8I_3 = 0$$

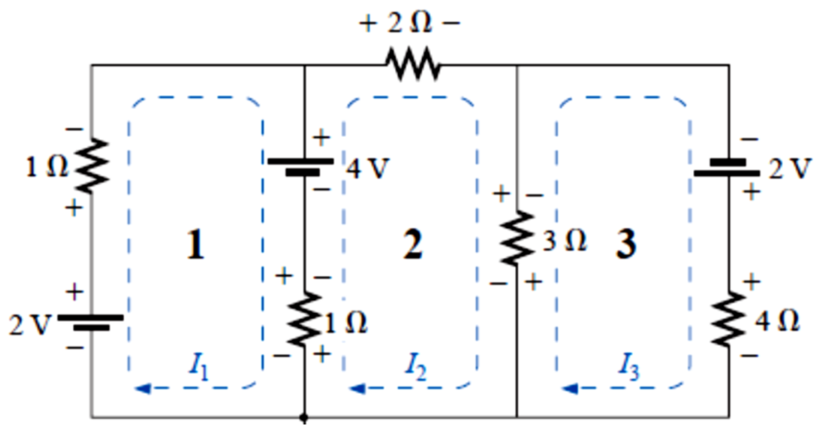
$$-2I_1 + 16I_2 - 8I_3 = 0 \ldots\ldots\ldots(2)$$

For loop (3)

$$I_3 = 8\,A \ldots\ldots\ldots(3)$$

Sub (1) &(3) in (2

Example: find the current of the source 2V at loop (1) for the following network:



Solution:

For loop (1):

$$(1+1)I_1 - I_2 - 0I_3 = 2 - 4$$

$$2I_1 - I_2 - 0I_3 = -2 \ldots\ldots\ldots(1)$$

For loop (2):

$$-I_1 + (1+2+3)I_2 - 3I_3 = 4$$

$$-I_1 + 6I_2 - 3I_3 = 4 \ldots\ldots\ldots(2)$$

For loop (3):

$$-0I_1 - 3I_2 + (3+4)I_3 = 2$$

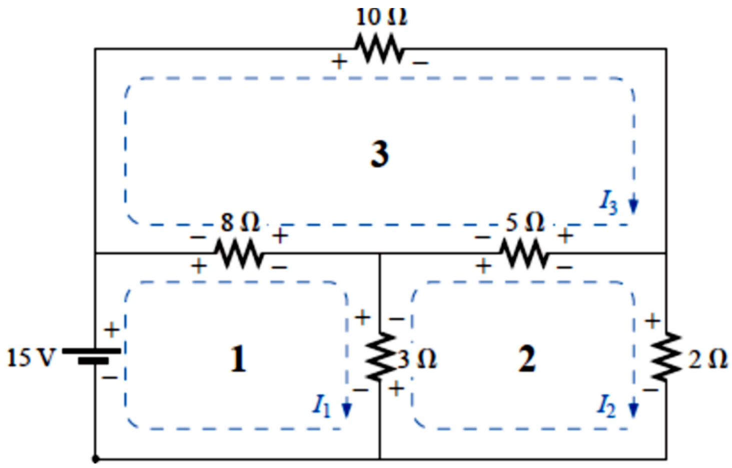
$$-0I_1 - 3I_2 + 7I_3 = 2 \ldots\ldots\ldots(3)$$

$$\Delta = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 6 & -3 \\ 0 & -3 & 7 \end{bmatrix}, \ \Delta_1 = \begin{bmatrix} -2 & -1 & 0 \\ 4 & 6 & -3 \\ 2 & -3 & 7 \end{bmatrix}$$

The current of source $2V = I_1$

$$I_1 = \frac{\begin{bmatrix} -2 & -1 & 0 \\ 4 & 6 & -3 \\ 2 & -3 & 7 \end{bmatrix}}{\begin{bmatrix} 2 & -1 & 0 \\ -1 & 6 & -3 \\ 0 & -3 & 7 \end{bmatrix}} = 0.542\ A$$

H.M: Find the current through the 10Ω resistor



2.7 Delta(Δ)-Star(Y)/Star-Delta transformation:

Circuit configurations are often encountered in which the resistors do not appear to be in series or parallel. Under these conditions, it may be necessary to convert the circuit from one form to another to solve for any unknown quantities if mesh or nodal analysis is not applied. Two circuit configurations that often account for these difficulties are the **(Y)** and **(Δ) configurations,** depicted in Fig. 2.24(a). They are also referred to as the **(T)** and **pi (Δ),** respectively, as indicated in Fig. 2.25(b). Note that the pi is actually an inverted delta.

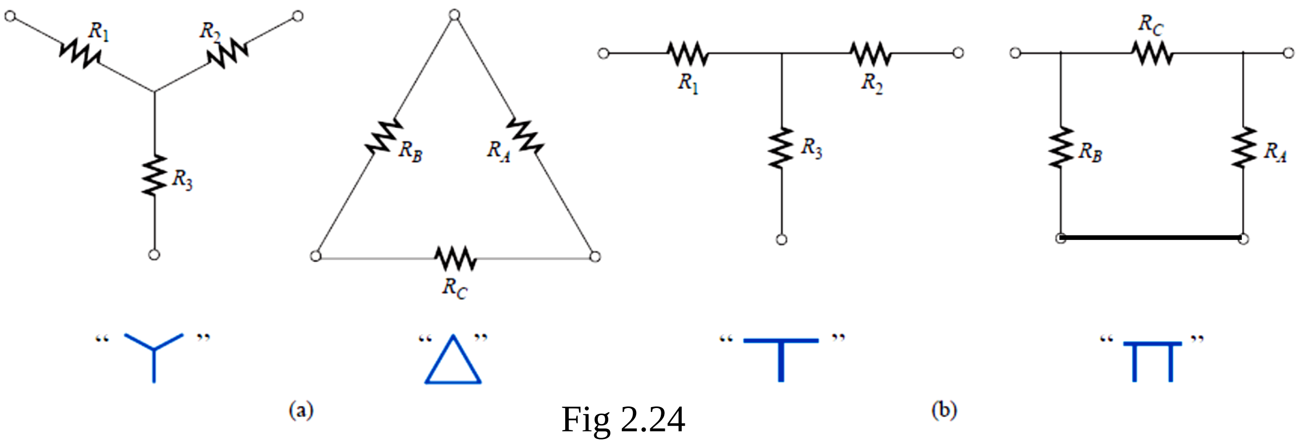


Fig 2.24

Y ➡ Δ:

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

$$R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$

$$R_C = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$

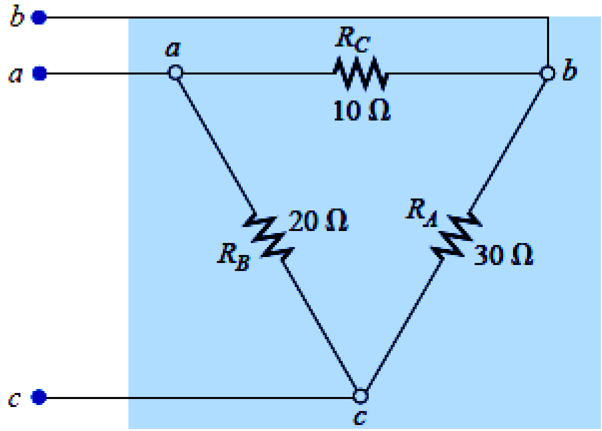
Δ ➡ Y:

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

Example: Convert the Δ to a Y.



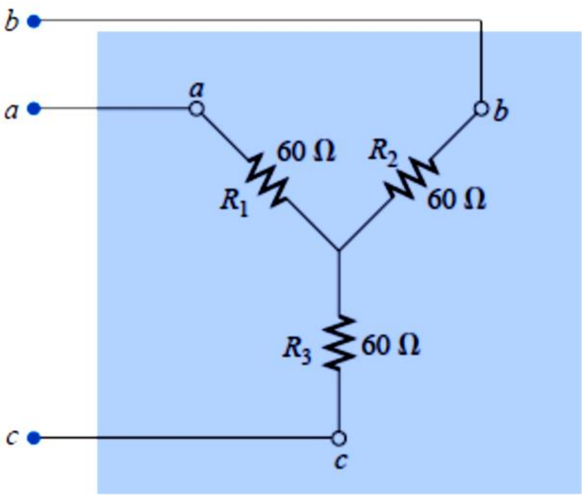
Solution:

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(20\ \Omega)(10\ \Omega)}{30\ \Omega + 20\ \Omega + 10\ \Omega} = \frac{200\ \Omega}{60} = 3\tfrac{1}{3}\ \Omega$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(30\ \Omega)(10\ \Omega)}{60\ \Omega} = \frac{300\ \Omega}{60} = 5\ \Omega$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(20\ \Omega)(30\ \Omega)}{60\ \Omega} = \frac{600\ \Omega}{60} = 10\ \Omega$$

Example: Convert the Y to a Δ.



Solution:

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

$$= \frac{(60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega)}{60\ \Omega}$$

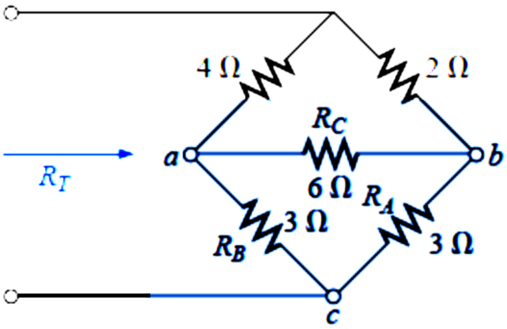
$$= \frac{3600\ \Omega + 3600\ \Omega + 3600\ \Omega}{60} = \frac{10{,}800\ \Omega}{60}$$

$$R_A = \mathbf{180\ \Omega}$$

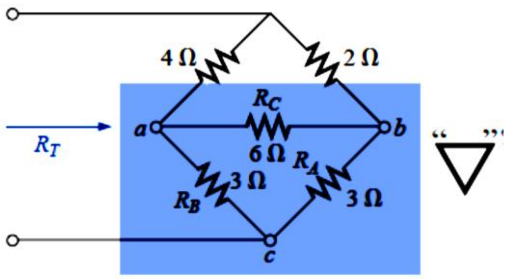
$$R_\Delta = 3R_Y = 3(60\ \Omega) = 180\ \Omega$$

and $$R_B = R_C = \mathbf{180\ \Omega}$$

Example: Find the total resistance of the networks



Solution:

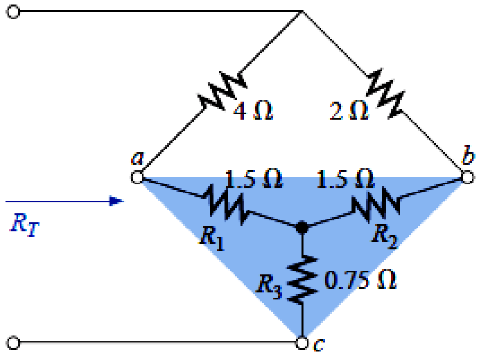


$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{12\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3\ \Omega)(3\ \Omega)}{12\ \Omega} = \frac{9\ \Omega}{12} = \mathbf{0.75\ \Omega}$$

Replacing the Δ by the Y, as shown



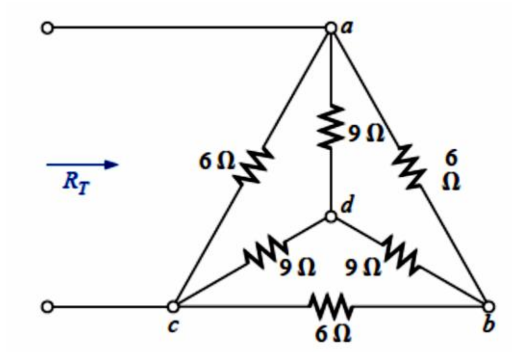
$$R_T = 0.75\ \Omega + \frac{(4\ \Omega + 1.5\ \Omega)(2\ \Omega + 1.5\ \Omega)}{(4\ \Omega + 1.5\ \Omega) + (2\ \Omega + 1.5\ \Omega)}$$

$$= 0.75\ \Omega + \frac{(5.5\ \Omega)(3.5\ \Omega)}{5.5\ \Omega + 3.5\ \Omega}$$

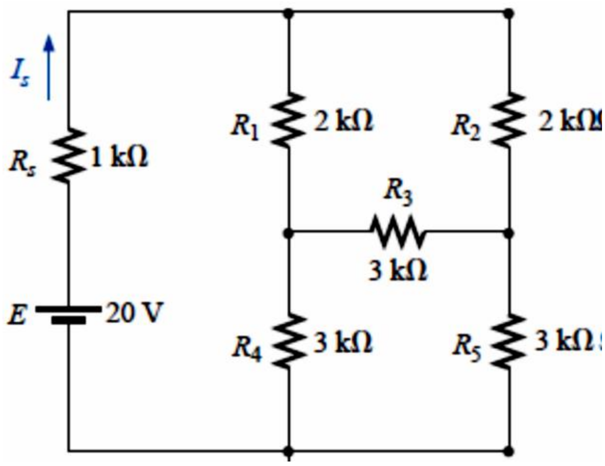
$$= 0.75\ \Omega + 2.139\ \Omega$$

$$R_T = \mathbf{2.889\ \Omega}$$

Example: Find the total resistance of the network



Example: find the current source



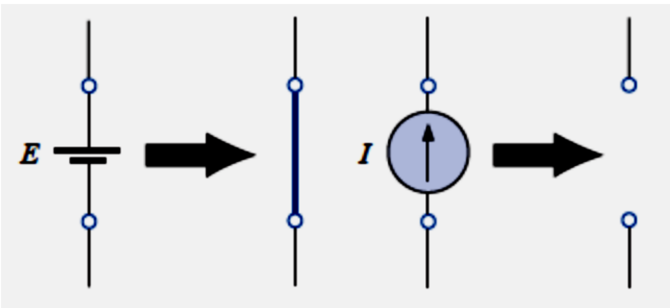
2.8 Superposition Theorem:

The superposition theorem states the following:

The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source.

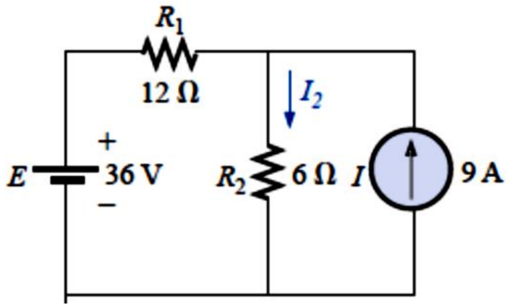
To consider the effects of each source independently requires that sources be removed and replaced without affecting the final result. To remove a voltage source when applying this theorem, the difference in potential between the terminals of the voltage source must be set to zero (short circuit); removing a current source requires that its terminals be opened (open circuit). Any internal resistance or conductance associated with the displaced sources is not eliminated but must still be considered. Figure 2.25 reviews the various substitutions required when removing any source.

Fig 2.25



Example: Using superposition, find the current through the 6Ω resistor of the network:

Fig 2.28



Solution:

Considering the effect of the 36-V source Fig 2.29

$$I'_2 = \frac{E}{R_T} = \frac{E}{R_1 + R_2} = \frac{36\text{ V}}{12\ \Omega + 6\ \Omega} = 2\text{ A}$$

Considering the effect of the 9-A source Fig 2.30

Applying the current divider rule,

$$I''_2 = \frac{R_1 I}{R_1 + R_2} = \frac{(12\ \Omega)(9\text{ A})}{12\ \Omega + 6\ \Omega} = \frac{108\text{ A}}{18} = 6\text{ A}$$

The total current through the 6-Ω resistor Fig 2.30 is

$$I_2 = I'_2 + I''_2 = 2\text{ A} + 6\text{ A} = \mathbf{8\text{ A}}$$

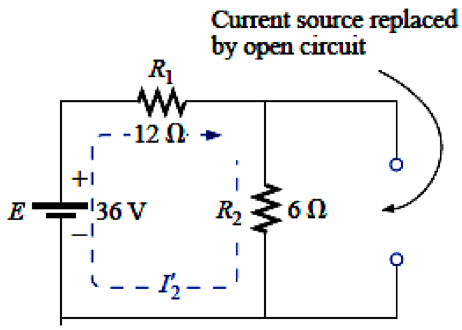


Fig 2.29

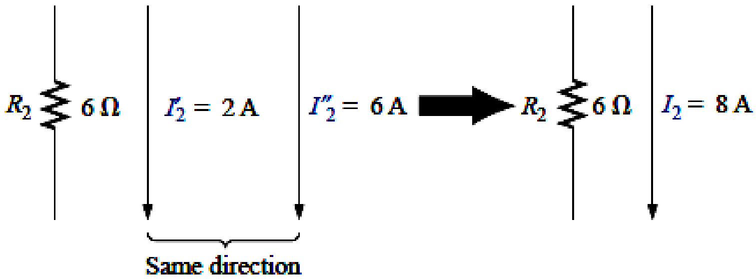


Fig 2.31

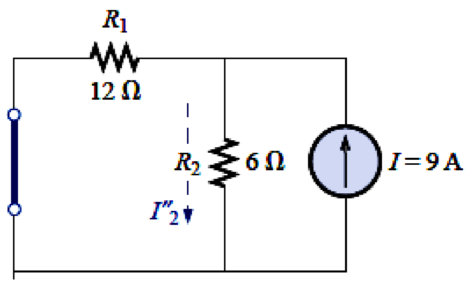
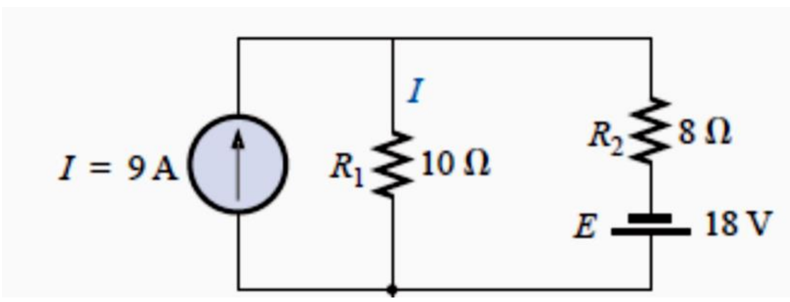
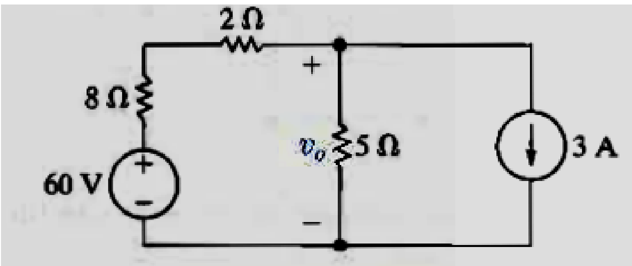


Fig 2.30

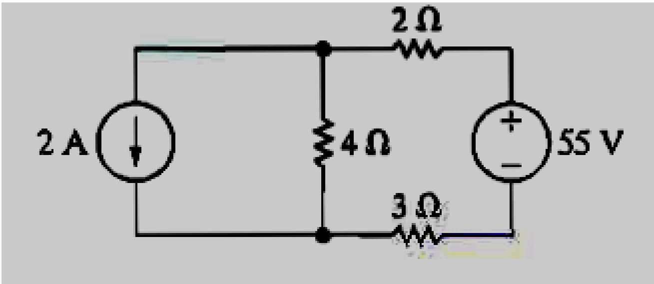
Example: Using superposition, find the current *I* through the 10Ω resistor



Example: Using Superposition Theorem find v_0



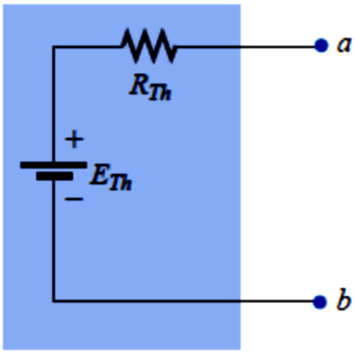
Example: Using Superposition Theorem find the current throw 3Ω resistor



2.9 Thevenin's Theorem:

Any two-terminal, in a network can be replaced by an equivalent circuit consisting of a voltage source and a series resistor, as shown in Fig.2.30

Fig 2.30



Preliminary:

1. Remove that portion of the network across which the Thévenin equivalent circuit is to be found. In Fig. 9.26(a), this requires that the load resistor R_L *be temporarily removed from the network.*

2. Mark the terminals of the remaining two-terminal network. (The importance of this step will become obvious as we progress through some complex networks.)

R_{Th}*:*

3. Calculate R_{Th} *by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)*

E_{Th}*:*

4. Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (This step is invariably the one that will lead to the most confusion and errors. In all cases, keep in mind that it is the open-circuit potential between the two terminals marked in step 2.)

Conclusion:

5. Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit. This step is indicated by the placement of the resistor RL between the terminals of the Thévenin equivalent circuit as shown in Fig. 2.31(b).

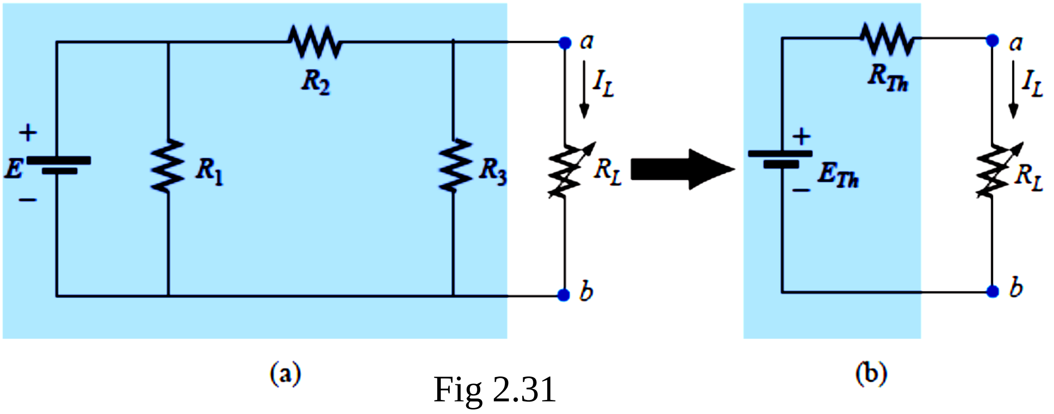


Fig 2.31

Example: Find Thévenin's equivalent circuit between a &b for the network in Fig. 2.32. Then find the current through R_L for values of 2 Ω, 10 Ω, and 100Ω**.**

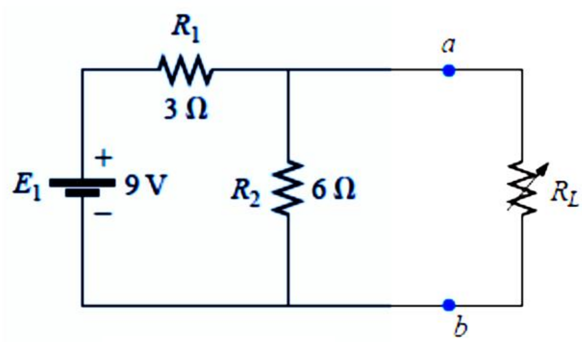


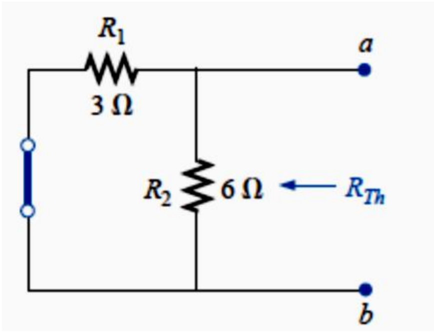
Fig 2.32

Steps 1 and 2 produce the network of Fig. 2.32. Note that the load resistor R_L has been removed and the two “holding” terminals have been defined as *a* and *b.*

Step 3: Replacing the voltage source E_1 with a short-circuit equivalent yields the network of Fig. 2.33, where

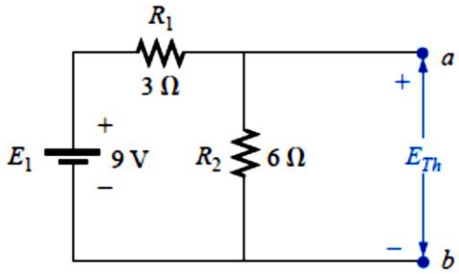
$$R_{Th} = R_1 \parallel R_2 = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = 2\ \Omega$$

Fig 2.33

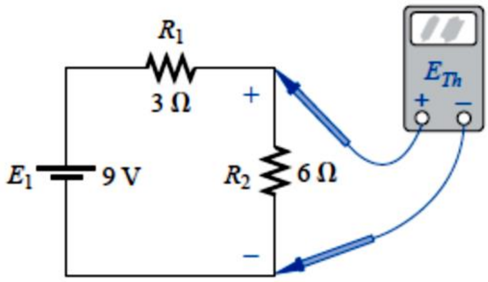


Step 4: Replace the voltage source (Fig. 2.34). For this case, the open circuit voltage E_{Th} is the same as the voltage drop across the 6Ω resistor. Applying the voltage divider rule,

Fig 2.34

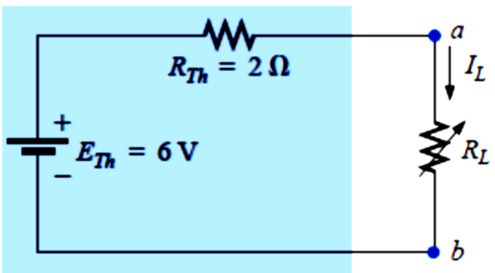


$$E_{Th} = \frac{R_2 E_1}{R_2 + R_1} = \frac{(6\ \Omega)(9\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{54\ \text{V}}{9} = \mathbf{6\ V}$$



Step 5 Fig 2.35

Fig 2.35

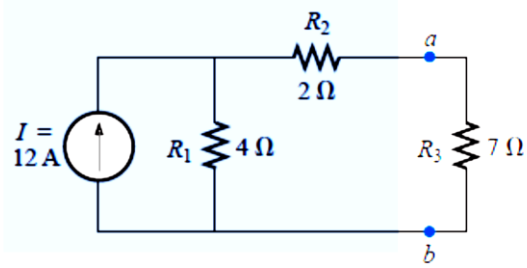


Example: Find the Thévenin equivalent circuit for the network in

the shaded area of the network of Fig. 2.36.
Solution:
Steps 1 and 2 are shown in Fig. 2.37.
Step 3 is shown in Fig. 2.38. The current source has been replaced with an open-circuit equivalent, and the resistance determined between terminals a and b.
In this case an ohmmeter connected between terminals a and b would send out a sensing current that would flow directly through R1 and R2 (at the same level). The result is that R1 and R2 are in series and the Thévenin resistance is the sum of the two.

$$R_{Th} = R_1 + R_2 = 4\ \Omega + 2\ \Omega = \mathbf{6\ \Omega}$$

Fig 2.36



Step 4 (Fig. 2.39):

In this case, since an open circuit exists between the two marked terminals, the current is zero between these terminals and through the 2Ω resistor. The voltage drop across R2 is, therefore,

$$V_2 = I_2R_2 = (0)R_2 = 0\ \text{V}$$
$$E_{Th} = V_1 = I_1R_1 = IR_1 = (12\ \text{A})(4\ \Omega) = \mathbf{48\ V}$$

Fig 2.37

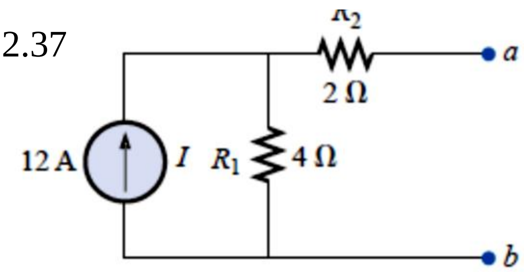


Fig 2.38

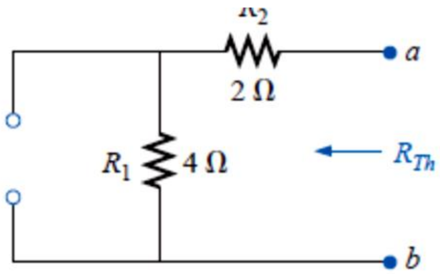
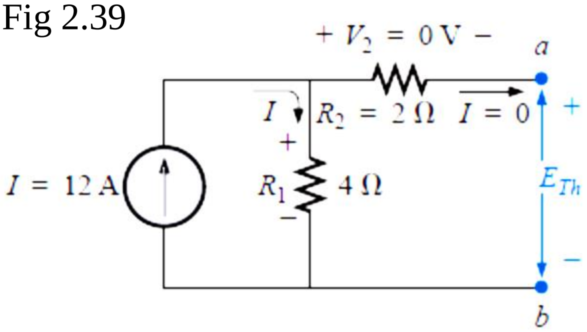
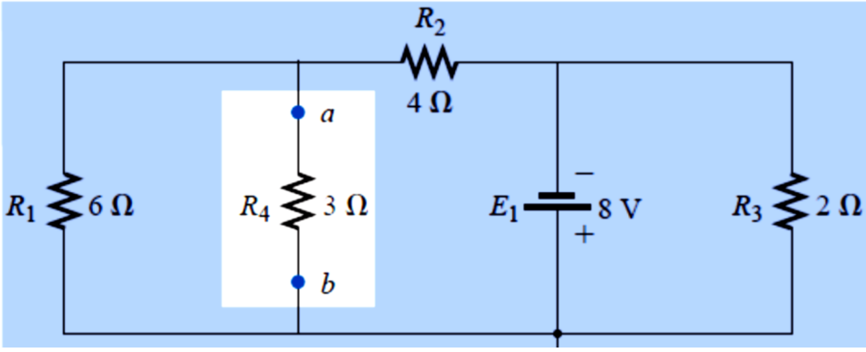


Fig 2.39



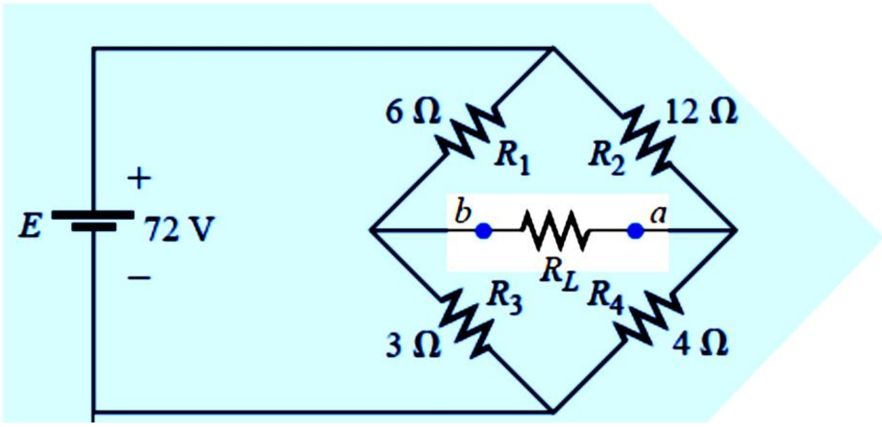
EXAMPLE : Find the Thévenin equivalent circuit for the network in the shaded area of the network of Fig. 2.40. Note in this example that

Fig 2.40



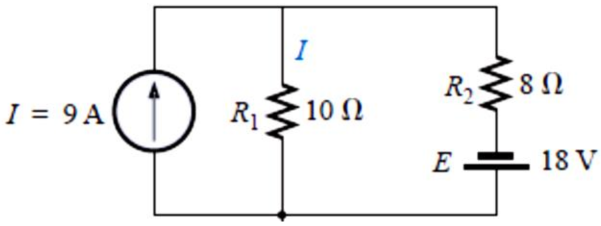
EXAMPLE : Find the Thévenin equivalent circuit for the network in the shaded area of the bridge network of Fig. 2.41.

Fig 2.41



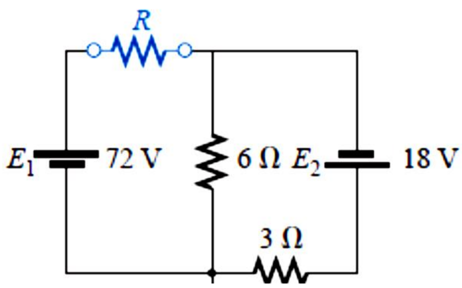
Example: Using superposition, find the current *I* through the 10Ω resistor for each of the networks of Fig. 2.42.

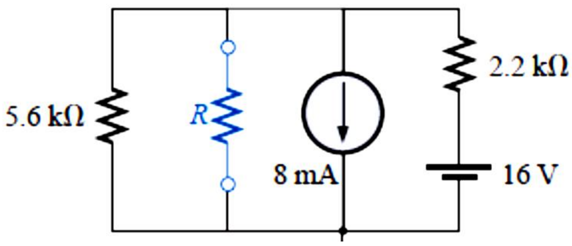
Fig 2.41



Example: Find the Thévenin equivalent circuit for the network external to the resistor *R* in each of the networks of Fig.2.41

Fig 2.41





Chapter three

Fundamental of Alternative Current (AC) Circuits

3.1 Introduction:

The analysis thus far has been limited to dc networks, networks in which the currents or voltages are fixed in magnitude except for transient effects. We will now turn our attention to the analysis of networks in which the magnitude of the source varies in a set manner. Of particular interest is the time-varying voltage that is commercially available in large quantities and is commonly called the *ac voltage.* (The letters *ac* are an abbreviation for *alternating current.*) To be absolutely rigorous, the terminology *ac voltage* or *ac current* is not sufficient to describe the type of signal we will be analyzing. Each waveform of Fig. 3.1 is an **alternating waveform** available from commercial supplies. The term *alternating* indicates only that the waveform alternates between two prescribed levels in a set time sequence (Fig. 3.1). To be

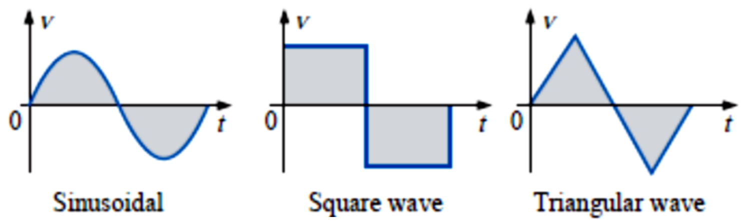


Fig 3.1

Alternating waveforms.

One of the important reasons for concentrating on the sinusoidal ac voltage is that it is the voltage generated by utilities throughout the world. Other reasons include its application throughout electrical, electronic, communication, and industrial systems.

3.2 SINUSOIDAL ac VOLTAGE CHARACTERISTICS AND DEFINITIONS Generation

Sinusoidal ac voltages are available from a variety of sources. The most common source is the typical home outlet, which provides an ac voltage that originates at a power plant; such a power plant is most commonly fueled by water power, oil, gas, or nuclear fusion. In each case an *ac generator* (also called an *alternator*), as shown in Fig. 3.2(a), is the primary component in the energy-conversion process.

The power to the shaft developed by one of the energy sources listed will turn a *rotor* (constructed of alternating magnetic poles) inside a set of windings housed in the *stator* (the stationary part of the dynamo) and will induce a voltage across the windings of the stator, as defined by Faraday's law,

$$e = N\frac{d\phi}{dt}$$

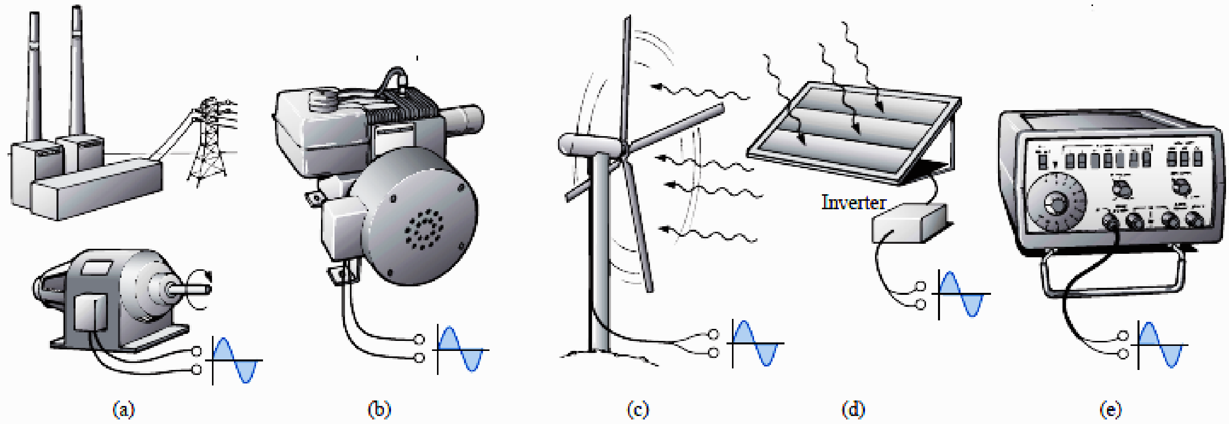


Fig 3.2
Various sources of ac power: (a) generating plant; (b) portable ac generator; (c) wind-power station; (d) solar panel; (e) function generator.

3.3 Definitions

The sinusoidal waveform of Fig. 3.1 with its additional notation will now be used as a model in defining a few basic terms. These terms, however, can be applied to any alternating waveform. It is important to remember as you proceed through the various definitions that the vertical scaling is in volts or amperes and the horizontal scaling is *always* in units of time.

Waveform: The path traced by a quantity, such as the voltage in Fig. 3.1, plotted as a function of some variable such as time (as above), position, degrees, radians, temperature, and so on.

Instantaneous value: The magnitude of a waveform at any instant of time; denoted by lowercase letters (e_1, e_2).
Peak amplitude: The maximum value of a waveform as measured from its *average*, or *mean*, value, denoted by uppercase letters (such as E_m for sources of voltage and V_m for the voltage drop across a load). For the waveform of Fig 3.3, the average value is zero volts, and E_m is as defined by the figure.
Peak value: The maximum instantaneous value of a function as measured from the zero-volt level. For the waveform of Fig 3.3, the peak amplitude and peak value are the same, since the average value of the function is zero volts.

Peak-to-peak value: Denoted by E_{p-p} or V_{p-p}, the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.

Periodic waveform: A waveform that continually repeats itself after the same time interval. The waveform of Fig 3.3 is a periodic waveform.

Period (T): The time interval between successive repetitions of a periodic waveform (the period $T_1 = T_2 = T_3$ in Fig 3.3), as long as successive *similar points* of the periodic waveform are used in determining T.

Cycle: The portion of a waveform contained in *one period* of time. The cycles within T_1, T_2, and T_3 of Fig 3.3 may appear different in Fig. 13.4, but they are all bounded by one period of time and therefore satisfy the definition of a cycle.

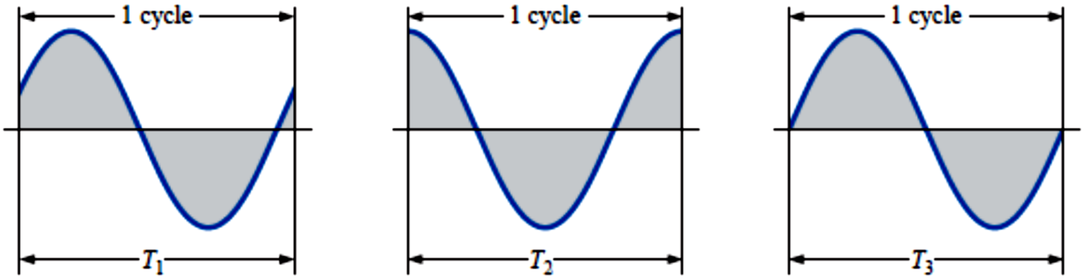


Fig 3.3

Defining the cycle and period of a sinusoidal waveform.

Frequency (f): The number of cycles that occur in 1 s. The frequency of the waveform of Fig 3.4 a is 1 cycle per second, and for Fig 3.4 b , 2½ cycles per second. If a waveform of similar shape had a period of 0.5 s Fig 3.4 c)], the frequency would be 2 cycles per second.

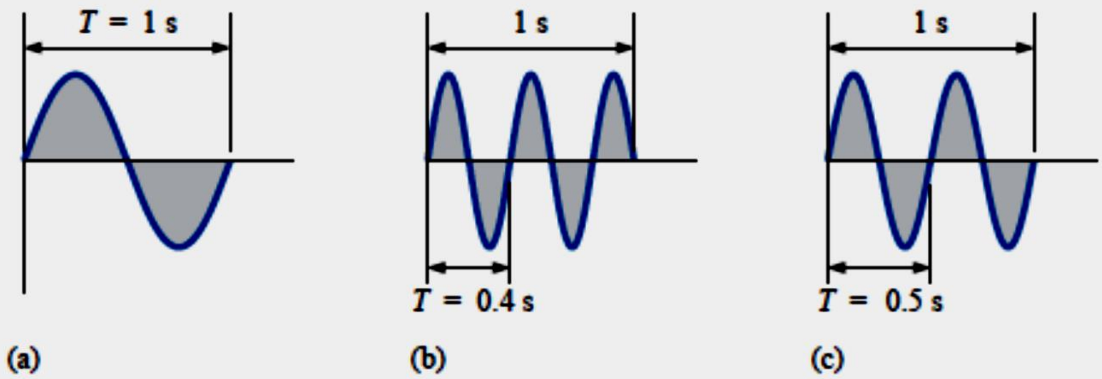


Fig 3.4

Demonstrating the effect of a changing frequency on the period of a sinusoidal waveform.

$$f = \frac{1}{T} \qquad \begin{aligned} f &= \text{Hz} \\ T &= \text{seconds (s)} \end{aligned}$$

or

$$T = \frac{1}{f}$$

EXAMPLE 3.1 Find the period of a periodic waveform with a frequency of
a. 60 Hz.
b. 1000 Hz.

Solutions:

a. $T = \frac{1}{f} = \frac{1}{60\text{ Hz}} \cong 0.01667\text{ s or } \mathbf{16.67\text{ ms}}$

(a recurring value since 60 Hz is so prevalent)

b. $T = \frac{1}{f} = \frac{1}{1000\text{ Hz}} = 10^{-3}\text{ s} = \mathbf{1\text{ ms}}$

EXAMPLE 3.2 Determine the frequency of the waveform of Fig. 3.5

Solution: From the figure, $T = (25\text{ ms} - 5\text{ ms}) = 20\text{ ms}$, and

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}\text{ s}} = \mathbf{50\text{ Hz}}$$

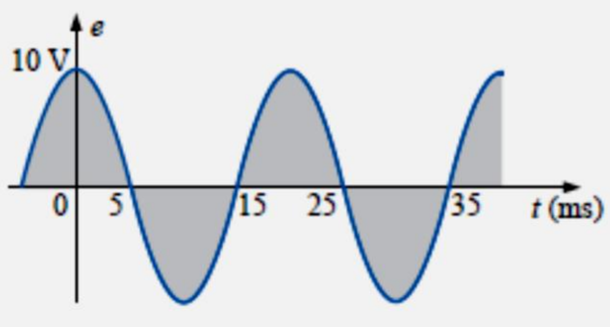


Fig 3.5

3.4 Sinusoids

A sinusoid is a signal that has the form of the sine or cosine function.

Consider the sinusoidal voltage

$$v(t) = V_m \sin \omega t \qquad \text{...... 3.1}$$

where

V_m = the *amplitude* of the sinusoid
ω = the *angular frequency* in radians/s
ωt = the *argument* of the sinusoid

The sinusoid is shown in Fig. 3.6 a as a function of its argument and in Fig. 3.6 b as a function of time. It is evident that the sinusoid repeats itself every T seconds; thus, T is called the *period* of the sinusoid. From the two plots in Fig 3.6 we observe that $\omega T = 2\pi$,

$$\boxed{T = \frac{2\pi}{\omega}} \qquad 3.2$$

The fact that $v(t)$ repeats itself every T seconds is shown by replacing t by $t + T$ in Eq. 3.1 We get

$$v(t+T) = V_m \sin \omega(t+T) = V_m \sin \omega \left(t + \frac{2\pi}{\omega}\right) \qquad 3.3$$

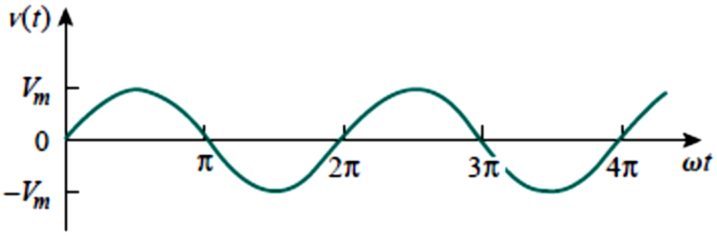
$$= V_m \sin(\omega t + 2\pi) = V_m \sin \omega t = v(t)$$

Hence,

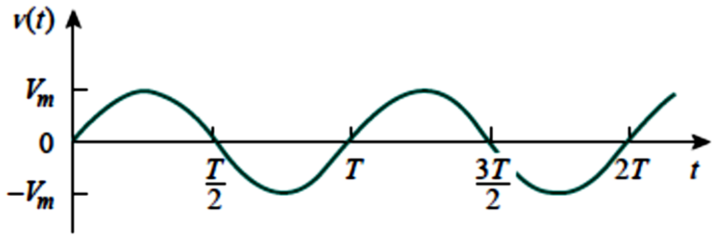
$$\boxed{v(t+T) = v(t)} \qquad 3.4$$

that is, v has the same value at $t + T$ as it does at t and $v(t)$ is said to be *periodic*. In general,

A periodic function is one that satisfies $f(t) = f(t + nT)$, for all t and for all integers n.



(a)



(b)

Figure 3.6 A sketch of $V_m \sin \omega t$: (a) as a function of ωt, (b) as a function of t.

As mentioned, the *period* T of the periodic function is the time of one complete cycle or the number of seconds per cycle. The reciprocal of this quantity is the number of cycles per second, known as the *cyclic frequency* f of the sinusoid. Thus,

$$\boxed{f = \frac{1}{T}} \quad 3.5$$

From Eqs. 3.2 and 3.5 it is clear that

$$\omega = 2\pi f \quad 3.6$$

While ω is in radians per second (rad/s), f is in hertz (Hz).

Let us now consider a more general expression for the sinusoid,

$$v(t) = V_m \sin(\omega t + \phi) \quad 3.7$$

where $(\omega t + \phi)$ is the argument and ϕ is the *phase*. Both argument and phase can be in radians or degrees.

Let us examine the two sinusoids

$$v_1(t) = V_m \sin \omega t \qquad \text{and} \qquad v_2(t) = V_m \sin(\omega t + \phi) \qquad (9.8)$$ 3.8

shown in Fig 3.8 The starting point of v_2 in Fig 3.8 occurs first in time. Therefore, we say that v_2 *leads* v_1 by ϕ or that v_1 *lags* v_2 by ϕ. If $\phi \neq 0$, we also say that v_1 and v_2 are *out of phase.* If $\phi = 0$, then v_1 and v_2 are said to be *in phase*; they reach their minima and maxima at exactly the same time. We can compare v_1 and v_2 in this manner because they operate at the same frequency; they do not need to have the same amplitude.

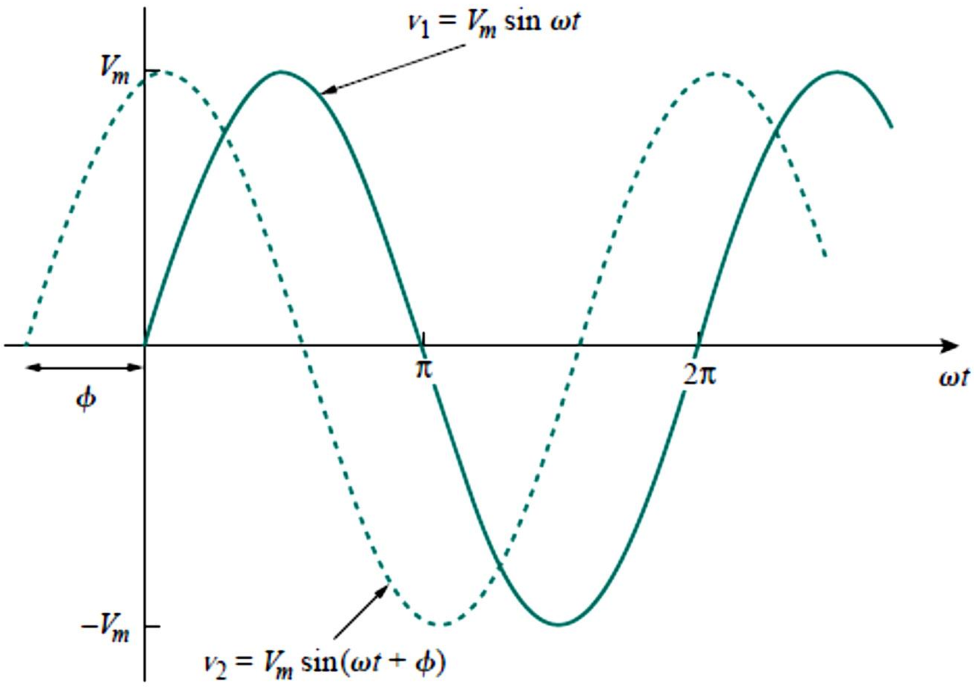


Figure 3.8 Two sinusoids with different phases.

A sinusoid can be expressed in either sine or cosine form. When comparing two sinusoids, it is expedient to express both as either sine or cosine with positive amplitudes. This is achieved by using the following trigonometric identities:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \qquad 3.9$$

With these identities, it is easy to show that

$$\begin{aligned}\sin(\omega t \pm 180^\circ) &= -\sin \omega t \\ \cos(\omega t \pm 180^\circ) &= -\cos \omega t \\ \sin(\omega t \pm 90^\circ) &= \pm \cos \omega t \\ \cos(\omega t \pm 90^\circ) &= \mp \sin \omega t\end{aligned} \qquad 3.9$$

Using these relationships, we can transform a sinusoid from sine form to cosine form or vice versa.

Example: Find the amplitude, phase, period, and frequency of the sinusoid

$$v(t) = 12\cos(50t + 10^\circ)$$

Solution:

The amplitude is $V_m = 12$ V.

The phase is $\phi = 10^\circ$.

The angular frequency is $\omega = 50$ rad/s.

The period $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{50} = 0.1257$ s.

The frequency is $f = \dfrac{1}{T} = 7.958$ Hz.

Example: Given the sinusoid $5\sin(4\pi t - 60^\circ)$, calculate its amplitude, phase, angular frequency, period, and frequency.

Answer: 5, -60°, 12.57 rad/s, 0.5 s, 2 Hz.

Example:

Calculate the phase angle between $v_1 = -10\cos(\omega t + 50^\circ)$ and $v_2 = 12\sin(\omega t - 10^\circ)$. State which sinusoid is leading.

Solution:

Let us calculate the phase in three ways. The first two methods use trigonometric identities, while the third method uses the graphical approach.

METHOD 1 In order to compare v_1 and v_2, we must express them in the same form. If we express them in cosine form with positive amplitudes,

$$v_1 = -10\cos(\omega t + 50^\circ) = 10\cos(\omega t + 50^\circ - 180^\circ)$$

$$v_1 = 10\cos(\omega t - 130^\circ) \quad \text{or} \quad v_1 = 10\cos(\omega t + 230^\circ) \qquad \text{a}$$

and

$$v_2 = 12\sin(\omega t - 10^\circ) = 12\cos(\omega t - 10^\circ - 90^\circ)$$

$$v_2 = 12\cos(\omega t - 100^\circ) \qquad \text{b}$$

It can be deduced from Eqs. (a) and (b) that the phase difference between v_1 and v_2 is 30°. We can write v_2 as

$$v_2 = 12\cos(\omega t - 130^\circ + 30^\circ) \quad \text{or} \quad v_2 = 12\cos(\omega t + 260^\circ) \qquad \text{c}$$

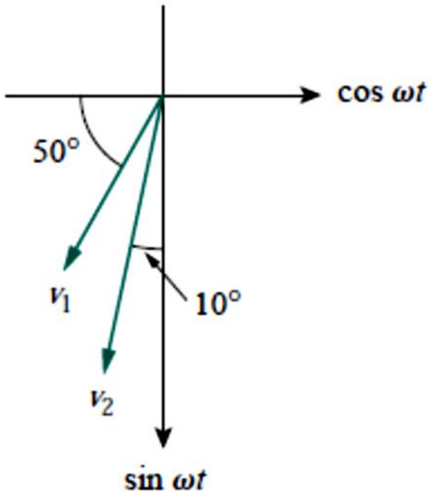
Comparing Eqs. (a) and (c) shows clearly that v_2 leads v_1 by 30°.

METHOD 2 Alternatively, we may express v_1 in sine form:

$$v_1 = -10\cos(\omega t + 50^\circ) = 10\sin(\omega t + 50^\circ - 90^\circ)$$

$$= 10\sin(\omega t - 40^\circ) = 10\sin(\omega t - 10^\circ - 30^\circ)$$

But $v_2 = 12\sin(\omega t - 10^\circ)$. Comparing the two shows that v_1 lags v_2 by 30°. This is the same as saying that v_2 leads v_1 by 30°.



Find the phase angle between

$$i_1 = -4\sin(377t + 25^\circ) \qquad \text{and} \qquad i_2 = 5\cos(377t - 40^\circ)$$

Does i_1 lead or lag i_2?

Answer: 155°, i_1 leads i_2.

3.5 Phasor s

resemblances between manipulating complex numbers and manipulating two-dimensional vectors.

The complex number z can also be written in polar or exponential form as

$$z = r\angle\phi = re^{j\phi}$$

where r is the magnitude of z, and ϕ is the phase of z. We notice that z can be represented in three ways:

$$z = x + jy \qquad \text{Rectangular form}$$
$$z = r\angle\phi \qquad \text{Polar form}$$
$$z = re^{j\phi} \qquad \text{Exponential form}$$

The relationship between the rectangular form and the polar form is shown in Fig. 9.6, where the x axis represents the real part and the y axis represents the imaginary part of a complex number. Given x and y, we can get r and ϕ as resemblances between manipulating complex numbers and manipulating two-dimensional vectors.

The complex number z can also be written in polar or exponential form as

$$z = r\angle\phi = re^{j\phi}$$

where r is the magnitude of z, and ϕ is the phase of z. We notice that z can be represented in three ways:

$z = x + jy$	Rectangular form	
$z = r\angle\phi$	Polar form	)
$z = re^{j\phi}$	Exponential form	

The relationship between the rectangular form and the polar form is shown in Fig. 9.6, where the x axis represents the real part and the y axis represents the imaginary part of a complex number. Given x and y, we can get r and ϕ as

$$r = \sqrt{x^2 + y^2}, \qquad \phi = \tan^{-1}\frac{y}{x}$$

On the other hand, if we know r and ϕ, we can obtain x and y as

$$x = r\cos\phi, \qquad y = r\sin\phi$$

$$r = \sqrt{x^2 + y^2}, \qquad \phi = \tan^{-1}\frac{y}{x}$$

On the other hand, if we know r and ϕ, we can obtain x and y as

$$x = r\cos\phi, \qquad y = r\sin\phi$$

Thus, z may be written as

$$\boxed{z = x + jy = r\angle\phi = r(\cos\phi + j\sin\phi)}$$

Addition and subtraction of complex numbers are better performed in rectangular form; multiplication and division are better done in polar form. Given the complex numbers

$$z = x + jy = r\angle\phi, \qquad z_1 = x_1 + jy_1 = r_1\angle\phi_1$$
$$z_2 = x_2 + jy_2 = r_2\angle\phi_2$$

the following operations are important.

Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

Multiplication:

$$z_1 z_2 = r_1 r_2 \underline{/\phi_1 + \phi_2}$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \underline{/\phi_1 - \phi_2}$$

Reciprocal:

$$\frac{1}{z} = \frac{1}{r} \underline{/-\phi}$$

Square Root:

$$\sqrt{z} = \sqrt{r} \underline{/\phi/2}$$

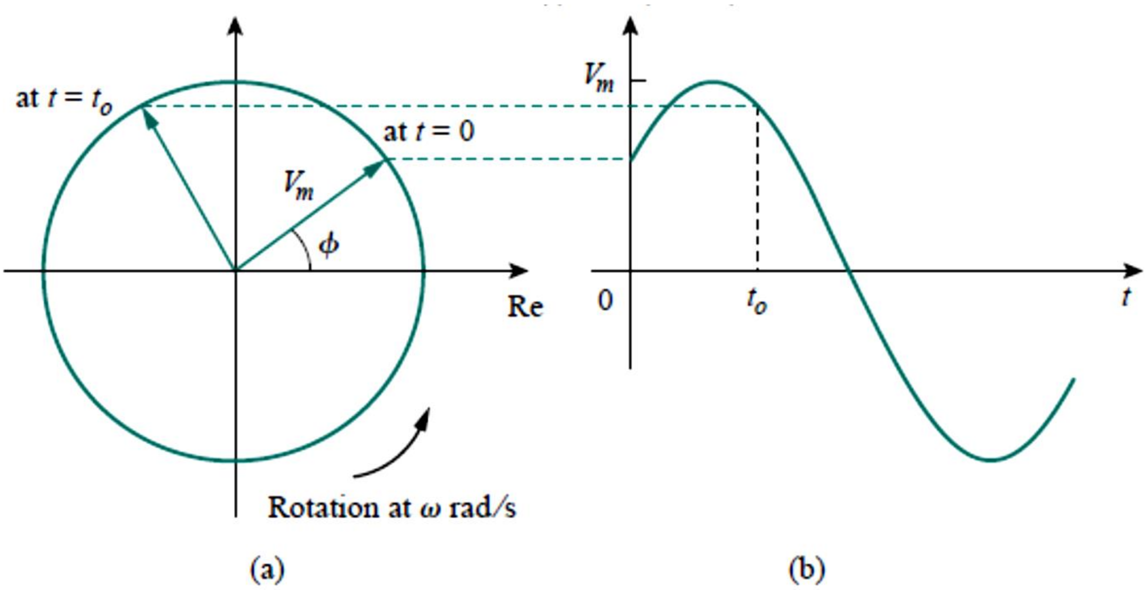


Fig 3.9

Phasor Representation

As a complex quantity, a phasor may be expressed in rectangular form, polar form, or exponential form. Since a phasor has magnitude and phase ("direction"), it behaves as a vector and is printed in boldface. For example, phasors $\mathbf{V} = V_m\angle\phi$ and $\mathbf{I} = I_m\angle -\theta$ are graphically represented in Fig. 3.9 Such a graphical representation of phasors is known as a *phasor diagram*.

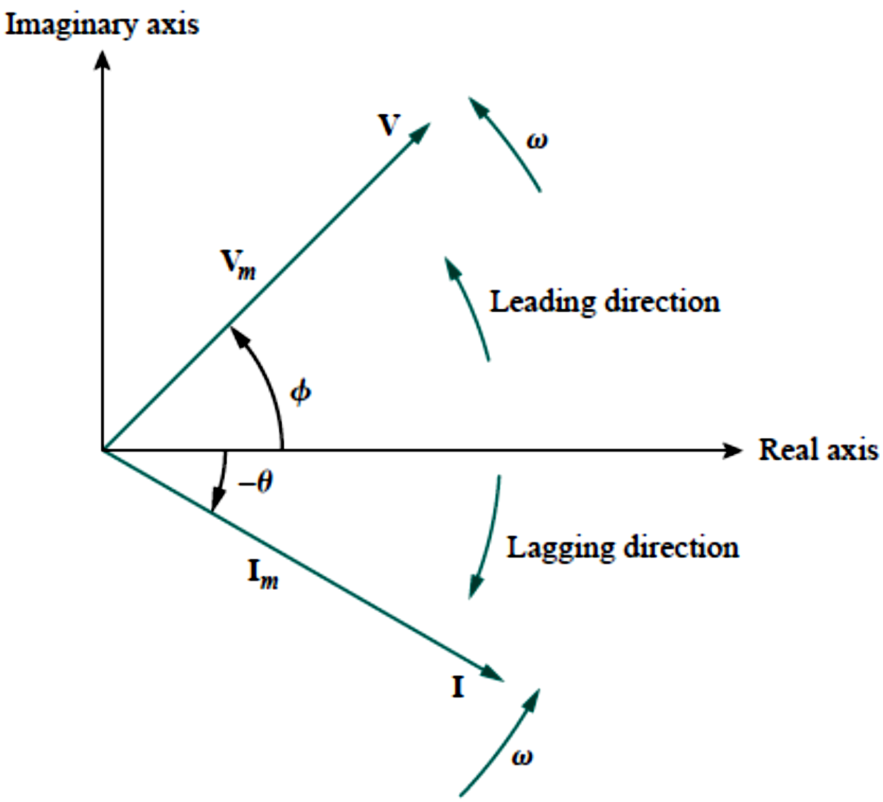


Figure 3.9 A phasor diagram showing $\mathbf{V} = V_m\angle\phi$ and $\mathbf{I} = I_m\angle -\theta$.

Example Evaluate these complex numbers:

(a) $(40\angle 50^\circ + 20\angle -30^\circ)^{1/2}$

(b) $\dfrac{10\angle -30^\circ + (3 - j4)}{(2 + j4)(3 - j5)^*}$

Solution:

(a) Using polar to rectangular transformation,

$$40\angle 50^\circ = 40(\cos 50^\circ + j\sin 50^\circ) = 25.71 + j30.64$$

$$20\angle -30^\circ = 20[\cos(-30^\circ) + j\sin(-30^\circ)] = 17.32 - j10$$

Adding them up gives

$$40\angle 50^\circ + 20\angle -30^\circ = 43.03 + j20.64 = 47.72\angle 25.63^\circ$$

Taking the square root of this,

$$(40\angle 50^\circ + 20\angle -30^\circ)^{1/2} = 6.91\angle 12.81^\circ$$

(b) Using polar-rectangular transformation, addition, multiplication, and division,

$$\frac{10\angle -30^\circ + (3 - j4)}{(2 + j4)(3 - j5)^*} = \frac{8.66 - j5 + (3 - j4)}{(2 + j4)(3 + j5)}$$

$$= \frac{11.66 - j9}{-14 + j22} = \frac{14.73\angle -37.66^\circ}{26.08\angle 122.47^\circ}$$

$$= 0.565\angle -160.31^\circ$$

Example

Evaluate the following complex numbers:

(a) $[(5 + j2)(-1 + j4) - 5\angle 60^\circ]^*$

(b) $\dfrac{10 + j5 + 3\angle 40^\circ}{-3 + j4} + 10\angle 30^\circ$

Answer: (a) $-15.5 - j13.67$, (b) $8.293 + j2.2$.

Example Transform these sinusoids to phasors:

(a) $v = -4\sin(30t + 50^\circ)$

(b) $i = 6\cos(50t - 40^\circ)$

Solution:

(a) Since $-\sin A = \cos(A + 90^\circ)$,

$$v = -4\sin(30t + 50^\circ) = 4\cos(30t + 50^\circ + 90^\circ)$$

$$= 4\cos(30t + 140^\circ)$$

The phasor form of v is

$$\mathbf{V} = 4\angle 140^\circ$$

(b) $i = 6\cos(50t - 40^\circ)$ has the phasor

$$\mathbf{I} = 6\angle -40^\circ$$

PRACTICE PROBLEM 9.4

Express these sinusoids as phasors:

(a) $v = -7\cos(2t + 40^\circ)$

(b) $i = 4\sin(10t + 10^\circ)$

Answer: (a) $\mathbf{V} = 7\angle 220^\circ$, (b) $\mathbf{I} = 4\angle -80^\circ$.

EXAMPLE 9.5

Find the sinusoids represented by these phasors:

(a) $\mathbf{V} = j8e^{-j20^\circ}$

(b) $\mathbf{I} = -3 + j4$

Solution:

(a) Since $j = 1\angle 90^\circ$,

$$\mathbf{V} = j8\angle -20^\circ = (1\angle 90^\circ)(8\angle -20^\circ) = 8\angle 90^\circ - 20^\circ = 8\angle 70^\circ \text{ V}$$

Converting this to the time domain gives

$$v(t) = 8\cos(\omega t + 70^\circ) \text{ V}$$

(b) $\mathbf{I} = -3 + j4 = 5\angle 126.87^\circ$. Transforming this to the time domain gives

$$i(t) = 5\cos(\omega t + 126.87^\circ) \text{ A}$$

Find the sinusoids corresponding to these phasors:

(a) $\mathbf{V} = -10\angle 30^\circ$

(b) $\mathbf{I} = j(5 - j12)$

Answer: (a) $v(t) = 10\cos(\omega t + 210^\circ)$, (b) $i(t) = 13\cos(\omega t + 22.62^\circ)$.

Given $i_1(t) = 4\cos(\omega t + 30^\circ)$ and $i_2(t) = 5\sin(\omega t - 20^\circ)$, find their sum.

Solution:

Here is an important use of phasors—for summing sinusoids of the same frequency. Current $i_1(t)$ is in the standard form. Its phasor is

$$\mathbf{I}_1 = 4\angle 30^\circ$$

We need to express $i_2(t)$ in cosine form. The rule for converting sine to cosine is to subtract 90°. Hence,

$$i_2 = 5\cos(\omega t - 20^\circ - 90^\circ) = 5\cos(\omega t - 110^\circ)$$

and its phasor is

$$\mathbf{I}_2 = 5\angle -110^\circ$$

If we let $i = i_1 + i_2$, then

$$\begin{aligned}\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 &= 4\angle 30^\circ + 5\angle -110^\circ \\ &= 3.464 + j2 - 1.71 - j4.698 = 1.754 - j2.698 \\ &= 3.218\angle -56.97^\circ \text{ A}\end{aligned}$$

Transforming this to the time domain, we get

$$i(t) = 3.218\cos(\omega t - 56.97^\circ) \text{ A}$$

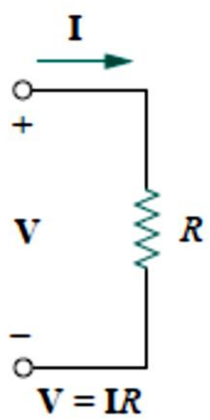
Of course, we can find $i_1 + i_2$ using Eqs. (9.9), but that is the hard way.

If $v_1 = -10\sin(\omega t + 30^\circ)$ and $v_2 = 20\cos(\omega t - 45^\circ)$, find $V = v_1 + v_2$.

Answer: $v(t) = 10.66\cos(\omega t - 30.95^\circ)$.

3.5 Phasor Relationship for Circuit Elements

1) Resistance

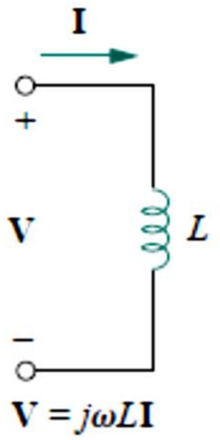


2) Inductance

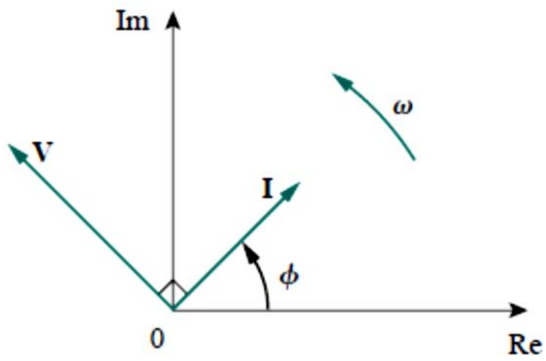
$V = jX_L\, I$

Where X_L: the inductance Reactance measured by (Ω)

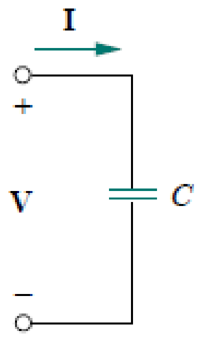
$$X_L = j\omega L$$



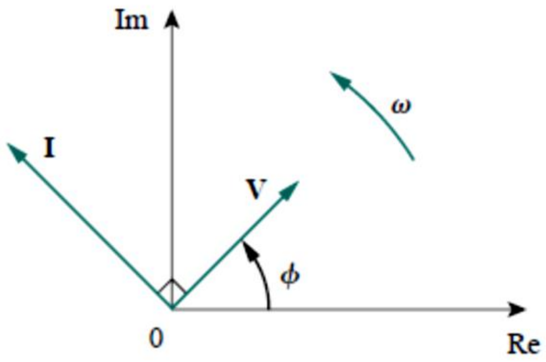
The current is lagging with the voltage of inductance by 90°



3) Capacitance
4) $V = -jX_c\, I$
5) Where X_c: the capacitance Reactance measured by (Ω)
6) $X_c = \frac{1}{\omega c}$



$$V = -j\frac{1}{\omega c}\, I$$



Example:

The voltage $v = 12\cos(60t + 45°)$ is applied to a 0.1-H inductor. Find the steady-state current through the inductor.

Solution:

For the inductor, $\mathbf{V} = j\omega L\mathbf{I}$, where $\omega = 60$ rad/s and $\mathbf{V} = 12\underline{/45°}$ V. Hence

$$\mathbf{I} = \frac{\mathbf{V}}{j\omega L} = \frac{12\angle 45^\circ}{j60 \times 0.1} = \frac{12\angle 45^\circ}{6\angle 90^\circ} = 2\angle -45^\circ \text{ A}$$

Converting this to the time domain,

$$i(t) = 2\cos(60t - 45^\circ) \text{ A}$$

PRACTICE PROBLEM 9.8

If voltage $v = 6\cos(100t - 30^\circ)$ is applied to a 50 μF capacitor, calculate the current through the capacitor.

Answer: $30\cos(100t + 60^\circ)$ mA.

3.6 The Impedance and Admitance:

The impedance **Z** of a circuit is the ratio of the phasor voltage **V** to the phasor current **I**, measured in ohms (Ω).

As a complex quantity, the impedance may be expressed in rectangular form as

$$\mathbf{Z} = R + jX \qquad 3.11$$

where $R = \text{Re}\,\mathbf{Z}$ is the *resistance* and $X = \text{Im}\,\mathbf{Z}$ is the *reactance*. The reactance X may be positive or negative. We say that the impedance is inductive when X is positive or capacitive when X is negative. Thus, impedance $\mathbf{Z} = R + jX$ is said to be *inductive* or lagging since current lags voltage, while impedance $\mathbf{Z} = R - jX$ is capacitive or leading because current leads voltage. The impedance, resistance, and reactance are all measured in ohms. The impedance may also be expressed in polar form as

$$\mathbf{Z} = |\mathbf{Z}|\angle\theta \qquad 3.12$$

Comparing Eqs. 3.11 and 3.12 , we infer that

$$\boxed{\mathbf{Z} = R + jX = |\mathbf{Z}|\angle\theta} \qquad 3.13$$

where

$$|\mathbf{Z}| = \sqrt{R^2 + X^2}, \qquad \theta = \tan^{-1}\frac{X}{R} \qquad 3.14$$

and

$$R = |\mathbf{Z}| \cos\theta, \qquad X = |\mathbf{Z}| \sin\theta \qquad 3.15$$

It is sometimes convenient to work with the reciprocal of impedance, known as *admittance*.

The admittance Y is the reciprocal of impedance, measured in siemens (S).

The admittance **Y** of an element (or a circuit) is the ratio of the phasor current through it to the phasor voltage across it, or

$$\boxed{\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{\mathbf{I}}{\mathbf{V}}} \qquad (9.46)$$

As a complex quantity, we may write **Y** as

$$\boxed{\mathbf{Y} = G + jB} \qquad 3.16$$

where $G = \text{Re}\,\mathbf{Y}$ is called the *conductance* and $B = \text{Im}\,\mathbf{Y}$ is called the *susceptance*. Admittance, conductance, and susceptance are all expressed in the unit of siemens (or mhos). From Eqs. (9.41) and (9.47),

$$G + jB = \frac{1}{R + jX} \qquad 3.17$$

By rationalization,

$$G + jB = \frac{1}{R + jX} \cdot \frac{R - jX}{R - jX} = \frac{R - jX}{R^2 + X^2} \qquad 3.18$$

Equating the real and imaginary parts gives

$$G = \frac{R}{R^2 + X^2}, \qquad B = -\frac{X}{R^2 + X^2} \qquad 3.19$$

showing that $G \neq 1/R$ as it is in resistive circuits. Of course, if $X = 0$, then $G = 1/R$.

Example:

Find $v(t)$ and $i(t)$ in the circuit shown in Fig. 9.16.

Solution:

From the voltage source $10 \cos 4t$, $\omega = 4$,

$$\mathbf{V}_s = 10\angle 0^\circ \text{ V}$$

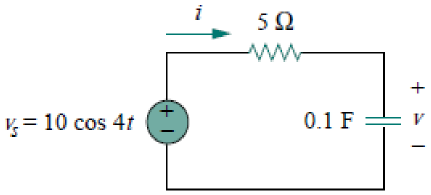


Figure 9.16 For Example 9.9.

The impedance is

$$\mathbf{Z} = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4 \times 0.1} = 5 - j2.5\ \Omega$$

Hence the current

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10\angle 0^\circ}{5 - j2.5} = \frac{10(5 + j2.5)}{5^2 + 2.5^2}$$
$$= 1.6 + j0.8 = 1.789\angle 26.57^\circ \text{ A} \qquad (9.9.1)$$

The voltage across the capacitor is

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_C = \frac{\mathbf{I}}{j\omega C} = \frac{1.789\angle 26.57^\circ}{j4 \times 0.1}$$
$$= \frac{1.789\angle 26.57^\circ}{0.4\angle 90^\circ} = 4.47\angle -63.43^\circ \text{ V} \qquad (9.9.2)$$

Converting $\mathbf{I}$ and $\mathbf{V}$ in Eqs. (9.9.1) and (9.9.2) to the time domain, we get

$$i(t) = 1.789 \cos(4t + 26.57^\circ) \text{ A}$$
$$v(t) = 4.47 \cos(4t - 63.43^\circ) \text{ V}$$

Notice that $i(t)$ leads $v(t)$ by 90° as expected.

PRACTICE PROBLEM 9.9

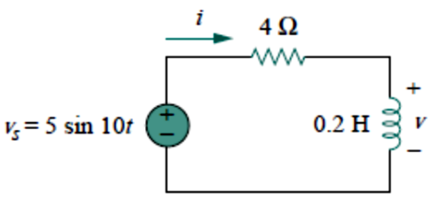


Figure 9.17 For Practice Prob. 9.9.

Refer to Fig. 9.17. Determine $v(t)$ and $i(t)$.

Answer: $2.236 \sin(10t + 63.43^\circ)$ V, $1.118 \sin(10t - 26.57^\circ)$ A.

Example:

Find the input impedance of the circuit in Fig. 9.23. Assume that the circuit operates at ω = 50 rad/s.

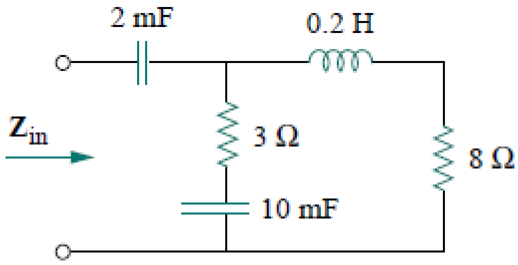


Figure 9.23 For Example 9.10.

Solution:

Let

$\mathbf{Z}_1$ = Impedance of the 2-mF capacitor

$\mathbf{Z}_2$ = Impedance of the 3-Ω resistor in series with the 10-mF capacitor

$\mathbf{Z}_3$ = Impedance of the 0.2-H inductor in series with the 8-Ω resistor

Then

$$\mathbf{Z}_1 = \frac{1}{j\omega C} = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10\ \Omega$$

$$\mathbf{Z}_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2)\ \Omega$$

$$\mathbf{Z}_3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j10)\ \Omega$$

The input impedance is

$$\mathbf{Z}_{in} = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3 = -j10 + \frac{(3 - j2)(8 + j10)}{11 + j8}$$

$$= -j10 + \frac{(44 + j14)(11 - j8)}{11^2 + 8^2} = -j10 + 3.22 - j1.07\ \Omega$$

Thus,

$$\mathbf{Z}_{in} = 3.22 - j11.07\ \Omega$$